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Review paper

Geodesy: General theory and methodology 2015–2018

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Abstract: The summary of research activities concerning general theory and methodology performed in Poland in the period of 2015–2018 is presented as a national report for the 27th IUGG (International Union of Geodesy and Geophysics) General Assembly. It contains the results of research on new or improved methods and variants of robust parameter estimation and their application, especially to control network analysis. Reliability analysis of the observation system and an integrated adjustment approach are also given. The identifiability (ID) index as a new measure for minimal detectable bias (MDB) in the observation system of a network, has been introduced. A new method of covariance function parameter estimation in the least squares collocation has been developed. The robustified version of the Shift-M_{split} estimation, termed as Shift-M^*_{split} estimation, which enables estimation of parameter differences (robustly), without the need of prior estimation of the parameters, has been introduced. Results on the analysis of geodetic time series, particularly Earth orientation parameter time series, geocenter time series, permanent station coordinates and sea level variation time series are also provided in this review paper. The entire bibliography of related works is provided in the references.

Keywords: Robust estimation, reliability, time series, polar motion, least squares collocation

1. Introduction

The general theory and methodology are not mentioned explicitly as the subject of any IAG (International Association of Geodesy) commission. However, this issue seems to be an important pillar for many research investigations in the framework of geodetic science. This review paper provides an outline of Polish researcher activities in the period from 2015 to 2018 that are related to general theory and methodology. It is a follow-up work of a previous report (Borkowski and Kosek, 2015) that covers the time span from 2011 to 2014. Two main topics are reported here: geodetic parameter estimation and geodetic time series analysis. The robust parameter estimation continues to be intensively developed. It seems to be the speciality of the group representing the University of Warmia and Mazury in Olsztyn. The M-estimation and the R-estimation have been deeply studied in the mentioned period. New variants of the Huber's approach have also been introduced. The well-established Baarda's concept of minimal detectable bias (MDB), i.e., the minimal magnitude of the error that can be detected as an outlier, has been revisited as well. Geodetic control networks are the main application area of the mentioned solutions. Some new aspects of the integrated adjustment of geodetic networks have been investigated as well.

The second part, comprising geodetic times series analysis, is related to the Commission 3 activities. Within the Polish researcher community, this topic is dominated by investigations related to Global Navigation Satellite System (GNSS) permanent station coordinates. Significant progress in Global Positioning System (GPS) position time series modelling has been made, especially by the group of researchers at the Military University of Technology in Warsaw. Various stochastic noise models and modelling approaches have been utilised to separate stochastic components of the time series. The most important achievements of the Polish researchers accomplished in the last four years are highlighted in the summary of this publication.

2. Geodetic parameter estimation

2.1. Robust estimation and its variants

Adjustment of the observational system biased by gross errors has been studied by many researchers since this issue was introduced by Baarda (1968). Baarda also introduced the concept of MDB that can be expressed by the following formula:

$$\mathrm{MDB}_i = \sigma_i \sqrt{\frac{\lambda}{r_i}}, \qquad r_i = \left\{\mathbf{H}^{\mathrm{T}} \mathbf{C}_{\mathrm{S}}^{-1} \mathbf{H}\right\}_{ii} \tag{1}$$

with MDB_i in the i-th observation, σ_i the standard deviation of the i-th observation, λ the non-centrality parameter (as in the global model test), r_i a generalised reliability number for the i-th observation, $\mathbf{H}$ the modified reliability matrix and $\mathbf{C}_{\mathrm{S}}$ the correlation matrix of observations (e.g., Prószyński, 2015).

Recently, this concept was revisited by Prószyński (2015). In the paper, he defined the terms detectable gross error, identifiable gross error and unidentifiable gross errors, and developed a method of evaluating the chances to identify a gross error. This method exploits the information contained in the modified reliability matrix of the Gauss–Markov model. Based on this matrix and conditional probability, the ID index was introduced. This index allows for an *a priori* analysis of the probability of identifying a gross error of MDB magnitude in the first adjustment iteration. Concerning pairs of observations, partial ID indices were introduced when one observation is contaminated by gross error and another observation is error free.

The most popular estimation method applied for adjustment of geodetic observational material contaminated by gross errors is the M-estimation based on the iterative least squares application with a robust weight function. This function allows for penalising of outlier contaminated observations. An alternative approach represents the class of the R-estimation based on ranks. One of the basic variants of R-estimation is the Hodges–Lehmann estimation. It is a relatively simple concept, for instance the Hodges-Lehman estimate of the shift, Δ, between two independent samples, $x_1, x_2, \ldots, x_m$ and $y_1, y, \ldots, y_n$, is given by

$$\Delta = med(y_i - x_j), \tag{2}$$

when the Wilcoxon test is applied and the samples are realisations of the random variables $X_j (1 \leq j \leq m)$ and $Y_i (1 \leq i \leq n)$, and *med* is a median.

However, since the R-estimation is based on ranks, the law of variance propagation cannot be applied and the accuracy assessment of estimated values is challenging. Usually, an assumption about the distribution of observation errors is necessary. Duchnowski and Wiśniewski (2017) studied the issue of the accuracy assessment of R-estimates by means of the Monte Carlo method under the assumption of particular error distribution. The accuracy results were computed in relation to the accuracy of the least squares estimates. The authors figured out that the accuracy of the Hodges–Lehmann estimates is similar to the least squares estimates. The differences between the corresponding standard deviations were between 5 and 12%. These results have been achieved for the normal distribution of observations. Taking into account distributions with kurtosis different from zero, the authors found that the accuracy of R-estimates is better than the accuracy of the least squares estimation in the case of leptokurtic distribution. The comparison is reverse in the case of the platykurtic distribution. Moreover, the study revealed that the accuracy of R-estimates becomes better in relation to the least squares accuracy when the number of observations increases.

M_p-estimation is the maximum likelihood solution based on a modified influence function and the explicit family of distribution. Also, the M-estimation is contained in the class of the M_p-estimation as one of its variants. Wiśniewski (2017) adopted this estimation to platykurtic sets of geodetic observations. The mentioned paper provides the solution of the optimisation problem received with the use of the Pearson platykurtic distribution of types I and II.

Properties of the solution have been studied as well. The weight function in the considered variant of the estimation is convex. It might lead to some estimation problems for small sets of observations. Moreover, the author found that the asymmetry of the empirical distribution of observation errors degrades the solution significantly. However, in most cases, the M_p-estimation with platykurtic distributions yields similar results like the M_p-estimation for leptokurtic distribution.

Several tens of robust M-estimation variants have been developed in the last decades. A robust weight function is that which distinguishes different variants from each other. Osada et al. (2017a) have proposed another realisation of the M-estimation that uses modified robust Huber's mean error function. In contrast to the original Huber's approach, the proposed implementation is based on the linearly modified standard deviation that consists of the sum of the standard deviation resulting from *a priori* observation errors and the distance, $v - r\sigma_v$. v and σ_v are the observation correction and its standard deviation, respectively. r is a constant value that defines the range of outliers, usually equal to 1.5. The authors demonstrated the proposed approach performance on the adjustment of the levelling network with possible unstable control points (Osada et al., 2017a). The same method, after the small modification, was used for successful robust adjustment of a precise planar network that was referenced to unstable control points (Osada et al., 2018). The performed numerical tests showed that the ability of outliers detection of the proposed approach is better than several other tested robust weight functions.

2.2. Control network analysis

Control network analysis, i.e., displacement and deformation analysis, has been one of the fundamental subjects of investigation within the field of engineering surveying for the last decades. This field has primarily concentrated on two problems, i.e., detecting and/or reducing impact of outlying observations on adjustment results in both horizontal and vertical networks, as well as detecting (and later eliminating) or damping instabilities of a reference system. Attempts of solution of the mentioned problems are dominated by robust estimation methods, particularly, by the most popular M-estimation, but also R-estimation and, rarely, other approaches. M-estimation is usually performed by means of iteratively reweighted least squares. It relies on using adaptive weights suppressing the impact of observations with large values of standardised residuals in subsequent iterations according to an adopted weighting function.

In their study on fitting a precise levelling network to the national control network, benchmarks, which are subject to vertical movements induced by underground mining (Osada et al., 2017a), construct and use a modified Huber's M-estimator as its classical version produces results that could not be accepted in deformation monitoring. Apart from possible gross measurement errors, which are significantly reduced by the use of modern measuring devices, the vertical movements of control points inevitably produce blunders that require a special adjustment strategy. The authors promote the adjustment strategy that reduces impact of both blunders in observations and blunders induced by unstable control points (the focus is on the latter case). The strategy includes a classical least squares adjustment, free network adjustment and robust adjustment separated by inspecting criteria based on standardised residuals. The newly introduced modification of Huber's estimator is also tested against other M-estimators. The same modification of Huber's M-estimator was used for the purpose of fitting a horizontal network to unstable control points within the area of a power plant (Osada et al., 2018). The authors emphasise that nearly half of the control points may be considered as outliers due to instability of reference coordinates. Such a number of suspected control points calls for the use of robust methods, but even with their use it may turn out to be a challenging task. It is claimed that the Huber's estimator may lead to unreliable results when accepting small values of initial reference coordinate standard errors. The modification of the estimator is free from this drawback and a comparison to other robust estimators proves the usefulness of this new method.

Another approach to control an impact of an unstable reference system on the adjustment results (and deformation analysis), is to check the stability of control points. Cymerman et al. (2016) use R-estimates (in particular Hodges–Lehmann estimates) for this purpose. In their work, the levelling network is an object under study. The presented R-estimate of a point displacement depends on model residuals which are secondary in relation to model parameters. Hence the authors' main objective in the study is to check how the selection of initial values of parameters (heights of points) affects the R-estimates of vertical displacements, and also to compare different strategies to initial point heights computations. A number of numerical tests, including Monte Carlo simulations, are provided. Taking all the results into account, it is concluded that adjustment of the first-epoch observations gives initial values of point heights that guarantee sufficient accuracy of the R-estimates of points' displacements.

For the same purpose, Zienkiewicz (2015) uses M_{split} estimation, originally introduced in works by Wiśniewski. M_{split} estimation is a generalization of M-estimation, hence the resistance to outliers is its natural feature. However, it is emphasised that the most characteristic and distinguishable feature of M_{split} estimation is the allocation of observations to one of two or more ($M_{(q)}$) competing functional models and the use of cross-weighting in the optimisation procedure. As for competing models for the purpose of deformation analysis, one may adapt functional models from two measurement epochs. This two-epoch deformation model may be extended with an additional one (virtual) consisting of observations that do not match either mentioned and, hence, clean the main two-epoch model from extraneous noises. The advocated method is compared with the least squares estimation and classical M-estimation. The results confirm that the constructed adjustment model may be an alternative to traditional displacement determination methods. Besides controlling the instability of a reference system, the method proved its usefulness in the presence of gross measurement errors in the observation set.

Wiśniewski and Zienkiewicz (2016) notice that a substantial part of computational algorithms applied in different aspects of deformation analysis are based on least squares estimation, which is known for its non-robustness to outlying data. Blunders in observations or instabilities of reference points pose a serious problem in the field of deformation analysis, hence, the outlier-resistant methods found its permanent place in deformation analysis and, among them, the mentioned M-estimation and its extensions and enhancements. Such extensions and enhancements are, e.g., M_{split} estimation and, particularly important in the field of deformation analysis, its variant termed Shift-M_{split} estimation. This variant was developed for direct estimating of a shift (a displacement vector) between parameters rather than parameters themselves. In fact, the evolution of M_{split} estimation concept may be investigated in the previous report (Borkowski and Kosek, 2015) apart from the original sources. The observation that squared Shift-M_{split} estimation is not robust to disturbances in observation set, led the authors to its robustification and this resulted in its extended version termed Shift*-M_{split} estimation. In fact, the robustification of Shift*-M_{split} estimation follows the same lines as in the aforementioned study (Zienkiewicz, 2015), i.e., it relies on introducing an additional functional model that absorbs outliers. He also presents a concept of control variable that allows for the determination of proper number of competitive models in $M_{split(q)}$ estimation.

Kwaśniak (2015), on the other hand, uses a different approach of identifying stable/unstable control points. He extends the usability of "all-pairs method" from studying vertical displacements (vertical networks) to horizontal displacements (horizontal networks). The method attempts to identify points that remained fixed from one measurement epoch to another one. The irrelevance of mutual displacements of points is checked in all combinations of point pairs on the basis of a two-step procedure. First, a criterion on the insignificance of displacements based on a distance change is checked generating a subset of possibly mutually fixed points. Second, the found subset is inspected component-wise (i.e., in x and y directions) based on a similar criterion as previously mentioned. The criteria are dependent on standard deviations and an adopted confidence level. The proposed method is simple conceptually and easy to implement computationally.

Nowel (2015) extends the applicability of the previously introduced method (by the same author) called Robust Estimation of Deformation from Observation Difference (REDOD), and limited in use for free control networks only. REDOD uses L_1-norm as a cost function in estimating vectors of displacements. The extension of the REDOD method, called Generalized REDOD (GREDOD), applies to both absolute (points inside and outside a deformable object) and relative (points inside a deformable object only) control networks and any loss function in the class of M-estimators. The author compares two approaches used in deformation analysis, namely, a coordinate-oriented one in which a displacement vector is obtained from differences of adjusted coordinates, and an observation-oriented one in which the displacement vector is determined from differences of unadjusted observations. The series of numerical tests conducted by the author proves that both approaches give the same results as to the displacement vector and its statistical significance test.

Nowel (2015) also recalls special cases when the GREDOD method outperforms the coordinate-oriented approach, i.e., when the magnitude of displacements only slightly exceeds the magnitude of measurement errors and when deformation measurements are contaminated by a systematic error (constant value and sign in both epochs). The same author (Nowel, 2016b), searches for the particular M-estimator (a weighting function) that will assure the highest efficiency of the newly introduced GREDOD method. The efficiency has been verified in a series of tests with different weighting functions and under different conditions as to the displacement vectors. The efficiency of the method itself was measured by the mean success rate (MSR) in thousands of simulation trials (Monte Carlo method). This measure is defined as a ratio of the number of sets where blunders were properly identified to the overall number of generated sets. Among tested methods, i.e., L_1-norm, Huber and Danish, the combination of Danish weighting function with GREDOD method reveals the highest MSR.

Nowel (2016a) took up a relevant subject in the field of deformation analysis concerned with statistical testing whether the estimates of displacements are the displacements themselves or just the effect of random errors. Conventionally, the F-test is used in this respect. This holds for both least squares based deformation analysis and M-estimation based deformation analysis. The author uses global and local F-tests. The global test answers the question whether all points in the network may be considered fixed. If this fails, the local test is performed answering the question which point is suspected of instability. The author finds that the use of F-tests in the M-estimation approach is defective and unreliable and cannot be satisfactorily applied in deformation analysis. He uses a Monte Carlo simulation as a remedy for the weakness of the F-test applied with M-estimators. He also advocates that problems with other conventional statistical tests (based on strong distributional assumptions, e.g., normality) may be solved in simulation way.

2.3. Reliability analysis of observation system

Since the late sixties of the 20^{th} century, a substantial literature has accumulated on the theory of design and optimisation of geodetic networks. The subject still remains a vital problem in geodesy and surveying engineering. Optimally-designed networks are reliable, i.e., possess the ability of proper detection of blunders or remaining resistant against undetected ones and a low cost product (comparing to other possible geometries). In a more detailed sense, network optimization may be explained following an intuitive definition given by Dermanis, i.e., how to best analyse data in their adjustment stage (zero order design, the datum problem), what to observe (first order design, the configuration problem), how to observe (second order design, the weight problem) and how to improve existing information with additional data (third order design, the improvement problem).

In their work, Pachelski and Postek (2016) use a computer simulation to optimize an observation plan, a task within the scope of the first order design of a geodetic network. The method is characterised by a low computational cost. They propose a strategy based on updating a covariance matrix (a carrier of accuracy of the network) after adding a new observation without the need of re-estimation of the adjustment model as a whole. The approach is borrowed from a sequential estimation used within the framework of Kalman filtering. After adding a single observation, its impact on the covariance matrix is verified and a decision is made whether the observation should be included or discarded.

Such a procedure is repeated in different configurations of observations and the one that satisfies, assumed accuracy criteria with a minimal number of observations, is accepted as an optimal observation plan for a given network.

Nowak and Odziemczyk (2018) concentrate on optimization of weighting scheme (problem belonging to the second order design) to make a geodetic network reactive to outlying observations. The ability to react to outliers is often measured by the diagonal elements of the residual marker matrix (reliability matrix containing redundancy numbers on the main diagonal). But since the residual marker matrix differs from the hat matrix only by a constant (identity) matrix the authors use the latter to shorten the formulas without loss of information. The authors examine an impact of changing the weight (standard deviation) of a single observation at a time on diagonal entries of hat matrix. They iteratively try to control a transfer mechanism which has a form one-to-many, i.e., a change of weight in a single observation takes effect in many entries of the hat matrix. The proposed procedure requires inversion of the normal equations matrix after every single change of observation standard deviation (weight). To avoid a computationally expensive matrix inversion, they use Sherman–Morrison–Woodbury formula to update a matrix inverse what makes the solution much less troublesome.

Prószyński and Kwaśniak (2016) study an effect of increase of observation correlation on the behaviour of various measures of geodetic network reliability. They consider the effect of increasing correlation on detectability and identifiability of a single gross error measured by MDB and ID index. They examine sensitivity of w-test for uncorrelated and correlated observations and also response-based measures of internal reliability under these circumstances. In order to study this effect in a controllable way the authors introduce a correlation matrix of a special structure, i.e., every off-diagonal entry is equal to a constant (arbitrariness of these constants is limited by the condition of positive-definiteness of the correlation matrix). They term this simplified structure a uniform correlation. Every considered reliability characteristic is presented as a function of a design matrix (representing the network structure), a vector of observations' uncertainty and, the most important from the standpoint of their work, a correlation matrix (representing dependencies among observations). By successively changing the magnitude of uniform correlation, i.e., a constant, the authors are able to draw some conclusions on the effect of the increasing observation correlation on the mentioned characteristics. They point out that the collected empirical material may become useful in discussion whether or to what degree the formula for a MDB, presented therein, can be a measure of blunder detectability in networks with correlated observations. By their numerical tests, they show that the increase of observation correlation strongly limits the ability of w-test for uncorrelated observations to identify a contaminated observation. On the other hand, they prove the usefulness of the test for correlated observations. Numerical experiments revealed also that the effect of increasing correlation on ID index and on response-based reliability measures is of negligible character.

In Prószyński and Kwaśniak (2018), the above discussion is extended to correlation matrices of any structure. They introduce a global measure of correlation among observations in a network that allows for a specific representation of the correlation matrix.

Since the global measure provides no information on the magnitudes of non-diagonal elements, the authors have also proposed the associated measures, based on off-diagonal elements, of such a matrix, i.e., maximum absolute value and their quadratic mean value. They proved that each positive-definite correlation matrix may be factorized into a scale factor and a so-called internal weight matrix. This factorization of the correlation matrix may be particularly helpful in investigating the impact of observation correlation on network reliability measures. Basic measures of network reliability (both internal and external) are easily rewritable in terms of newly introduced factorization and it is easy to verify which measures are dependent on the global correlation index and which are not. The examples presented therein refer only to MDB, reliability number and a variance of standardised least squares residual, quantities that are dependent on the global correlation coefficient. The authors classify the strength of the global correlation coefficient into particular intervals, i.e., up to 0.3 (weak), from 0.3 to 0.6 (moderate) and from 0.6 to 1.0 (strong).

2.4. Integrated adjustment

The integrated adjustment of a geodetic network comprised of both terrestrial and satellite observations have been studied for decades. Several alternative approaches are possible to build the functional model of the combined adjustment. One of them, quite typical, is the conversion of GNSS vectors and other observations into so-called geodetic pseudo-observations on a reference ellipsoid (azimuth, length). Kadaj (2016a), in his recent investigation, found out that such an adjustment can be biased by systematic errors that exceed the level of stochastic errors of observations. Moreover, he has shown that this problem can be avoided when original GNSS vectors are adjusted in the ellipsoidal space. Also, corresponding modified ellipsoidal observation equations for terrestrial observation are provided in the mentioned publication. The numerical tests have been performed on the territory of Poland using reference stations of the national GNSS permanent network.

The issue of observation reductions from the physical observation space to the mathematical (adjustment) space has been investigated in the work of Kadaj (2016b). The author figured out that the known algorithms and formulas for reduction and mapping onto cartographic space are not suitable for very long GNSS vectors. Therefore he proposed an empirical method for geodetic observation reduction. This method replaces the classical multistage approach and is realised in one step iteratively using Gauss-Newton procedure. Approximate station coordinates of the network are needed for this process. Several numerical examples for different kind of geodetic observations are given in (Kadaj, 2016b).

Least squares collocation is used frequently for filtering of geodetic and geophysical data or for investigating the ratio between signal and noise. Jarmołowski (2015) studied the estimation of *a priori* errors associated with non-correlated noise within one dataset. The author proposed a method that comprises cross-validation based on leaveone-out technique and restricted maximum likelihood estimation of *a priori* noise for different groups of observations. It means that the individual *a priori* variances are estimated data point by data point when only one single data point is omitted. To solve maximum likelihood equations numerically, the fast Fisher scoring technique is used. The method has been tested on the U.S. gravity database. Many observations, with errors several times larger than the average error value, have been found in the dataset. The fast estimation of covariance parameters in least squares collocation by means of the Fisher scoring and the Levenberg–Marquardt optimisation has been further investigated by Jarmołowski (2017). The author performed several numerical tests and figured out that the Fisher scoring technique optimised through Levenberg–Marquardt and applied in the parametrisation of the least squares collocation is much faster than any other technique. Moreover, this approach can be implemented in a fully automatic way. The mentioned paper provides also an extended review of techniques and approaches applied for covariance model parametrisation in least squares collocation in geodesy and geophysics.

The issue of integrated geocentric positioning has been studied by Osada et al. (2017b). The authors proposed an approach integrating total station measurements, GNSS positioning and plumb line direction calculated from the Global Earth Gravity Model EGM 2008. This approach allows for precise positioning in the global geocentric coordinate frame. The Gauss–Markov adjustment model is implemented for observation and data integration and for precise 3D coordinate determination. Numerical tests have been performed for several variants of observation combination and integration in order to determine coordinates of a spatial traverse. These tests demonstrated that the use of plumb line deflection parameters improves the coordinate quality significantly, i.e., up to 26 cm in the case study in question.

The mentioned approach has been further developed and applied to direct georeferencing of terrestrial laser scanners (Osada et al., 2017c). To determine orientation parameters of point clouds in the global reference points, GNSS observations and plumb line vertical deflection, components are utilised and integrated within the Gauss–Helmert adjustment model. The minimum number of GNSS measurements is the special feature of the proposed approach. Only two reference points have to be measured by GNSS in order to allow the point cloud transformation into a well-defined global reference frame.

2.5. Least fourth powers adjustment

Minimising the sum of squares is the most popular objective function utilised for adjustment of geodetic observation. A robust estimation aims at the parameter estimation and, meanwhile, penalisation of observations biased by gross errors. Cellmer (2015) proposed a method that is in opposition to robust estimation and favours outliers. These properties have the solution of the optimisation problem $\mathbf{v}^{\mathrm{T}}\mathbf{v} \rightarrow \min$ with $\mathbf{v}^{\mathrm{T}} = [v_1^2, v_2^2, \ldots, v_n^2]^{\mathrm{T}}$ where v_i are residuals of observations. The solution of this problem faces some numerical problems that are considered in the mentioned contribution. The proposed approach can be applied in particular engineering applications when a geometric figure has to be fitted to a set of points taking into account the worst scenario.

3. Geodetic time series analysis

3.1. Earth orientation parameters

The differences between pole coordinates data and their least squares (LS) extrapolation and autoregressive (AR) predictions increase with prediction length and depend mostly on starting prediction epochs. The time series of differences for 2, 4 and 8 weeks in the future between pole coordinates data and their LS+AR predictions were analysed by the Fourier Transform Band Pass Filter (FTBPF). The FTBPF amplitude spectra revealed some power in the frequency band corresponding to the prograde Chandler and annual oscillations. This means that the increase of pole coordinates data prediction errors is partly caused by the residual Chandler and annual oscillations due to mismodelling them by the LS extrapolation model and chaotic oscillations with periods less than about 200 days (Brzezinski et al., 2016). This means that longer term polar motion prediction cannot be improved and shorter term prediction can possibly be improved by taking into account atmospheric and oceanic excitation.

3.2. Geocenter time series

The geocenter motion model computed from the center of mass coordinates data determined from the Satellite Laser Ranging (SLR), GNSS and DORIS (Doppler Orbitography and Radiopositioning Integrated on Satellite) observations was used to compute the corrections to the sea level anomaly (SLA) data due to center of Earth mass variations. To compute this stochastic geocenter motion model, the centre of mass coordinates data were filtered by the wavelet based semblance filtering, which allows one to designate common signals in two time series. This kind of correction to SLA data of the order of a few millimetres should be applied to altimetric measurements to refer them to the International Terrestrial Reference Frame (ITRF) (Kosek et al., 2015a).

3.3. Permanent station coordinates

For various GNSS applications, it is necessary to know the values of tropospheric delay in real time. To provide such estimates, the paper (Wilgan, 2015) presents a statistical approach to predicting a short-term zenith tropospheric delay (ZTD) from long time series. Several models have been used, such as AR model or autoregressive moving average model (ARMA). Fitting the stochastic correlated part (signal) into these models allows predicting the ZTD values (together with estimation of the deterministic trend). Depending on the purpose of the forecasts, different time series lengths and various prediction horizons have been considered (from 1 to 24 hours). Predictions were included in both global and local mode. In the local mode, each test station has a separate prediction model (degree and order). In global mode, one statistical model was provided for all the stations simultaneously. For the 5-hour local forecasts, the statistical models have the average bias close to 0 with standard deviations of $5 \div 10$ mm with respect to the actual GNSS time series. Accuracy of the global mode is similar to the local one. The average bias for the global mode ranges from $-2 \div 2$ mm with standard deviations at the level of $6 \div 8$ mm. It can be concluded that there is no need to use the local mode, only the global mode, which makes it much easier for the automation of the prediction process for all stations simultaneously (Wilgan, 2015).

Bogusz et al. (2016) presented the existence of long-range dependencies within the stochastic part of GPS position time series. They employed 130 Polish GPS permanent stations and analyzed them using rescaled-range method with Hurst exponent and detrended fluctuation analysis. Both results proved that there is a clear dependence between consecutive values of GPS residuals, indicating a power-law noise presence.

Gruszczynska et al. (2017) examined common seasonal time-varying signal for a set of European stations. They used the Multichannel Singular Spectrum Analysis (MSSA) and proved that common seasonal curves are better-fitted to the original series than the least squares estimates. Moreover, employing the MSSA approach leads to no reduction in the time series power, which constitutes another advantage of this methodology.

Similar to the daily GPS position time series (Klos et al., 2016a), the weekly-sampled data are characterized by power-law noise as shown by (Klos et al., 2015). However, due to their sparser sampling, the amplitudes of weekly observations are smaller than for the daily time series.

The impact that the pre-analysis has on the noise estimates, has been demonstrated by (Klos et al., 2016b) for the outliers. The authors focused on various methods to identify and remove values outlying from others, followed by noise analysis. They concluded that the outliers have to be identified and removed as reliably as possible, to provide the best estimates of noise character.

Klos et al. (2018a) focused on the estimates of noise character basing on the DORIS position time series. The authors divided the time span that the DORIS stations have been operating within into three different periods. For each of them, they estimated the character of noise. It was noticed, that this character changed thorough years from autoregressive process into pure power-law noise, with the quality of data significantly improved.

Klos et al. (2019) introduced into geodetic community, a completely new methodology to estimate the time-varying seasonal signals including the character of the original time series. This methodology is named as the Adaptive Wiener Filter (AWF). For the synthetic series, AWF has been confronted with the commonly employed Kalman Filter, Singular Spectrum Analysis, Wavelet Decomposition and least squares methods, demonstrating that it provides the accurate estimates for time-varying seasonalities, leaving the noise character intact. In this way, no artificial impact on the velocity estimates is noticed.

For the first time, the character of Zenith Wet Delay (ZWD) tropospheric series has been examined by (Klos et al., 2018b). The authors presented the appropriateness of various noise models to describe the ZWD residuals. They noticed that the first-order autoregressive noise process combined along with white noise is preferred over the widely employed white-noise-only approach. They also found that the ZWD trend uncertainty is largely underestimated (by $5 \div 14$ times) using the white-noise-only assumption.

Klos et al. (2018c) provided a General Dilution of Precision (GDP) estimates being the ratio of two uncertainties of velocities. Both uncertainties are determined from two different deterministic models while accounting for stochastic noise at the same time. The authors proved that adding more and more seasonal terms to the series, we increase the bias of the velocity uncertainties. They estimated that 9 and 17 years of continuous daily observations is needed for, respectively, flicker and random-walk noise to make the GDP decrease below 5%.

Analyses of seasonal signals in the GNSS coordinate time series using the iterative Least Squares Estimation approach (iLSE) have been presented by (Kaczmarek and Kontny, 2018a). Additionally, the correlation between coordinates and deformations of the Earth's crust from geophysical models provided by the BKG center was examined. Analysis has shown that the iLSE method is a good tool for detecting periodic components in time series, which was confirmed by the FFT (Fast Fourier Transform) method. In addition, the correlation coefficient between the deformations of the Earth's crust and the changes in coordinates is high for the Up component. However, for horizontal components, the correlation coefficient is low due to the phase shift between the deformation and coordinate signals (the cause is currently unknown to the authors). The final conclusion is that caution should be taken carefully to introduce deformation corrections from geophysical models to coordinates, and the annual and semi-annual periods are not constant for the analyzed time series (Wavelet analysis).

The methods of identifying the noise model in the time series of GNSS station coordinates using two methods: signal reconstruction using coefficients from Continuous Wavelet Transform (CWT), as well as classical modeling of the least squares estimation signal for annual and semi-annual period have been presented by Kaczmarek and Kontny (2018b). The spectral index was used to determine the type of noise. Analyzes have shown that the signal modeling method does not affect the type of noise occurring in the coordinate time series (the difference of the spectral index between the estimation methods is $0 \div 0.2$ for the analyzed GNSS stations) and the character of the noise is colored (flicker noise).

3.4. Sea level variations and hydrometeorology

Using the FTBPF variable broadband seasonal and subseasonal oscillations were computed as a function of geographic location in the SLA data. The FTBPF analysis revealed that the annual frequency has a broadband character which creates oscillations with the frequencies being integer multiplicities of this frequency. The maxima of the annual and semi-annual oscillation amplitudes are located in almost the same geographic regions. The irregular amplitude and phase variations of the broadband annual oscillation computed by a combination of the FTBPF and the Hilbert transform occur in the same geographic regions where prediction errors of the SLA data for two weeks in the future reach the highest values. These predictions of the SLA were computed by a combination of the polynomial-harmonic model with the AR prediction as well as with the threshold AR model (Kosek et al., 2015b).

The research topic comprising sea level change problems with a particular emphasis put on the analysis of satellite altimetric data and sea floor modelling have been investigated by the group led by Tomasz Niedzielski. The Prognocean Plus system has been developed to predict altimetric SLA in real time. Three deterministic-stochastic data-based models are employed within a dedicated system which is implemented and run on the PLGrid infrastructure (grid supercomputing solutions for Polish science). The results have been compared with the established MyOceansystem and the previous version of Prognocean (Świerczyńska et al., 2016). The new method for reconstructing the depth-age curve has been proposed (Niedzielski et al., 2016) and the novel approach to estimate the reference ocean depth has been developed (Jurecka et al., 2016). The two methodological findings are important for modelling long-term sea level variation due to changes of ocean floor. The overview of different prediction methods in marine studies has been published by Niedzielski (2017).

4. Diverse algorithms

The planetary cartography uses predominantly a triaxial ellipsoid as a reference surface in order to map celestial objects with irregular shapes. Appropriate for that mapping functions are usually represented as functions of planetographic or planetocentric coordinates. Projections are complicated and comprise several steps. Pędzich (2017) described triaxial ellipsoid and map projections by means of reduced coordinates. The advantage of this approach is that the calculating of reduced coordinates is performed using the single algorithm that is based on solution of the elliptic integral of the second kind. In the paper in question, mapping functions based on reduced coordinates are provided for cylindrical, azimuthal and pseudocylindrical map projections.

5. Summary and conclusions

This review contribution provides an overview through the activities of the Polish researchers in the field of geodetic general theory and methodology in the period from 2015 to 2018. The investigations reported in this review paper are to some extent continuation of studies initiated in the previous years. The report (Borkowski and Kosek, 2015) preceding this one, covers the period from 2011 to 2014 and provides an outline of research activities for the respective period. Thus, a recapitulation can be found in the previous report.

In the last four years, several essential accomplishments have been achieved by researchers representing the Polish geodetic community. To sum up, the following issues can be emphasised as highlights:

- Revisiting the concept of minimal detectable bias and introducing the ID index that allow for an *a priori* analysis the probability of identifying a gross error (Prószyński, 2015).
- Improvement of the Huber's approach by introducing the linear weight function. It allows for fitting of geodetic network to unstable reference points (Osada et al., 2017a, 2018).
- Development of the covariance function parametrisation approach that is based on the Fisher scoring technique and the Levenberg–Marquardt optimisation (Jarmołowski, 2015, 2017).
- Development of various strategies of network adjustment in the presence of gross errors in observation sets and instabilities of reference system (Osada et al., 2017a, 2018; Zienkiewicz, 2015; Nowel, 2015, 2016a, 2016b).
- Introduction of robustified version of Shift-M_{split} estimation termed as Shift-M^*_{split} estimation, which enables to estimate parameter differences (robustly) without the need of prior estimation of parameters themselves (Wiśniewski and Zienkiewicz, 2016).
- Development of analytic tools for investigating the increase of observation correlation on network reliability measures. Introduction of new measures of observation correlation (Prószyński and Kwaśniak, 2016, 2018).
- Finding that stochastic models characterizing geodetic observations may vary depending on the type of observations one collect (Kadaj, 2016a, 2016b).
- Concluding that the stochastic model of geodetic observations evolves together with the quality of observations; the more precise observations are, the closer the noise is to white noise (Klos et al., 2016a, 2016b, 2018a, 2018b, 2019).
- Longer term polar motion prediction accuracy cannot be better and more emphasis should be put on shorter term prediction by taking into account atmospheric and oceanic excitation (Brzezinski et al., 2016).
- Showing that some data-based prediction methods can serve well the purpose of forecasting sea level anomalies, with higher accuracies than the physically-based method implemented in the European MyOcean system and, concurrently, lower correlations than the MyOcean-based predictions (Świerczyńska et al., 2016; Niedzielski, 2017).
- The approach to modelling the time series of GNSS coordinates does not affect the type of noise occurring in these data (Kaczmarek and Kontny, 2018a, 2018b).
- Forecasting the values of zenith total delays of GNSS signal is recommended by statistical autoregressive (AR) and autoregressive moving average (ARMA) models (Wilgan, 2015).

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