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ANALYSIS OF THE INFLUENCE OF SHAFT LOAD ON THE VALUE OF ACCEPTABLE PRELOAD IN A SYSTEM OF ANGULAR BALL BEARINGS

ANALIZA WPŁYWU OBCIĄŻENIA WAŁU NA WARTOŚĆ DOPUSZCZALNEGO ZACISKU WSTĘPNEGO W UKŁADZIE ŁOŻYSK KULKOWYCH SKOŚNYCH

Key words: angular ball bearing, preload, shaft loading, durability of bearing, elasticity of bearing.

Abstract: In standard solutions, reactions of the supports and bending moments in bearing shafts are calculated without taking into consideration the elasticity of the bearings themselves. They are treated as perfectly stiff supports of articulate character. Whereas, in reality, the majority of bearings do not fulfil conditions of being classified as joints, because the angular tilt of inner bearing ring in relation to the outer one causes inner elastic deformations in elements of bearings and the resulting reaction moments. These moments have an impact on reactions of bearings.
The aim of the research is to determine how shaft loading influences the value of preload by using the criterion of the durability of the bearing and taking into consideration the elasticity of bearings.

Słowa kluczowe: łożysko kulkowe skośne, zacisk występny, obciążenie wału, trwałość łożyskowania, sprężystość łożysk.

Streszczenie: W standardowych rozwiązaniach oblicza się reakcje podpór i momenty gnące w łożyskowanych wałach bez uwzględnienia sprężystości samych łożysk. Są one traktowane jako doskonale sztywne podpory o charakterze przegubowym. Tymczasem w rzeczywistości większość rodzajów łożysk nie spełnia warunków przegubu, bowiem kątowe wychylenie pierścienia wewnętrznego względem zewnętrznego wywołuje sprężyste odkształcenia wewnętrzne w elementach łożysk i wynikające z tego momenty reakcyjne. Momenty te mają wpływ na reakcje łożysk.
Celem przeprowadzonych badań jest określenie, jak wpływa obciążenie wału na wielkość zacisku wstępnego, kierując się kryterium trwałości łożyskowania i z uwzględnieniem sprężystości łożysk.

INTRODUCTION

Depending on the type of a bearing, a preload can be radial or angular. For example, cylindrical bearings, due to their construction, can be solely a subject to radial preload, whereas thrust ball bearings and thrust roller bearings – to axial preload. Single row angular bearings (**Fig. 1**) are usually placed in a system of two bearings of the same type in divergent layout 'O' or in convergent layout 'X'. The above is possible thanks to the use of preload via an offset shift. Preload concerns both divergent and convergent systems, but they are achieved with the use of a bit different technical methods for "O" and for "X" systems.

Regular ball bearings can also be axially preloaded. To achieve that, these bearings should have an inner radial clearance of a higher value than the regular one, so that, under the impact of an axial force, the obtained angle of work is much higher than zero, similarly to the case of angular ball bearings **[L. 10]**.

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While studying a shaft supported by angular ball bearings, it is common to assume that support reaction is cumulated in the nodal point of angular bearing (NP point in **Fig. 1**). In the case of angular ball bearings, the distance between nodal points is longer than the distance L between centres of bearings when the bearings are in divergent system (**Fig. 1a**), and it is shorter when they are in convergent system (**Fig. 1b**). The above means that bearings in divergent system "O" can acquire relatively high overturning moment, even if distance between the bearings in relatively small. Transversal forces emerging as a result of loading with overturning moment and deformations caused by them in bearings in a divergent system are weaker than in the case of a convergent system **[L. 2, 10]**.

As given in **[L. 12]**, while determining the preload force, one should first calculate the target force of preload, which will ensure optimum combination of stiffness, exploitation time and the reliability of work of bearing. Then, the force of preload to be applied during assembling of the bearings has to be calculated. During assembly, the bearings should have the temperature of the environment and should not be treated with the working load.

In the normal temperature of work, a correct preload depends on loading of a bearing. Angular ball bearings are capable of carrying radial and axial load. With axial load, a new force, working in axial direction, will act in the bearing. This force has to be overtaken by the second bearing placed in opposite direction to the first one.

In a bearing consisting of two angular ball bearings in divergent or convergent system, every bearing has to overtake axial forces from the other one.

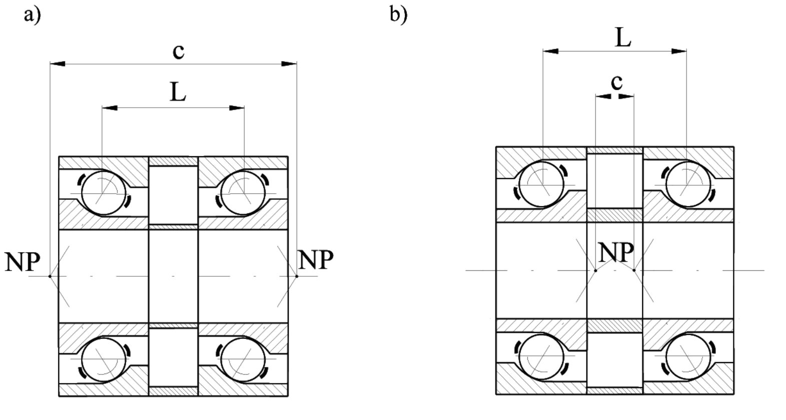


Fig. 1. Angular ball bearings: a) in divergent system (system "O"), b) in convergent system (system "X") [L. 12]
Rys. 1. Łożyska kulkowe skośne: a) w układzie rozbieżnym (układ „O"), b) w układzie zbieżnym (układ „X") **[L. 12]**

Preload is understood as an axial displacement of one bearing's ring in relation to the second one in such a direction that the inner force in the bearings increases. This displacement is measured starting from the neutral state. The neutral state is defined as one taking place when a system of bearings is not treated with any outer force (even the gravity), and all rolling parts adjoin to both rings of every bearing without contact force. As a result, a preload with a positive value has to cause the appearance of contact pressure of rolling parts, even with no outer force operating. Preload with negative value causes rolling parts to recede from the bearing track.

If bearings are loaded with radial forces, preload with positive value causes the increase of the number of rolling parts under the contact pressure, whereas the preload with negative value causes this number to decrease. The higher the pressure (in the absolute values), the stronger is the phenomenon.

It was necessary to establish how, in the first approximation, the axial displacements in bearings depend on preload. It is obvious that preload has a simultaneous impact on both bearings, but in general, when a bearing system is a subject to axial force, these impacts are varied. Higher axial load can appear both in the left and right bearing, which depends on the direction in which the shaft is "pushed" together with outer rings of bearings. The resultant of forces working axially is decisive here. The forces are the following:

- Total of outer forces directed axially: ΣF_x
- Inner longitudinal force in the left bearing: (S_A),
- Inner longitudinal force in the right bearing: (S_B).

A simplified analysis of the impact of preload on axial deformations in bearings is presented in **Fig. 2**. Small circles represent the part of the bearing that is bound with the node (inner ring + rolling parts). They are denoted as model balls. Vertical lines in the proximity of balls represent the outer ring. The distance between the balls and these vertical lines represents the total clearance in bearing. If a model ball penetrates the vertical line, the dimension of this penetration represents the total joint deformation of all working surfaces in bearing (f). Part "a" in the figure schematically illustrates a system of two ball bearings with a supporting shaft, with the assumption that axial forces with the resultant ΣF_x directed towards bearing B act on the shaft. The value of preload is 0; consequently, the distance between the edge surfaces of model balls K is equal to the distance between abutment surfaces P. As a result, balls in the right bearing are pressed into the surface of abutment track to the depth of f_{Bxw}, while the balls of the left bearing are moved away to the same distance from its abutment surface. The value of f_{Bxw} deformation is calculated with the following formula deriving from the work **[L. 1]**:

$$f_{Bxw} = 4.4 \cdot 10^{-4} \left(\frac{\Sigma F_x + S_A}{Z_B} \right)^{0.667} D_{kB}^{-0.333} \left(\sin \alpha_{0B} \right)^{-1.667}$$

f_{Bxw} represents initial axial deformation resulting from the total of forces ΣF_x and S_A acting on the bearing B without preload. f_{Ax} and f_{Bx} represent axial deformations in the subsequent states of preload.

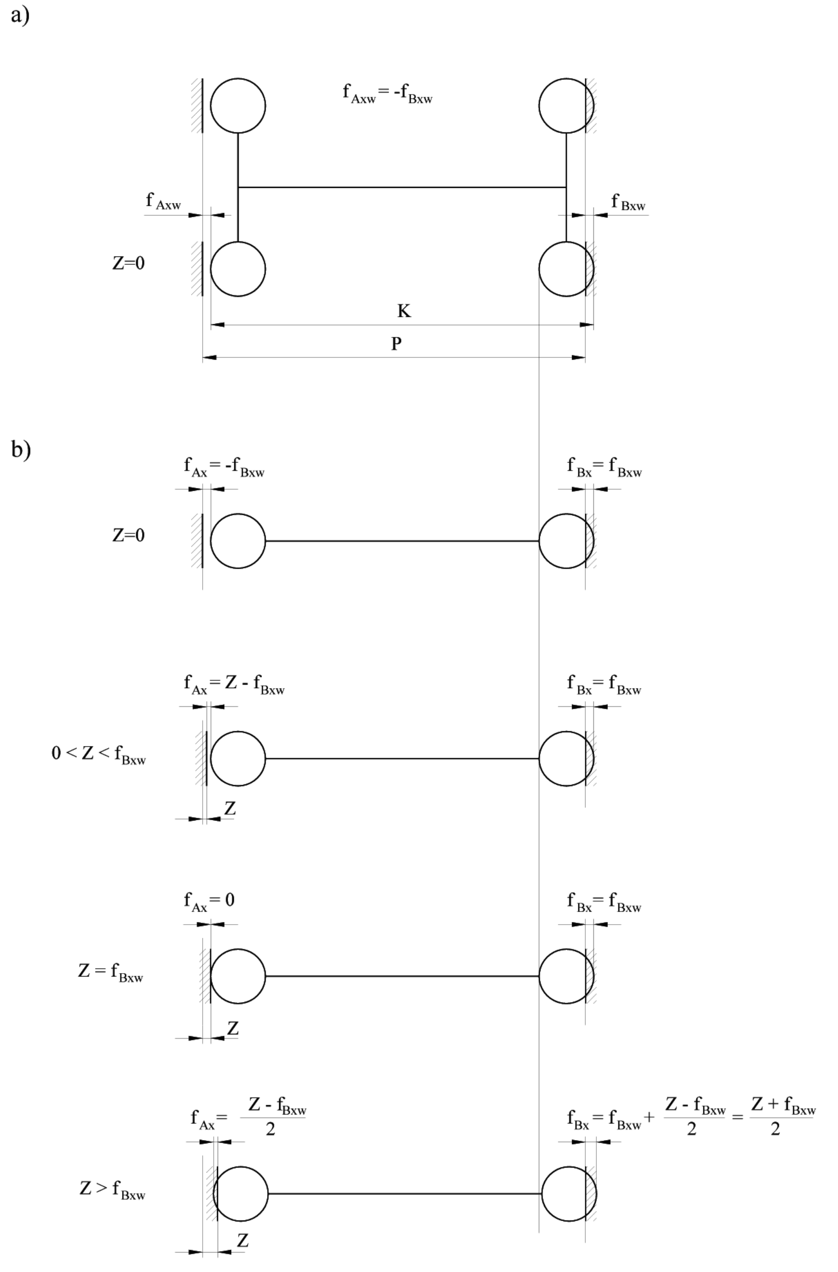


Fig. 2. Model used for the analysis of impact of preload on axial deformations in bearings
Rys. 2. Model do analizy wpływu zacisku wstępnego na przesunięcia poosiowe w łożyskach

Part “b” of **Fig. 2** presents subsequent situations occurring in this setting with the increasing value of preload. The drawing model is more simplified, because it shows only one ball out of each bearing. In every following case, distances f_{Ax} and f_{Bx} in left and right bearings are described. A positive value of each of these parameters means deep deformation, and a negative value means clear space between the ball and the track. The first case (Z = 0) is a repetition of the situation illustrated in Fig. “a”. In the second case, a preload is observed, yet it has a value lower from the one of the deformation f_{Bxw}. This low preload causes only a decrease in the absolute value of f_{Ax} distance for the unchanged f_{Bx}. In the next case, the Z value is equal to f_{Bxw}. As a result, f_{Ax} distance becomes equal to 0, but f_{Bx} remains unchanged. The last case illustrates a situation caused by high preload, exceeding the deep initial deformation in bearing B. As a result, deep deformations in both bearings must take place. If the bearings are identical, as the result of preload, the increase in deformation is distributed equally between the two bearings. Thus, half of the value of preload is assigned to the deep deformation. That distribution is evident in this case in f_{Ax} and f_{Bx}.

It is evident that the sum of values f_{Ax} and f_{Bx} is unchanged and equal to preload Z, which proves the correctness of the solution.

The course of the variability of values f_{Ax} and f_{Bx} in relation to preload Z as determined above is illustrated in **Fig. 3**. It is easy to notice that after crossing the abscissa axis Z = f_{Bx} both lines are rising at the angle, the tangent of which is equal to 0.5, which is consistent with the rule formulated above, according to which $f_{Ax} + f_{Bx} = Z$.

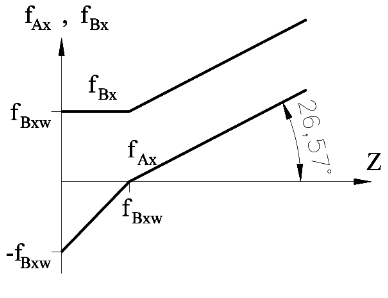


Fig. 3. Approximate impact of preload on axial dislocations in bearings

Rys. 3. Przybliżony wpływ zacisku wstępnego na przesunięcia poosiowe w łożyskach

In the situation when the resultant of forces acting on a system of bearings ΣF_x is oriented in the opposite direction, i.e. towards the bearing A, the procedure has to be reversed. The depth of deformation in the left bearing becomes (f_{Axw}) the base value, and the further procedure is analogous, while consequently maintaining the opposite symbol.

Knowledge of values of dislocations in bearings enables determining deformations in contact points of balls and tracks, which, in consequence, leads to the calculation of contact forces, and then of full reactions in bearings of the system in subject.

The presented method of determining dislocations in bearings in the function of preload is needed only in the initial (approximate) stage of calculations. In later, more exact calculations, dislocations are determined in iteration process based on the condition of the fulfilment of equilibrium equations in bearings.

In standard solutions known from literature, reactions of supports and bending moments in bearing shafts are calculated without taking into consideration the elasticity of the bearings themselves. They are treated as perfectly stiff supports having the character of joints. Meanwhile, in reality, the majority of types of bearings (except for the self-aligning ones) do not fulfil the conditions of a joint, because angular deflection of inner ring in relation to the outer ring (**Fig. 4**) evokes elastic inner deformations in elements of bearings.

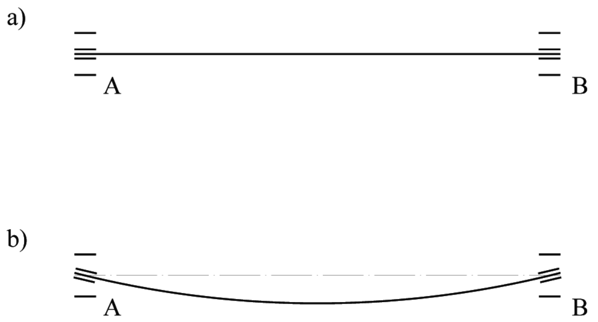


Fig. 4. Influence of shaft bending line on deflection of bearing rings: a) state before shaft deflection, b) state after shaft deflection

Rys. 4. Wpływ linii ugięcia wału na wychylenie pierścieni łożyska: a) stan przed ugięciem wału, b) stan po ugięciu wału

A situation in an angular ball bearing after deflection of the inner ring in relation to the outer one is presented in **Fig. 5**. It is clearly evident that, in consequence of the deflection of inner ring in relation to the outer one, certain deformations of balls and track occur, which generates additional inner forces between these elements. This is manifested by the increase in normal forces of the Q pressure of balls on the track. Moreover, the directions of the operation of Q forces change. As a result of deflection of the inner ring by the angle θ (compare with **Fig. 5**), lines of operation of vectors Q’ and Q” (inner forces on the bottom and upper ball) pass the nodal point of nominal reactions (W point) on the right side, i.e. they create moments with the same sign.

Lines of acting of forces originating from all balls of the bearing deflect analogously, but to a different degree, depending on the location of a ball on the circumference of the bearing. As a result of these force deflections, a resultant moment in axial plane of bearing and shaft appears. This moment emerges together with the increase of Q forces compared to the state without the deflection. In the analysis of shaft statics, this moment has a bending character. Reaction bending moments appear in both bearings. They are presented in a simplified form in **Fig. 6**.

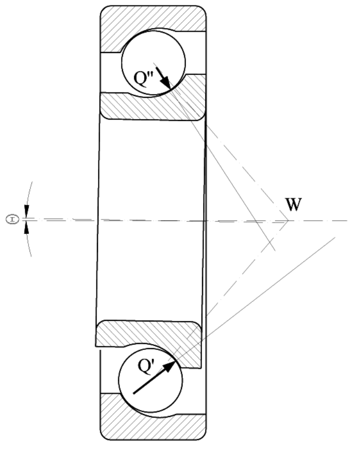


Fig. 5. Directions of inner forces acting after deflection of inner ring
Rys. 5. Kierunki działania sił wewnętrznych po wychyleniu pierścienia wewnętrznego

Fig. 6. Emerging of reaction bending moments in bearings
Rys. 6. Powstawanie reakcyjnych momentów gnących w łożyskach

It is evident that the angular deflection of a shaft in the points of support is restrained by reaction bending moments, which are a response of the bearing to angular deflection. A counteraction to the bending (deflection) of a shaft occurs here. In consequence, bearings are loaded differently than it could result from a simple model built with the use of outer forces and reactions of supports. Moreover, the reaction bending moment is included among loads of the shaft, which influences the line of shaft deflection, which in turn has an impact on the distribution of the bending moment and reaction of supports.

So in general terms shaft is a subject to the following loads:
- Outer forces and moments,
- Reaction forces of supports, and
- Reaction moments of M_A, M_B supports.

In the case of a shaft loaded with a number of forces working in different planes, a shaft deflection line is a spatial one with relatively complicated shape. Therefore, defining arrows and angles of deflection with the use of simple formulas used in technical guides does not apply to the considered case.

In practice, two methods are used in relation to so complicated cases. For many simple shaft loads, it is possible to use the rule of superposition, which involves putting together of the bending originating from particular forces. The second method used in more complicated cases of load is Mohr's method, which is based on the analogy of differential equations describing geometric parameters of deformation and dependencies between loads.

CALCULATING METHOD

In order to solve the problem the following issues have been connected:
1. The machine shaft bending line with complex outer load,
2. The displacement of inner bearing rings in relation to the outer ones as a result of loads and preload,
3. Contact elastic deformations in contact points of roller parts and tracks in both bearings of the system,
4. Calculations of contact forces in bearings on the basis of contact deformations,
5. The balance between inner (contact) forces in bearings and the outer load acting on the whole bearing system, and
6. Calculations of bearings durability based on these contact forces.

The adoption of a suitable calculation model is a vital step on the way to a theoretical solution of the issue. Not only the degree of approximation to reality and the range of considered phenomena, but also the laboriousness of the solution depend on the model. In publications concerning phenomena occurring in roller bearings, calculation models are rarely revealed in detail. This study uses the method of modelling developed by the authors and applied in works **[L. 3–11]** and others.

CALCULATIONS

Angular ball bearings work in systems and have to be considered in systems, especially in terms of preload consideration. Forces acting in bearings depend on numerous factors, for example, on the load per each bearing in a system, or on shaft deflection. The loads acting on bearings depend, among others, on the type, size, and number of transmission wheels mounted on a shaft or on the location of cooperating wheels. The possibilities are infinite; therefore, a specific construction has been adopted.

A model shaft was adopted as the construction (**Fig. 7**). The shaft has two angular ball bearings 7212B with dynamic capacity as in [L. 13] C=57200 N.

The bearing placed at the end of the shaft on the left side was marked as "A" and the one on the right – "B".

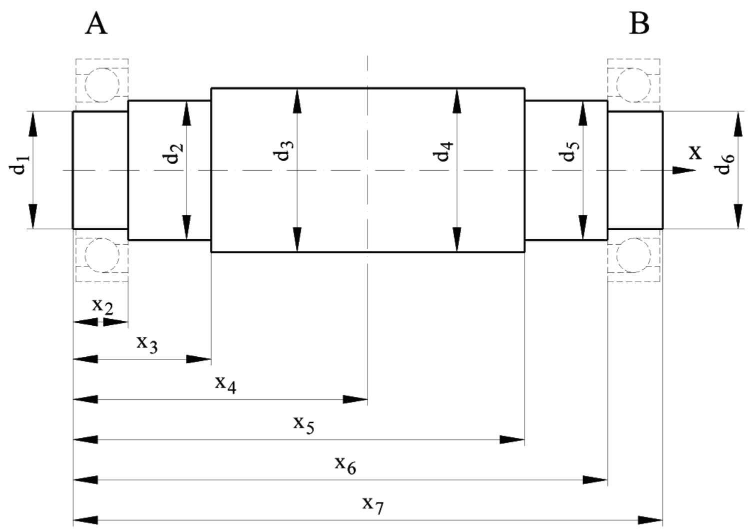


Fig. 7. Draft of a model shaft
Rys. 7. Szkic modelowego wału

Dimensions of the model shaft are the following: x_2 = 22 mm, x_3 = 100 mm, x_4 = 200 mm, x_5 = 300 mm, x_6 = 371 mm, x_7 = 400 mm, d_1 = 60 mm, d_2 = 67 mm, d_3 = 75 mm, d_4 = 75 mm, d_5 = 67 mm, and d_6 = 60 mm. Calculations were performed for the bearing with loads of different values and locations. The variations of locations of loads are presented in **Fig. 10**. In the first variation of location, it is assumed that the load is applied on two sides of one gearwheel placed at x_L distance from the beginning of the shaft. In the second variation, the load is placed on two gearwheels located at the distance of x_{L1} and x_{L2} from the beginning of the shaft. The locations of the points of load application are defined with the angles β_1 and β_2.

Locations of the planes of loads were adopted in defined relations to the length of the shaft L_w, equal to dimension x_7:

For the first variation of location (**Fig. 8**): $x_L = 0.3\ L_w$ (120 mm), $x_L = 0.4\ L_w$ (160 mm), $x_L = 0.5\ L_w$ (200 mm), $x_L = 0.6\ L_w$ (240 mm) or $x_L = 0.7\ L_w$ (280 mm).

For the second variation of location (**Fig. 8**): $x_{L1} = 0.4\ L_w$ (160 mm) , $x_{L2} = 0.6\ L_w$ (240 mm), for the angles: $\beta_1 = 90º$, $\beta_2 = 90º$ and $\beta_1 = 90º$, $\beta_2 = 180º$ or $\beta_1 = 90º$, $\beta_2 = 270º$.

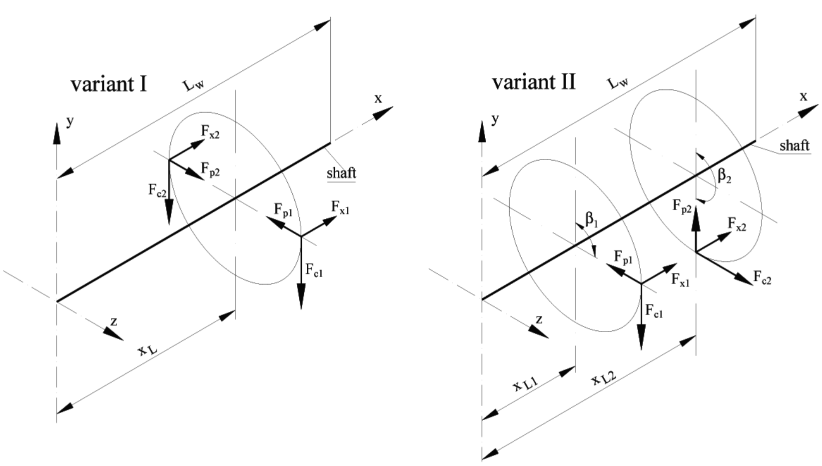


Fig. 8. Models of location of bearing loads
Rys. 8. Modele umiejscowienia obciążeń łożyskowania

Diameter of the running wheel D_t = 200 mm.

It has been assumed that loads it both the points presented in **Fig. 8** are identical ($F_{c1} = F_{c2}$, $F_{p1} = F_{p2}$, $F_{x1} = F_{x2}$) and that the diameters of wheels are also identical.

It has also been determined that the circumferential force on the presumed gearwheel F_{c1} will depend on dynamic capacity and will be adopted on three levels: as 0.075 C, 0.1 C, or as 0.125 C.

The main goal of the research is to determine the influence of shaft load on the value of preload, taking into consideration only durability of bearing without considering its stiffness and the moment of friction. It is vital to underline that preload has its impact on all the above elements, which will be the topic of further research. The durability of the bearing will be defined in relative way by determining the indicator of durability, defined with the following formula [**L. 7**]:

$$W_T = \frac{L_{hA}}{L_{hA0}} \cdot \frac{L_{hB}}{L_{hB0}},$$

where

- L_{hA} – fatigue life of bearing A in specific conditions with preload given,
- L_{hB} – fatigue life of bearing B in the same conditions with preload given,
- L_{hA0} – fatigue life of bearing A in the same conditions without preload given,
- L_{hB0} – fatigue life of bearing B in the same conditions without preload given.

RESULTS AND CONCLUSIONS

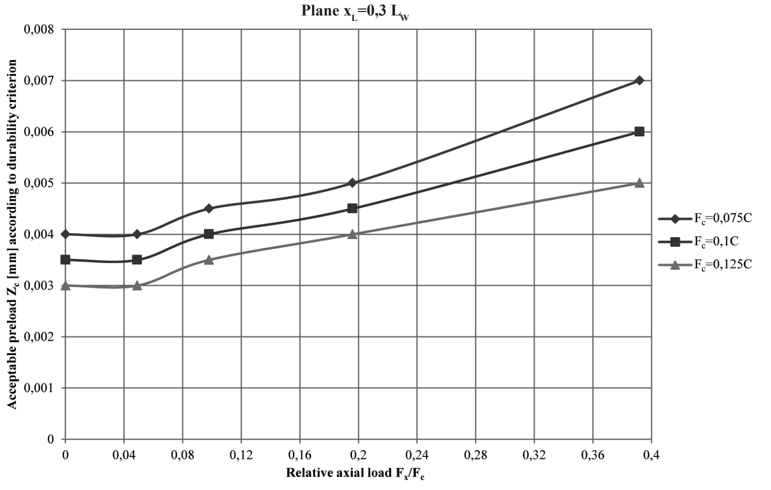
Summary diagrams have been made in order to determine the beneficial range of preload values. The diagrams were based on agreed limit points. The limit points were the values Z_c, for which characteristics of W_T index have the value 0.98. It has been assumed that, when aiming at the increase of longitudinal stiffness of bearing, the indicator decrease will be 2%.

Summary diagrams for the limit (acceptable) preload depending on the shaft load are presented in **Figs. 9** and **10.**

Based on the received characteristics, the following observations have been made:

- When x_L = 0.3 L_w, very low value of acceptable preload, equal to 3–5 µm, has been obtained. Consequently, in this position of a load plane, the preload is not recommended with regards to total durability of bearing.
- When x_L = 0.4 L_w, the acceptable values of preload are very low, and they are on the level of circa 6–7 µm. Higher values correspond with higher axial load, nevertheless they do not increase much. In reality such low values of preload are difficult to maintain.

a)



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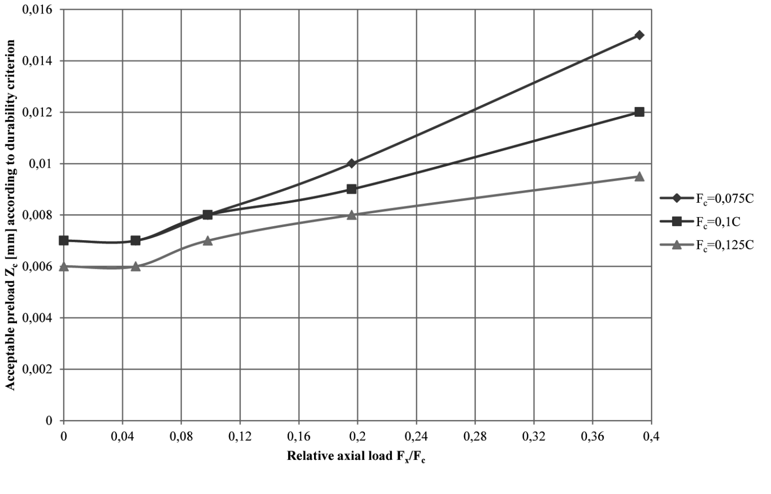

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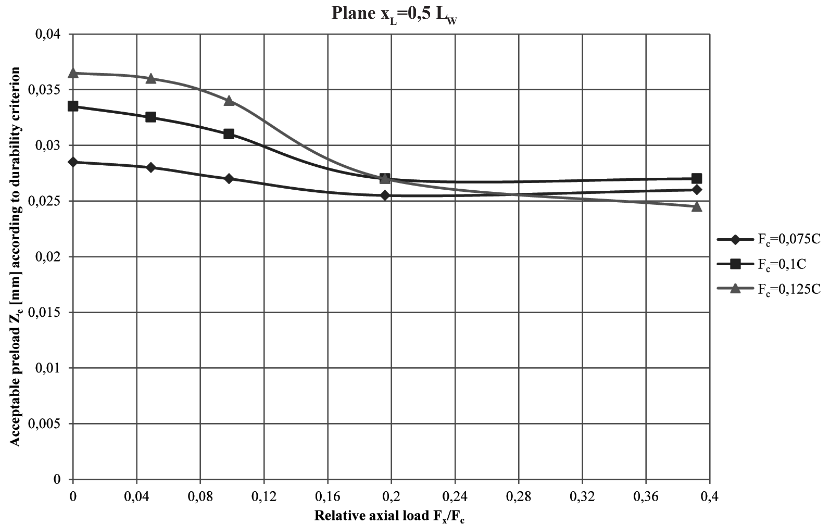

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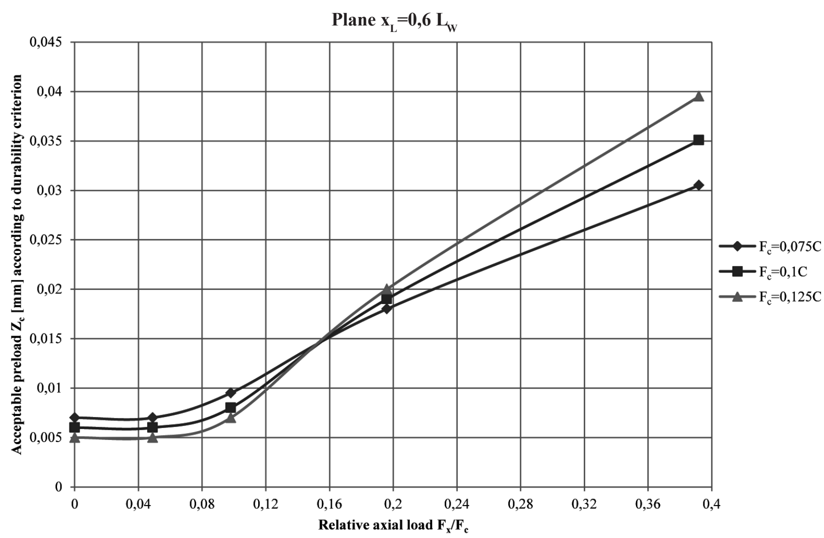

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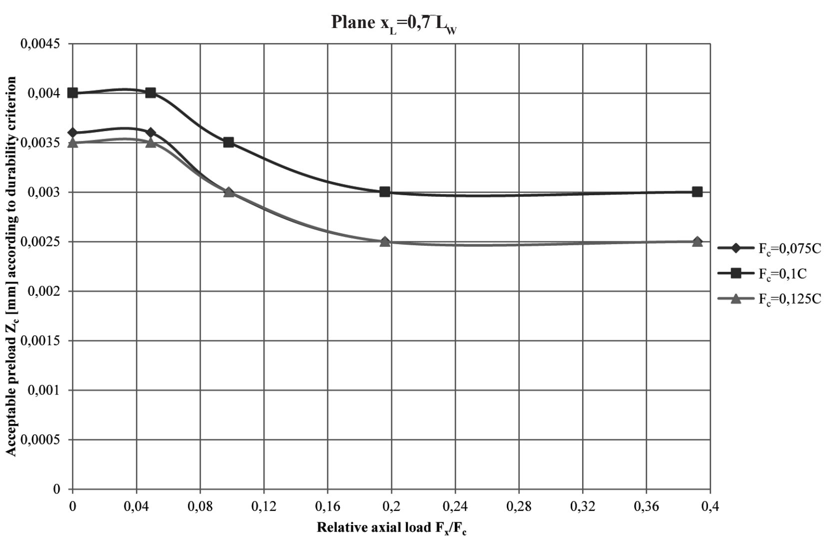
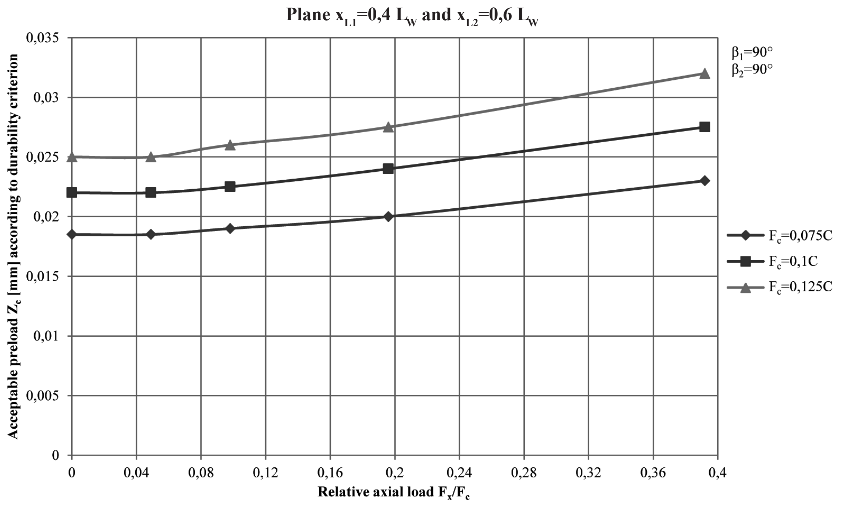


Fig. 9. Limit value of preload for the 1st variation of load

Rys. 9. Graniczny zacisk wstępny dla I wariantu obciążenia a)



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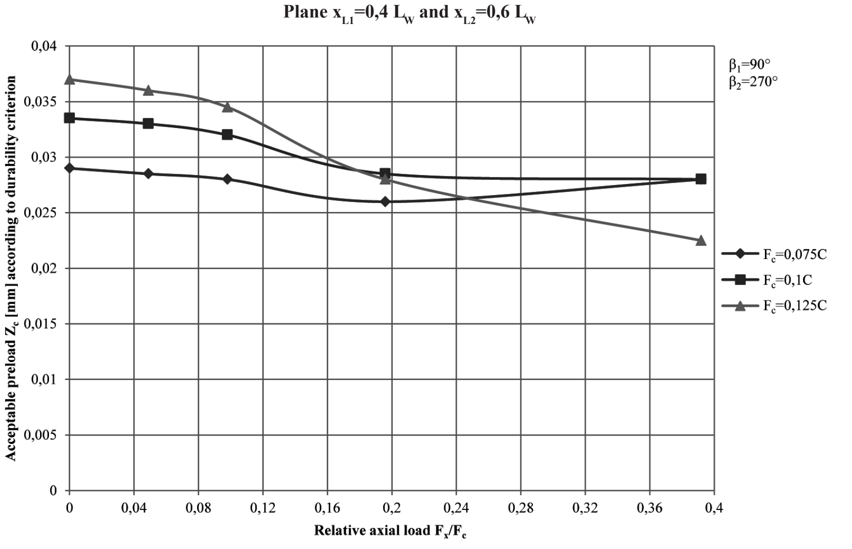

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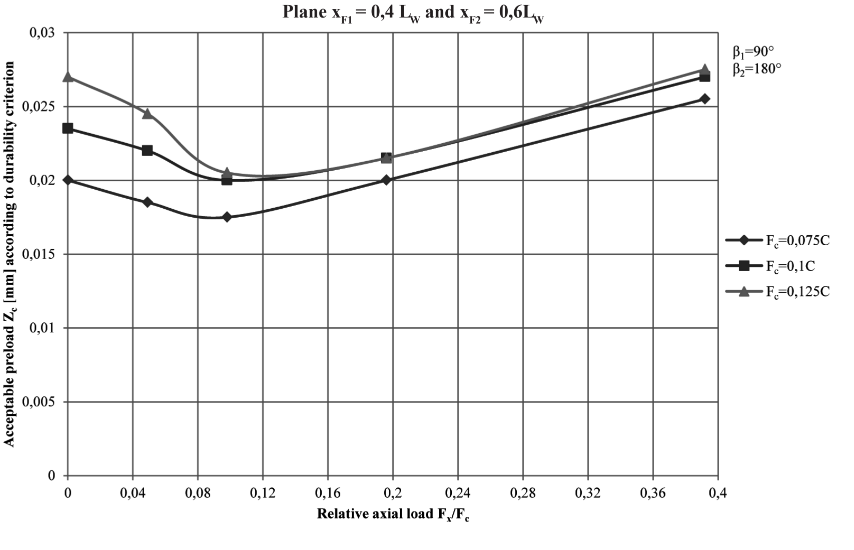

Fig. 10. Limit value of preload for the 2nd variation of load

Rys. 10. Graniczny zacisk wstępny dla II wariantu obciążenia

- For the central position of a plane, in the case of a low value of transverse load in a bearing (0.075 C), border values of preload hardly depend on shaft force and are about 26–28 μm, and they decrease with the stronger axial force. With medium transverse load (0.1 C), the beneficial value of preload ranges from 27 μm when axial force is equal to 0.4 F_c and up to 33 μm when axial force is equal to 0. When applying a higher value of transversal load (0.125 C), the border value of preload ranges from 24 μm when the axial force is equal to 0.4 F_c, up to 36 μm with no axial force operating.
- When $x_L = 0.6\ L_w$, the border value of preload ranges from 4–7 μm when the sum of shaft forces is 0, up to 30–38 μm when $F_x/F_c \approx 0.4$.
- When $x_L = 0.7\ L_w$, a very low level of border value of preload, equal to 2.5–3–5μm, is observed. Practically, this value is on the error level for preload adjustment. Therefore, in this position of load plane, preload is not recommended if total durability of the bearing is taken into consideration.

With reference to the 2nd variation of load location, where $x_L = 0.4$ and $0.6\ L_w$, the following observations have been made:

- When β_1=90º and β_2=90º, the course of Z_c index is somewhat flat, which proves its small dependence on axial force. Yet, what is more important is that these characteristics run on a considerably high level of preload, i.e. at over 18μm.
- When β_1=90º and β_2=180º, the course of curves for the border Z_c, has been observed as quite flat, i.e. with flat courses. All the lines run above the level of 17.5μm (practically at 18 μm). The system of lines is almost ordered, i.e. characteristics run subsequently at increasing height for the lowest, medium, and highest transverse load in the bearing.
- When β_1=90º and β_2=270º, the course of curves of the Z_c index is also somewhat flat; however, compared to the previous cases, they run at the highest level of over 23μm.

CONCLUSIONS

Characteristics corresponding to the location of load plane $x_L = 0.3\ L_w$ are very similar to the characteristics received earlier for $x_L = 0.4\ L_w$, and they allow for a very low preload if considering the criterion $W_T \geq 0.98$. At the same time, characteristics for $x_L = 0.7\ L_w$ bear no similarity to characteristics obtained for $x_L = 0.6\ L_w$. They rather resemble the characteristics obtained when $x_L \approx (0.3–0.4)\ L_w$. As a result, it should be recognized that only in the range $x_L = (0.5–0.6)\ L_w$ can a high preload be used while maintaining the criterion $W_T \geq 0.98$, and even with some additional restrictions for $x_L > 0.5\ L_w$.

It can be concluded from the overview of characteristics made for the 2nd variation of the load location that, in each of the studied cases, regardless of the angle determining the point of load on the load plane, the use of preload remains beneficial. It is a common denominator for these cases that transversal loads in both bearings are much alike, which is similar to the case when there exists only one plane of load placed in the middle between the bearings ($x_L = 0.5\ L_w$).

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