Int. J. Appl. Math. Comput. Sci., 2019, Vol. 29, No. 3, 541–553
DOI: 10.2478/amcs-2019-0040

EVENT–BASED FEEDFORWARD CONTROL OF LINEAR SYSTEMS WITH INPUT TIME–DELAY

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This paper proposes a new method for the analysis of continuous and periodic event-based state-feedback plus static feedforward controllers that regulate linear time invariant systems with time delays. Measurable disturbances are used in both the control law and triggering condition to provide better disturbance attenuation. Asymptotic stability and L_2-gain disturbance rejection problems are addressed by means of Lyapunov–Krasovskii functionals, leading to performance conditions that are expressed in terms of linear matrix inequalities. The proposed controller offers better disturbance rejection and a reduction in the number of transmissions with respect to other robust event-triggered controllers in the literature.

Keywords: time delay systems, linear systems, process control, control system design.

1. Introduction

Traditionally, control systems in digital platforms are implemented following a periodic control paradigm, that is, signals are sent with a fixed sampling period. However, periodic control may lead to an unnecessary waste of resources, and some forms of aperiodic control have emerged lately to overcome this drawback (Tabuada, 2007; Lunze and Lehmann, 2010). In self-triggered control (see, e.g., Velasco *et al.*, 2003; Wang and Lemmon, 2008; Mazo *et al.*, 2010) a prediction of the evolution of the system is used to determine the instances of time in which the control loop is closed. The main problem of this paradigm is that, since the sensors are not monitored during the inter-event time, conservative sampling intervals may be necessary to properly address unknown phenomena such as disturbances. On the contrary, in event-based control, the control loop is closed when a triggering condition based on the current value of the system state is satisfied. The triggering condition can be checked continuously (Lunze and Lehmann, 2010; Lehmann and Lunze, 2011; Dimarogonas *et al.*, 2012), leading to the so-called continuous event-triggered control (CETC), or at prefixed instances of time (Heemels *et al.*, 2013; Peng and Han, 2013; Aranda-Escolástico *et al.*, 2016; Ma *et al.*, 2018), i.e., periodic event-triggered control (PETC). The analysis of PETC controllers is closely related to the study of networked control systems, in which an unreliable communication network is usually treated as a time-variable dead time (Yue *et al.*, 2005; 2013; Millán *et al.*, 2010).

Even though the benefits of event-triggering have been demonstrated, it might occur that an event-triggered control system that performs properly in the absence of disturbances becomes rather ineffective in the presence of disturbances, even if these are small (Borgers and Heemels, 2013). Hence, the design of control schemes that take into account disturbances and deal with them effectively is one of the open problems in event-based control.

Traditionally, three control strategies are used to reduce the effects of disturbances: local feedback, direct feedforward and prediction-based feedforward (Åström

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and Wittenmark, 1997). The last two require precise information on the process. The direct feedforward strategy consists in supplying a complementary control signal that is computed from the current measured value of the disturbance. The prediction-based feedforward strategy estimates the value of the disturbance using an internal model. The choice of the most convenient technique depends on the process characteristics, e.g., whether the disturbance can be measured or is affected by a dead time.

In general, feedforward control has proved to improve the performance, especially in process control regarding chemical or agricultural systems, but also in robotic manipulators, servo systems or disk drive systems; see, e.g., the works of Chen (2004), Guzmán and Hägglund (2011), Li *et al.* (2016), and the references therein. Note that event-based control has itself a feedforward control aspect in the sense that the feedback control is closed only at event times while remaining open between consecutive events (Miskowicz, 2015). In spite of that, the response against the disturbance normally relies on feedback, that is, until there is not a change in the output due to the disturbance, the information is not transmitted and no new input signals are computed. In our work, we use information on measurable disturbances in order to reduce their effect over the system in advance, and this is done by considering also an event-triggering mechanism to decide whether or not there is a significant change in the disturbance.

Several strategies have been proposed in the literature to improve the disturbance rejection in the context of event-triggered control. In the work of Lehmann and Lunze (2011), a disturbance estimation is proposed. The disturbance is estimated based on the difference between the measured state and the predicted state given by a model. Though the number of events can be reduced by this approach, the effect of the disturbance over the system cannot be anticipated. Several works (see, e.g., Peng and Han, 2013; Yue *et al.*, 2013; Orihuela *et al.*, 2014; Heemels *et al.*, 2016) propose robust controllers to guarantee a certain level of disturbance rejection. Recently, event-triggered sliding mode control has been studied (Wu *et al.*, 2017; Behera and Bandyopadhyay, 2017; Behera *et al.*, 2018; Chu and Li, 2018). However, none of the works employs disturbance measurements in order to improve the system's performance. In this paper, the effect of disturbances is addressed by a novel event-based feedforward strategy. To the best of our knowledge, the analysis of direct feedforward control of dead-time systems within an event-based control paradigm has not been treated in the literature, remaining an interesting open line of research (Lunze, 2015).

Asymptotic stability and L_2-gain disturbance rejection conditions are ensured for an event-based state-feedback plus static feedforward controller. Specifically, given feedback and feedforward gains with a desired disturbance attenuation level, the proposed approach, which is based on linear matrix inequalities (LMIs), provides a trigger function design that guarantees the L_2 stability of the system. In addition, a joint method for the design of the feedback and feedforward gains with time-varying delays and the proposed event-triggering mechanism is developed. That is, the use of a time-delay system framework (see, e.g., Zhang *et al.*, 2013; 2015; Qi *et al.*, 2018) allows, in one step, the inference of the most adequate design of the triggering policy and the controller depending on the networked system characteristics (maximum delay) and desired performance (attenuation level). Additionally, when compared with other robust event-triggered control strategies (Peng and Han, 2013; Yue *et al.*, 2013) in an example, the proposed design allows a further reduction in the number of transmissions as well as a reduced integral square error (ISE). The proposed controller also performs better than conventional feedback CETC or PETC, as expected. Another contribution of this work is the joint analysis for PETC and CETC, which has not been addressed before. Indeed, the solution in terms of LMIs holds for both strategies by just redefining few parameters.

The remainder of the paper is organized as follows. Section 2 is devoted to stating the control problem, the proposed control law, and triggering condition. In Section 3 the main results are presented. Asymptotic stability and L_2-gain disturbance rejection conditions of the CETC and PETC systems are guaranteed through a set of LMIs, and the method to design the feedback and feedforward gains is provided. Section 4 describes a case study to prove the effectiveness of the proposed controller. Finally, Section 5 contains the main conclusions of the work.

2. Preliminaries

2.1. Notation. Throughout this paper a scalar is denoted by italic letters ($x \in \mathbb{R}$), a vector by bold italic letters ($\boldsymbol{x} \in \mathbb{R}^n$) and a matrix by upper-case italic letters ($A \in \mathbb{R}^{n\times m}$). The notation $\|\cdot\|$ stands for the Euclidean vector norm or the induced matrix 2-norm, appropriately. $\boldsymbol{x}^T$ and A^T denote the transpose of a vector and a matrix, respectively. $\|\cdot\|_p$ refers to the L_p-norm. The maximum and minimum eigenvalues of a symmetric real matrix A are denoted by $\lambda_M(A)$ and $\lambda_m(A)$, respectively. For an arbitrarily real matrix B and two real symmetric matrices A and C, $\left(\begin{smallmatrix} A & \star \\ B & C \end{smallmatrix}\right)$ denotes a real symmetric matrix, where $\star$ stands for the entries implied by symmetry. For a symmetric positive-definite matrix $A \in \mathbb{R}^{n\times n}$ we write $A > 0$, whereas $A \geq 0$, $A < 0$ and $A \leq 0$ refer to symmetric positive-semidefinite, negative-definite and negative-semidefinite matrices, respectively. Finally, $\mathbb{C}_{n,\tau} = \mathbb{C}([-\tau, 0], \mathbb{R}^n)$ denotes the Banach space of continuous vector functions mapping the interval $[-\tau, 0]$ into $\mathbb{R}^n$ with the topology of uniform convergence and with the norm $\|\cdot\|_\infty$ of an element $\phi \in \mathbb{C}([-\tau, 0], \mathbb{R}^n)$ as $\|\phi\|_\infty = \sup_{\theta\in[-\tau,0]} \|\phi(\theta)\|$.

2.2. Past results. Time-delay systems are traditionally studied considering two different approaches based on Lyapunov–Razumikhin functions or Lyapunov–Krasovskii functionals, respectively. The former exploits the knowledge of the current state whereas the latter proposes the use of functionals that divide the delay interval into segments. In general, the application of Lyapunov–Krasovskii functionals usually leads to less conservative results (Jiang and Han, 2006) and is more suited for the study of event-based control systems. Throughout this paper, the Lyapunov–Krasovskii theorem is used to prove the stability and performance properties of the proposed control strategies.

Lyapunov–Krasovskii theorem. (Krasovskii, 1956) *Consider the system* $\dot{\boldsymbol{x}}(t) = f(t, \boldsymbol{x}_t(t))$*, where* $\boldsymbol{x}_t(t)$ *denotes* $\boldsymbol{x}$ *in the interval* $[t-\delta, t]$*,* δ *being the delay. Define* $\boldsymbol{x}_t(\theta) = \phi(t+\theta), \forall\theta \in [-\delta, 0]$*. Let* $f : \mathbb{R}_+ \times \mathbb{C}_{n,\delta} \to \mathbb{R}^n$ *map bounded sets of* $\mathbb{C}_{n,\delta}$ *in bounded sets of* $\mathbb{R}^n$ *and let* $\alpha, \beta, \chi : \mathbb{R} \to \mathbb{R}$ *be continuous non-decreasing functions, where additionally* $\alpha(s)$ *and* $\beta(s)$ *are positive for* $s > 0$*, and* $\alpha(0) = \beta(0) = 0$*. If there exists a continuous differentiable functional* $V : \mathbb{R} \times \mathbb{C} \to \mathbb{R}$ *such that*

$$\alpha(\|\phi(0)\|) \le V(t, \phi) \le \beta(\|\phi\|_\infty)$$

and

$$\dot{V}(t, \phi) \le -\chi(\|\phi(0)\|),$$

then the trivial solution of the system is uniformly stable. If $\chi(s) > 0$ *for* $s > 0$*, then it is uniformly asymptotically stable.*

In addition, if $\lim_{s\to\infty}\alpha(s) = \infty$*, then it is globally uniformly asymptotically stable.*

A drawback of using Lyapunov–Krasovskii functionals is that some unknown terms usually appear in their time derivatives. A common solution to this problem is to upper bound them using some inequalities, e.g., Jensen's inequality, which is stated below.

Jensen's inequality. (*Gu* et al.*, 2003*) Let $M \in \mathbb{R}^{n\times n}$ be a symmetric positive-definite matrix, $\gamma > 0$ a scalar and $\boldsymbol{\omega} : [0, \gamma] \to \mathbb{R}^n$ a vector function. Then

$$\gamma \int_0^\gamma \boldsymbol{\omega}^T(\beta) M \boldsymbol{\omega}(\beta)\,\mathrm{d}\beta \ge \left(\int_0^\gamma \boldsymbol{\omega}(\beta)\,\mathrm{d}\beta\right)^T M \int_0^\gamma \boldsymbol{\omega}(\beta)\,\mathrm{d}\beta.$$

Furthermore, the use of a periodic event-triggered mechanism together with a control law that makes use of an exogenous signal, i.e., the measurable disturbance, leads to the appearance of some terms that we upper bounded using the Hardy inequality.

Hardy inequality. (*Kufner* et al.*, 2007*) Let $\boldsymbol{f} : (0, \infty) \to \mathbb{R}^n$ be in L^p. Let $\boldsymbol{F}(x) = \frac{1}{x}\int_0^x \boldsymbol{f}(\alpha)\,\mathrm{d}\alpha$. Then $\boldsymbol{F} \in L^p$ and

$$\|\boldsymbol{F}\|_p \le \frac{p}{p-1}\|\boldsymbol{f}\|_p .$$

2.3. Problem statement. Consider the LTI system given by

$$\dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t) + B\boldsymbol{u}(t - \delta_u(t)) + B_d\boldsymbol{d}(t), \tag{1}$$
$$\boldsymbol{y}(t) = C\boldsymbol{x}(t), \tag{2}$$
$$\boldsymbol{x}(t) = \phi(t), \quad t \in [t_0 - \bar{\delta}_u, t_0], \tag{3}$$

where $\boldsymbol{x}(t) \in \mathbb{R}^{n_x}$ and $\boldsymbol{u}(t) \in \mathbb{R}^{n_u}$ are the system state vector and the control input, respectively, $\boldsymbol{d}(t) \in \mathbb{R}^{n_d}$ are the measurable disturbances, $\delta_u(t)$ is a time-variable dead time for $\boldsymbol{u}(t)$ such that $0 \le \delta_u(t) \le \bar{\delta}_u$, A,B,B_d and C are the parameter matrices with appropriate dimensions such that the pair (A, B) is assumed to be controllable, and $\phi(t)$ denotes the initial conditions.

Hereafter, an event-based state-feedback plus static feedforward controller is considered:

$$\boldsymbol{u}(t) = K\boldsymbol{x}(t_k) + K_d\boldsymbol{d}(t_k),$$

for $t \in [t_k + \delta_u(t_k), t_{k+1} + \delta_u(t_{k+1}))$, $k \in \mathbb{N}_0$, where t_k are the transmission instants.

In discrete-time control, every sampling time is computed from the previous one in accordance with, $t_{k+1} = t_k + h$, where $h > 0$ is the sampling time. On the contrary, in event-based control, an event-triggering condition determines when the control signal is updated. In this paper, the transmission instants $\{t_k\}_{k\in\mathbb{N}}$ are determined recursively by $t_{k+1} = \inf\{t : t > t_k, f(\boldsymbol{e}(t), \boldsymbol{s}(t)) > 0\}$, where $f(\boldsymbol{e}(t), \boldsymbol{s}(t))$ is an event-triggering condition, which depends on $\boldsymbol{e}^T(t) = [\boldsymbol{e}_x^T(t) \quad \boldsymbol{e}_d^T(t)]$, $\boldsymbol{e}_x(t) = \boldsymbol{x}(t_k) - \boldsymbol{x}(t)$, $\boldsymbol{e}_d(t) = \boldsymbol{d}(t_k) - \boldsymbol{d}(t)$, $\boldsymbol{s}^T(t) = [\boldsymbol{x}^T(t) \quad \boldsymbol{d}^T(t)]$. Specifically, we consider a quadratic triggering condition in the form

$$f(\boldsymbol{e}(t), \boldsymbol{s}(t)) = \boldsymbol{e}^T(t)\Omega\boldsymbol{e}(t) - \sigma^2\boldsymbol{s}^T(t)\Omega\boldsymbol{s}(t) \tag{4}$$

with $\sigma > 0$ and

$$\Omega = \begin{pmatrix} \Omega_1 & \star \\ \Omega_2 & \Omega_3 \end{pmatrix} > 0.$$

Thus, the error remains bounded as $\boldsymbol{e}^T(t)\Omega\boldsymbol{e}(t) \le \sigma^2\boldsymbol{s}^T(t)\Omega\boldsymbol{s}(t)$.

Next the main characteristics of the proposed controller are discussed.

3. Main results

In this section, for a given disturbance attenuation level γ for $\boldsymbol{d}(t)$, we design an event-based state-feedback plus static-feedforward controller such that the closed-loop system meets the following requirements:

1. The closed-loop system is asymptotically stable with $\boldsymbol{d}(t) = 0$.
2. Under zero initial conditions, the controlled output $\boldsymbol{y}(t)$ satisfies $\|\boldsymbol{y}(t)\|_2 \leq \gamma \|\boldsymbol{d}(t)\|_2$ for any nonzero $\boldsymbol{d}(t) \in L_2[t_0, \infty)$.

Let us consider a PETC paradigm such that the triggering condition (4) is only satisfied at fixed instances of time, i.e., $0, h, 2h, \dots$, for a given sampling time $h > 0$. The results obtained in this framework will also hold for the continuous case (CETC) with a slight modification, as remarked below.

Thus, let us define the discretized error as

$$\boldsymbol{e}(lh) = \boldsymbol{s}(t_k) - \boldsymbol{s}(lh),$$

lh being the last sampling time such that

$$\delta(t) = t - lh$$

for $t \in [lh + \delta_u(lh), (l+1)h + \delta_u((l+1)h))$. Obviously,

$$0 \leq \delta(t) \leq h + \bar{\delta}_u \triangleq \bar{\delta}. \tag{5}$$

Consequently, we modify the triggering condition (4) as follows:

$$\begin{aligned} f(\boldsymbol{e}(lh), \boldsymbol{s}(lh)) =& \boldsymbol{e}^T(lh)\Omega \boldsymbol{e}(lh) \\ & - \sigma^2 \boldsymbol{s}^T(lh)\Omega \boldsymbol{s}(lh). \end{aligned} \tag{6}$$

We further define the difference of the delayed disturbance signals

$$\boldsymbol{d}_\Delta(t) = \boldsymbol{d}(t - \delta(t)) - \boldsymbol{d}(t). \tag{7}$$

In addition, we make the following assumption.

Assumption 1. The L_2-norm of the difference of the delayed disturbance signals $\boldsymbol{d}_\Delta(t)$ is bounded,

$$\|\boldsymbol{d}_\Delta(t)\|_2 \leq \beta \|\boldsymbol{d}(t)\|_2, \tag{8}$$

with $\beta > 0$.

Assumption 1 basically implies that $\|\boldsymbol{d}_\Delta(t)\|_2$ should be finite, i.e., $\|\boldsymbol{d}_\Delta(t)\|_2 < \infty$. However, it may be difficult to find an analytically a value of β.

For some types of disturbances, the following proposition can be stated.

Proposition 1. *If the disturbance $\boldsymbol{d}(t)$ applied to the plant is bounded as*

$$\|\dot{\boldsymbol{d}}(t)\| \leq \tilde{\beta} \|\boldsymbol{d}(t)\|$$

for some $\tilde{\beta} > 0$, then Assumption 1 is satisfied for $\beta = 2\bar{\delta}\tilde{\beta}$.

Proof. Equation (7) can be written as

$$\boldsymbol{d}_\Delta(t) = -\int_{t-\delta(t)}^{t} \dot{\boldsymbol{d}}(\alpha)\, \mathrm{d}\alpha.$$

Consequently,

$$\frac{\boldsymbol{d}_\Delta(t)}{\delta(t)} = -\frac{1}{\delta(t)} \int_{t-\delta(t)}^{t} \dot{\boldsymbol{d}}(\alpha)\, \mathrm{d}\alpha.$$

Applying the Hardy inequality and (5), we obtain

$$\frac{\|\boldsymbol{d}_\Delta(t)\|_2}{\bar{\delta}} \leq 2\tilde{\beta} \|\boldsymbol{d}(t)\|_2,$$

and the proposition is proved. ■

Finally, the continuous system (1)–(3) is replaced by

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= A\boldsymbol{x}(t) + BK\boldsymbol{x}(t - \delta(t)) \\ &\quad + BK\boldsymbol{e}_x(t - \delta(t)) + (B_d + BK_d)\boldsymbol{d}(t) \\ &\quad + BK_d\boldsymbol{d}_\Delta(t) + BK_d\boldsymbol{e}_d(t - \delta(t)), \end{aligned} \tag{9}$$

$$\boldsymbol{y}(t) = C\boldsymbol{x}(t), \tag{10}$$

$$\boldsymbol{x}(t) = \phi(t), \quad t \in [t_0 - \bar{\delta}, t_0]. \tag{11}$$

Asymptotic stability and L_2 disturbance rejection of (9)–(11) with the triggering condition (6) can be guaranteed with the following theorem.

Theorem 1. *For given σ, γ, $\bar{\delta}$, state-feedback gain K and static-feedforward gain K_d, the system described by (9)–(11) with the triggering condition (6) is asymptotically stable with the H_∞ norm bound γ for $\boldsymbol{d}(t)$ if there exist matrices $P > 0$, $Q > 0$, $R > 0$, $\Omega_1 > 0$, Ω_2 and $\Omega_3 > 0$ of appropriate dimensions and a scalar $n > 0$ such that*

$$\begin{pmatrix} \Omega_1 & \star \\ \Omega_2 & \Omega_3 \end{pmatrix} > 0, \quad M < 0,$$

where M is given by (12).

Proof. Construct a Lyapunov–Krasovskii functional as

$$\begin{aligned} V(t) =& \boldsymbol{x}^T(t)P\boldsymbol{x}(t) \\ & + \int_{t-\bar{\delta}}^{t} \boldsymbol{x}^T(\alpha)Q\boldsymbol{x}(\alpha)\, \mathrm{d}\alpha \\ & + \bar{\delta} \int_{t-\bar{\delta}}^{t} \int_{\alpha}^{t} \dot{\boldsymbol{x}}^T(\theta)R\dot{\boldsymbol{x}}(\theta)\, \mathrm{d}\theta\, \mathrm{d}\alpha \end{aligned} \tag{13}$$

where P, Q and R are symmetric positive-definite matrices. Taking the time derivative of $V(t)$ yields

$$\begin{aligned} \dot{V}(t) =& 2\dot{\boldsymbol{x}}^T(t)P\boldsymbol{x}(t) + \boldsymbol{x}^T(t)Q\boldsymbol{x}(t) \\ & - \boldsymbol{x}^T(t - \bar{\delta})Q\boldsymbol{x}(t - \bar{\delta}) + \bar{\delta}^2\dot{\boldsymbol{x}}^T(t)R\dot{\boldsymbol{x}}(t) \\ & - \bar{\delta} \int_{t-\bar{\delta}}^{t} \dot{\boldsymbol{x}}^T(\theta)R\dot{\boldsymbol{x}}(\theta)\, \mathrm{d}\theta. \end{aligned} \tag{14}$$

$$M = \begin{pmatrix} M_{11} & \star & \star & \star & \star & \star & \star \\ 0 & M_{22} & \star & \star & \star & \star & \star \\ M_{31} & 0 & M_{33} & \star & \star & \star & \star \\ M_{41} & 0 & M_{43} & M_{44} & \star & \star & \star \\ M_{51} & 0 & M_{53} & M_{54} & M_{55} & \star & \star \\ M_{61} & 0 & M_{63} & M_{64} & M_{65} & M_{66} & \star \\ M_{71} & 0 & M_{73} & M_{74} & M_{75} & M_{67} & M_{77} \end{pmatrix}, \tag{12}$$

$$\begin{aligned}
M_{11} &= A^T P + PA + Q + \bar{\delta}^2 A^T RA - R + C^T C, & M_{61} &= K^T B^T P + \bar{\delta}^2 K^T B^T RA,\\
M_{22} &= -Q - R, & M_{63} &= \bar{\delta}^2 K^T B^T RBK,\\
M_{31} &= K^T B^T P + \bar{\delta}^2 K^T B^T RA + R, & M_{64} &= \bar{\delta}^2 K^T B^T R(B_d + BK_d),\\
M_{33} &= \bar{\delta}^2 K^T B^T RBK - 2R + \sigma^2 \Omega_1, & M_{65} &= \bar{\delta}^2 K^T B^T RBK_d,\\
M_{41} &= (B_d^T + K_d^T B^T) PA + \bar{\delta}^2 (B_d^T + K_d^T B^T) R, & M_{66} &= \bar{\delta}^2 K^T B^T RBK - \Omega_1,\\
M_{43} &= \bar{\delta}^2 (B_d^T + K_d^T B^T) RBK + \sigma^2 \Omega_2, & M_{71} &= K_d^T B^T P + \bar{\delta}^2 K_d^T B^T RA,\\
M_{44} &= \bar{\delta}^2 (B_d^T + K_d^T B^T) R(B_d + BK_d) - \gamma^2 (1-n) I + \sigma^2 \Omega_3, & M_{73} &= \bar{\delta}^2 K_d^T B^T RBK,\\
M_{51} &= K_d^T B^T P + \bar{\delta}^2 K_d^T B^T RA, & M_{74} &= \bar{\delta}^2 K_d^T B^T R(B_d + BK_d),\\
M_{53} &= \bar{\delta}^2 K_d^T B^T RBK + \sigma^2 \Omega_2, & M_{75} &= \bar{\delta}^2 K_d^T B^T RBK_d,\\
M_{54} &= \bar{\delta}^2 K_d^T B^T R(B_d + BK_d) + \sigma^2 \Omega_3, & M_{76} &= \bar{\delta}^2 K_d^T B^T RBK - \Omega_2,\\
M_{55} &= \bar{\delta}^2 K_d^T B^T RBK_d - \gamma^2 nI + \sigma^2 \Omega_3, & M_{77} &= \bar{\delta}^2 K_d^T B^T RBK_d - \Omega_3.
\end{aligned}$$

We define the extended state

$$\begin{aligned}\boldsymbol{\xi}^T = [&\boldsymbol{x}^T(t) \quad \boldsymbol{x}^T(t-\bar{\delta}) \quad \boldsymbol{x}^T(t-\delta(t)) \\ &\boldsymbol{d}^T(t) \quad \boldsymbol{d}_\Delta^T(t) \quad \boldsymbol{e}_x^T(t-\delta(t)) \quad \boldsymbol{e}_d^T(t-\delta(t))]\end{aligned}$$

of dimension $n_\xi \triangleq 4n_x + 3n_d$, such that

$$\dot{\boldsymbol{x}}(t) = \begin{pmatrix} A & 0 & BK & B_d + BK_d & BK_d & BK & BK_d \end{pmatrix} \boldsymbol{\xi}.$$

We need to bound the integral term in (14). To this end, we separate it into two intervals

$$\begin{aligned}-\bar{\delta}\int_{t-\bar{\delta}}^{t} \dot{\boldsymbol{x}}^T(\theta) R \dot{\boldsymbol{x}}(\theta)\,\mathrm{d}\theta = &-\bar{\delta}\int_{t-\bar{\delta}}^{t-\delta(t)} \dot{\boldsymbol{x}}^T(\theta) R \dot{\boldsymbol{x}}(\theta)\,\mathrm{d}\theta \\ &-\bar{\delta}\int_{t-\delta(t)}^{t} \dot{\boldsymbol{x}}^T(\theta) R \dot{\boldsymbol{x}}(\theta)\,\mathrm{d}\theta\end{aligned}$$

and then we apply Jensen's inequality to both terms (taking into account (5)), such that

$$\begin{aligned}&-\bar{\delta}\int_{t-\bar{\delta}}^{t-\delta(t)} \dot{\boldsymbol{x}}^T(\theta) R \dot{\boldsymbol{x}}(\theta)\,\mathrm{d}\theta \\ &\quad\le \big(\boldsymbol{x}(t-\delta(t)) - \boldsymbol{x}(t-\bar{\delta})\big)^T R\big(\boldsymbol{x}(t-\delta(t)) - \boldsymbol{x}(t-\bar{\delta})\big)\end{aligned}$$

and

$$\begin{aligned}&-\bar{\delta}\int_{t-\delta(t)}^{t} \dot{\boldsymbol{x}}^T(\theta) R \dot{\boldsymbol{x}}(\theta)\,\mathrm{d}\theta \\ &\quad\le \left(\boldsymbol{x}(t) - \boldsymbol{x}(t-\delta(t))\right)^T R \left(\boldsymbol{x}(t) - \boldsymbol{x}(t-\delta(t))\right),\end{aligned}$$

and, consequently,

$$\begin{aligned}&-\bar{\delta}\int_{t-\bar{\delta}}^{t} \dot{\boldsymbol{x}}^T(\theta) R \dot{\boldsymbol{x}}(\theta)\,\mathrm{d}\theta \\ &\quad\le \boldsymbol{\xi}^T \begin{pmatrix} -R & \star & \star & \star \\ 0 & -R & \star & \star \\ R & R & -2R & \star \\ 0 & 0 & 0 & 0 \end{pmatrix} \boldsymbol{\xi}.\end{aligned} \tag{15}$$

We now introduce in (14) the null terms

$$\begin{aligned}
0 &= \gamma^2 n \boldsymbol{d}_\Delta^T(t)\boldsymbol{d}_\Delta(t) - \gamma^2 n \boldsymbol{d}_\Delta^T(t)\boldsymbol{d}_\Delta(t),\\
0 &= \gamma^2\left(1 - n\beta^2\right)\boldsymbol{d}^T(t)\boldsymbol{d}(t) - \gamma^2\left(1 - n\beta^2\right)\boldsymbol{d}^T(t)\boldsymbol{d}(t),\\
0 &= \boldsymbol{x}^T(t) C^T C \boldsymbol{x}(t) - \boldsymbol{y}^T(t)\boldsymbol{y}(t),\\
0 &= \boldsymbol{e}^T(t-\delta(t))\Omega\boldsymbol{e}(t-\delta(t)) - \boldsymbol{e}^T(t-\delta(t))\Omega\boldsymbol{e}(t-\delta(t)),
\end{aligned}$$

and use the inequalities (15) and

$$\begin{aligned} &\boldsymbol{e}^T(t-\delta(t))\Omega \boldsymbol{e}(t-\delta(t)) \\ &\quad \le \sigma^2 \boldsymbol{s}^T(t-\delta(t))\Omega \boldsymbol{s}(t-\delta(t)), \end{aligned}$$

for $t \in [lh + \delta_u(lh), (l+1)h + \delta_u((l+1)h))$. Then

$$\begin{aligned} \dot V(t) \le &\boldsymbol{\xi}^T M \boldsymbol{\xi} + \gamma^2 \left(1 - n\beta^2\right) \boldsymbol{d}^T(t)\boldsymbol{d}(t) \\ &+ \gamma^2 n \boldsymbol{d}_\Delta^T(t)\boldsymbol{d}_\Delta(t) - \boldsymbol{y}^T(t)\boldsymbol{y}(t). \end{aligned} \tag{16}$$

If M is negative definite, the following inequality holds trivially:

$$\begin{aligned} \dot V(t) \le &\gamma^2 \left(1 - n\beta^2\right) \boldsymbol{d}^T(t)\boldsymbol{d}(t) + \gamma^2 n \boldsymbol{d}_\Delta^T(t)\boldsymbol{d}_\Delta(t) \\ &- \boldsymbol{y}^T(t)\boldsymbol{y}(t). \end{aligned}$$

Integrating both the sides, provided that $V(t)$ is continuous in t since $\cup_{k=0}^{\infty}[t_k, t_{k+1}) = [t_0, \infty)$, we obtain

$$\begin{aligned} V(t) \le V(t_0) &+ \int_{t_0}^{t} \gamma^2 \left(1 - n\beta^2\right) \boldsymbol{d}^T(\alpha)\boldsymbol{d}(\alpha)\,\mathrm{d}\alpha \\ &+ \int_{t_0}^{t} \gamma^2 n \boldsymbol{d}_\Delta^T(\alpha)\boldsymbol{d}_\Delta(\alpha)\,\mathrm{d}\alpha - \int_{t_0}^{t} \boldsymbol{y}^T(\alpha)\boldsymbol{y}(\alpha)\,\mathrm{d}\alpha. \end{aligned}$$

Then, letting $t \to \infty$, introducing the upper bound (8) for $\|\boldsymbol{d}_\Delta(t)\|_2$, and taking into account the zero initial condition $V(t_0) = 0$ and the positive definitiveness of the functional (Millán *et al.*, 2010), it can be shown that

$$\int_{t_0}^{\infty} \boldsymbol{y}^T(\alpha)\boldsymbol{y}(\alpha)\,\mathrm{d}\alpha \le \int_{t_0}^{\infty} \gamma^2 \boldsymbol{d}^T(\alpha)\boldsymbol{d}(\alpha)\mathrm{d}\alpha, \tag{17}$$

i.e., $\|\boldsymbol{y}(t)\|_2 \le \gamma \|\boldsymbol{d}(t)\|_2$.

Note that, with $\boldsymbol{d}(t) = 0$ (and $\gamma = 0$), we obtain

$$\dot V(t) \le \boldsymbol{\xi}^T M \boldsymbol{\xi} - \boldsymbol{y}^T(t)\boldsymbol{y}(t),$$

i.e., $\dot V(t) \le -\rho \|\boldsymbol{\xi}(t)\| \le -\rho \|\boldsymbol{x}(t)\|$ for a sufficiently small $\rho > 0$. Thus, the asymptotic stability of the system (9)–(11) is ensured in the absence of disturbances. ■

Remark 1. The same LMI system is obtained for the CETC case if it is noted that, if $h = 0$, then $\delta(t) = \delta_u(t)$, i.e., $\bar\delta = \bar\delta_u$.

Remark 2. For simplicity, we only consider the existence of delays in the input signal. However, it is possible to take into account intrinsic delays of the system expanding the construction of the Lyapunov function (13) with new integral terms corresponding to these delays.

Remark 3. Observe that the feasibility of the LMI conditions in Theorem 1 depends on σ, γ and $\bar\delta$. Hence, there is a trade-off between the event-triggering condition, the disturbance attenuation level, the sampling period and the delay. If time delays are large, either the event-triggering condition or the sampling period must be conservative to guarantee stability. Analogously, the disturbance attenuation level which can be guaranteed is limited by the input delay.

Naturally, the LMIs conditions proposed in Theorem 1 are more easily solvable for a smaller sampling period, delays and event-triggering parameter. This limit case of feasibility is described in the following corollary.

Corollary 1. *For sufficiently small sampling period h, delay $\bar\delta_u$ and event-triggering parameter σ, as well as a sufficiently large γ and in the absence of disturbances, Theorem 1 is feasible if*

$$(A + BK)^T P + P(A + BK) + C^T C < 0. \tag{18}$$

Proof. Assume no disturbance and $\bar\delta \to 0$ and $\sigma \to 0$. Suppose that Q and Ω_2 approach the zero matrix, i.e., $\lambda_M(Q) \to 0$ and $\lambda_M(\Omega_2) \to 0$. Then

$$M \to G = \begin{pmatrix} G_{11} & \star & \star & \star & \star & \star & \star \\ 0 & -R & \star & \star & \star & \star & \star \\ K^T B^T P + R & R & -2R & \star & \star & \star & \star \\ (B_d^T + K_d^T B^T)P & 0 & 0 & G_{44} & \star & \star & \star \\ K_d^T B^T P & 0 & 0 & 0 & -\gamma^2 n I & \star & \star \\ K^T B^T P & 0 & 0 & 0 & 0 & -\Omega_1 & \star \\ K_d^T B^T P & 0 & 0 & 0 & 0 & 0 & -\Omega_3 \end{pmatrix},$$

with $G_{11} = A^T P + PA - R + C^T C$, $G_{44} = -\gamma^2 \left(1 - n\beta^2\right) I$. We explore the conditions for $G < 0$. Applying the Schur complement iteratively,

$$\begin{aligned} -G > 0 \Leftrightarrow &- A^T P - PA - C^T C + R \\ &- \frac{P(B_d + BK_d)(B_d + BK_d)^T P}{\gamma^2(1 - n\varepsilon^2)} \\ &+ \frac{PBK_d K_d^T B^T P}{\gamma^2 n} - PBK\Omega_1^{-1}K^T B^T P \\ &- PBK_d\Omega_3^{-1}K_d^T B^T P - R - PBK \\ &- PBKR^{-1}K^T B^T P - K^T B^T P > 0 \end{aligned}$$

and other trivial constraints. Since we can take R, Ω_1 and Ω_3 as large as we need, the condition can be reduced to proving that

$$(A + BK)^T P + P(A + BK) + C^T C < 0.$$

■

With this information, we are in a position to design the state feedback and feedforward controller for the closed-loop system (9).

Theorem 2. *For a given σ, γ, the system described by (9)–(11) with the triggering condition (6) is asymptotically stable with the H_∞ norm bound γ for $\boldsymbol{d}(t)$, the feedback gain $K = YX^{-1}$, and the static-feedforward gain K_d, if there exist matrices $X > 0$, $\tilde Q > 0$, $\tilde R > 0$, $\Omega_1 > 0$, Ω_2 and $\Omega_3 > 0$ of appropriate dimensions and a scalar $n > 0$ such that*

$$\begin{pmatrix} \Omega_1 & \star \\ \Omega_2 & \Omega_3 \end{pmatrix} > 0, \quad \tilde M < 0,$$

$$M=\begin{pmatrix} \tilde{M}_{11} & \star & \star & \star & \star & \star & \star & \star & \star \\ 0 & \tilde{M}_{22} & \star & \star & \star & \star & \star & & \\ \tilde{M}_{31} & 0 & \tilde{M}_{33} & \star & \star & \star & \star & \star & \star \\ \tilde{M}_{41} & 0 & \tilde{M}_{43} & \tilde{M}_{44} & \star & \star & \star & \star & \star \\ \tilde{M}_{51} & 0 & \tilde{M}_{53} & \tilde{M}_{54} & \tilde{M}_{55} & \star & \star & \star & \star \\ \tilde{M}_{61} & 0 & 0 & 0 & 0 & \tilde{M}_{66} & \star & \star & \star \\ \tilde{M}_{71} & 0 & 0 & 0 & 0 & \tilde{M}_{76} & \tilde{M}_{77} & \star & \star \\ \tilde{M}_{81} & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{M}_{88} & \star \\ \tilde{M}_{91} & 0 & \tilde{M}_{93} & \tilde{M}_{94} & \tilde{M}_{95} & \tilde{M}_{96} & \tilde{M}_{97} & 0 & \tilde{M}_{99} \end{pmatrix}, \tag{19}$$

$$\begin{aligned}
&\tilde{M}_{11}=AX+XA^T+\tilde{Q}-\tilde{R}, && \tilde{M}_{71}=-(BK_d)^T,\\
&\tilde{M}_{22}=-\tilde{Q}-\tilde{R}, && \tilde{M}_{76}=-\tilde{\Omega}_{21},\\
&\tilde{M}_{31}=-Y-\tilde{R}, && \tilde{M}_{77}=-\Omega_3,\\
&\tilde{M}_{33}=\tilde{R}, && \tilde{M}_{81}=CX,\\
&\tilde{M}_{41}=B_d^T-(BK_d)^T, && \tilde{M}_{88}=-I,\\
&\tilde{M}_{43}=\sigma^2\tilde{\Omega}_{21}, && \tilde{M}_{91}=\bar{\delta}AX,\\
&\tilde{M}_{44}=-\gamma^2(1-n\beta^2)I-\sigma^2\Omega_3, && \tilde{M}_{93}=\bar{\delta}BY,\\
&\tilde{M}_{51}=-(BK_d)^T, && \tilde{M}_{94}=\bar{\delta}B_d+\bar{\delta}BK_d,\\
&\tilde{M}_{53}=\sigma^2\tilde{\Omega}_{21}, && \tilde{M}_{95}=\bar{\delta}BK_d,\\
&\tilde{M}_{54}=\sigma^2\Omega_3, && \tilde{M}_{96}=\bar{\delta}BY,\\
&\tilde{M}_{55}=-\gamma^2 nI+\sigma^2\bar{\delta}^2 I, && \tilde{M}_{97}=\bar{\delta}BK_d,\\
&\tilde{M}_{61}=-YB^T, && \tilde{M}_{99}=-\mu^{-1}X,\\
&\tilde{M}_{66}=-\tilde{\Omega}_{11}.
\end{aligned}$$

where $\tilde{M}$ *is given by* (19).

Proof. Define $X=P^{-1}$, $K=YX^{-1}$, $\tilde{Q}=XQX$, $\tilde{R}=XRX$, $Z=R^{-1}$, and assume that $\tilde{R}<\mu X$ for $\mu>0$. Then

$$-X\tilde{R}^{-1}X<-\frac{1}{\mu}. \tag{20}$$

Applying the Schur complement three times to (12), pre- and post-multiplying the result by $\operatorname{diag}(X,X,X,I,I,I,I,Z,Z)$ and using (20), the inequality (19) is obtained. ■

4. Illustrative examples

Example 1. (*Comparison of periodic control, CETC and PETC with and without feedforward control*) Consider the SISO system of two interconnected tanks that is depicted in Fig. 1. Its state-space representation around a desired operating point is given by (Lehmann and Lunze, 2011)

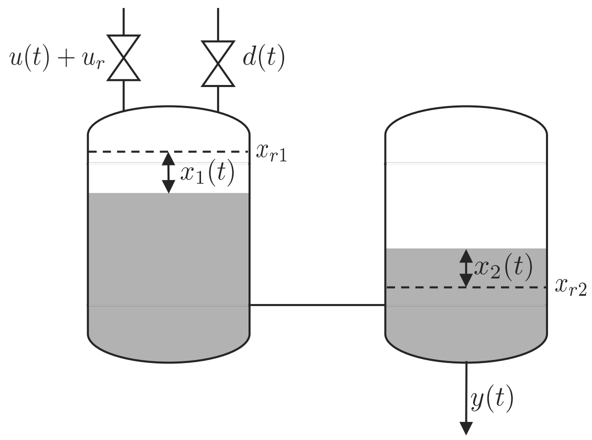

Fig. 1. Diagram of the SISO system of two interconnected tanks.

$$\begin{aligned}\dot{\boldsymbol{x}}(t)&=\begin{pmatrix}-0.1 & 0.1\\ \frac{1}{12} & \frac{-1}{8}\end{pmatrix}\boldsymbol{x}(t)+\begin{pmatrix}0.1\\0\end{pmatrix}u(t-10)\\ &\quad+\begin{pmatrix}0.1\\0\end{pmatrix}d(t),\end{aligned} \tag{21}$$

$$y(t)=\begin{pmatrix}0 & 0.5\end{pmatrix}\boldsymbol{x}(t), \tag{22}$$

where $x_1(t)$, $x_2(t)$ are the difference between the operating point x_r and the level of the first and second tank, respectively, which are assumed small in order to maintain the validity of the linear model, $y(t)$ is the outflow of the second tank, and $u(t)$ is the difference between the inlet flow and the reference input u_r, i.e., the necessary inlet to maintain the stationary state at the operating point. Thus, we assume that this steady state set point of the stirred tank system corresponds to the positive constant inflow through the control input and the null inflow through the disturbance input, and results in positive constant water levels. Note that $u(t)$ is affected by a constant time delay that is considered for demonstrative purposes. A time delay may appear in practical applications if a fluid has to be transported from a distant source or if control signals are transmitted through a communication network. This implies that the reaction against disturbances is applied later to the process and, consequently, the process returns to the operating point later. Besides, if the value of the disturbance changes too quickly, then the reaction might be "outdated", and consequently, the benefits of feedforward control are reduced.

Consider a disturbance $\boldsymbol{d}_1(t) = ae^{bt}$, with $a = 1$ and $b = -0.01$, such that Proposition 1 can be applied with $\tilde{\beta} = 0.01$, a sampling time of 1 [s] and a maximum dead time of 10 [s], which gives the upper bound $\|\boldsymbol{d}_{1\Delta}(t)\| \leq 0.22\,\|d(t)\|$ (see Fig. 2). Then, solving the LMI system of Theorem 1, a feedback gain $K = -[0.64 \quad 0.41]$, a feedforward gain $K_d = -0.75$, and an event-triggering matrix

$$\Omega = \begin{pmatrix} 0.0224 & \star & \star \\ 0.0234 & 0.0343 & \star \\ 0.0019 & 0.0023 & 1.1337 \end{pmatrix} \qquad (23)$$

are obtained for $\sigma = 0.3$ and $\gamma = 0.35$.[1]

The state response of the plant, control signals and events generated are depicted in Figs. 3 and 4. Both CETC and PETC approaches yield a significant reduction in the number of events with respect to the periodic controller with the same sampling time (see Table 1). Furthermore, the proposed controllers keep the Euclidean norm of the state lower than the pure feedback controller most of the time, as shown in Fig. 5 and Table 1. In Table 1, the results are also compared with the classical feedback controller based on the Lyapunov–Krasovskii method described by Fridman (2014, Proposition 5.3)). Finally, note that a greater value of σ can be considered with the CETC controller, i.e., it may be possible to reduce even more the number of events with this approach.

In the case of a disturbance which does not satisfy the assumptions of Proposition 1, the value of β can

[1]Note that in the CETC case a different result may be obtained but the performance condition is also guaranteed with the chosen value of Ω.

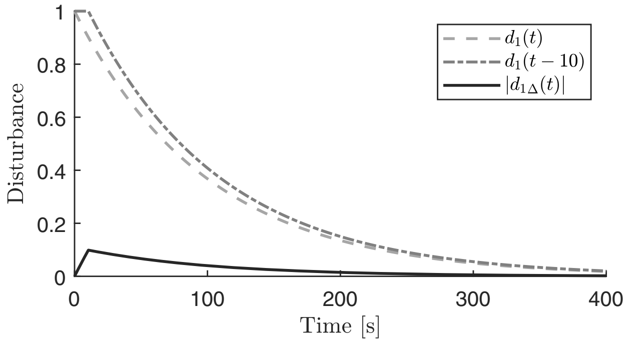

Fig. 2. Evolution of the disturbance signals considering a fixed input delay of 10 [s] and a sampling period of 1 [s].

be numerically computed. Consider, for example, the disturbance presented in Fig. 6. The value of β for that disturbance is $\beta = 0.8$. The behavior of the system under this disturbance can be observed in Figs. 7–9. Similarly to the first example, the feedforward control allows reducing the effect of the disturbance. The number of transmissions is also clearly reduced in the CETC and PETC cases in comparison with the periodic control (Table 2). ♦

Example 2. (*Comparison of the prosed method and other robust controllers in the literature*) Consider the inverted pendulum system proposed by Wang and Lemmon (2009) and used as an example in different robust controllers (e.g., Peng and Han, 2013; Yue *et al.*, 2013). The plant state-space representation is given by the matrices

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -mg/M & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & g/l & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 1/M \\ 0 \\ -1/Ml \end{pmatrix},$$
$$B_d^T = C = \begin{pmatrix} 1 & 1 & 1 & 1 \end{pmatrix},$$

where $M = 10$ is the cart mass and $m = 1$ is the mass of the pendulum, $l = 3$ is the length of the pendulum link and $g = 10$ is the gravitational acceleration. The four states of the system are the cart position, the cart velocity, the pendulum angle and the pendulum angular velocity. The eigenvalues of the system matrix are $(0, 0, \pm 1.8257)$, so the system is clearly unstable.

Let us consider also the disturbance $d(t) = \operatorname{sgn}(\sin t)$ if $t \in [0, 10]$, otherwise $d(t) = 0$ as proposed by Yue *et al.* (2013). For a sampling period $h = 0.01$ s, $\gamma = 200$, $\sigma = 0.1$, applying Theorem 2, we obtain the feedback gain $K = \begin{pmatrix} 3.29 & 12.93 & 317.33 & 177.16 \end{pmatrix}$, the feedforward gain $K_d = 9.80$ and

$$\Omega = 10^4 \cdot \begin{pmatrix} 0.02 & 0.05 & 1.35 & 0.75 & -0.06 \\ 0.05 & 0.16 & 4.19 & 2.33 & -0.22 \\ 1.35 & 4.19 & 115.05 & 64.08 & -1.10 \\ 0.75 & 2.33 & 64.08 & 35.70 & -0.66 \\ -0.06 & -0.22 & -1.10 & -0.66 & 34.14 \end{pmatrix}.$$

♦

Table 1. Comparison of event parameters between the CETC and PETC controllers with disturbance $\boldsymbol{d}_1(t)$.

Controller	Number of events	Average inter-event time	ISE of $\|x(t)\|$
Periodic (Fridman, 2014) ($K_d = 0$)	400	1.00 [s]	451.71
Periodic (Theorem 2) ($K_d = -0.75$)	400	1.00 [s]	146.90
CETC ($K_d = 0$)	17	23.52 [s]	173.62
CETC ($K_d = -0.75$)	18	22.22 [s]	164.33
PETC ($K_d = -0.75$)	18	22.22 [s]	157.27

Table 2. Comparison of event parameters between the CETC and PETC controllers with disturbance $\boldsymbol{d}_2(t)$.

Controller	Number of events	Average inter-event time	ISE of $\|x(t)\|$
Periodic (Fridman, 2014) ($K_d = 0$)	400	1.00 [s]	$2.13 \cdot 10^4$
Periodic (Theorem 2) ($K_d = -0.75$)	400	1.00 [s]	$1.51 \cdot 10^3$
CETC ($K_d = 0$)	23	17.39 [s]	$1.26 \cdot 10^4$
CETC ($K_d = -0.75$)	22	18.18 [s]	$2.80 \cdot 10^3$
PETC ($K_d = -0.75$)	20	20.00 [s]	$2.97 \cdot 10^3$

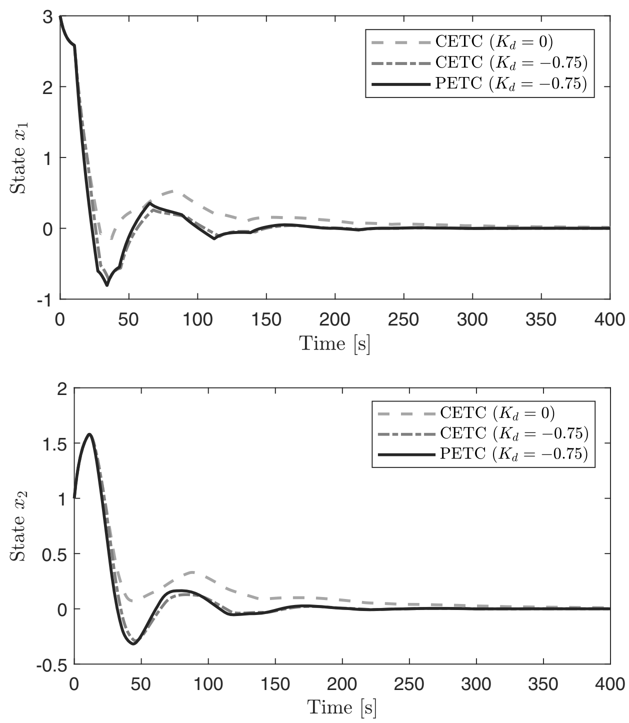

Fig. 3. Evolution of the states of the system (21)–(22) with the proposed CETC and PETC controllers and with disturbance $\boldsymbol{d}_1(t)$.

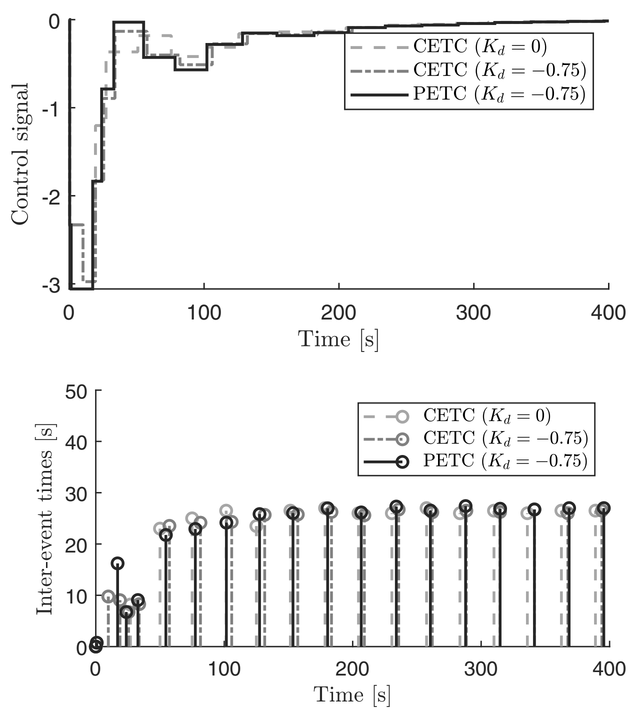

Fig. 4. Control signals and events produced by the event generator for the system (21)–(22) with the proposed CETC and PETC controllers and with disturbance $\boldsymbol{d}_1(t)$.

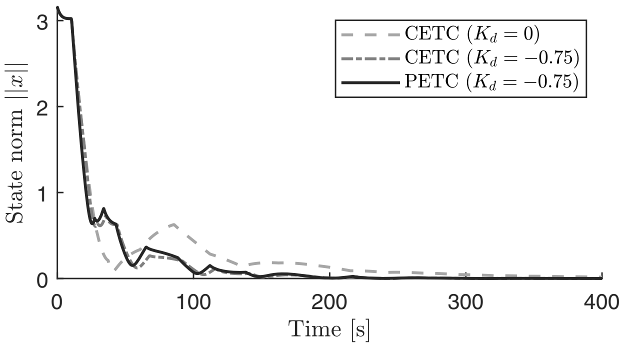

Fig. 5. Evolution of the Euclidean norm of the state for the system (21)–(22) with the proposed CETC and PETC controllers and with disturbance $\boldsymbol{d}_1(t)$.

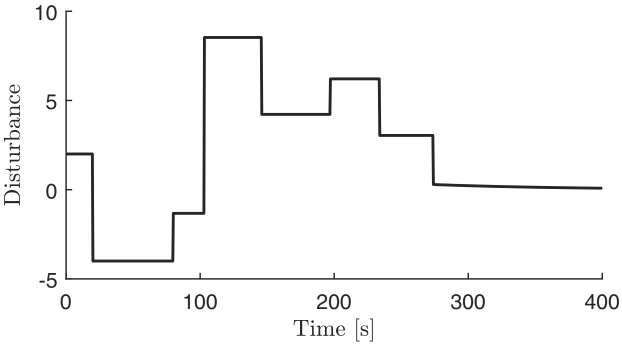

Fig. 6. Evolution of the disturbance signal $\boldsymbol{d}_2(t)$.

If we compare the positions of the pendulum and the arm in the different schemes (Fig. 10), we observe that the use of the feedforward controller can improve the response against disturbance. We can measure the ISE of the output and we obtain an improvement of 15% with respect to Peng and Han (2013), of 21% with respect to Yue *et al.* (2013) but, in addition, we obtain it by reducing the transmitted information by 16% with respect to Peng and Han (2013) and by 22% with respect to Yue *et al.* (2013), as can be observed in Table 3.

Besides, in Fig. 10, we can observe the effect of minimizing γ. If we maintain the rest of the parameters but we choose $\gamma = 80$, then the feedback and feedforward gains are $K = \begin{pmatrix} 8.34 & 25.59 & 417.20 & 234.93 \end{pmatrix}$, with a feedforward gain of $K_d = 45.65$. Naturally, this implies better disturbance attenuation, but also faster changes in the state of the process and, consequently, more events triggered, as shown in Table 3.

5. Conclusions

In this paper, a novel approach to the analysis of dead-time linear systems controlled by means of state-feedback plus static feedforward controllers was introduced. The criteria for both, the CETC and PETC paradigms with quadratic

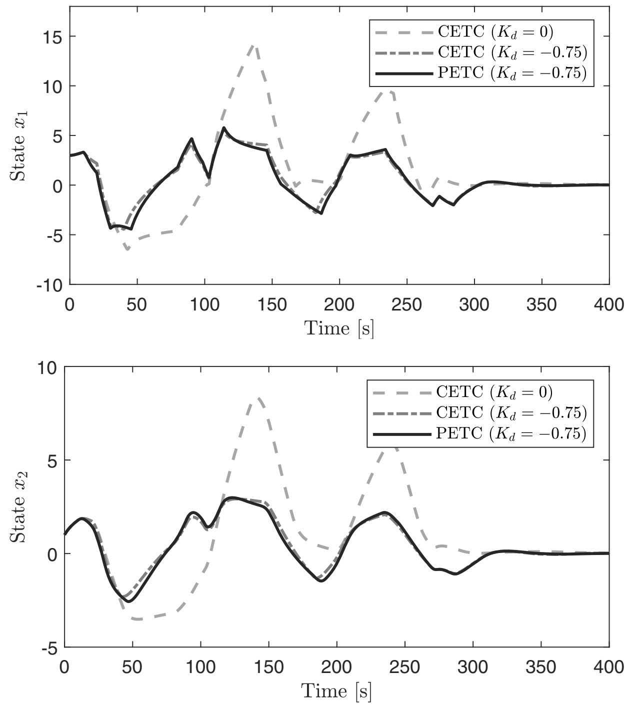

Fig. 7. Evolution of the states of the system (21)–(22) with the proposed CETC and PETC controllers and with disturbance $\boldsymbol{d}_2(t)$.

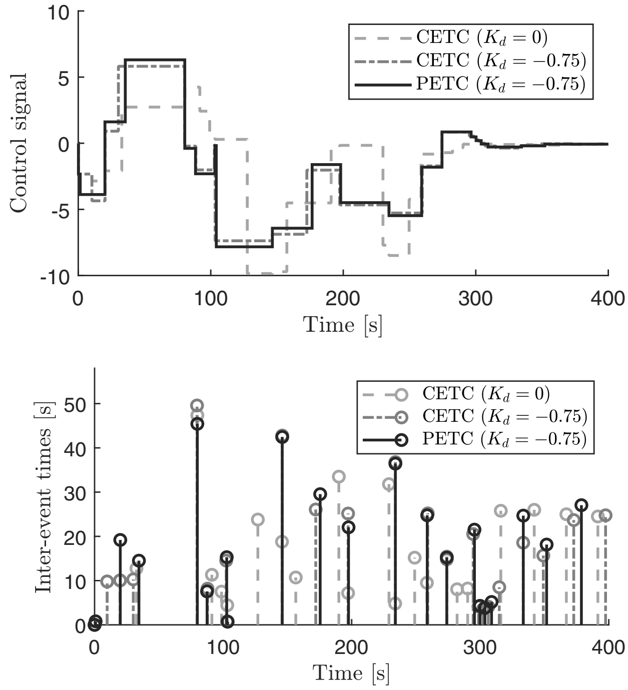

Fig. 8. Control signals and events produced by the event generator for the system (21)–(22) with the proposed CETC and PETC controllers and with disturbance $\boldsymbol{d}_2(t)$.

Table 3. Comparison with other robust controllers.

Controller	Number of events	Average inter-event time	ISE of $y(t)$
Controller by Peng and Han (2013)	249	0.12 [s]	$1.59{\cdot}10^4$
Controller by Yue *et al*. (2013)	270	0.11 [s]	$1.69{\cdot}10^4$
Proposed controller with $\gamma = 200$	209	0.14 [s]	$1.34{\cdot}10^4$
Proposed controller with $\gamma = 80$	271	0.11 [s]	$6.88{\cdot}10^3$

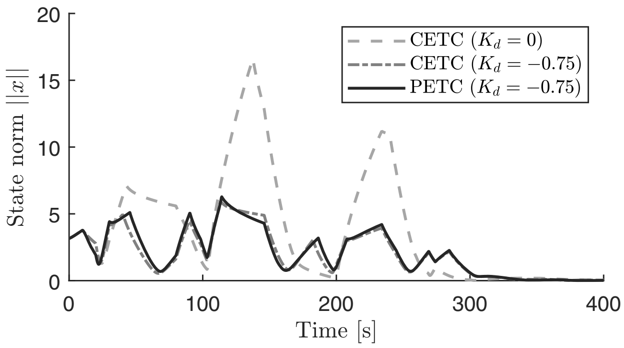

Fig. 9. Evolution of the Euclidean norm of the state for the system (21)–(22) with the proposed CETC and PETC controllers and with disturbance $\boldsymbol{d}_2(t)$.

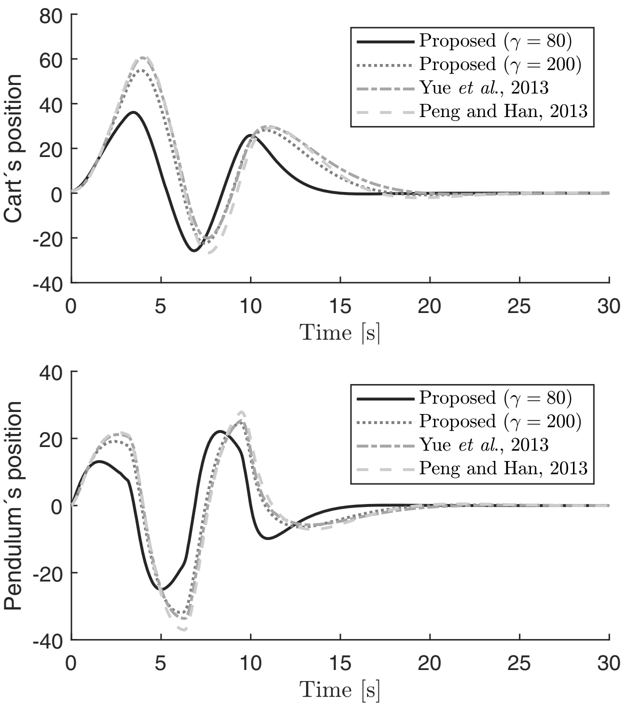

Fig. 10. Evolution of the positions of the cart and the pendulum.

event-triggering conditions. Were derived making use of the same LMI system. It was shown that the use of an event-based feedforward action can improve the disturbance response even under the presence of dead times. Furthermore, synthesis of controllers was also addressed.

Future works will include extending the proposed approach to cope with additional phenomena, e.g., parametric uncertainties and unmeasurable states or disturbances, and to other control laws, e.g., output-feedback plus feedforward.

Acknowledgment

This work was supported in part by the Spanish Ministry of Economy and Competitiveness (MINECO) under the projects DPI2012-31303, DPI2014-55932-C2-1-R, DPI2014-55932-C2-2-R, DPI2017-84259-C2-1-R and DPI2017-84259-C2-2-R.

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Received: 11 October 2018
Revised: 23 January 2019
Re-revised: 11 March 2019
Accepted: 30 March 2019