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ANALYSIS OF THE VALUES OF HYDRODYNAMIC PRESSURE AND LOAD CARRYING CAPACITIES FOR VARIOUS METHODS OF SOLVING A REYNOLDS TYPE EQUATION

ANALIZA WARTOŚCI CIŚNIENIA HYDRODYNAMICZNEGO I SIŁY NOŚNEJ PRZY RÓŻNYCH METODACH ROZWIĄZYWANIA RÓWNANIA TYPU REYNOLDSA

Key words: small parameter method, successive approximation method, Reynolds type equation, load carrying capacity, friction force, coefficient of friction, hydrodynamic pressure.

Abstract Calculations of the hydrodynamic pressure distribution in the slide bearing gap occur most often on the basis of ready-made computer programs based on CFD methods or one's own calculation procedures based on various numerical methods. The use of one's own calculation procedures and, for example, the finite difference method, allows one to include in the calculations of various additional non-classical effects on the lubricant (e.g., the influence of the magnetic field on ferrofluid, the influence of pressure or temperature on viscosity changes, non-Newtonian properties of lubricant or various non-classical models of dynamic viscosity changes). The aim of the authors' research is to check how large the differences in results may be obtained using the two most frequently used methods of solving a Reynolds type equation. In this work, the authors use the small parameter method and the method of subsequent approximations to determine the distribution of hydrodynamic pressure. For numerical calculations, the finite difference method and our own calculation procedures and Mathcad 15 software were used. With both methods, identical conditions and parameters were assumed and the influence of pressure and temperature on viscosity change was taken into account. In the hydrodynamic pressure calculations, a laminar flow of the lubricating liquid and a non-isothermal lubrication model of the slide bearing were adopted. The classic Newtonian model was used as a constitutive equation. A cylindrical-type slide bearing of finite length with a smooth pan with a full wrap angle was accepted for consideration. In the thin layer of the oil film, the density and thermal conduction coefficient of the oil were assumed to remain unchanged.

Słowa kluczowe: metoda małego parametru, metoda kolejnych przybliżeń, równanie typu Reynoldsa, siła nośna, siła tarcia, współczynnik tarcia, ciśnienie hydrodynamiczne.

Streszczenie Obliczanie rozkładu ciśnienia hydrodynamicznego w szczelinie łożyska ślizgowego następuje najczęściej na podstawie gotowych programów komputerowych opartych na metodach CFD lub własnych procedur obliczeniowych opartych na różnych metodach numerycznych. Zastosowanie własnych procedur obliczeniowych i np. metody różnic skończonych pozwala na uwzględnienie w obliczeniach różnych dodatkowych nieklasycznych oddziaływań na czynnik smarujący (np. pola magnetycznego na ferrociecz, wpływu ciśnienia lub temperatury na zmianę lepkości, właściwości nienewtonowskich czynnika smarującego, różnych nieklasycznych modeli zmian lepkości dynamicznej). Celem badań autorów jest sprawdzenie, jak duże różnice w wynikach uzyskuje się, stosując dwie często wykorzystywane metody rozwiązywania równania typu Reynoldsa. W niniejszej pracy autorzy wykorzystują metodę małego parametru oraz metodę kolejnych przybliżeń w celu wyznaczenia rozkładu ciśnienia hydrodynamicznego. Do obliczeń numerycznych wykorzystano metodę różnic skończonych, własne procedury obliczeniowe oraz oprogramowanie typu Mathcad 15. Przy obu metodach stosuje się identyczne warunki i parametry oraz uwzględnia się wpływ ciśnienia i temperatury na zmianę lepkości. W obliczeniach ciśnienia hydrodynamicznego przyjęto laminarny przepływ cieczy smarującej oraz nieizotermiczny model smarowania łożyska ślizgowego. Jako równanie konstytutywne zastosowano klasyczny model newtonowski. Do rozważań przyjęto walcowe łożysko ślizgowe o skończonej długości z gładką panewką o pełnym kącie opasania. W cienkiej warstwie filmu olejowego przyjęto niezmienność gęstości i współczynnika przewodzenia ciepła oleju od temperatury.

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INTRODUCTION

The load carrying capacity in the slide bearing balances the load of bearing from external forces. It is determined on the basis of hydrodynamic pressure distribution. The hydrodynamic pressure is determined by various numerical methods **[L. 1–4]**. We have the ability to use many ready-made computer programs to perform the numerical calculations for standard classic sliding bearing lubrication conditions (ANSYS CFD, Numeca, Fluent, Autodesk Simulation CFD or others). The most popular methods of numerical calculations are based on one of the following: finite elements, finite volumes, or boundary elements **[L. 4–7]**. One can also use the finite difference method for calculations. Each of these methods has its own advantages and disadvantages. The main disadvantage of ready-made programs is that one cannot usually introduce unconventional elements into moment equations or one's own models of dynamic viscosity changes. Appropriate methods for solving partial differential equations and our own calculation procedures allow one to eliminate these inconveniences.

The value of hydrodynamic pressure is influenced, among others, by the height of the lubrication gap, dimensions of the bearing, changes in dynamic viscosity, bearing operating conditions, including angular velocity **[L. 8–10]**. Changes in dynamic viscosity are affected by pressure, temperature, and shear rates as well as the life time of a given lubricant **[L. 8–10]**. Taking into account changes in viscosity to changes in hydrodynamic pressure is very difficult, because there are functions of dynamic viscosity depending on the pressure or temperature in the equations of momentum and equations of energy. The solution to this problem requires the use of the small parameter method or the method of subsequent approximations. Both methods simplify the system of equations to some extent.

The method of successive approximations assumes the determination in the first computational step of the distributions of hydrodynamic pressure and temperature for a constant and is independent of dynamic viscosity. In subsequent calculation steps, changes in dynamic viscosity depending on, e.g., hydrodynamic pressure or temperature in the form of a matrix of values based on previously determined (in the previous calculation step) values of pressure and temperature that are assumed. The calculation is repeated until the convergence of the results.

The method of the small parameter consists in the fact that the functions of hydrodynamic pressure, velocity vector components and temperature in momentum equations, stream continuity, and energy equations are introduced interchangeably in a uniformly convergent power series developed in relation to successive powers of small dimensionless parameters **[L. 9, 10]**.

The functions of the dynamic viscosity of the lubricant, depending on the pressure or temperature, should also be expanded into a series relative to the dimensionless small parameters. Basically, the small parameter method disengages a non-linear system of partial differential equations, forming several linear systems of equations. The first system of equations allows determining flow parameters for classic non-isothermal Newton lubrication without taking into account the influence of pressure and temperature on the change of the viscosity of the lubricant. The other systems of equations allow determining, the so-called "correction of velocity vector," hydrodynamic pressure, and temperature components resulting from taking into account the effect of pressure or temperature on changing the dynamic viscosity of the lubricant. Additionally, this method allows isolating and then analysing the influence of temperature, hydrodynamic pressure, and non-Newtonian properties on the values of operating parameters.

In this paper, the authors use the small parameter method and the next approximation method to determine the hydrodynamic pressure distribution and then the carrying capacity, friction force, and friction coefficient. For numerical calculations, the finite difference method, our own calculation procedures, and Mathcad 15 software were used. Both methods use identical conditions and parameters, and the influence of pressure and temperature on viscosity change is taken into account.

The aim of the authors' research is to check how large are the differences in results obtained using two frequently used methods of solving a Reynolds type equation.

ANALITYCAL CALCULATIONS

In order to determine the distribution of hydrodynamic pressure and then the carrying capacity, the equations of the conservation of the momentum, stream continuity, and energy conservation were estimated and dimensioned. The calculations were made for the classical case of stationary non-isothermal lubrication with Newtonian oil. Equations of the conservation of the momentum, stream continuity, energy conservation, and functions of viscosity changes and some characteristic numbers take the following form [**L. 8–10**]:

$$0 = -\frac{\partial p_1}{\partial \phi} + \frac{\partial}{\partial r_1}\left[\eta_1 \frac{\partial v_1}{\partial r_1}\right] \tag{1}$$

$$0 = \frac{\partial p_1}{\partial r_1} \tag{2}$$

$$0 = -\frac{\partial p_1}{\partial z_1} + \frac{\partial}{\partial r_1}\left[\eta_1 \frac{\partial v_3}{\partial r_1}\right] \tag{3}$$

$$\frac{\partial v_1}{\partial \phi} + \frac{\partial v_2}{\partial r_1} + \frac{1}{L_1^2}\frac{\partial v_3}{\partial z_1} = 0 \tag{4}$$

$$\frac{\partial^2 T_1}{\partial r_1^2}+\eta_1\left[\left(\frac{\partial v_1}{\partial r_1}\right)^2+\frac{1}{L_1^2}\left(\frac{\partial v_3}{\partial r_1}\right)^2\right]=0 \qquad (5)$$

$$\eta_1 \equiv \eta_{1T}(T_1)\eta_{1p}(p_1),\ \eta_{1p}(\phi,z)=e^{\varsigma\cdot p_o\cdot p_1}=e^{\varsigma_p p_1},\quad \eta_{1T}(\phi,z,r)\equiv e^{-\delta_T(T-T_o)}=e^{-Q_{Br}T_1} \qquad (6)$$

$$p_o \equiv \frac{RU\eta_o}{\varepsilon^2},\ \psi\equiv\frac{\varepsilon}{R}\cong 10^{-3},\ \varsigma_p=\varsigma\cdot p_o,\ Br\equiv\frac{U^2\eta_o}{\kappa_o T_o},$$
$$Q_{Br}\equiv BrT_o\delta_T,\ T=T_o+T_oBrT_1,\ p=p_op_1, \qquad (7)$$
$$v_\phi=Uv_1,\ v_r=U\psi v_2,\ v_z=\frac{U}{L_1}v_3,\ L_1\equiv\frac{b}{R},$$
$$0\le r_1<h_{c1}=1+\lambda\cdot\cos\phi+a_\gamma\cdot z_1\cdot\cos\phi,\ 0\le\phi<\phi_k,\ -1\le z_1<+1$$

where Br – dimensionless Brinkman's number,
L_1 – dimensionless length of bearing,
Q_{Br} – dimensionless coefficient of viscosity changes depends on temperature,
R – radius of journal [m],
T_o – dimensional value of temperature [K],
T_1 – dimensionless value of temperature,
$U = w \cdot R$ – dimensional value of peripheral velocity [$m\cdot s^{-1}$],
a_γ – misalignment factor,
$2b$ – length of bearing [m],
h_{c1} – dimensionless total height of the lubrication gap,
p_o – dimensional characteristic value of hydrodynamic pressure [Pa],
p_1 – dimensionless hydrodynamic pressure value,
r_1 – dimensionless radial coordinate,
z_1 – dimensionless longitude coordinate,
γ – angle of misalignment,
δ_T – dimensional material factor taking into account viscosity changes from the temperature T [K^{-1}],
$\varepsilon = R' - R$ – radial clearance [m],
ς – dimensional material piezo-factor of viscosity [Pa^{-1}],
ς_p – dimensionless material piezo-factor of viscosity,
η_{1p} – dimensionless dynamic viscosity depended on pressure p_1,
η_{1T} – dimensionless dynamic viscosity depended on temperature T_1,
η_o – dimensional value of dynamic viscosity for $T = T_o$; $p = p_{at}$; [$Pa\cdot s$],
κ_o – dimensional heat transfer coefficient of lubricant [$W\cdot m^{-1}\cdot K^{-1}$],
κ_1 – dimensionless heat transfer coefficient of lubricant,
$\lambda = OO'/\varepsilon$ – relative eccentricity,
ρ_o – dimensional value of density of lubricant [$kg\cdot m^{-3}$],
ρ_1 – dimensionless value of density of lubricant,
ψ – dimensionless value of radial relative clearance,
ω – angular velocity of journal [s^{-1}].

Integrating Equations (1) and (3) twice and assuming that the changes in dynamic viscosity relative layer thickness are insignificant under classic boundary conditions, we get two components of the velocity vector. Then, by integrating once the stream continuity Equation (4) and applying the appropriate boundary condition for the radial component on the pivot, the third component of the velocity vector can be obtained. Applying the second boundary condition for the radial component on the bearing, a Reynolds type equation is obtained, based on which the hydrodynamic pressure distribution will be determined [**L. 8–10**]:

$$\frac{\partial}{\partial\phi}\left[\frac{h_{c1}^3}{\eta_1}\left(\frac{\partial p_1}{\partial\phi}\right)\right]+\frac{1}{L_1^2}\frac{\partial}{\partial z_1}\left[\frac{h_{c1}^3}{\eta_{1B}}\left(\frac{\partial p_1}{\partial z_1}\right)\right]=6\frac{\partial h_{c1}}{\partial\phi} \qquad (8)$$

Assuming as before that the dynamic viscosity relative the thickness of the lubricant layer does not change and by integrating Equation (5) twice and then using the classic boundary conditions, we can obtain a dimensionless temperature distribution in the following form [**L. 9, 10**]:

$$T_1(r_1,\phi,z_1)=1+\frac{1}{2}\eta_1(1-2s)-q_{1c}h_{c1}s+$$
$$-\frac{1}{6}h_{c1}^2\left(\frac{\partial p_1}{\partial\phi}\right)s(3-3s+s^2)+\frac{1}{2}\eta_1\left[(v_1)^2+\frac{1}{L_1^2}(v_3)^2\right]+ \qquad (9)$$
$$-\frac{1}{24\eta_1}h_{c1}^4\left[\left(\frac{\partial p_1}{\partial\phi}\right)^2+\frac{1}{L_1^2}\left(\frac{\partial p_1}{\partial z_1}\right)^2\right]s^3(s-2)$$

where $s \equiv r_1/h_{c1}$ and $0\le s\le+1$, $0\le\phi<2\pi$, $-1\le z_1\le+1$.

Assuming s = 1, Formula (9) gives an unknown function of the temperature distribution on the pan f_{1p} depending on both the angle of wrap and the length of the bearing.

In the case of a journal slide bearing lubricated with Newtonian oil, the dimensional carrying capacity of the bearing C_Σ is determined from the known formula [**L. 8–10**]:

$$C_\Sigma = C_{1\Sigma}\cdot bR\eta_o\omega/\psi^2 \qquad (10)$$

And the dimensionless value of carrying capacity of he bearing $C_{1\Sigma}$ may be calculated from the following dependence [**L. 8–10**]:

$$C_{1\Sigma}=\sqrt{\left(\int_{-1}^{+1}\left(\int_0^{\phi_k}p_1\cos\gamma\sin\phi\,d\phi\right)dz_1\right)^2+\left(\int_{-1}^{+1}\left(\int_0^{\phi_k}p_1\cos\gamma\cos\phi\,d\phi\right)dz_1\right)^2} \qquad (11)$$

where symbol g means angle of misalignment.

The total dimensional friction force Fr_Σ and the total dimensionless friction force Fr_1 in the gap of the journal sliding bearing show the following dependencies [**L. 8–10**]:

$$Fr_\Sigma = Fr_1 \cdot bR\eta_o\omega/\psi \quad (12)$$

$$Fr_1 = \int_{-1}^{+1}\left[\int_0^{\varphi}\left(\eta_1\frac{\partial v_1}{\partial r_1}\right)_{r_1=h_{p1}} d\varphi\right]dz_1 = \quad (13)$$

$$= \int_{-1}^{+1}\left[\int_0^{2\pi}\left(\eta_1\frac{\partial v_{1s}}{\partial r_1}\right)_{r_1=h_{p1}} d\varphi\right]dz_1 + \int_{-1}^{+1}\left[\int_0^{\varphi_k}\left(\eta_1\frac{\partial v_{1p}}{\partial r_1}\right)_{r_1=h_{p1}} d\varphi\right]dz_1$$

In Equation (13), the peripheral component of the velocity vector v_1 is separated into the pressure-dependent part v_{1p} and a part dependent on the rotational movement of the journal v_{1s}.

The method of subsequent approximations

In the case of the method of subsequent approximations, Equations (8) and (9) can be used directly without special transformations. In the first computational step, it is assumed in these equations that the dimensionless dynamic viscosity η_1 is constant and equal to 1. After determining the pressure and temperature from the first calculation step, the obtained values are inserted into the viscosity function in Equations (8) and (9). Next, the corrected hydrodynamic pressures are determined and then are substituted into Equation (9). The calculations are repeated in the next calculation steps.

The small parameter method

In order to apply the small parameter method, we should introduce the uniform convergent power series developed in relation to successive powers of small dimensionless parameters into the system of Equations (1) - (6) [**L. 9, 10**]:

$$v_i = v_i^{(0)} + Q_{Br}v_{i0}^{(1)} + \ldots + Q_{Br}^j v_{i0}^{(j)} + \ldots + \varsigma_p v_{i1}^{(1)} + \ldots + \varsigma_p^j v_{i1}^{(j)} + \ldots$$

$$p_1 = p_1^{(0)} + Q_{Br}p_{10}^{(1)} + \ldots + Q_{Br}^j p_{10}^{(j)} + \ldots + \varsigma_p p_{11}^{(1)} + \ldots + \varsigma_p^j p_{11}^{(j)} + \ldots \quad (14)$$

$$T_1 = T_1^{(0)} + Q_{Br}T_{10}^{(1)} + \ldots + Q_{Br}^j T_{10}^{(j)} + \ldots + \varsigma_p T_{11}^{(1)} + \ldots + \varsigma_p^j T_{11}^{(j)} + \ldots$$

$$\eta_{1p} \equiv \exp\left(\varsigma_p p_1\right) = 1 + \varsigma_p p_1 + \frac{1}{2!}\varsigma_p^2 p_1^2 + \ldots + \frac{1}{n!}\varsigma_p^n p_1^n + \ldots$$

$$\eta_{1T} \equiv \exp\left(-Q_{Br}T_1\right) = 1 - Q_{Br}T_1 + \frac{1}{2}Q_{Br}^2 T_1^2 - \ldots - \frac{1}{n!}Q_{Br}^n T_1^n + \ldots$$

for i = 1, 2, 3 j = 1, 2... .

The above mentioned series are multiplied by using the Cauchy method, and then equated with the coefficients with the same power of small parameters Q_{Br} and ς_p. In this way, successive systems of partial differential equations can be obtained. Based on these systems, unknown functions and their corrections should be determined.

The equations based on which the hydrodynamic pressure distributions and its adjustments were determined, and they are as follows [**L. 9, 10**]:

$$\frac{\partial}{\partial\varphi}\left[h_{c1}^3\left(\frac{\partial p_1^{(0)}}{\partial\varphi}\right)\right] + \frac{1}{L_1^2}\frac{\partial}{\partial z_1}\left[h_{c1}^3\left(\frac{\partial p_1^{(0)}}{\partial z_1}\right)\right] = 6\frac{\partial h_{c1}}{\partial\varphi} \quad (15)$$

$$\frac{\partial}{\partial\phi}\left[h_{c1}^3\left(\frac{\partial p_{10}^{(1)}}{\partial\phi}\right)\right] + \frac{1}{L_1^2}\frac{\partial}{\partial z_1}\left[h_{c1}^3\left(\frac{\partial p_{10}^{(1)}}{\partial z_1}\right)\right] =$$
$$= 12\left\{\frac{\partial}{\partial\phi}\left[\left(\int_0^{h_{c1}}\left(\int_0^{r_1} T_1^{(0)}\frac{\partial v_1^{(0)}}{\partial r_1}dr_1\right)dr_1 - \int_0^{h_{c1}}\frac{r_1}{h_{c1}}\left(\int_0^{h_{c1}} T_1^{(0)}\frac{\partial v_1^{(0)}}{\partial r_1}dr_1\right)dr_1\right)\right] + \right. \quad (16)$$
$$\left. + \frac{1}{L_1^2}\frac{\partial}{\partial z_1}\left[\int_0^{h_{c1}}\left(\int_0^{r_1} T_1^{(0)}\frac{\partial v_3^{(0)}}{\partial r_1}dr_1\right)dr_1 - \int_0^{h_{c1}}\frac{r_1}{h_{c1}}\left(\int_0^{h_{c1}} T_1^{(0)}\frac{\partial v_3^{(0)}}{\partial r_1}dr_1\right)dr_1\right]\right\}$$

$$\frac{\partial}{\partial\phi}\left[h_{c1}^3\left(\frac{\partial p_{11}^{(1)}}{\partial\phi}\right)\right] + \frac{1}{L_1^2}\frac{\partial}{\partial z_1}\left[h_{c1}^3\left(\frac{\partial p_{11}^{(1)}}{\partial z_1}\right)\right] = \quad (17)$$
$$= 12\left\{\frac{\partial}{\partial\phi}\left[\int_0^{h_{c1}}\frac{r_1}{h_{c1}}\left(\int_0^{h_{c1}} p_1^{(0)}\frac{\partial v_1^{(0)}}{\partial r_1}dr_1\right)dr_1 - \int_0^{h_{c1}}\left(\int_0^{r_1} p_1^{(0)}\frac{\partial v_1^{(0)}}{\partial r_1}dr_1\right)dr_1\right] + \right.$$
$$\left. + \frac{1}{L_1^2}\frac{\partial}{\partial z_1}\left[\int_0^{h_{c1}}\frac{r_1}{h_{c1}}\left(\int_0^{h_{c1}} p_1^{(0)}\frac{\partial v_3^{(0)}}{\partial r_1}dr_1\right)dr_1 - \int_0^{h_{c1}}\left(\int_0^{r_1} p_1^{(0)}\frac{\partial v_3^{(0)}}{\partial r_1}dr_1\right)dr_1\right]\right\}$$

Where $p_1^{(0)}$ – dimensionless hydrodynamic pressure without taking into account changes in dynamic viscosity,

$p_{10}^{(1)}$ – dimensionless pressure correction resulting from consideration of viscosity changes with temperature,

$p_{11}^{(1)}$ – dimensionless pressure correction resulting from consideration of viscosity changes with pressure,

$v_1^{(0)}$ – dimensionless peripheral component of velocity vector without taking into account changes in dynamic viscosity,

$v_3^{(0)}$ – dimensionless longitudinal component of velocity vector without taking into account changes in dynamic viscosity,

$T_1^{(0)}$ – dimensionless temperature without taking into account changes in dynamic viscosity.

The functions, on the basis of which the temperature distributions and their corrections were determined are as follows [**L. 9, 10**]:

$$
\begin{aligned}
T_1^{(0)}(r_1,\varphi,z_1) &= 1+\frac{1}{2}(1-2s)-q_{1c}^{(0)}h_{c1}s-\\
&-\frac{1}{6}h_{c1}^2\left(\frac{\partial p_1^{(0)}}{\partial\varphi}\right)s\,(3-3s+s^2)+\\
&-\frac{1}{2}\left[\left(v_1^{(0)}\right)^2+\frac{1}{L_1^2}\left(v_3^{(0)}\right)^2\right]+\\
&-\frac{1}{24}h_{c1}^4\left[\left(\frac{\partial p_1^{(0)}}{\partial\varphi}\right)^2+\frac{1}{L_1^2}\left(\frac{\partial p_1^{(0)}}{\partial z_1}\right)^2\right]s^3\left(s-2\right)
\end{aligned}
\tag{18}
$$

$$
\begin{aligned}
T_{10}^{(1)} &= \int_0^{r_1}\int_0^{r_1} T_1^{(0)}\left[\left(\frac{\partial v_1^{(0)}}{\partial r_1}\right)^2+\frac{1}{L_1^2}\left(\frac{\partial v_3^{(0)}}{\partial r_1}\right)^2\right]dr_1dr_1+\\
&-2\int_0^{r_1}\int_0^{r_1}\left(\frac{\partial v_{10}^{(1)}}{\partial r_1}\right)\left(\frac{\partial v_1^{(0)}}{\partial r_1}\right)+\frac{1}{L_1^2}\left(\frac{\partial v_{30}^{(1)}}{\partial r_1}\right)\left(\frac{\partial v_3^{(0)}}{\partial r_1}\right)dr_1dr_1-q_{10c}^{(1)}r_1+f_{10c}^{(1)}
\end{aligned}
\tag{19}
$$

$$
\begin{aligned}
T_{11}^{(1)} &= -\eta_{1B}\int_0^{r_1}\int_0^{r_1} p_1^{(0)}\left[\left(\frac{\partial v_1^{(0)}}{\partial r_1}\right)^2+\frac{1}{L_1^2}\left(\frac{\partial v_3^{(0)}}{\partial r_1}\right)^2\right]dr_1dr_1+\\
&-2\eta_{1B}\int_0^{r_1}\int_0^{r_1}\left(\frac{\partial v_{11}^{(1)}}{\partial r_1}\right)\left(\frac{\partial v_1^{(0)}}{\partial r_1}\right)+\frac{1}{L_1^2}\left(\frac{\partial v_{31}^{(1)}}{\partial r_1}\right)\left(\frac{\partial v_3^{(0)}}{\partial r_1}\right)dr_1dr_1-q_{11}^{(1)}r_1+f_{11c}^{(1)}
\end{aligned}
\tag{20}
$$

Substituting the hydrodynamic pressure $p_1 = p_1^{(0)} + Q_{Br}p_{10}^{(1)} + \varsigma_p p_{11}^{(1)}$ to Equation (11) and then developing it into the Taylor series, surrounded by zero values of small parameters, we obtain the appropriate formulas for the carrying capacity and the correction of the carrying capacity [**L. 9–11**]:

$$
C_1^{(0)} = \sqrt{\left(\int_{-1}^{+1}\left(\int_0^{\varphi_k} p_1^{(0)}\cos\gamma\sin\varphi\,d\varphi\right)dz_1\right)^2+\left(\int_{-1}^{+1}\left(\int_0^{\varphi_k} p_1^{(0)}\cos\gamma\cos\varphi\,d\varphi\right)dz_1\right)^2}
\tag{21}
$$

$$
C_{10}^{(1)} = \sqrt{\left(\int_{-1}^{+1}\left(\int_0^{\varphi_k} p_{10}^{(1)}\cos\gamma\sin\varphi\,d\varphi\right)dz_1\right)^2+\left(\int_{-1}^{+1}\left(\int_0^{\varphi_k} p_{10}^{(1)}\cos\gamma\cos\varphi\,d\varphi\right)dz_1\right)^2}
\tag{22}
$$

$$
C_{11}^{(1)} = \sqrt{\left(\int_{-1}^{+1}\left(\int_0^{\varphi_k} p_{11}^{(1)}\cos\gamma\sin\varphi\,d\varphi\right)dz_1\right)^2+\left(\int_{-1}^{+1}\left(\int_0^{\varphi_k} p_{11}^{(1)}\cos\gamma\cos\varphi\,d\varphi\right)dz_1\right)^2}
\tag{23}
$$

$$
C_1 = C_1^{(0)} + Q_{Br}C_{10}^{(1)} + \varsigma_p C_{11}^{(1)}
\tag{24}
$$

The total dimensional friction force Fr_Σ or the total dimensionless friction force Fr_1 in the gap of the journal sliding bearing presents the following relationship [**L. 9, 10**]:

$$
\begin{aligned}
Fr_\Sigma &= Fr_1\cdot bR\eta_o\omega/\psi =\\
&=\left(Fr_1^{(0)}+Q_{Br}Fr_{10}^{(1)}+\varsigma_p Fr_{11}^{(1)}\right)\cdot bR\eta_o\omega/\psi
\end{aligned}
\tag{25}
$$

The dimensionless friction force for classic Newtonian oil and the correction of the friction force resulting from taking into account changes in dynamic viscosity from temperature and pressure are determined on the basis of the following dependencies [**L. 9–11**]:

$$
\begin{aligned}
Fr_1^{(0)} &= \int_{-1}^{+1}\left[\int_0^{\varphi}\left(\eta_{1B}\frac{\partial v_1^{(0)}}{\partial r_1}\right)_{r_1=h_{p1}}d\varphi\right]dz_1=\\
&=\int_{-1}^{+1}\left[\int_0^{2\pi}\left(\eta_{1B}\frac{\partial v_{1s}^{(0)}}{\partial r_1}\right)_{r_1=h_{p1}}d\varphi\right]dz_1+\\
&-\int_{-1}^{+1}\left[\int_0^{\varphi_k}\left(\eta_{1B}\frac{\partial v_{1p}^{(0)}}{\partial r_1}\right)_{r_1=h_{p1}}d\varphi\right]dz_1
\end{aligned}
\tag{26}
$$

$$
\begin{aligned}
Fr_{10}^{(1)} &= \int_{-1}^{+1}\left[\int_0^{\varphi}\left(\eta_{1B}\frac{\partial v_{10}^{(1)}}{\partial r_1}\right)_{r_1=h_{p1}}d\varphi\right]dz_1=\\
&=\int_{-1}^{+1}\left[\int_0^{2\pi}\left(\eta_{1B}\frac{\partial v_{10s}^{(1)}}{\partial r_1}\right)_{r_1=h_{p1}}d\varphi\right]dz_1+\\
&+\int_{-1}^{+1}\left[\int_0^{\varphi_k}\left(\eta_{1B}\frac{\partial v_{10p}^{(1)}}{\partial r_1}\right)_{r_1=h_{p1}}d\varphi\right]dz_1
\end{aligned}
\tag{27}
$$

$$Fr_{11}^{(1)} = \int_{-1}^{+1}\left[\int_{0}^{\varphi_k}\left(\eta_{1B}\frac{\partial v_{11}^{(1)}}{\partial r_1}\right)_{r_1=h_{p1}} d\varphi\right]dz_1 = \int_{-1}^{+1}\left[\int_{0}^{2\pi}\left(\eta_{1B}\frac{\partial v_{11s}^{(1)}}{\partial r_1}\right)_{r_1=h_{p1}} d\varphi\right]dz_1 + \int_{-1}^{+1}\left[\int_{0}^{\varphi_k}\left(\eta_{1B}\frac{\partial v_{11p}^{(1)}}{\partial r_1}\right)_{r_1=h_{p1}} d\varphi\right]dz_1 \quad (28)$$

Different integration ranges for the velocity components depending on the pressure ($v_{1p}^{(0)}, v_{10p}^{(1)}, v_{11p}^{(1)}$) and rotational motion of the journal ($v_{1s}^{(0)}, v_{10s}^{(1)}, v_{11s}^{(1)}$) in the Equations (26) - (28) were used.

The total coefficient of friction and correction of the coefficient of friction were determined from the following formulas [**L. 9, 10**]:

$$\left(\frac{\mu}{\psi}\right)_{\Sigma} = \frac{Fr_{\Sigma}}{\psi C_{\Sigma}} = \frac{\left[Fr_1^{(0)} + Q_{Br}Fr_{10}^{(1)} + \varsigma_p Fr_{11}^{(1)} + De_{\alpha}Fr_1^{(1)} + O\left(Q_{Br}^2\right) + O\left(\varsigma_p^2\right)\right]\ bR\eta_o\omega/\psi}{\left[C_1^{(0)} + Q_{Br}C_{10}^{(1)} + \varsigma_p C_{11}^{(1)} + De_{\alpha}C_1^{(1)} + O\left(Q_{Br}^2\right) + O\left(\varsigma_p^2\right)\right]\ bR\eta_o\omega/\psi} =$$

$$= \left(\frac{\mu}{\psi}\right)_1^{(0)} + Q_{Br}\left(\frac{\mu}{\psi}\right)_{10}^{(1)} + \varsigma_p\left(\frac{\mu}{\psi}\right)_{11}^{(1)} + O(Q_{Br}) + O(\varsigma_p) + O(Q_{Br}) + O\left(Q_{Br}^2\right) + O\left(\varsigma_p^2\right) \quad (29)$$

$$\left(\frac{\mu}{\psi}\right)_1^{(0)} = \frac{Fr_1^{(0)}}{C_1^{(0)}},\quad \left(\frac{\mu}{\psi}\right)_{10}^{(1)} = \frac{\left[Fr_1^{(0)} + Q_{Br}Fr_{10}^{(1)}\right]}{Q_{Br}\cdot\left[C_1^{(0)} + Q_{Br}C_{10}^{(1)}\right]},\quad \left(\frac{\mu}{\psi}\right)_{11}^{(1)} = \frac{\left[Fr_1^{(0)} + \varsigma_p Fr_{11}^{(1)}\right]}{\varsigma_p\cdot\left[C_1^{(0)} + \varsigma_p C_{11}^{(1)}\right]} \quad (30)$$

RESULTS OF NUMERICAL CALCULATIONS

In the analytical and numerical calculations, laminar flow of the lubricating liquid and the non-isothermal lubrication model of the slide bearing were adopted. A cylindrical slide bearing of finite length with a smooth pan with a full wrap angle was also accepted for consideration. It was assumed that the density and thermal conduction coefficient of the oil to remain unchanged from temperature and pressure in the thin layer of the oil film. Numerical calculations of hydrodynamic pressure and lifting force, friction force, and friction coefficient were made for relative eccentricity from $\lambda = 0.1$ to $\lambda = 0.9$ and dimensionless bearing length $L_1 = 1/4$ and $L_1 = 1$ and the angle between the shaft axis and the axis $\gamma = 0$. Numerical calculations were made using the Mathcad 15 program using our own calculation procedures. Both methods use the same calculation grid with dimensions of 20×50 points (50 points around the perimeter, 20 points on the longitudinal direction). Reynolds boundary conditions were used to determine the hydrodynamic pressure. In order to calculate the values of small parameters and dimensional quantities, the following values of coefficients and characteristic dimensional physical quantities were adopted: angular velocity of journal $\omega = 400\ s^{-1}$, characteristic dimensional value of dynamic viscosity $h_o = 0.01546$ Pas, dimensionless value of radial relative clearance $\psi = 0.002$, radius of journal R = 0.020 m, dimensional heat transfer coefficient of lubricant $k_o = 0.15$ W/mK, dimensional material factor taking into account viscosity changes from the temperature $\delta T = 0.04267\ K^{-1}$, dimensional material factor taking into account viscosity changes from the pressure $\varsigma = 0.87 \cdot 10^{-8}\ Pa^{-1}$, dimensionless heat flow reaching into the journal $q_{1c} = -0.5$, characteristic dimensional value of temperature $T_o = 363$ K, small parameter including corrections of pressure impact $\varsigma_p = 0.01345$, small parameter including corrections of temperature impact $Q_{Br} = -0.28146$, and a dimensional characteristic value of hydrodynamic pressure $p_o = 1.546 \cdot 10^6$ Pa. The characteristics of changes of dimensionless carrying capacity for the above data are shown in **Fig. 1**, dimensionless values of friction force are shown in **Fig. 2**, while the friction coefficient is shown in **Fig. 3**.

When analysing the obtained values presented in **Fig. 1**, it can be concluded that, for bearings with a dimensionless bearing length $L_1 = 1/4$, the changes are insignificant (1% – 8%) over the range of relative eccentricity, while, for bearing with a dimensionless bearing length $L_1 = 1$, significant changes are observed only for relative eccentricity $\lambda = 0.9$ (64%). The large difference in the values of the carrying capacity, for the relative eccentricity $\lambda = 0.9$, results from the adopted exponential model of viscosity changes from pressure. In the small parameter method, the exponential function is linearized by assuming only the first two

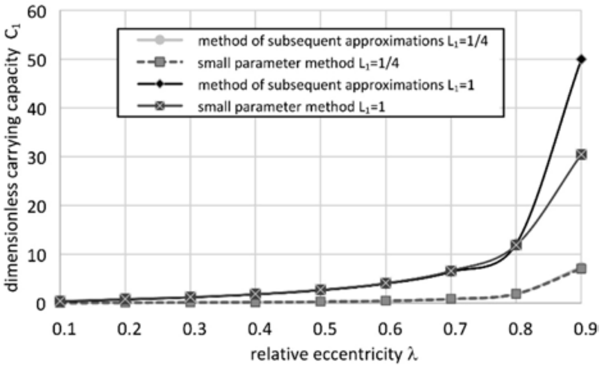

Fig. 1. Dimensionless carrying capacity as a function of relative eccentricity for various dimensionless bearing lengths and numerical methods

Rys. 1. Bezwymiarowa siła nośna w funkcji mimośrodowości względnej dla różnych bezwymiarowych długości łożyska i metod numerycznych

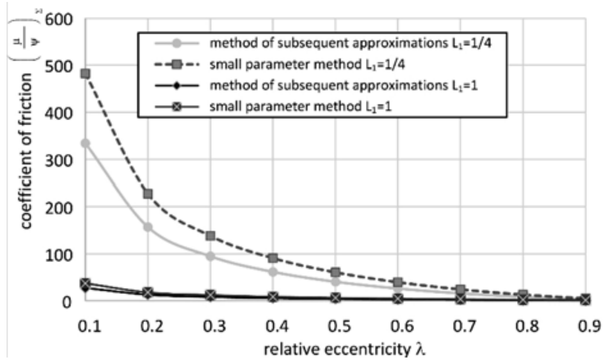

Fig. 3. The coefficient of friction as a function of relative eccentricity for various dimensionless bearing lengths and numerical methods

Rys. 3. Umowny współczynnik tarcia w funkcji mimośrodowości względnej dla różnych bezwymiarowych długości łożyska i metod numerycznych

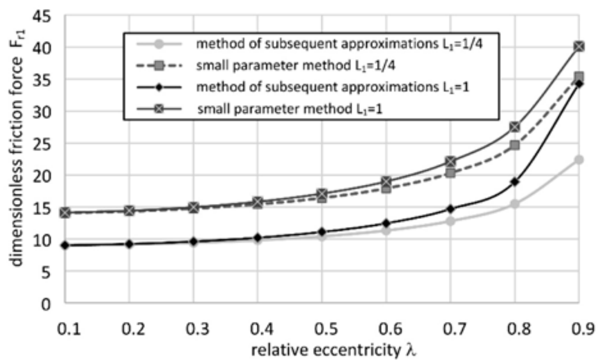

Fig. 2. Dimensionless friction force as a function of relative eccentricity for various dimensionless bearing lengths and numerical methods

Rys. 2. Bezwymiarowa siła tarcia w funkcji mimośrodowości względnej dla różnych bezwymiarowych długości łożyska i metod numerycznych

members of the Taylor series in the calculation $(14)_{4,5}$. The results obtained for the frictional forces shown in **Fig. 2** are significantly different. Changes in the whole range of relative eccentricity are of the order of magnitude of 31% to 37%. Only for dimensionless bearing length L_1=1 and relative eccentricity $\lambda = 0.9$, we have changes of the order of 14%. When analysing the values of the friction coefficient shown in **Fig. 3**, it can be stated that the changes are large in the entire range of relative eccentricity changes, ranging from 26% to 51% for the assumed data.

The percentage changes of the discussed dimensionless values are presented in **Table 1**. The values given in the table have been calculated as follows:

$$\frac{\text{value in the small parameter method - value in the subsequent approximations}}{\text{value in the small parameter method}} \cdot 100\%$$

Table 1. Percentage changes of the carrying capacity, friction force and coefficient of friction obtained by the method of subsequent approximations in relation to the value of carrying capacity, friction force and coefficient of friction obtained by the small parameter method

Tabela 1. Zmiany procentowe wartości siły nośnej, siły tarcia i umownego współczynnika tarcia uzyskanych metodą kolejnych przybliżeń w stosunku do wartości siły nośnej, siły tarcia i umownego współczynnika tarcia uzyskanych metodą małego parametru

	Relative eccentricity								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	Percentage changes of carrying capacity								
L_1=1/4	6.90	7.94	7.48	6.47	5.54	4.21	3.00	1.01	-4.97
L_1=1	13.26	13.43	5.59	4.90	3.84	2.60	2.51	-1.93	-64.12
	Percentage changes of friction force								
L_1=1/4	36.26	36.27	36.33	36.44	36.60	36.78	36.98	37.19	36.71
L_1=1	36.13	36.01	35.79	35.51	35.09	34.41	33.53	31.17	14.51
	Percentage changes of friction coefficient								
L_1=1/4	30.76	31.02	31.50	32.17	33.04	34.01	35.21	36.90	41.17
L_1=1	26.22	26.15	32.09	32.41	32.84	33.14	32.61	33.80	50.43

OBSERVATIONS AND CONCLUSIONS

When analysing the obtained results of numerical calculations, it can noted that significantly different values are obtained in the case of friction forces and friction coefficients calculated by the method of subsequent approximations and the small parameter method. In the small parameter method, only corrections multiplied by small parameters in the first power were taken into account, and the remaining corrections were omitted. This is justified when the value of small parameters is actually small, in the order of hundredth parts. In case when small parameters are in the order of tenths of parts (0.3–0.6), omitting corrections multiplied by small parameters into the second and higher power can cause large calculation errors.

For the small parameter method, the calculation of only the first correction in the calculation linearizes the changes in viscosity from temperature and pressure. The method of subsequent approximations is devoid of this disadvantage. Unfortunately, the adoption of the exponential function as the viscosity changes from pressure gives higher results of the carrying capacity and the coefficient of friction for high values of relative eccentricity.

According to the authors, the method of subsequent approximations is a more precise method, especially when the models of viscosity changes from the pressure and temperature of the lubricating oil are accurate.

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