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PARTICIPATION FACTOR IN MODAL ANALYSIS OF POWER SYSTEMS STABILITY

This paper provides a brief overview on existing approaches for defining participation factor in modal analysis which characterizes the interaction between modes and state variables of power system. We calculated participation factors using different methods and compared the results obtained with the expressions derived from mode evolution. For cases of complex eigenvalues of linear differential equations characteristic matrix it was discovered incorrect existing approaches for defining participation factor of state variable in mode. Modern software applications designed to analyze power system stability widely deploy the approaches discussed that provide incorrect results of modal analysis and pose risks to the operation of real power systems. Therefore the problem of calculating the participation factor remains as important as ever.

KEYWORDS: modal analysis, participation factor, static stability, power system

1. INTRODUCTION

The Modal Analysis is the most modern method currently used to analyse power system stability. This method involves decomposition of power system oscillations to separate components. Considering complex systems, the process of merging power systems is the most difficult to simulate. First, the main problem is the size of such system as it consists of hundreds of generators connected with thousands of power lines, bushes, and hundreds of load centers. Secondly, complex nature of network physical processes causes problems due to physical values with different time dynamics (electrical changes usually occur faster than mechanical change of generator rotor position). As is known the good enough mathematical model for study of oscillating static stability for power system, is system of linearized differential equations that describe behaviour of system oscillations caused by minor disturbances.

The one of main concept of modal analysis is participation factor. Participation factor is a scalar value that determines the degree of system

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parameters influence in formation of oscillations mode for linear systems. The theory of modal analysis was presented by scientist Perez-Arriaga and others in the works [1, 2]. Since then this technique was developed greatly and applied to problems of power systems stability. In a series of papers [3-5] authors (E.H. Abed, W.A. Hashlamoun, and M.A. Hassouneh) reviewed the concept of participation factor and showed that it is actually determined using different formulas for calculating mode-in-state and state-in-mode participation. In other words, the difference between the mode-in-state and state-in-mode concepts was introduced. And for this case they used the stochastic nature of the input state vector. Our goal are comparing different approaches to calculating the participation factor of linear time-invariant systems and investigate their correctness. The comparison will be conduct based on numerical examples.

2. PARTICIPATION FACTOR

2.1. Initial equations

Today it is known two different approaches to the participation factor of state variable in mode: the approach of scientists Perez-Arriaga and Verghese, and approach of group of scientists led by E.H. Abed. Before we consider these approaches let's briefly overview of the history of participation factor problematics.

The power system under study consists of a number of N synchronous machines connected by a large number of connections. In the linear approximation, the description of this power system can be presented with the system of differential equations:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}, \tag{1}$$

where $\mathbf{x}$ – column vector of state variables, $\mathbf{A}$ – characteristic matrix of differential equations system by which power system is described in the linear approximation. In the case of power system description matrix A is real, that is, $\mathbf{A}^* = \mathbf{A}$. In general, the matrix A has N different eigenvalues, some of which are a complex conjugate:

$$\lambda_i = \sigma_i \pm j\omega_i, \tag{2}$$

where σ_i – real part of eigenvalue which characterizes state stability margin of power system, ω_i – imaginary part of eigenvalue which determines fluctuation frequency of power system mode. By stability margin we shall basically mean real part module of eigenvalue. Left and right eigenvectors that correspond to eigenvalue λ_i are defined by expressions:

$$\begin{aligned} \mathbf{A}\mathbf{r}_i &= \lambda_i \mathbf{r}_i, \\ \mathbf{l}_i \mathbf{A} &= \mathbf{l}_i \lambda_i. \end{aligned} \tag{3}$$

The notation is following: $\mathbf{r}_i$ – right column eigenvector, $\mathbf{l}_i$ – left row eigenvector. Left and right eigenvectors are normalized to the symbol Kronecker:

$$\mathbf{l}_i\,\mathbf{r}_j = \delta_{ij}. \tag{4}$$

The solution of equation (1) with initial condition $\mathbf{x}_0 = \mathbf{x}(0)$ is an expression:

$$\mathbf{x}(t) = \mathbf{R} \cdot e^{\mathbf{\Lambda} t} \cdot \mathbf{L} \cdot \mathbf{x}_0 \,. \tag{5}$$

The notation of expression (5) is following: $\mathbf{R}$ – right eigenvectors matrix, each column of which is the right eigenvector; $\mathbf{L}$ – left eigenvectors matrix, each column of which is the left eigenvector; $\mathbf{\Lambda}$ – diagonal matrix, the main diagonal of which contains the eigenvalues of matrix A**.** Condition of normalization between the left and right eigenvectors in this case would look like this:

$$\mathbf{L} \times \mathbf{R} = \mathbf{1}.$$

To determine participation factor of k-state variable in i-mode you need to actually decompose i-mode on the basis of the state vector. That is, we need to consider the equation:

$$\mathbf{z}(\mathrm{t}) = \mathbf{L}\mathbf{x}(\mathrm{t}). \tag{6}$$

Components of vector $\mathbf{z}(\mathrm{t})$ represent the evolution of mode associated with the corresponding eigenvalue. Substituting the expression (5) in equation (6) we get the evolution of k-mode:

$$\mathbf{z}_k(t) = \mathbf{l}_k \mathbf{x}_0 e^{\lambda_k t} \,. \tag{7}$$

In this approach, the biggest problem is the choice of initial conditions $\mathbf{x}_0$.

Then we proceed to consider the approach proposed by scientists Perez-Arriaga i Verghese. In the paper [1] as initial conditions the authors chose the right eigenvectors, $\mathbf{x}_0 = \mathbf{r}_\mathrm{i}$, which is not quite correct. In this case the participation factor of k-state variable in the i-mode is determined by the formula:

$$p_i^k = l_i^k r_i^k \tag{8}$$

Please note that for such definition of participation factor it is a complex value in cases complex eigenvalue. This leads to the inapplicability of this formula as complex numbers cannot be compared with each other. To avoid this problem we can of course slightly modify the expression as follows:

$$p_i^k = \left| l_i^k r_i^k \right|. \tag{9}$$

In this case the participation factor will always be positive real value, as it should be. Note that for the convenience participation factor analysis can be normalized to:

$$\sum_{k=1}^{N} p_i^k = 1 \,. \tag{10}$$

Further we will consider one of the newest approaches (Abed and others) for calculating the participation factor of k-state variable in the formation of i-mode, which is offered in a series of papers [3-5]. In the paper [3] the authors use a set-theoretic formulation for calculating the participation factor of k-state variable in the formation of i-mode. In the next paper [4], to avoid the problem of choosing the initial conditions probabilistic description of initial conditions is applied by using the mathematical expectation:

$$p_i^k = E\left(\frac{\left(l_i^k + l_i^{k^*}\right)x_0^k}{z_0^i + z_0^{i^*}}\right). \tag{11}$$

This formula is proposed for the cases of real and complex eigenvalue λ_i. After calculating the mathematical expectation, the authors obtained the following expression of participation factor of k-state variable in the formation of i-mode:

$$p_i^k = \frac{\left[\mathrm{Re}(l_i^k)\right]^2}{\mathrm{Re}(l_i) \times \left(\mathrm{Re}(l_i)\right)^T}. \tag{12}$$

2.2. Examples showing the inadequacy of expressions for determining participation factor

In general, the matrix **A** is an arbitrary real matrix of size N, depending on the system, which we describe using differential linear equations. Let's study the adequacy of expressions for participation factor obtained by different authors, on specific examples.

Example 1. Consider the two-dimensional system, which state vector is $\mathbf{x}$(t) = [x_1(t), x_2(t)]. As a partial case of matrix A, we choose it to be the following [4]:

$$A = \begin{pmatrix} a & b \\ 0 & d \end{pmatrix}. \tag{13}$$

Values a, b, d – nonzero real constants, and $a \neq d$. Equation (1) for this two-dimensional case this will look like:

$$\begin{pmatrix} \dot{\mathbf{x}}_1 \\ \dot{\mathbf{x}}_2 \end{pmatrix} = \begin{pmatrix} a & b \\ 0 & d \end{pmatrix}\begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{pmatrix}.$$

The eigenvalues of matrix **A** equal $\lambda_1 = a$, $\lambda_2 = d$. After calculating the left and right eigenvectors using formulas (3) we obtain the following vectors:

$$\mathbf{r}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix},\ \mathbf{r}_2 = \begin{pmatrix} 1 \\ (d-a)/b \end{pmatrix},\ \mathbf{l}_1 = \begin{pmatrix} 1 & \dfrac{b}{a-d} \end{pmatrix},\ \mathbf{l}_2 = \begin{pmatrix} 0 & \dfrac{-b}{a-d} \end{pmatrix}. \tag{14}$$

It is easy to see that the left and right eigenvectors are normalized to the symbol Kronecker.

Let's explore the mode associated with eigenvalues λ_1 and determine participation factor of each component of the state vector in the formation of mode. Using the approach (9) we obtain:

$$p_1^1 = 1, \quad p_1^2 = 0. \tag{15}$$

Hence we see that mode formation associated with eigenvalues λ_1 is determined only by the first component of the state vector x_1 with a weight of 1.

Then determine the participation factor for the components of the state vector through approach [4] by the formula (6):

$$\begin{aligned} p_1^1 &= \frac{(a-d)^2}{(a-d)^2 + b^2}, \\ p_1^2 &= \frac{b^2}{(a-d)^2 + b^2}. \end{aligned} \tag{16}$$

Comparing the results for the participation factor (15) and (16) we see that they differ. A natural question arises: which results are correct? To answer this question should we should investigate the evolution of mode under study that is associated with eigenvalue λ_1 using expression (7):

$$\mathbf{z}_1(t) = \left(x_0^1 + \frac{b}{a-d} x_0^2 \right) e^{\lambda_1 t}. \tag{17}$$

From the formula (17) we can see that participation factors are nonzero for the components of the state vector $\mathbf{x}_0$. Expression (17) is actually the decomposition of studied mode on the basis of states x_0^1, x_0^2, then the value that are near x_0^1, x_0^2 is nothing other than the amplitude of weight. The weight is defined as the square of the amplitude:

$$\begin{aligned} p_1^1 &= \frac{1}{1 + b^2 / (a-d)^2} = \frac{(a-d)^2}{(a-d)^2 + b^2}, \\ p_1^2 &= \frac{b^2 / (a-d)^2}{1 + b^2 / (a-d)^2} = \frac{b^2}{(a-d)^2 + b^2}. \end{aligned} \tag{18}$$

Note that participation factors calculated using (18) are normalized to 1.

As a result, it can be concluded that the participation factor (15), which is defined by the formula (9) returns incorrect results because from the expression (17) for the evolution of mode we can see that mode formation is affected by both components of the state. Comparing expressions (16) and (18) we can conclude that the approach proposed in [4] for determining factor participation by the formula (12) provides correct results. Note that matrix $\mathbf{A}$ has real eigenvalues, so you need to investigate the validity of the approach [4] for the cases of complex eigenvalues of the input matrix **A.**

Example 2. Let's study the factor participation using approach (12) in the case of complex eigenvalues of input matrix. For simplicity of calculating eigenvalues of matrix **A** we choose it as follows:

$$\mathbf{A} = \begin{pmatrix} 0 & -c \\ b & 0 \end{pmatrix}. \tag{19}$$

Values b, c – constants, and $b > 0$, $c > 0$. After calculating eigenvalues of matrix **A** we obtain:

$$\lambda_1 = j\sqrt{bc}\,, \qquad \lambda_2 = -j\sqrt{bc}\,.$$

For each eigenvalue using equation (3) let's determine the left and right eigenvectors:

$$\mathbf{l}_1 = \begin{pmatrix} -\frac{j}{2}\sqrt{b/c} & \frac{1}{2} \end{pmatrix},\ \mathbf{l}_2 = \begin{pmatrix} \frac{j}{2}\sqrt{b/c} & \frac{1}{2} \end{pmatrix},$$

$$\mathbf{r}_1 = \begin{pmatrix} j\sqrt{c/b} \\ 1 \end{pmatrix},\ \mathbf{r}_2 = \begin{pmatrix} -j\sqrt{c/b} \\ 1 \end{pmatrix}. \tag{20}$$

The left and right eigenvectors satisfy the normalization condition.

To study participation factor for the mode associated with eigenvalues λ_1. First, consider the evolution of mode under study using the expression (7):

$$\mathbf{z}_1(t) = \left(-\frac{j\sqrt{b/c}}{2} x_0^1 + \frac{1}{2} x_0^2 \right) e^{\lambda_1 t}\,. \tag{21}$$

As we can see from the expression (21), state variables x_0^1 and x_0^2 are included in the expression (21) unequally, then it can be concluded that the participation factors of variables x_0^1, x_0^2 in the formation of investigated mode are different and nonzero.

Then let's proceed to the calculation participation factors using approaches (9) and (12). Once again, using the formula (9) to calculate the participation factor of state variable in mode, we obtain the following results:

$$p_1^1 = \frac{1}{2},\ p_1^2 = \frac{1}{2}\,. \tag{22}$$

Hence we see that the weight of influences of state variables x_0^1 and x_0^2 on the formation investigated mode associated with eigenvalue λ_1, are equal, that contradicts the findings obtained from the expression for the evolution of this mode (21). This shows the incorrectness of approach of scientists Perez-Arriaga and Verghese for determination of participation factor of state variable in mode for complex eigenvalues of the input matrix.

Our next step is to calculate the participation factor through approach proposed by a group of scientists led by Abed [4]. Applying the formula (12) to calculate participation factors in the formation of state variables of investigated mode we get:

$$p_1^1 = 0, \quad p_1^2 = 1. \tag{23}$$

The fact, that indicating the incorrectness of approach (12) is that the participation factor of component x_0^1 in the formation of investigated mode associated with value λ_1 is not equal to zero, as seen from the expression for the investigated mode evolution (21) and got through formula (12) – $p_1^1 = 0$ (23).

Another proof of the incorrectness of the formula (12) in case complex eigenvalues is the ambiguity of left and right eigenvectors as for phase: if we multiply the left vector and divide the right vector to the same complex number, the result of normalization condition (4) between them does not change, but the value of left and right vector components will change. Indeed, let's consider the normalization condition (4) for i-mode: $\mathbf{l}_i \times \mathbf{r}_i = 1$. Suppose z is arbitrary complex number other than zero $|z| \neq 0$. Let us make the following changes: $\mathbf{l}'_i \rightarrow \mathbf{l}_i/z$, $\mathbf{r}'_i \rightarrow \mathbf{r}_i \times z$. Further we will consider condition of normalization between vectors $\mathbf{l}'_i$ and $\mathbf{r}'_i$:

$$\mathbf{l}'_i \times \mathbf{r}'_i = \mathbf{l}_i / z \times \mathbf{r}_i \times z = 1.$$

Eigenvectors $\mathbf{l}'_i$ and $\mathbf{r}'_i$ satisfy the equation for the eigenvalues and eigenfunctions (3) with the same eigenvalue λ_i, as vectors $\mathbf{l}_i$ and $\mathbf{r}_i$. The difference between eigenvectors $\mathbf{l}_i$, $\mathbf{r}_i$ and $\mathbf{l}'_i$, $\mathbf{r}'_i$ is the values of their components: the values of real and imaginary parts. Therefore, actually there is an ambiguity (inadequacy) of formula (12) for determining participation factor for cases of complex eigenvalues.

Thus, in the case of real eigenvalues of initial matrix the approach proposed in paper [4] (formula (12)) provides correct results for determining participation factor of state variable in mode, but for the cases of complex eigenvalues of initial matrix, this approach provides incorrect results.

3. CONCLUSION

We analyzed different approaches to determining participation factor of state variable in mode formation: original approach [1, 2] and suggested in the paper [4] based on probabilistic method of setting the initial conditions. It was shown the inadequacy of these approaches for determining the participation factor of state variable in mode formation associated with eigenvalue of characteristic matrix of linear differential equations system. The approach of scientists led by Abed provides correct results only for real eigenvalues of the input matrix, but for the cases of complex eigenvalues we get incorrect results.

Thus, the problem of determining the participation factor of state variable in power system mode formation remains important as participation factor plays a key role in modal analysis. Note that a great number of modern software applications designed to analyze power system stability deploy the approach of Perez-Arriaga and Verghese, causing incorrectness of modal analysis and as a result risks to the operation of real power systems.

REFERENCES

[1] Perez-Arriaga I. J., Verghese G. C., and Schweppe F. C. Selective modal analysis with applications to electric power systems. Part I: Heuristic introduction – IEEE Transactions on Power Apparatus and Systems, No. 9, Vol. 101, 3117–3125, 1982.

[2] Verghese G. C., Perez-Arriaga I. J. and Schweppe F. C. Selective modal analysis with applications to electric power systems. Part II: The dynamic stability problem – IEEE Transactions on Power Apparatus and Systems, No. 9, Vol. 101, 3126–3134, 1982.

[3] Abed E. H., Lindsay D. and Hashlamoun W. A. On participation factors for linear systems, Automatica. Vol. 36, 1489–1496, 2000.

[4] Hashlamoun W. A., Hassouneh M. A. and Abed E. H. New results of modal participation factors: revealing a previously unknown dichotomy, IEEE Transactions on Automatic Control, No. 7, Vol. 54, 1439–1449, 2009.

[5] Sheng L., Abed E. H., Hassouneh M. A., Yang H. and Saad M. S. Mode in Output Participation Factors for Linear Systems, American Control Conference, Baltimore, 956–961, 2010.

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