

A transformer Class E amplifier*

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Abstract: In a high-efficiency Class E ZVS resonant amplifier a matching and isolation transformer can replace some or even all inductive components of the amplifier thus simplifying the circuit and reducing its cost. In the paper a theoretical analysis, a design example and its experimental verification for a transformer Class E amplifier are presented. In the experimental amplifier with a transformer as the only inductive component in the circuit high efficiency $\eta_{\text{MAX}} = 0.95$ was achieved for supply voltage $V_I = 36$ V, maximum output power $P_{\text{OMAX}} = 100$ W and the switching frequency $f = 300$ kHz. Measured parameters and waveforms showed a good agreement with theoretical predictions. Moreover, the relative bandwidth of the switching frequency was only 19% to obtain output power control from 4.8 W to P_{OMAX} with efficiency not less than 0.9 in the regulation range.

Key words: Class E amplifier, resonant inverter, dc/dc converter, transformer, leakage inductance

1. Introduction

High efficiency and a high degree of miniaturization achieved in switching resonant Class E amplifiers increase the scope of their applications including presently such areas as wireless energy transfer, induction and dielectric heating, dc/dc converters, plasma drivers, electro-surgical generators, electronic ballasts as well as r.f. transmitters. Among different types of Class E amplifiers much research effort is devoted to optimise a Class E ZVS (zero-voltage switching) amplifier with a power supply choke and a series resonant circuit [1-5]. One of possible improvements in this amplifier is the replacement of its supply choke with a finite dc-feed inductance. This also allows increasing load resistance and decreasing the amplitude of the current in the series resonant circuit for given output power and a supply voltage consequently reducing power losses in the circuit [2-5]. The improvement is useful e.g. in on-chip UHF Class E amplifiers, where small inductances of nH range are usually required and discrete

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coils are often replaced by inductances made of bond wires or copper tracks on silicon substrate [3].

In many applications of Class E amplifiers it is necessary to ensure galvanic isolation of the amplifier output from its input power supply source. This can be made by applying a transformer in the amplifier as proposed e.g. in [5]. The transformer also matches the amplifier load impedance to ensure nominal or off-nominal operation of the amplifier for its maximum output power. The primary winding inductance of the transformer can be utilized as a finite dc-feed inductance in the amplifier eliminating the need to use a separate dc-feed inductance (Fig. 1a). This reduces the number of separate inductive components in the circuit. However, the transformer also introduces leakage inductances, which modifies the operation of the amplifier (Fig. 1b). These leakage inductances have to be taken into account in the design of the amplifier. If a transformer with high leakage inductances is used then also the inductance of the series resonant circuit in the amplifier can be replaced by the leakage inductance L_2 of the transformer secondary winding ($L_{SR} = 0$ in Fig. 1b). Thus in the isolated Class E ZVS amplifier of Figure 1a the application of the transformer with high enough leakage inductances can reduce the number of used inductive components from three (a dc-feed inductance, a resonant inductance and a transformer) to only one – the transformer. Such a reduction of the inductive components simplifies the amplifier and lowers its cost. This is essential for the circuits operating in kilohertz and megahertz frequency range where discrete inductive components are usually with a magnetic core, and therefore are often bulky and costly.

However, the transformer Class E amplifier requires an analysis to find explicit relations between the parameters of the transformer, the amplifier nominal operation and obtained parameters of the amplifier. Published papers on Class E amplifiers with a transformer have not provided so far direct design rules or equations for computing parameters and component values for the lumped-component circuit of Figure 1a operating in kilohertz and megahertz frequency range [5-6]. The transformer Class E amplifier can find applications in low-cost isolated resonant dc/dc converters [7] and inverters. Some applications for the amplifier can be also predicted in emerging VHF power circuits with air core transformers, which are usually characterized by high leakage inductances [8-9].

The paper presents a theoretical analysis of the transformer Class E amplifier, a design example as well as experimental results for the designed and built circuit.

2. Principle of the circuit operation

Figure 1a shows a simplified diagram of the transformer Class E amplifier. The circuit comprises a bi-directional transistor switch $T1D1$ driven by a rectangular gate waveform $v_{GS}(t)$ with the duty cycle $D = 0.5$ and the frequency f , a resonant circuit with the capacitors C_1 , C_{SR} , the inductance L_{SR} and the inductances of the transformer Tr with the winding turn ratio $1:n$. The secondary leakage inductance L_2 of the transformer Tr along with the external inductance L_{SR} , the capacitor C_{SR} and load resistance R_L form a series resonant circuit $L_2-L_{SR}-C_{SR}-R_L$ (Fig. 1b) with a high loaded quality factor to ensure a sinusoidal shape of the output current i_O .

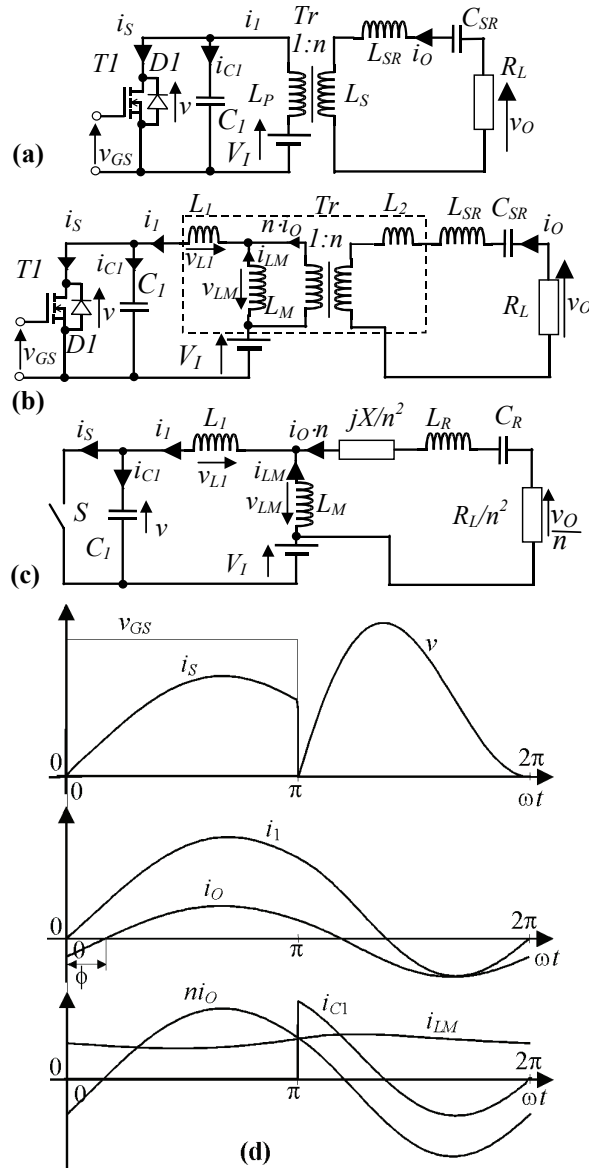


Fig. 1. The circuit diagram (a), equivalent circuits (b), (c) and theoretical waveforms (d) for the transformer Class E amplifier; L_P , L_S – inductances of the transformer Tr primary and secondary windings, respectively; L_M , L_1 , L_2 – magnetizing inductance as well as primary and secondary leakage inductances of the transformer Tr , respectively; V_I – supply voltage

The primary winding inductance L_P acts as a finite dc-feed inductance and conducts the supply current i_{LM} as well as the transformed output current $n i_O$. Both currents i_{LM} and $n i_O$ compose the current i_1 which either is conducted by the transistor switch $T1D1$ as the current i_s or flows through the capacitor C_1 as the current i_{C1} .

The analysis of the transformer Class E amplifier is based on the following assumptions:

- The equivalent circuit of the amplifier is given in Figure 1c, where the transformer Tr is modelled by its T -equivalent network with the winding turn ratio $n = (L_S/L_P)^{1/2}$,
- the operation of the amplifier is nominal for the frequency $f = \omega/2\pi$, load resistance R_L and the switch S ON duty cycle $D = 0.5$,
- the components of the amplifier are ideal,
- the sinusoidal output current is given by $i_O(\omega t) = I_m \sin(\omega t + \phi)$, where I_m , ϕ – the amplitude and the phase shift of the i_O waveform, respectively.

The loaded quality factor of the series resonant branch $L_R-C_R-R_L/n^2$ (Fig. 1c) is

$$Q_R = \frac{\omega L_R}{R_L/n^2} = \frac{1}{\omega C_R R_L/n^2}. \quad (1)$$

For the frequency f the reactance X of the $L_2-L_{SR}-C_{SR}-R_L$ circuit referred to the primary side is equal to

$$\frac{X}{n^2} = \frac{\omega(L_2 + L_{SR})}{n^2} - \frac{1}{\omega C_{SR} n^2} = \omega L_R - \frac{1}{\omega C_R} + \frac{X}{n^2}, \quad (2)$$

where $\omega L_R - 1/(\omega C_R) = 0$ – for the frequency f .

The following parameters are defined: p , q [3, 5] and k (the coupling coefficient of the transformer Tr)

$$p = \frac{\omega L_M n I_m}{V_I}, \quad (3)$$

$$q = \frac{1}{\omega \sqrt{(L_1 + L_M) C_1}} = \frac{1}{\omega \sqrt{L_P C_1}}, \quad (4)$$

$$k = \frac{L_M}{L_1 + L_M} = \frac{L_M}{L_P}. \quad (5)$$

The current $i_{C1}(\omega t)$ flowing through the capacitor C_1 during the interval $\pi < \omega t \leq 2\pi$ is given by

$$\begin{aligned} i_{C1}(\omega t) &= \omega C_1 \frac{dv(\omega t)}{d\omega t} = i_{LM}(\omega t) + n i_O(\omega t) = \\ &= \frac{1}{\omega L_M} \int_{\pi}^{\omega t} (V_I - v_{L1}(\omega t) - v(\omega t)) d\omega t + i_{LM}(\pi) + n I_m \sin(\omega t + \phi). \end{aligned} \quad (6)$$

For the same ωt interval the voltage $v_{L1}(\omega t)$ across the leakage inductance L_1 is found as

$$v_{L1}(\omega t) = \omega L_1 \frac{di_{C1}(\omega t)}{d\omega t} = \omega^2 L_1 C_1 \frac{d^2 v(\omega t)}{d(\omega t)^2}. \quad (7)$$

By differentiating both sides of Equation (6), then substituting Equation (7) to (6) and using the parameters given by Formulas (3), (4), (5) one arrives at the second order differential equation for the voltage waveform $v(\omega t)$ across the switch S [3, 5] for the interval $\pi < \omega t \leq 2\pi$

$$\frac{d^2 v(\omega t)}{d(\omega t)^2} + q^2 (v(\omega t) - V_I (1 + p \cos(\omega t + \phi))) = 0. \quad (8)$$

When the switch S is on ($0 < \omega t \leq \pi$) the switch current $i_S(\omega t)$ is equal to the current $i_{L1}(\omega t)$ of the inductance L_1

$$i_S(\omega t) = i_{L1}(\omega t) = \frac{V_I \omega t}{\omega(L_1 + L_M)} + \frac{\omega L_M}{\omega(L_1 + L_M)} (nI_m \sin(\omega t + \phi) + i_{LM}(0)). \quad (9)$$

For $\omega t = 0$ there is

$$i_S(0) = i_{L1}(0) = i_{LM}(0) + nI_m \sin \phi = 0 \Rightarrow i_{LM}(0) = -nI_m \sin \phi. \quad (10)$$

Hence substituting $i_{LM}(0)$ found from Equation (10) to (9) and using (3) and (4) Equation (9) is now expressed as

$$i_S(\omega t) = i_{L1}(\omega t) = \frac{knI_m}{p} (\omega t + p(\sin(\omega t + \phi) - \sin \phi)). \quad (11)$$

The dc value of the supply current I_I flowing through the voltage source V_I is found from Equation (11) as

$$I_I \frac{1}{2\pi} \int_0^\pi i_S(\omega t) d\omega t = \frac{nI_m k B_1}{2p\pi}, \quad (12)$$

where

$$B_1 = 2p \cos \phi + \pi(\pi/2 - p \sin \phi). \quad (13)$$

From Equations (11), (12) and (13) the normalized switch current $i_S(\omega t)/I_I$ equals

$$\frac{i_S(\omega t)}{I_I} = \begin{cases} \frac{2\pi(\omega t + p(\sin(\omega t + \phi) - \sin \phi))}{B_1} & \text{for } 0 < \omega t \leq \pi \\ 0, & \text{for } \pi < \omega t \leq 2\pi \end{cases}. \quad (14)$$

For the nominal operation of the amplifier the switch is turned on in ZVS and ZDS (zero-derivative switching) conditions (15), and it is turned off in ZVS conditions (16)

$$v(\omega t)|_{\omega t=2\pi} = 0 \wedge \frac{dv(\omega t)}{d\omega t}|_{\omega t=2\pi} = 0. \quad (15)$$

$$v(\pi) = 0 \wedge i_S(\pi) = i_{L1}(\pi) = \omega C_1 \frac{dv(\omega t = \pi)}{d\omega t} = \frac{V_I}{\omega L_P} (\pi - 2p \sin \phi). \quad (16)$$

By solving Equation (8) for the condition (16) the normalized $v(\omega t)/V_I$ voltage waveform across the switch S is found

$$\frac{v(\omega t)}{V_I} = \begin{cases} 0, & \text{for } 0 \leq \omega t < \pi \\ A_1 \cos q\omega t + A_2 \sin q\omega t + 1 - \frac{q^2 p}{1-q^2} \cos(\omega t + \phi), & \text{for } \pi \leq \omega t < 2\pi \end{cases} \quad (17)$$

where

$$A_1 = \frac{-1}{q^2 - 1} (A_3 q \sin \pi q - A_4 \cos \pi q), \quad A_2 = \frac{1}{q^2 - 1} (A_3 q \cos \pi q - A_4 \sin \pi q), \quad (18)$$

$$A_3 = (q^2 - 1)(\pi - p \sin \phi) - pq^2 \sin \phi, \quad A_4 = 1 - q^2 + pq^2 \cos \phi. \quad (19)$$

By substituting Equation (17) along with Equations (18) and (19) to the condition (15) a system of trigonometric equations is obtained that is used to compute ϕ and p for a given $q \in < 0; 2 > - \{1\}$

$$\begin{cases} 1 + \frac{1}{q^2 - 1} \begin{pmatrix} pq^2 \cos \phi + \sin 2\pi q (A_3 q \cos \pi q + A_4 \sin \pi q) \\ -\cos 2\pi q (A_3 q \sin \pi q - A_4 \cos \pi q) \end{pmatrix} = 0 \\ -pq \sin \phi + \cos 2\pi q (A_3 q \cos \pi q + A_4 \sin \pi q) + \sin 2\pi q (A_3 q \sin \pi q - A_4 \cos \pi q) = 0. \end{cases} \quad (20)$$

The system (20) can be solved numerically e.g. by using the Newton-Raphson method. For $q = 1$ the system has no solutions, but it can be solved for q close to 1 (q can differ from 1 even less than 10^{-6} , and (20) still can be solved), which is enough for practical circuits. For $q > 2$ the amplitudes of currents circulating in the amplifier components become increasingly high, and power losses increase making the circuit impractical [3-5].

If power losses in the amplifier are neglected then the output power P_O equals the input power P_I

$$P_O = \frac{I_m^2 R_L}{2} = V_I I_I = P_I. \quad (21)$$

Substituting Equation (12) to (21) the amplitude I_m of the output current i_O can be expressed as

$$I_m = \frac{knV_I B_1}{R_L p \pi}. \quad (22)$$

Using (21) and (22) the output power P_O is

$$P_O = \frac{I_m^2 R_L}{2} = \frac{1}{2R_L} \left(\frac{knV_I B_1}{p\pi} \right)^2. \quad (23)$$

From Equations (3) and (5) the normalized inductance $n^2 \omega L_p / R_L$ of the primary winding of the transformer Tr is given by

$$\frac{n^2 \omega L_P}{R_L} = \frac{p^2 \pi}{k^2 B_1}. \quad (24)$$

The normalized value $\omega C_1 R_L / n^2$ of the capacitance C_1 can be found from Equations (4) and (24) as

$$\frac{\omega C_1 R_L}{n^2} = \frac{1}{q^2} \frac{R_L}{\omega L_P n^2} = \frac{1}{q^2} \frac{k^2 B_1}{p^2 \pi}. \quad (25)$$

From Equation (3) and (9) the voltage $v_{L1}(\omega t)$ across the leakage inductance L_1 for the interval $0 < \omega t \leq \pi$ is found as

$$v_{L1}(\omega t) = \omega L_1 \frac{di_{L1}(\omega t)}{d\omega t} = (1-k)V_I(1+p \cos(\omega t + \phi)). \quad (26)$$

For the interval $\pi < \omega t \leq 2\pi$ the voltage $v_{L1}(\omega t)$ is obtained from Equations (8) and (7) in the form

$$v_{L1}(\omega t) = (1-k)V_I(1+p \cos(\omega t + \phi) - v(\omega t)). \quad (27)$$

The input voltage v_1 of the series resonant circuit $X/n^2 - L_R - C_R - R_L/n^2$ at the operating frequency f is the fundamental component of the sum $v(\omega t) + v_{L1}(\omega t)$, and is given by

$$v_1 = v_R + v_X = V_R \sin(\omega t + \phi) + V_X \cos(\omega t + \phi), \quad (28)$$

where V_R and V_X are found using the Fourier trigonometric series formula as follows

$$V_R = \frac{1}{\pi} \int_0^{2\pi} (v(\omega t) + v_{L1}(\omega t)) \sin(\omega t + \phi) d\omega t, \quad (29)$$

$$V_X = \frac{1}{\pi} \int_0^{2\pi} (v(\omega t) + v_{L1}(\omega t)) \cos(\omega t + \phi) d\omega t. \quad (30)$$

Hence the normalized series reactance X/R_L of the series resonant circuit $X/n^2 - L_R - C_R - R_L/n^2$ at the operating frequency f is equal to

$$\frac{X}{R_L} = \frac{V_X}{V_R}. \quad (31)$$

The amplitudes V_R and V_X are found by substituting Equations (17-19, 26, 27) to Equations (29) and (30). Next, using V_R and V_X Equation (31) provides the normalized reactance X/R_L as

$$\frac{X}{R_L} = \frac{\pi \left(\frac{2p(1-k)}{k} + \frac{p\pi q^2 + 4(q^2 - 1)\sin\phi}{\pi(q^2 - 1)} \right) \times \left(\frac{\cos\frac{\pi q}{2}}{\pi(q^2 - 1)^2} \times \left(\begin{aligned} & \left((1+q)\cos\left(\frac{3\pi q}{2} - \phi\right) + (q-1)\cos\left(\frac{3\pi q}{2} + \phi\right) \right) \\ & 2\pi q(q^2 - 1)\cos\pi q - 2(q^2 - 1)\sin\pi q \\ & + pq\left((2q^2 + q - 1)\sin(\pi q - \phi) + (1 + q - 2q^2)\sin(\pi q + \phi) \right) \end{aligned} \right) \right.}{\left. \left(\begin{aligned} & 2(q^2 - 1)\cos\pi q - q \left(\begin{aligned} & (2q^2 + q - 1)p\cos(\pi q - \phi) \\ & - (q-1)\left(\begin{aligned} & (1+2q)p\cos(\pi q + \phi) \\ & + 2\pi(1+q)\sin\pi q \end{aligned} \right) \end{aligned} \right) \right) \\ & \times \left((1+q)\sin\left(\frac{3\pi q}{2} - \phi\right) + (q-1)\sin\left(\frac{3\pi q}{2} + \phi\right) \right) \end{aligned} \right)} \right) \quad (32)$$

$$+ \frac{\cos\frac{\pi q}{2}}{(q^2 - 1)^2} \times \left(\begin{aligned} & -4\cos\phi \\ & \left(\begin{aligned} & 2\pi q(q^2 - 1)\cos\pi q - 2(q^2 - 1)\sin\pi q \\ & + pq\left((2q^2 + q - 1)\sin(\pi q - \phi) + (1 + q - 2q^2)\sin(\pi q + \phi) \right) \end{aligned} \right) \\ & \times \left((q+1)\sin\left(\frac{3\pi q}{2} - \phi\right) - (q-1)\sin\left(\frac{3\pi q}{2} + \phi\right) \right) \\ & + \left((q+1)\cos\left(\frac{3\pi q}{2} - \phi\right) - (q-1)\cos\left(\frac{3\pi q}{2} + \phi\right) \right) \\ & \times \left(-2(q^2 - 1)\cos\pi q - q \left(\begin{aligned} & p(2q^2 + q - 1)\cos(\pi q - \phi) \\ & - (q-1)\left(\begin{aligned} & p(2q+1)\cos(\pi q + \phi) \\ & + 2\pi(q+1)\sin\pi q \end{aligned} \right) \end{aligned} \right) \right) \end{aligned} \right)$$

A high value of the loaded quality factor $Q_R (\geq 5)$ that ensures a sinusoidal waveform of the output current i_O [1] can be also achieved for $L_{SR} = 0$ if the value of the secondary leakage inductance L_2 is high enough. Because the inductance L_2 is related to k and L_p by $L_2 = (1 - k)L_S = (1 - k)n^2L_p$ the value of Q_R that can be obtained for given k and q ($X \leq 0$) is

$$Q_R = \frac{\omega L_2}{R_L} = \frac{(1-k)p^2\pi}{k^2 B_1}. \quad (33)$$

Even though q does not explicitly occurs in Equation (33) the value of q is necessary to compute B_1 and p as can be seen from (13) and (20).

By rearranging Equation (33) one can estimate the value of the coupling coefficient k of the transformer Tr for the given q and the assumed value of Q_R when $L_{SR} = 0$

$$k = p \frac{\sqrt{p^2\pi^2 + 4\pi B_1 Q_R - p\pi}}{2B_1 Q_R}. \quad (34)$$

Normalized parameters of the transformer Class E amplifier are plotted in Figures 2 and 3. The dashed lines in the plots show the amplifier parameters for which the assumed value of Q_R is constant (here $Q_R = 5$, $Q_R = 10$) and can be obtained by means of the secondary leakage inductance L_2 without the external inductor ($L_{SR} = 0$). The arrows in the plots point the direction where higher values of Q_R are achieved with $L_{SR} = 0$. The diamond symbols in the plots mark the locations of the parameters of the circuit from the design example.

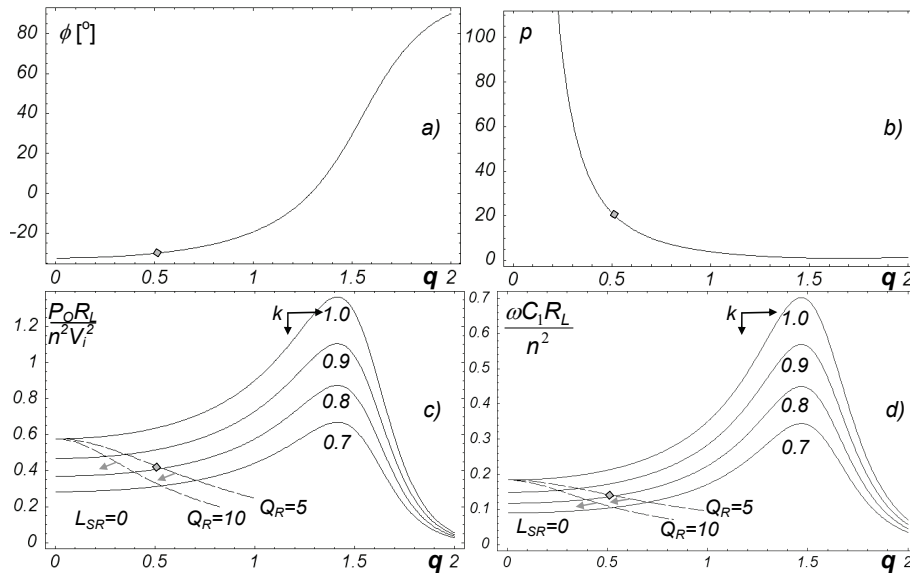


Fig. 2. Theoretical parameters of a transformer Class E amplifier versus q for various values of the coupling coefficient k of the transformer Tr ; a) ϕ – phase shift of the output current i_o , b) p parameter as defined by (3), c) normalized output power, d) normalized parallel capacitance C_1 of the switch S

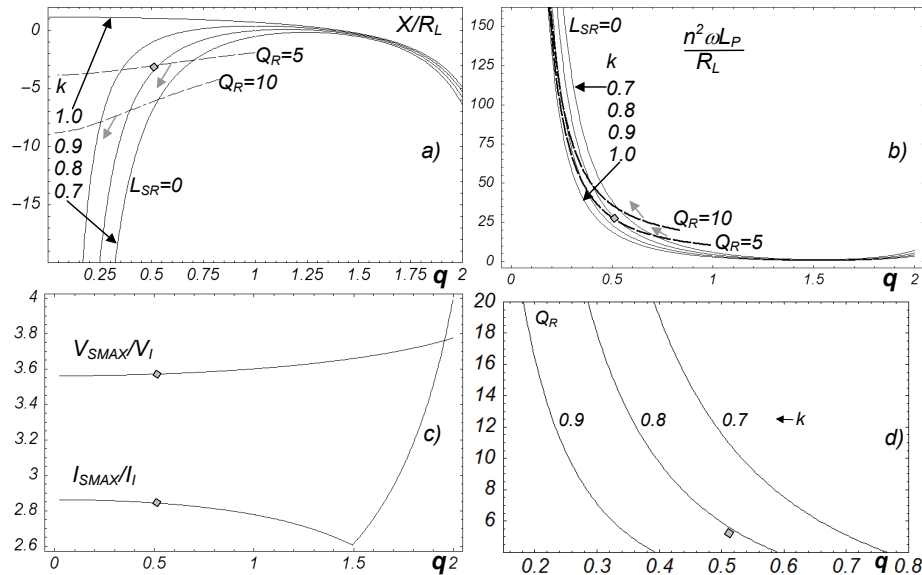


Fig. 3. Theoretical parameters of a transformer Class E amplifier versus q for various values of the coupling coefficient k of the transformer Tr ; a) normalized series reactance for the frequency f of the resonant circuit $X/n^2 - L_R - C_R - R_I/n^2$, b) normalized reactance of the transformer primary winding inductance L_P , c) normalized peak voltage and current of the switch S , d) attainable values of Q_R versus q at constant k for $L_{SR} = 0$

Figure 2c shows the plots of normalized output power $P_{OR_L}/(n^2V_I^2)$ versus q for various values of k . The maximum value of $P_{OR_L}/(n^2V_I^2)$ for different k occurs at $q = 1.412$ [3-5]. From Figure 2c it can also be noticed that as the coupling coefficient k decreases the output power of the amplifier is reduced as well (see also Eq. (23)). Then to obtain a required level of output power it is necessary to increase the amplitude I_m of the output current i_O , which in turn, can enlarge power losses. Thus, on the whole, a transformer Class E amplifier with $k < 1$ can experience higher power losses if compared to its transformerless version ($k = 1$) for the same operating conditions. This issue is important for the special case of a transformer Class E amplifier when $L_{SR} = 0$ and it is expected then that k is rather low (typically there can be $0.7 \leq k \leq 0.9$ for a transformer Tr with a ferrite core). The low value of k (when $L_{SR} = 0$) is necessary to achieve a high enough value of the leakage inductance L_2 and to obtain a high Q_R , and consequently the sinusoidal shape of the output current i_O .

Figure 2d displays the plots of normalized capacitance C_1 of the switch S versus q for given k . The maximum values of $\omega C_1 R_L/n^2$ for various k occur at $q = 1.468$. If q is increased from 0 to 1.468 the value of capacitance C_1 rises allowing the application in the amplifier transistor switches with higher output capacitance.

Figure 3a depicts normalized series reactance X/R_L of the series resonant circuit $X/n^2 - L_R - C_R - R_L/n^2$ against q for given k . For $L_{SR} = 0$ to obtain a reasonable high value of Q_R (≥ 5) the value of k is rather low and then $X < 0$ as shown by the dashed lines of X values for constant Q_R .

Plots of normalized reactance of the transformer primary winding inductance $n^2\omega L_P/R_L$ versus q for various k are shown in Figure 3b. For q below 0.5 there is a fast increase in the value of $n^2\omega L_P/R_L$, which also means that then the value of the primary inductance L_P becomes high. A high-value primary inductance combined with a significant dc magnetizing component in the supply current conducted by the primary winding can cause that the size of the transformer Tr with a magnetic core can become large for low q and low k .

In Figure 3c, the peak values of the switch voltage $V_{S\text{MAX}}$ and the switch current $I_{S\text{MAX}}$ normalized to the supply voltage V_I and the supply current I_I are shown, respectively. For the case of $L_{SR} = 0$ the attainable values of the loaded quality factor Q_R versus q for various k are plotted in Figure 3d. As it can be noticed the Q_R values can be even above 10 for low k and/or low q .

3. Experimental circuit – design and measurements

To verify results of the theoretical analysis an experimental transformer Class E amplifier for $L_{SR} = 0$ was designed and built. Next, its basic parameters were measured. The given design example illustrates how the equations from the theoretical analysis can be used in the amplifier design. The experimental amplifier was designed for the nominal operation with the matching circuit $C_O - R_O$ (Fig. 4a) and for the following assumptions: supply voltage $V_I = 36$ V, maximum output power $P_{O\text{MAX}} = 100$ W, the operating frequency $f = 300$ kHz, amplifier drain efficiency $\eta_A = 0.94$, supply power $P_{I\text{MAX}} = P_{O\text{MAX}}/\eta_A = 100/0.94 = 106.4$ W, load

resistance $R_O = 50 \Omega$, the duty cycle of the gate voltage waveform $D = 0.5$. The transformer Tr was built with a gapped ETD39/20/13-3F3 ferrite core (gap – 1.32 mm) from Ferroxcube and a bi-sectional bobbin. Each of the two windings of the transformer was wound in the separate sections of the bobbin with a Litz wire 270×0.071 . The primary winding was $N_P = 20$ turns, its inductance $L_P = 73.8 \mu\text{H}$ and the secondary winding was $N_S = 22$ turns, and its inductance $L_S = 91.3 \mu\text{H}$, which gave the winding turn ratio $n = (L_S/L_P)^{1/2} = (91.3/73.8)^{1/2} = 1.1123$. The coupling coefficient k of the transformer Tr estimated from measurements was $k = 0.811$ [12]. From Equations (23) and (24) one computes the parameter $B_1 = 2\pi\omega L_P P_{MAX}/V_I^2 = 2\pi \times 2\pi \times 300 \times 10^3 \times 73.8 \times 10^{-6} \times 106.4/36^2 = 71.76$, which permits calculating from Equations (13) and (20) $q = 0.5121$, $p = 20.224$ and $\phi = -0.5244$ rad. From Equation (24) for the circuit of Figure 4a the load resistance R_L for the nominal operation is equal to $R_L = (R_{loss} + R_S) = n^2\omega L_P k^2 B_1 / (p^2\pi) = 1.1123^2 \times 2\pi \times 300 \times 10^3 \times 73.8 \times 10^{-6} \times 0.811^2 \times 71.76 / (20.224^2 \times \pi) = 6.309 \Omega$, where R_S – series load resistance, R_{loss} – series loss resistance modelling all power losses in the amplifier. The output current amplitude is $I_m = (2P_{MAX}/R_L)^{1/2} = (2 \times 106.4 / 6.309)^{1/2} = 5.808$ A, the series loss resistance $R_{loss} = 2(P_{MAX} - P_{OMAX})/I_m^2 = 2(106.4 - 100)/5.808^2 = 0.38 \Omega$ and the series load resistance $R_S = R_L - R_{loss} = 6.309 - 0.38 = 5.93 \Omega$. From Equation (25) the capacitance $C_1 = n^2 k^2 B_1 / (q^2 p^2 \omega \pi R_L) = 1.1123^2 \times 0.811^2 \times 71.76 / (0.5121^2 \times 20.224^2 \times 2\pi \times 300 \times 10^3 \times \pi \times 6.309) = 14.57$ nF. The parallel capacitance C_O is used to match resistance R_O to R_S , and the value of C_O is given by $C_O = (R_O/R_S - 1)^{1/2} / (\omega R_O) = (50/5.93 - 1)^{1/2} / (2\pi \times 300 \times 10^3 \times 50) = 28.92$ nF. From (32) the normalized series reactance for the frequency f is $X/R_L = -3.12$, which gives $X = -3.12 \times R_L = -3.12 \times 6.309 = -19.68 \Omega$. The series equivalent reactance X_S in the circuit in Fig. 4a is found from $X_S = R_O(R_O/R_S - 1)^{1/2} / (R_O/R_S) = 50(50/5.93 - 1)^{1/2} / (50/5.93) = -16.166 \Omega$. Taking into account X_S the series capacitance C_{SR} is given by $C_{SR} = 1 / (\omega(Q_R R_L + X_S - X)) = 1 / (2\pi \times 300 \times 10^3 (5.16 \times 6.309 - 16.166 - (-19.68))) = 14.71$ nF. The secondary leakage inductance is $L_2 = (1 - k)L_S = (1 - 0.811) \times 91.3 \times 10^{-6} = 17.26 \mu\text{H}$. Hence the loaded quality factor Q_R is computed as $Q_R = \omega L_2 / R_L = 2\pi \times 300 \times 10^3 \times 17.26 \times 10^{-6} / 6.309 = 5.16$. The theoretical peak values of the switch voltage $v(\omega t)$ and current $i_S(\omega t)$ are found from the waveform Equations (14), (17) as $V_{SMAX} = 3.572 \times V_I = 3.572 \times 36 = 128.6$ V and $I_{SMAX} = 2.843 \times I_I = 2.843 \times P_{MAX}/V_I = 8.403$ A. As the switch S an IR MOSFET transistor IRFB4115 was used. Polypropylene capacitors FKP1 from Wima were applied as C_1 , C_{SR} and C_O , and their measured values were $C_1 = 13.94$ nF, $C_{SR} = 14.35$ nF, $C_O = 28.83$ nF. The value of C_1 in the experimental circuit had to be lowered from its theoretical value to obtain nominal operation of the circuit because of the output capacitance of the applied MOSFET switch. The obtained maximum output power in the experimental amplifier was $P_{OMAX} = 99.97$ W with the efficiency $\eta_{MAX} = 0.95$.

The measured peak values of the transistor switch voltage and current were $V_{SMAX} = 136$ V, $I_{SMAX} = 8.2$ A, respectively. The designed amplifier was also simulated using LTSpice with an ideal switch and component values as in the design example. The simulation results were $P_{OMAX} = 103.5$ W, $V_{SMAX} = 134.5$ V, $I_{SMAX} = 8.33$ A for a steady state (after a hundred periods). For higher values of Q_R (>10) the output current i_O contains a lower level of harmonics and the differences between parameters of the simulated amplifier and the analytically designed circuit decrease.

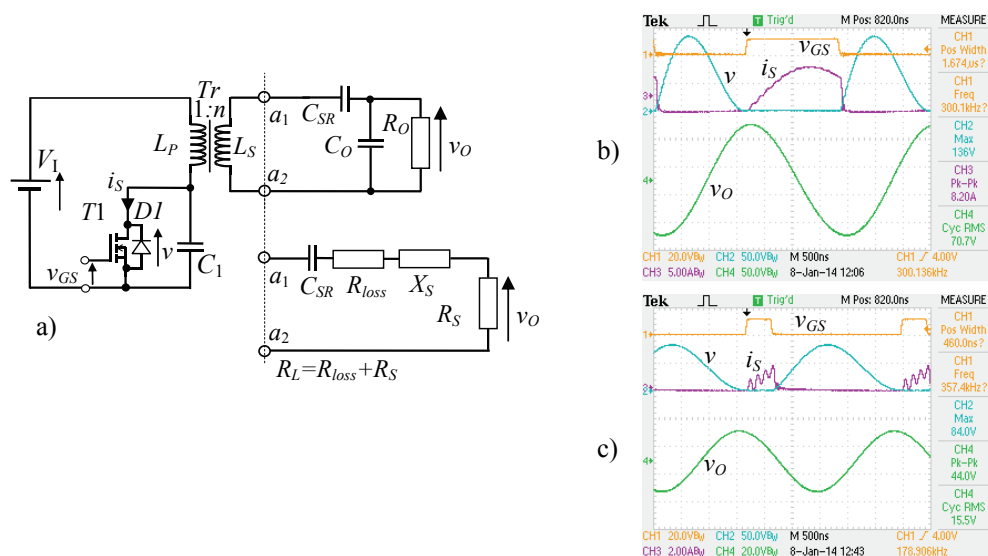


Fig. 4. Experimental amplifier with a matching circuit and its equivalent circuit (a), measured waveforms in the experimental amplifier for $P_{O,MAX}$ (b) and for $P_O = 4.8$ W (c)

The experimental amplifier was also tested with respect to FM control of its output power for $R_O = 50 \Omega = \text{const.}$ and $V_I = 36$ V. The operating frequency of the amplifier was varied from 300 kHz to 357.4 kHz and simultaneously the on-time of the switch was adjusted (from 1.67 μs to 0.46 μs) to assure its ZVS operation. The output power in the experimental circuit was regulated from its maximum value to 4.8 W with the efficiency decreasing from 0.95 to 0.9, respectively. Thus this wide range of output power control in the amplifier required only 19% change of the operating frequency.

4. Conclusions

The results of the presented detailed analysis of the Class E amplifier have shown how the amplifier parameters depend on some of the parameters of the applied transformer. A design procedure for a transformer Class E amplifier with only one inductive component has been proposed as well as verified both experimentally and by simulation. The theoretical and experimental results have agreed well enough to use the design procedure and the analysis results for engineering purposes. The analysis can be also used to design transformer Class E amplifiers both with an external inductor in the series resonant circuit and without the inductor ($L_{SR} = 0$). It is worth noticing that the analysis can be used to further optimise the transformer Class E amplifier by solving the presented equations for $D \neq 0.5$ and the nominal or off-nominal operation of the amplifier. Because the number of inductive components in the trans-

former Class E amplifier can be much reduced the circuit can find applications in low-cost, isolated miniaturised resonant dc/dc converters [7, 10, 11, 13, 14] and h.f. power generators [15, 16].

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