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The Use of Dynamic Modeling for Optimization of Machine Design on the Example of a Toothed Gear

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Abstract The paper presents a dynamic model of a test stand with gears operating in a circulating power system as an example of the possibility of using dynamic modeling to optimize the gear construction. The presented model is one of two advanced tools developed within the research conducted for the optimization and diagnosis of various types of gear transmission systems. Both of them were based on the assumption that simulations of system operation at variable loads and rotational speeds could be carried out and that an advanced description of gear tooth interaction could be included. The developed models, after their tuning to real objects, were used as tools for generating vibration signals for various technical states of gears. This allowed determining the influence of various constructional, technological, and operational factors on the vibroactivity of a toothed gear, which is undoubtedly helpful for designers both during the construction of new toothed gears, as well as optimization of the existing ones.

Keywords: dynamic model, toothed gear, power transmission system, transport.

1. Introduction

Designing machines requires extensive knowledge about the constructed device and its operating conditions. Not taking into account the operating conditions at the design stage, often leads to a significant error in the estimated durability of the machine and may be the cause of unplanned downtime of the machine or a serious accident. Analysis of real operating conditions is therefore particularly important for any device, whose production cost is high or whose failure may lead to significant economic losses or the emergence of a threat to human life safety.

In many situations, effective actions leading to the fulfillment of the set criteria and design optimization, seen as e.g. minimization of dynamic phenomena, are possible to a large part due to simulation studies. Models used for this purpose allow determining the influence of various factors on dynamic phenomena occurring in designed devices, which often, due to a large number, cannot be determined experimentally in the whole requested range. Conducting research with the use of models allows to reduce the number of experimental tests to a necessary minimum and is thus economically justified. Extended models allow also to reflect the cause-effect relationship between vibration and noise of devices and their technical condition, which is the basis of vibroacoustic diagnostics, and thus facilitate the task of their diagnosis.

Undoubtedly, the toothed gear, which is one of the most important elements of power transmission systems, is a device that requires detailed knowledge of the occurring phenomena and processes to be properly designed and diagnosed. High constructional, operational, and economic requirements set before toothed gears result in the necessity of solving a complicated task by constructors. Design optimization taking into account often a large number of contradictory criteria becomes possible as a result of using electronic calculation techniques.

Numerous studies on gear transmission modeling can be found in the literature. In [1] a dynamic model for determining vibrations in involute helical gears is presented. A number of works by Bartelmus [2, 3] and Zimroz [19] discuss dynamic modelling for the purpose of diagnosing toothed gears, including planetary ones. Batko and Korbiel [4] also deal with the issue of determining the technical condition. Early damage detection is also a problem discussed in works [9, 10, 13], where a dynamic model of a toothed gear serves as one of the tools. Dynamic modeling of spur and planetary gears is also a subject of numerous works undertaken by the author [14, 15].

Regardless of the scope of conducted numerical investigations, it is necessary to determine all taken account parameters of the used model. The model tuning allows to obtain the conformity of simulation results with the results of experimental tests on a real object, and thus influence the usefulness of the model to solve the considered problem. The quality of model tuning is also strongly under the influence of the progress in the development of measurement methods. From the point of view of the presented model, the modern technique, e.g. non-contact measurement of vibrations of rotating elements, makes it possible to avoid the impact of complicated transmittances.

2. Vibroacoustics of toothed gears

In an ideal single-stage toothed gear with parallel axes, the vibroacoustic signal contains only the mesh frequency and its harmonics [12]. It is described by the following relation:

$$x(t) = \sum_{k=1}^{K} A_k \cdot \cos(2 \cdot \pi \cdot k \cdot f_z \cdot t + \varphi_k), \tag{1}$$

where k is harmonic of the mesh frequency, (k = 1, 2, 3, ..., K), A_k – amplitude of the k-th harmonic of the mesh frequency, f_z – mesh frequency, φ_k – initial phase of the k-th harmonic of the mesh frequency.

In real toothed gears, the power of the vibroacoustic signal is contained mainly in the frequency band of meshing, and due to the occurrence of involute error also in the bands of its initial harmonics. Moreover, there are frequencies f_{01}, f_{02} of shaft rotations and their harmonics, resulting, among others, from eccentric seating or unbalance of wheels. They are visible in the low-frequency part of the spectrum as well as in the form of sidebands created from the modulation of the interlocking frequency and its harmonics:

$$f = k \cdot f_z \pm k_{01} \cdot f_{01} \pm k_{02} \cdot f_{02} \tag{2}$$

where k_{01}, k_{02} – successive harmonics of the rotational frequency of the pinion and the wheel, k_{01}, k_{02} = 1, 2, 3, …, f_{01} – frequency of rotation of the pinion shaft, f_{02} – frequency of rotation of the wheel shaft.

The spectrum of gears also contains several harmonics with frequencies associated with the tooth mating cycle. Disturbances in the kinematic conditions of tooth mating along the mesh line result in vibrations with frequencies associated with the radial contact ratio ε_α.

Numerous works show that rolling bearings are also an additional source of modulation. The causes of vibrations generated by a bearing can be divided into structural, manufacturing, and operational. In the first group, there is a variable bearing stiffness, resulting from changing number of rolling elements in the load transmission zone. Manufacturing factors include shape and dimensional deviations obtained during production and assembly errors. Operational factors include wearing processes occurring in the bearing elements [5]. The problem of modeling and simulation a toothed gear operating with a defective bearing has been addressed in [17].

The dynamic interaction in the mesh zone modulated by the series connected rolling bearing interaction causes vibrations in the gear body. The body is characterized by a complex resonant structure and is the main emitter of noise. The convolution of the signal coming from the mesh and the source-to-receiver transition function forms the basis for evaluating the vibroacoustic activity of the toothed gear.

In addition to the aforementioned spectral components, there are many designs and operational factors in toothed gears that force vibration, which can modulate the amplitude and frequency of the signal generated in the gear mesh. Selected of these are circumferential backlash, radial and overlapcontact ratio, bearing type and mounting, wheel tooth pitch deviations, outline and profile line deviations, backlash angle error, condition and roughness of tooth working surfaces, wheel runout and eccentricity, misalignment, wobble and axle distance deviation, load magnitude and rotational speed.

Vibroactivity of toothed gears is also influenced by wear phenomena and possible damage of elements of gear and power transmission system. Wear of teeth causes changes in their initial contours, including their shift, which consequently contributes to an increase in circumferential clearances, and thus to the occurrence of impacts and an increase in dynamic forces in the meshing. Wear simulation using a FEM model of a helical tooth wheel was realized in the work [8]. Investigations based on Hertz's theory showed greater sensitivity to load distribution than in straight tooth wheels. Based on the results of numerous experimental studies, it was found that wear leads to an increase in gear dynamics only in the initial phases, after which no further increase in inter-tooth forces is observed. On the other hand, simulation studies show that the dynamic loads at the initial stage of toothed gear operation are mainly influenced by errors in tooth outlines, and at a later stage of operation by changes in the outline caused by wear. Wear of rolling bearings increases the clearance between teeth, whereas wear of load-bearing elements, couplings, and connections causes disturbances in the torque transmission. All these wear processes result in the randomness of amplitude and frequency of elementary events resulting from the rotation of a wheel. Damage to gear elements may contribute to excitation to resonance vibrations of all structural elements of the toothed gear, the conducted research is usually directed at using vibration signals to find symptoms of damage in the early stages of its development.

3. Dynamic model of a toothed gear operating in a power transmission system

Numerous research and development centers are engaged in modeling gears, perceiving the developed models as useful tools in the design and dynamic analysis of gears. The creation of new models and extension of existing ones is supported by the continuous development of computational hardware and software, which makes it possible to include subsequent factors in the models. Simulation tests remove inconveniences of tests performed on real objects, they are repeatable and give a possibility of performing complicated dynamic tests, which in many cases cannot be realized to the required degree in real conditions.

The dynamic model of the laboratory test stand with toothed gears operating in the circulating power system is an example of an extended model, which allows detailed investigations of dynamic phenomena occurring both in gears and in other elements of the system. It was developed to obtain a tool useful for both designers at the stage of toothed gear design and optimization of its structure as well as for people involved in the monitoring of the technical condition and toothed gear diagnostics. A decision on its construction was made as a result of an analysis of its usefulness in a very wide range of research than originally intended by the creators of the stand. Two laboratory test stands at the author's disposal allowing to fine-tune the model and determine the conformity of simulation test results with laboratory tests. Comprehensive research made it possible to determine the influence of a wide range of constructional, technological, and operational factors on the vibroactivity of gears and contributed to the determination of the methodology of designing gears with reduced vibroactivity.

The developed dynamic model of the FZG stand is shown in Fig. 1. The model takes into account an electric motor, two single-stage spur gears (test and closing gear) connected by tensioning and torsion shafts. The tensioning shaft and the motor with the input shaft of the closing gear are connected by couplings.

The vibration excitation of the mass elements is the result of complex interactions of both internal vibration sources, such as varying mesh stiffness, damping in the mesh, and mesh deviations and external ones, resulting from changes in speed and shaft torques.

As in the model presented in work [11], the pinion and wheel of each gear and coupling are treated as rigid solids with known moments of inertia, connected by weightless elements modeling elasticity and damping. The masses of the other gear elements were reduced to masses centered on the bearing.

Taking into account that the *x*-axis is directed according to the direction of the mesh force, the *y*-axis is directed according to the direction of the friction force in the mesh, the z-axis coincides with the direction of the axis of the stand shafts, as a result of applied simplifications, the system is described assuming the following generalized coordinates q_i:

- angular displacement about the *z*-axis: rotor φ_s, couplings φ_{sp}, φ_{wn} and wheels φ_B, φ_E, φ_H, φ_K,
- displacements in all bearings in *x*-axis direction: x_A, x_C, x_D, x_F, x_G, x_I, x_J, x_L,
- displacements in all bearings in *y*-axis direction: y_A, y_C, y_D, y_F, y_G, y_I, y_J, y_L.

The phenomena occurring in toothed gears are described according to one of the most known concepts - the spatial palisade model of Müller. The application of the spatial model of gearing in the form of a palisade of springs moving between two surfaces makes it possible to carry out tests of, among others, wheels with tooth line modification as well as wheels with helical teeth. By shaping the surfaces it is possible to take into account in the model periodical deviations of the outline, modifications of the outline, deviations, and modifications of the tooth line as well as wear of tooth working surfaces. The wheels are modeled as elementary gears with straight teeth, which in the case of helical meshing are shifted in phase with each other depending on the angle β_b of inclination of the tooth line on the base diameter. The meshing stiffness of each elementary gear is then determined as in the case of the spur gear, but with the geometric parameters of the helical gear in the transverse plane.

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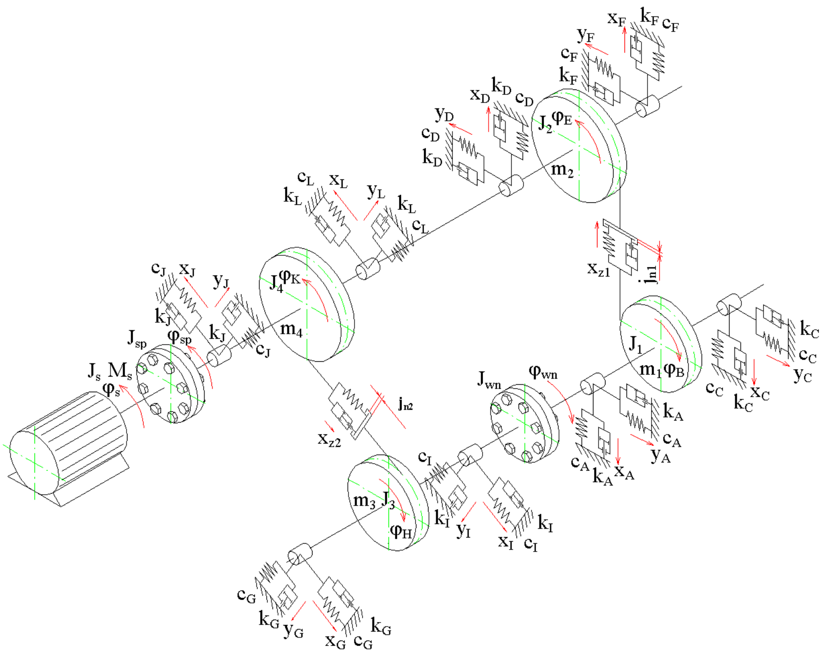

Figure 1. Dynamic model of FZG stand.

The model makes it possible to carry out dynamic calculations for toothed gears with HCR wheels, characterized by a value of the radial contact ratio ε_α greater than 2. On the other hand, it is assumed that the minimum value may be 1 and only locally, in the case of tooth damage, may be smaller. As a result, the number of pairs of teeth in the meshing of each elementary gear wheel, depending on the value, can be up to 3. The possibility of a temporary loss of contact between the mating teeth as a result of dynamic loads was also taken into account. If the distance at which the teeth move away from each other is greater than the normal mesh clearance j_n, imact of the teeth by their opposite sides will occur, as well as impact on re-entry into the normal cooperation. However, if the distance is less than the mesh clearance, there will only be a loss of contact and a one-sided impact when returning to the normal cooperation. To determine the mesh stiffness of one pair of teeth, among the different methods presented in [16], it was decided to use in the model the method described in the paper [6], adapted from the study of Niemann and Baethge. The vibration damping in the mesh was modeled using the dimensionless damping coefficient described in [6, 11].

To model the bearing stiffness, the basic version of the test stand dynamic model uses the methods known from the literature for determining nonlinear characteristics of the displacement of the shaft pivot in the direction of the mesh force [11]. They allow taking into account the bearing stiffness, which on the tested stands was of the same order as the gear stiffness. For detailed studies of the influence of bearings on the dynamics of phenomena in the power transmission system, a description of the bearing acting based on characteristics of its rigidity determined using FEM was used.

The presented system is driven by an electric motor. Two alternative descriptions from the literature were also used to model the system, which differs in detail. One version of the motor model focuses exclusively on its mechanical parameters, while the other taken from [18] is extended and include electrical factors, which have a noticeable influence on the accuracy of the results obtained in transient states of the workstation. During the simulation of stationary electric motor operation, the results obtained from both models are in good agreement, but the calculations performed with the use of a motor model that also takes into account its electrical parameters are more time-consuming. Therefore, in such cases, a simpler model taking into account only mechanical parameters is chosen for the simulation. In other situations, a mechanical-electrical motor model is used. The advantage of the extended description is also the inclusion of the power supply system - the modeled motor is powered by a frequency converter operating based on pulse width modulation PWM, which further approximates the construction of the model to the real test stands.

4. Dynamic model of a toothed gear as a tool supporting its design and optimization

The developed model has been applied in a wide range of research, the first group of which are tests allowing for toothed gear structure optimization already at the design stage. With its use, it is possible to shape the expected characteristics of a toothed gear, which goes beyond basic actions verifying the geometric correctness and strength of a structure. Tuning of the model was performed based on experimental research, literature analysis, and using advanced CAD environments. The use of the model in numerical investigations of the effects of selected constructional, technological, and operational factors on the course of dynamic phenomena and vibration behavior of gears was influenced by the complexity of the model, due to which it did not omit significant parameters describing the gear construction, which is not possible with less developed models.

The parameters included in the model, which are very easy to determine at the stage of constructing a new toothed gear include geometric parameters of gears, mass and moments of inertia of elements, manufacturing deviations of tooth pitches, which can be drawn according to the assumed distribution and accuracy class. The rest of the set of parameters constituting input data for simulation calculations must be entered into the program based on e.g. literature data or be determined based on experience with other designs.

The results of simulation tests conducted using the presented model, tuned to two significantly different FZG stands, showed very high agreement with the results of tests conducted on these stands using several different pairs of gears. Figure 2 shows the diagrams of RMS values of velocities of transverse vibrations of the gear wheel shaft of the tested stand (point D – Fig. 1) obtained from computer calculations and during laboratory measurements conducted with the use of a laser vibrometer. In the gear were mounted spur gears.

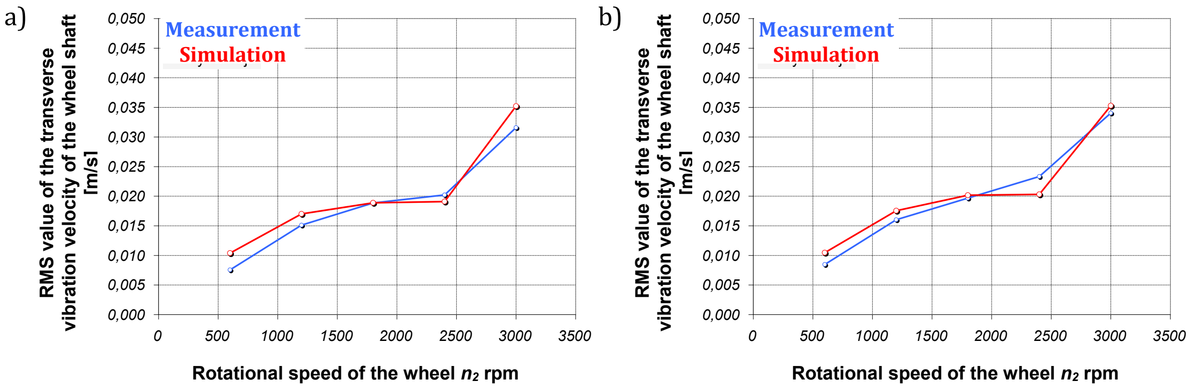

Figure 2. Comparison of RMS values of velocity of transverse vibrations of the wheel shaft (point D) of the tested toothed gear working as a reducer, determined on the laboratory stand and using the model: a) unit load Q = 1.5 [MPa], b) unit load Q = 2.0 [MPa].

To indicate the influence of the accuracy of determining one of the parameters - deviations of the gear pitch on the compatibility of the numerical results with the results of the laboratory tests, Fig. 3 presents an example comparison of the RMS values of transverse vibrations velocities in another measuring point, located on the pinion shaft of the closing gear (point I – Fig. 1). For this point the deviations of the gear pitch were not measured, the values for each tooth were randomly generated. In this case, the deviations did not exceed the maximum value read from the standards for the specified accuracy class and had a normal distribution.

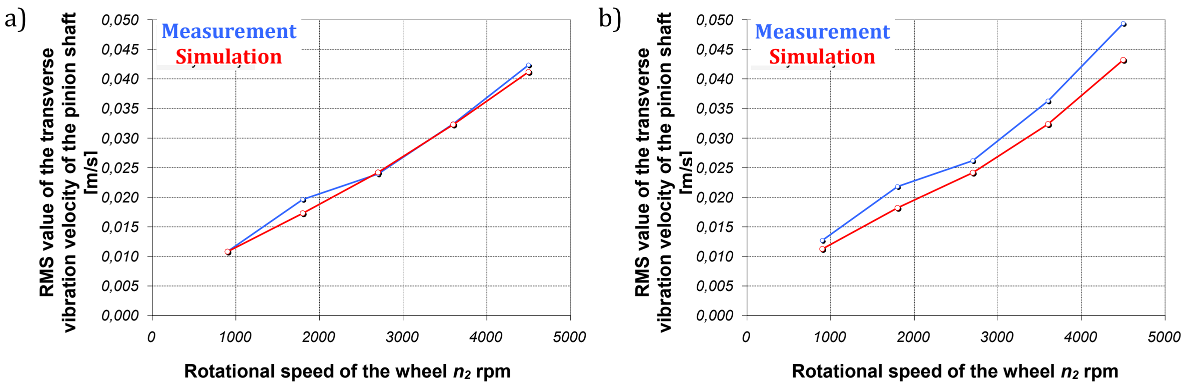

Figure 3. Comparison of RMS values of velocities of transverse vibrations of pinion shaft (point I) of a closing gear working as a multiplier, determined on a laboratory test stand and with the use of a model: a) unit load Q = 1.5 [MPa], b) unit load Q = 2.0 [MPa].

On the other hand, Fig. 4 presents a comparison of results obtained from simulation calculations and laboratory measurements for the second test stand, in the measuring point located on the wheel shaft of the tested gear. At the presented stage of research, the distribution of deviations of helical tooth gears was not determined, nor was the estimation of the class of their manufacturing accuracy.

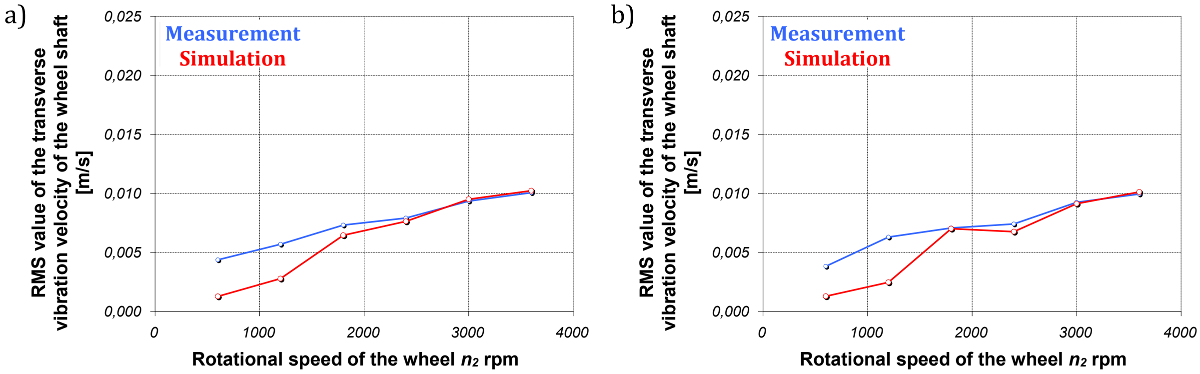

Figure 4. Comparison of RMS values of velocity of transverse vibrations of the wheel shaft (point D) of the tested toothed gear working as a reducer, determined on the laboratory bench and with the use of the model: a) unit load Q = 1.5 [MPa], b) unit load Q = 2.0 [MPa].

One of the simplest examples of using a model when designing or optimizing a toothed gear is determining the effect of gear accuracy on the vibroactivity of this gear. The values of dynamic excess significantly depend on the gear manufacturing accuracy class, therefore Fig. 5 presents the results of a numerical test carried out in a wide range of rotational speed changes, load, and for several assumed manufacturing accuracy classes. The model allows considering the influence of manufacturing accuracy class on the dynamics of gear operation for different values of deviations of pitch and position of tooth outline. The values of unit load were Q = 1.5 ÷ 4.0 [MPa] in 0.5 [MPa] steps. For each value of the unit load, calculations were performed for twelve rotational speeds of the wheel shaft in the range $n_2 \approx 300 \div 3600$ rpm with the step of 300 rpm, which corresponded to speeds of the pinion shaft in the range $n_1 \approx 450 \div 5400$ rpm.

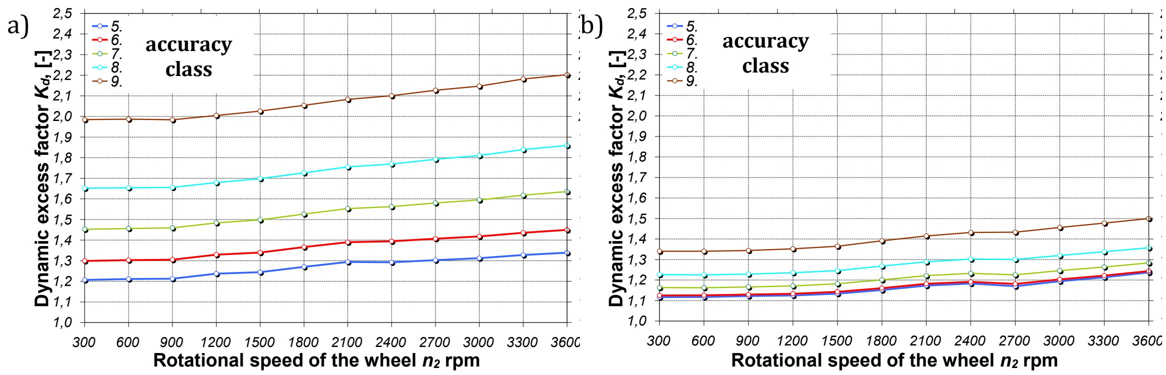

Figure 5. Influence of accuracy class and rotational speed on the values of the dynamic excess factor K_d, unit load: a) Q = 1.5 [MPa], b) Q = 4.0 [MPa].

The calculations were carried out for the values of deviations defined by the accuracy classes from 5^{th} to 9^{th}, i.e. the most commonly used in gears. The same distribution of tooth pitch deviations was assumed for each accuracy class, which made it possible to maintain comparability of results and constitutes a certain advantage over laboratory tests.

The influence of accuracy class and rotational speed can also be determined on the RMS values of the transverse vibration velocity of the shafts. As an example, Fig. 6 shows the obtained in simulation studies RMS values of the transverse vibration velocity of the wheel shaft at point D (according to the Fig. 1).

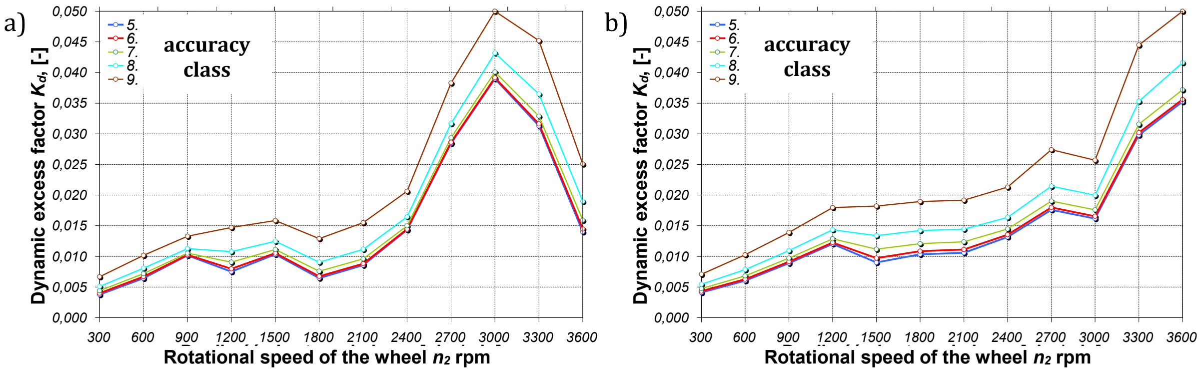

Figure 6. Influence of accuracy class and rotational speed on the RMS values of the transverse vibration velocity of the wheel shaft (point *D*), unit load: a) Q = 1,5 [MPa], b) Q = 4,0 [MPa].

Using the model, it is relatively simple to get a comprehensive view of the factor being analyzed. An example can be the research, in which the model was used to determine the influence of the value of the radial contact ratio ε_α and rotational speed on the values of dynamic excesses in the meshing. Figure 7 shows the obtained image of the value of this factor, the analysis of which allows indicating unfavorable areas of the toothed gear operation, determined by a pair of parameters - rotational speed (in the shown case of the gear wheel) and the radial contact ratio ε_α.

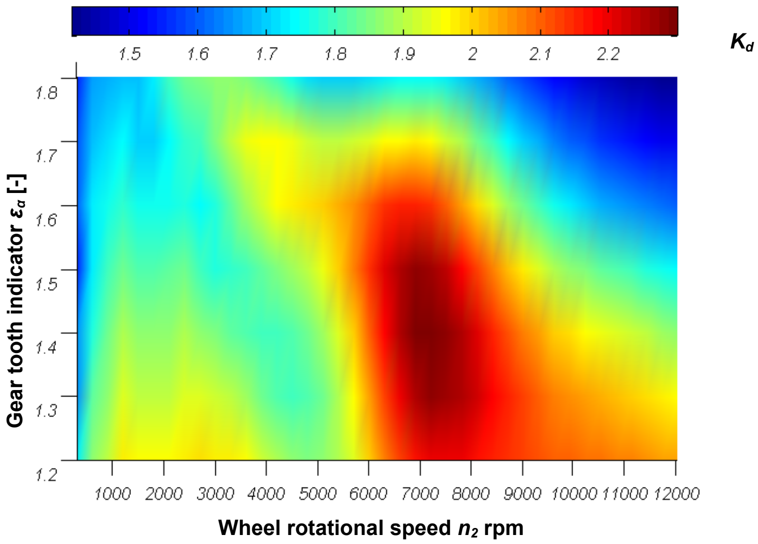

Figure 7. Influence of radial contact ratio ε_α and wheel rotational speed on the values of dynamic excess ratio K_d, toothed gear operating under load Q = 4.0 [MPa], 6th manufacturing accuracy class.

5. Conclusions

As a result of simulation calculations with the use of the presented model, it is possible to obtain the time courses of, among others, mesh forces in both gears, mesh friction forces, forces in bearings, accelerations, speeds, and angular displacements of wheels and couplings and linear shaft journals in bearings, as well as torques, gear efficiency, voltages and currents in motor windings. Tests can be performed over a wide range of speeds and loads, which often may not be possible under laboratory conditions.

To tune and then verify the correctness of the model, tests were carried out on two different test stands using nine pairs of gears interchangeably mounted, differing in geometric parameters and manufacturing accuracy class. This allowed us to determine the usefulness of the model for further research, including research on the effects of the selected constructional, technological, and operational factors on the level of dynamic phenomena based on the values of mesh forces determined during calculations, coefficients of dynamic excess forces in the gears, and speeds of transverse vibrations in the bearing nodes. The determined time courses of forces in bearings can be input data to FEM models, which allow for analyzing the influence of body design on the vibroactivity of toothed gears, which was realized in one of the studies.

Additional information

The author declare no competing financial interests.

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