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Automatic simplification of the geometry of a cartographic line using contractive self-mapping – illustrated with an example of a polyline band

Abstract. The present article is another attempt to adapt map geometry to automatic digital cartography. The paper presents a method of digital polyline generalisation that uses contractive self-mapping. It is a method of simplification, not just an algorithm for simplification. This method in its 1996 version obtained a patent entitled "Method of Eliminating Points in the Process of Numerical Cartographic Generalisation" – Patent Office of the Republic of Poland, No. 181014, 1996. The first results of research conducted using the presented method, with clearly defined data (without singular points of their geometry), were published in the works of the authors in 2021 and 2022.

This article presents a transition from the DLM (Digital Landscape Model) to the DCM (Digital Cartographic Model). It demonstrates an algorithm with independent solutions for the band axis and both its edges. The presented example was performed for the so-called polyline band, which can represent real topographic linear objects such as rivers and boundaries of closed areas (buildings, lakes, etc.). An unambiguous representation of both edges of the band is its axis, represented in DLM, which can be simplified to any scale. A direct consequence of this simplification is the shape of the band representing the actual shape of both edges of the object that is classified in the database as a linear object in DCM.

The article presents an example performed for the so-called polyline band, which represents real topographic linear objects (roads, rivers) and area boundaries. The proposed method fulfils the following conditions: the Lipschitz condition, the Cauchy condition, the Banach theorem, and the Salichtchev's standard for object recognition on the map. The presented method is objective in contrast to the previously used approximate methods, such as generalisations that use graph theory and fractal geometry, line smoothing and simplification algorithms, statistical methods with classification of object attributes, artificial intelligence, etc. The presented method for changing the geometry of objects by any scale of the map is 100% automatic, repeatable, and objective; that is, it does not require a cartographer's intervention.

Keywords: digital generalisation, contractive self-mapping, Salichtchev's minimum measures, geometry of a polyline in the binary tree structure, Lipschitz's contraction triangle, Banach theorem

1. Introduction

Generalisation of map content is subject to specific rules and is one of the biggest challenges of cartography. This problem has puzzled researchers since E. von Sydow defined the three reefs of cartography in *Drei Karten-Klippen* in 1866. The problem of generalisation has not been solved yet. Automating the objective generalisation of object geometry and generalising topographic map content is a problem that is and will remain without a satisfactory solution (Maudet et al., 2017; Zhou et al., 2018).

In the 1960s, the first attempts were made to automate the process of generalisation in order to ensure its objectivity. Many strategies have been proposed for implementing the generalisation process (Brassel & Weibel, 1988; McMaster & Shea, 1992). The first algorithms for automatic cartographic generalisation (Salichtchev, 1967; Yan, 2019) that used computer technology were developed, among others, by Perkal (1966), Tobler (1966), Lang (1969). Initially, research focused on generalising lines (Pannekoek, 1962; Perkal, 1958; Srnka, 1970; Tobler, 1964;). Research on simplifying line geometry in computer cartographic generalisation has been conducted for many years (Kozioł, 2013). Simplification algorithms have been developed, among others, by McMaster (1987), Bjørke (1996, 2003), Chrobak (1999, 2003, 2007), Chrobak et al. (2017), Opheim (1982), Robinson et al. (1988), Visvalingam and Whyatt (1991), Weibel (1996). A comprehensive overview of the development of algorithms, starting with Perkal's work (1966), was provided by Li (2007). Every year, numerous new line generalisation algorithms are developed, and many existing ones are modified. One example is the Douglas-Peucker algorithm from 1973 (Douglas & Peucker, 1973; Li, 2007), which was the basis for various modifications of the line generalisation algorithm (Berg et al., 1995, 1998; Li & Openshaw, 1992; McMaster, 1986; McMaster & Shea, 1992; Muller & Mouwes, 1990; Saalfeld, 1999; Visvalingham & Whyatt, 1990; Wang & Muller 1998; White, 1985; Zheng & Tian, 1997). The purpose of each of these algorithms was to reduce the number of points of the line and to simplify its original shape.

Generalisation algorithms have also been supplemented with graphs theories and fractal geometry (Arora et al., 2018; Dupuis et al., 2023; Wu, 1997).

Another group of line algorithms is for smoothing a line. Various techniques have been employed such as Gaussian convolution, Fourier transform (Boutoura, 1989; Plazanet et al., 1995), wavelet transform (Balboa & Lopez, 2000), snakes (Borkowski & Keller, 2003; Burghardt & Meier, 1997, 2005; Kass et al., 1987; Trinder, 1995).

New solutions to the problem of simplifying objects on maps are still appearing in the literature. In 2020, Kronenfeld et al. developed an algorithm to simplify polylines, Area Preserving Segment Collapse (APSC), that preserves the area of the object. Procedurally, APSC is similar to Visvalingam and Whyatt's (1993) so-called effective area (VEA) algorithm, but instead of deleting a single vertex at each step, two vertices are removed and one new vertex is added. Yan et al. (2022, 2023) developed a formula to express the change in spatial similarity that follows the change in the scale of the map. The proposed quantitative method provides the basis for using spatial similarity as a limitation during the generalisation of the road network, through the control of generalisation procedures and the assessment of the quality of generalisation.

Statistical methods used in generalisation are based on the selection of attributes concerning semantic, geometric, and topological properties (Ajdacka & Karsznia, 2022; Cebrykow, 2017; Jiang & Claramount, 2004; Karsznia et al., 2022; Li & Choi, 2002; Sester et al., 1998; Touya, 2007; Weiss & Weibel, 2014; Zheng et al., 2021; Zhang, 2004). They are necessarily burdened with a significant component of subjectivity.

In the last 20 years, artificial intelligence has also been used in the generalisation of objects on maps (Balboa & Lopez, 2008; Courtial et al., 2020; Du et al., 2022; Jepsen et al., 2022; Karsznia et al., 2022; Lagrange et al., 2000; Touya et al., 2019; Werschlein & Weibel, 1994; Xiao et al., 2023; Yan, 2023; Zhou & Li, 2014). Machine learning supports the application of generalisation algorithms to large sets, allows for the use of new test areas and experimenting with various machine learning models.

As Sester et al. (2018) have observed, however, this research, although interesting, only confirms that the use of machine learning in cartography is possible. Research so far has focused on topographic databases and large-scale maps and has not yet been used more widely.

The increasing computing capabilities of computers and the rapid development of GIS technologies have allowed cartographers to make various attempts at systematising and clarifying the principles of generalisation which allow for greater objectivity and repeatability in mapping. GIS software providers such as ESRI, Intergraph, Spatial, Laser-Scan, Axes Systems AG, and others add to their products procedures and functions that enable generalising objects on the map. Examples of such generalization operators include algorithms for the simplification, smoothing, aggregation, amalgamation, merging, collapse, refinement, exaggeration, enhancement and displacement of cartographic features (McMaster & Shea, 1992).

In computer cartography, many rules for simplifying lines have been developed, but these solutions are most often to some extent subjective. Object simplification methods used in GIS, such as those by Douglas, Peucker, Jenks, Lang or Reumann-Witkam and others, as well as those based on fractal theory or artificial intelligence, return the result after a few or a dozen or so iterations.

Generalisation services can be used in various application scenarios, for example as an intermediary software component that extends an Internet map with adaptive zoom, or as a stand-alone service that supports topographic production of maps developed by national cartographic agencies (Burghardt, 2005; Burghardt et al., 2005; Burghardt & Shmid, 2009; Neun et al., 2009).

Manual, semi-automatic, and automatic generalisation were compared in detail by Li and Su (1995). Automatic generalisation has become an effective way to create cartographic documents at different scales based on a single spatial database containing information of high spatial timeliness and accuracy. National Mapping Agencies (NMA) are involved in the process of map generalisation, also for the purpose of seeking financial gains. Scientists agree on the need for complete automation of the generalisation process (among others, Burghardt et al., 2008; Chaundhry & Mackaness, 2008; Kilpeläinen & Sarjakoski, 1995; Liu & Li, 2019; Regnauld, 2015; Stoter et al., 2009a, 2009b, 2016; Weibel, 1995; Weiss & Weibel, 2014).

In cartography, generalisation is the most critical transformation, resulting in the modification of the shape of objects and – sometimes – partial or complete elimination of spatial information. Generalisation modelling aims to control the process of generalisation and has been a field of extensive research since 1990 (Blana et al., 2023; Jiang et al., 2013; Mackaness et al., 2007; Weibel & Dutton, 1998). Assessment of generalisation quality has been identified as an inherent part of generalisation models since the first attempts in this regard (João, 1998; McMaster & Shea, 1992; Weibel, 1995).

The applied method of changing the geometry of objects to any map scale is 100% automatic. It is the first simplification method, not just an algorithm for simplification. This method depends on the fulfillment of the following conditions:

– Lipschitz's condition $p > h$ for contraction triangles created on a polyline in a binary tree system,

− K. A. Salichtchev's ($p>h$, where: $p = 0.7M$ and $h = 0.3M$ in [mm]) recognisability of the measures of triangles defined in the metric space.

The recognition norm introduced into the process of generalisation gives positive results as it causes the smallest possible changes in the geometry of the polyline (Chrobak, 1999, 2003, 2007; Chrobak et al., 2017).

The article presents a scheme of stages of automatic data generalisation illustrated with an example of a polyline that is ordered in a binary tree structure and belongs to the metric space. For polyline simplification tests, random scales were selected, in which simplified object geometry was obtained automatically (Banasik et al., 2022; Barańska et al., 2021). Mathematical theorems are applied in the process of simplifying the geometry of an object and after its verification process. The accuracy of generalisation of the geometry of objects on the map after simplification is verified by the recognition metric of Salichtchev (2003).

The research was concluded with a sample generalisation of real data (topographic database BDOT10k) for the Jałowica River. It was proved that it is possible to automatically simplify the shape of a polyline by significantly reducing the number of its points. Simplification is achieved by reducing polyline points that are not recognisable to the human eye. The applied generalisation method is characterised by the use of original, real points of a polyline (from a topographic database) and not creating new, non-existent points in the process of generalisation of a polyline. The only value determining the degree of simplification is the minimal recognisability of the shape of the polyline, which results from the resolution capabilities of the human eye. This makes it possible to generalise a polyline at the target scale of the map, including non-standard scales. This method can be used to change the geometry of polyline-shaped objects.

2. Definitions of mathematical spaces and theorems used in the publication for automatic generalisation

In the subsequent sections, the following theorems in the field of mathematical spaces are used, as well as concepts that can be employed in automatic generalisation:

1) A necessary and sufficient condition for a number a to be the limit of a sequence (a_n) is that for each number $\varepsilon > 0$ there exists such a number n_0 that for natural numbers $m,k > n_0$ there exists the inequality $|a_m - a_k| < \varepsilon$ (Cauchy condition) (Bronsztejn et al., 2011),

2) Metric space – a set with a given metric, i.e. a function that determines the distance between each pair of elements of this set. Metric spaces form the most general class of sets which use the concept of distance modelled on the distance known from Euclidean spaces (line, plane or three-dimensional space) (Bronsztejn et al., 2011),

3) Banach's fixed-point theorem (the contraction theorem) (Dziubiński, 1982, p. 535),

4) Lipschitz contraction or contraction mapping – the transformation of f from metric space (X,g_x) into metric space (Y) whose real constant for $\alpha \in (0,1)$ is such that for any $(x_1,x_2) \in X$ there exists the inequality: $g_y(f(x_1), f(x_2)) \leq \alpha g_x(x_1,x_2)$.

In the works by Barańska et al. (2021) and Banasik et al. (2022), of which the present paper is a continuation, three types of triangles were used:

1) TB – base triangles, constructed from points of the object in different coordinate systems, transformed to a single geodetic coordinate system. They are the basis for creating the first TK contraction triangles on the left and right leg of TB. The distinguishing feature of these triangles are their bases, which are at the same time the longest side of TK triangles because they connect the beginning and the end in each section.

2) TK – contraction triangles, built in polyline envelopes, as representations of contractive self-mapping, whereby:

− The first TK triangle of each polyline envelope in contractive self-mapping has the longest base and height; they meet the condition that the base is greater than the height,

− Subsequent TK triangles with a common side are created according to the binary tree scheme. Their sides are shorter than the sides of the preceding triangles.

3) TG – a limiting triangle, or a special contraction triangle whose height is zero. The limiting triangle ends the process of contractive self-mapping.

3. Adjusting polyline geometry to automatic generalisation at any scale *s < 1*

In the automatic generalisation of linear objects, the sequences of geodetic points that form the polyline must be converted from Euclidean to metric space. This ensures that in the process of simplifying the polyline to the $s < 1$ scale, its continuity will be maintained. Maintaining continuity also determines the unambiguous result of the simplification process. Distances between object points are measured. They are used to verify the recognisability of an object, after applying contractive self-mapping and comparing lengths with the metric of Salichtchev (2003). Contraction triangles created in a binary tree system on a polyline have the bases and heights measured to verify the condition of the Lipschitz contraction. For each contraction triangle built on a polyline, it is confirmed whether the base of the triangle is greater than its height. In contractive self-mapping, the triangles formed on the original polyline are verified by the recognition norm defined by Salichtchev (2003). The result of simplification with self-contractive mapping is verified by the source data of the polyline with the imposed condition of Lipschitz contraction. In turn, a polyline generalised to the $s < 1$ scale is verified by the recognition norm of Salichtchev. This is done by examining the polyline result after applying contractive self-mapping to polyline points for the $s = 1$ scale.

4. Transformation of polyline data from Euclidean to metric space

The process of automatic digital simplification of the cartographic geometry of an ordered polyline – P_O in $0 < s < 1$ scales requires that, in contractive self-mapping performed according to the principle of "from the general to the specific", the coordinates of the points be identical to the source data. In the metric space of the polyline, the continuity of point sequences in contractive self-mapping guarantees a single result. This was shown in article by Banasik et al. (2022). In comparison to approximate methods, the result of a simplified object obtained by means of contractive self-mapping gives a single result consistent with the source data. The justification for the comparison of results is set out in article by Barańska et al. (2021). Positive results of the study on the exact method used for polylines made it possible to apply it to simplify a polyline band $(LRA)_O$. Such a band consists of the left edge – L_O and the right edge – R_O (both are polylines). The vertices of the L_O and R_O edges are used to generate the A_O band axis. Therefore, the base triangles $TB_O{}^L$, $TB_O{}^R$ can be built on either of the edges or on the axis as a P_O polyline. This operation does not affect the condition of Lipschitz contraction. A set of TK triangles of P_O polyline of contractive self-mapping, when s = 1, is a generalisation standard for the $s < 1$ scale. This way, the sides of generalised polylines are verified by measures of distances between the vertices of the source polylines.

4.1. The concept of polyline band and its generalisation

The term polyline band is understood as three polylines of the same shape (Figure 1). Two of them are its natural edges – left (L) and right (R). The third polyline is a specially created band axis (A). Each polyline has "n" matching vertices: (L_1,R_1,A_1), (L_2,R_2,A_2),… (L_n,R_n,A_n). The axis (A) of the web is secondary in relation to the edges (L) and (R) because the coordinates of the axis vertices are derived from the coordinates of the edge vertices. The band (L_i,R_i,A_i) defined in this way can be a flat model of a real topographic object, e.g. a road, river, etc. The band includes a special type of vertices: the so-called geodetic points, which include the vertices of the ends of the band and other vertices whose location is independent of the results of simplification because it is not changed in the process. The width of the band is determined by $2d_1$, $2d_2$,…$2d_n$ values, whereby the successive d_i values can differ (Figure 1).

Such a band is recorded on the plane with X,Y coordinates (m) of the vertices of both edges and their numbers, for example:

Vertex 1:	L_1	$X_L = 0.0$	$Y_L = -4.0$
	R_1	$X_R = 0.0$	$Y_R = 4.0$
Vertex 2:	L_2	$X_L = 19.0$	$Y_L = 1.0$
	R_2	$X_R = 15.0$	$Y_R = 7.9$

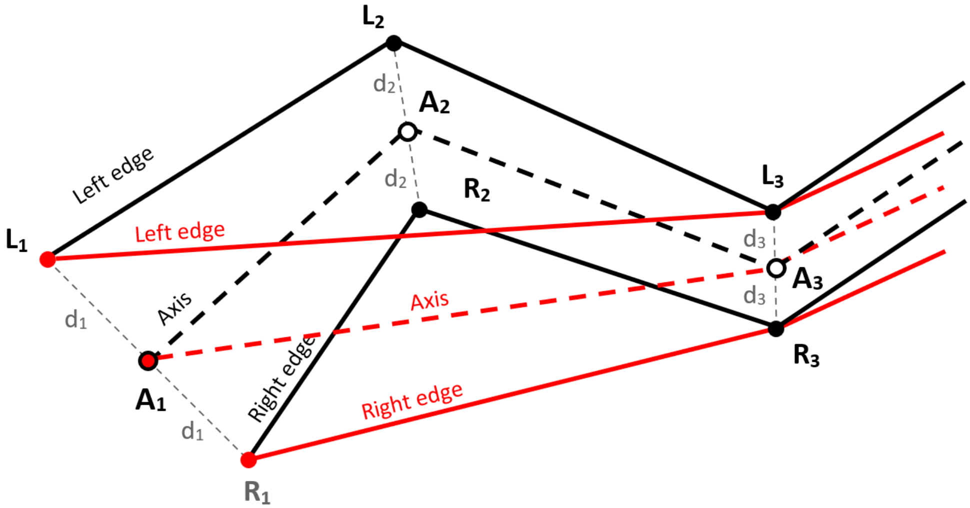


Figure 1. Sample band $(L,R,A)_i$ with vertices i = 1, 2…n (black: primary band; red: simplified band)

Vertex 3:	L_3	$X_L = 28.1$	$Y_L = 17.2$
	R_3	$X_R = 21{,}0$	$Y_R = 19.8$
Vertex n-1:	L_{n-1}	$X_L = 15.4$	$Y_L = 131.8$
	R_{n-1}	$X_R = 12.5$	$Y_R = 127.8$
Vertex n:	L_n	$X_L = 0.0$	$Y_L = 137.0$
	R_n	$X_R = 0.0$	$Y_R = 133.8$

The coordinates X_A,Y_A of the i-th vertex of the band axis (A) are calculated from the coordinates X_L,Y_L,X_R,Y_R of the corresponding vertices of both edges, according to the formula (1):

$$X_{A,i} = \frac{X_{L,i} + X_{R,i}}{2}; \qquad Y_{A,i} = \frac{Y_{L,i} + Y_{R,i}}{2} \quad (1)$$

The band is generalised according to the following rules:

1) The band defined by the edges (L) and (R) becomes reduced to the axis (A). The axis of the band represents the band in the process of its simplification.

2) The band axis should be generalised to the new *s < 1* scale in accordance with the principles of generalising a polyline set out in (Banasik et al., 2022). These include: identification of singular vertices of the polyline, construction of its binary tree, simplification of the polyline according to the given scale of reduction.

3) The identified singular vertex of the axis does not meet the Lipschitz condition necessary for contractive self-mapping (Barańska et al., 2021). Such a vertex, together with the neighbouring vertices, forms a so-called singular triangle. The vertices of the singular triangle are excluded from the subsequent stages of generalisation and are given a status similar to the status of geodetic vertices.

4) The construction of the binary tree of a polyline is performed only on those polyline sub-segments that are located between geodetic or singular vertices. The binary tree structure contains the metric information necessary to simplify the polyline. The branches of the binary tree are triangles formed from the vertices of the polyline, starting from the base triangle and ending with the side of the polyline as a zero-height triangle, maintaining the Lipschitz condition (Barańska et al., 2021).

5) The simplification of the fragments of the polyline is carried out according to the recognisability conditions specified by Salichtchev (2003) for a given scale of reduction (M) The shape of the polyline is changed by removing some polyline vertices. The polyline vertices that did not meet the recognisability conditions, i.e. the condition of the minimum height of the vertex above the base of the triangle from the binary tree (h_{min} = 0.3M), or the condition of the minimum triangle base length (p_{min} = 0.7M), are removed.

6) The simplified polyline becomes the axis of the simplified band. To obtain a generalised band, the vertices of both edges of the band need to be reconstructed. The reconstruction only applies to the vertices remaining in the axis after its simplification.

As a result of simplifying the band axis from the original polyline (A), a polyline with a reduced number of vertices (A_i) remains (Figure 1).

Leaving or removing the k-th vertex (A_k) of the axis after its simplification means at the same time leaving or removing the corresponding vertices of both edges: (L_k) and (R_k). In Figure 1, the axis vertex A_2 was removed, and as a result, the generalised ribbon retains the vertices L_1, R_1 (as fixed geodetic vertices) and the vertices L_3, R_3. The vertices L_2, R_2 were also removed due to the deletion of the corresponding A_2. After generalisation, the original band was simplified from (L_1,R_1,A_1), (L_2,R_2,A_2), (L_3,R_3,A_3),… (L_n,R_n,A_n) to (L_1,R_1,A_1), (L_3,R_3,A_3),… (L_n,R_n,A_n).

As a result of the generalisation of the band to the *s < 1* scale, the width of the band will be smaller. If the width is not recognisable and the band must be visible, it should be marked using a conventional symbol.

Figure 2 (a–c) illustrates an example of simplification of a 20-vertex ribbon (the numbers of left and right edge vertices are marked on the band axis). Vertices 1 and 20 are the vertices of the endpoints of the band, and vertex 10 is a geodetic (fixed) vertex, the furthest from the base of the TB triangle. All these vertices (marked in red) are not simplified due to their status and cannot be omitted from the band geometry. The band was generalised by simplifying its axis. As a result of the phase of identification of singular vertices, two such vertices were found: 4 and 18. Together with the neighbouring vertices 3 and 5, 17 and 19, they formed so-called singular triangles (Figure 2b). Singular vertices do not meet the conditions of contractive self-mapping (Barańska et al., 2021) and are therefore excluded from the successive stages of simplification, as are the geodetic vertices.

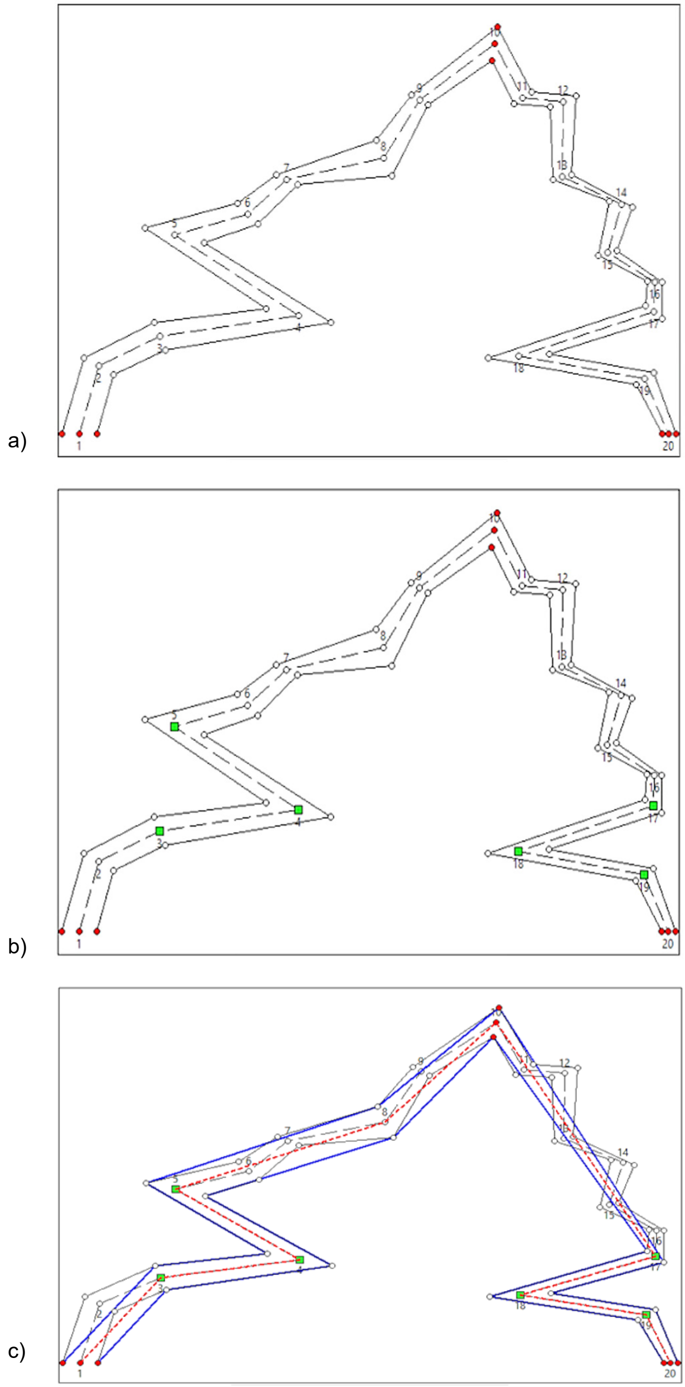


Figure 2. Examples of successive stages of band generalisation $(L_{1\text{-}20}, R_{1\text{-}20}, A_{1\text{-}20})_i$ with vertices i = 1,2…20 from 1:1 scale to 1:50 scale: a) the ribbon before generalisation (in black – the axis and edges of the band, in red – geodetic vertices); b) the band with localised singular vertices (in green); c) the band after generalisation (in red – band axis, blue – band edges)

The subsequent stages of the axis generalisation process, i.e. the construction of a binary tree and the simplification of TK triangles according to Salichtchev criterion of recognisability (for the adopted scale of reduction s = 1:50) are carried out on fragments between geodetic and singular vertices. These fragments are 1–3, 5–10, and 10–16. The values adopted as the criterion of recognisability are: for the triangle base p_{max} = 0.7M, and for the triangle height h_{max} = 0.3M, where M is the reduction value (in this case M = 50). The result of simplifying the shape of the above-mentioned axis fragments is the rejection of vertices 2, 6, 7, 9 and all vertices from 11 to 15 (Figure 2c). From the twenty vertices of the original band axis, after it was reduced 50-fold, 10 vertices remained after simplification. Both edges of the band after generalisation were recreated from the axis vertices that remained after simplification. The shape of the band after simplification is shown in Figure 2c. The assessment of the accuracy of the achieved simplification was carried out using the method proposed in Banasik et al. (2022), by analysing the heights of the rejected vertices in triangles. The standard deviation values for simplified fragments on the left and right edges and on the axis were respectively: $\delta_{1\text{-}3}$ = (4.30; 3.41; 3.85), $\delta_{5\text{-}8}$ = (1.31; 2.86; 1.77), $\delta_{8\text{-}10}$ = (1.24; 2.73; 2.08), $\delta_{10\text{-}17}$ = (4.06; 3.51; 3.75). These values are less than the limit values for the recognisability of length according to the Salichtchev norm, which are equal to p_{max} = 35 and h_{max} = 15, respectively, in the case of reduction to a scale of 1:50.

5. Description of the generalisation of the Jałowica River

The Jałowica River is a tributary of Konradka, which is a left tributary of the Biała Lądecka River. These tributaries belong to the drainage basin of the Nysa Kłodzka River in Poland. The Jałowica River has a south-north orientation, from the source (ϕ = 50°56′06.32″, λ = 15°45′04.96″) to the mouth (ϕ = 50°55′04.69″, λ = 15°49′13.62″) (Figure 3ab).

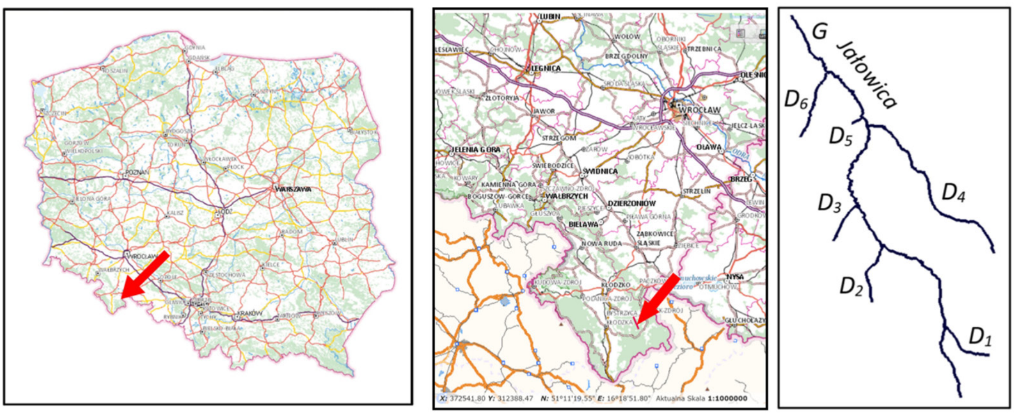
In the BDOT10k (Rozporządzenie Ministra Rozwoju, Pracy i Technologii [MRPiT], 2021) topographic database, the location of this river was defined by a set of points that mark river bends. The x,y coordinates of these points are given in the PL-1992 system (EPSG 2180, 2022). These points form polylines that are the axes of polyline bands (left and right banks of the river and its tributaries). For generalisation purposes, the axes of these bands are marked as follows (Figure 3c):

− G polyline – the main current of the river, consisting of 380 points,

− D_1, D_2, D_3, D_5, D_6 polylines – left tributaries of the river, consisting of 5–52 points,

− D_4 polyline – the right tributary of the river – a polyline consisting of 134 points.

The total number of points in all polylines is 650. A detailed description of all the above-mentioned polylines can be found in Table 1.



a) b) c)

Figure 3. Location of the Jałowica River: a) on the map of Poland, b) in the southern part of the Lower Silesia voivodeship), c) division of the river into subsequent polylines (base map: geoportal.gov.pl, 2023)

Table 1. Characteristics of polylines before generalisation based on BDOT10k data

Polyline	Number of points	Total length in [m]	Minimum side length in [m]	Maximum side length in [m]	Average side length in [m]
G	380	6194.2	3.1	72.1	16.4
D_1	25	731.8	2.7	71.2	30.5
D_2	30	782.4	2.6	67.4	27.0
D_3	22	552.1	2.0	64.7	26.4
D_4	134	2524.8	2.4	77.1	19.0
D_5	7	153.9	15.9	45.4	25.7
D_6	52	887.1	3.2	35.7	14.4

Each of the polylines was generalised in selected scales from 1:10,000 to 1:100,000. As a result of the generalisation and reduction of the number of polyline points, its shape was simplified. According to the principles of this generalisation method, all points were subject to reduction apart from the ends of the polyline and other so-called geodetic points (Banasik et al., 2022). Polyline G has six such geodetic points; they are simultaneously the ends of the tributaries of the river (D_1-D_6). Detailed results of generalisation are given in Table 2. A visualisation of the generalised polylines is shown in Figure 4.

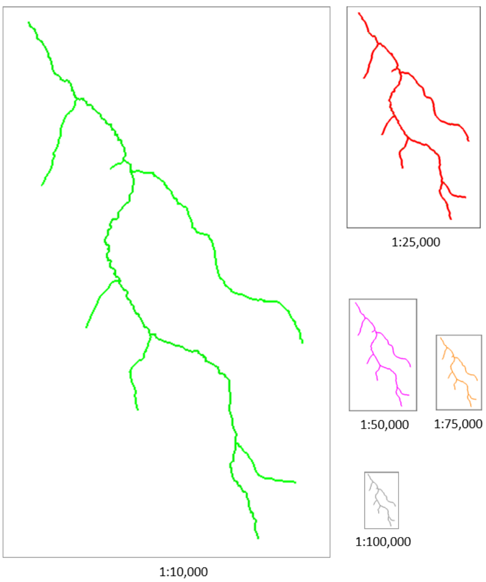


Figure 4. The shape of the Jałowica River with its tributaries obtained as a result of generalisation in selected scales

Table 2 shows that the higher the number in the scale denominator, the greater the reduction of polyline points. For the smallest scale, 1:100,000, it reaches values from 70% to 90%. At this scale, the longest polyline (G), originally consisting of 380 points, was simplified to a polyline of 38 points (reduction by 90%). The greatest reduction in the number of points occurred in generalisations at the 1:25,000 scale and at the 1:50,000 scale. This is the result of the use of Salichtchev's recognition norms (0.7M and 0.4M, where M – the denominator of the scale), which are the only values determining the simplification of the shape of the polylines (Banasik et al., 2022). In the first case, there is a 2.5-fold reduction; in the second – 2-fold.

The shortest polyline D_5, originally consisting of 7 points, was reduced in a scale of 1:75,000 (and smaller scales) to a segment with an initial and final point. This section is 152 m long. On a map with a scale of 1:75,000, it will be 2 mm long, which allows for it to be recognised. This section will also be recognisable at the scale of 1:100,000.

When assessing the shape of generalised polylines (Figure 4), it should be noted that despite a significant reduction in the number of points (Table 2), the shape of polylines is not deformed. Polylines retain their characteristic bends at any scale, both at the scale of 1:10,000 and 1:100,000. It is also worth pointing out that polylines at any scale are created only from

Table 2. Polyline generalisation results in selected scales

Polyline	1:10,000	1:25,000	1:50,000	1:75,000	1:100,000
	Number of points reduced (in %)				
G	43 (11)	199 (52)	295 (78)	331 (87)	342 (90)
D_1	5 (20)	12 (48)	13 (52)	17 (68)	20 (80)
D_2	2 (7)	9 (30)	18 (60)	22 (73)	23 (77)
D_3	3 (14)	8 (36)	16 (73)	18 (82)	19 (86)
D_4	14 (10)	62 (46)	100 (75)	108 (81)	114 (85)
D_5	0 (0)	1 (14)	4 (57)	5 (71)	5 (71)
D_6	8 (15)	21 (40)	37 (71)	42 (81)	45 (87)
A total	75 (12)	312 (48)	483 (74)	543 (84)	568 (87)

the points of the original polyline, i.e. from the points of the BDOT10k topographic database. In this method, no other new points are added to the polyline. It is worth noting that the use of the above-mentioned coefficients of the Salichtchev's norm, dependent only on the scale, makes it possible to generalise a polyline in any non-standard scale.

6. Conclusions

The article is in line with the topic of spatial data processing contained in the INSPIRE Directive (Directive 2007/2/EC of the European Parliament and of the Council of 14 March 2007 establishing an Infrastructure for Spatial Information in the European Community, 2007). The article presents a solution to the problem of generalisation, as one of the three "reefs of cartography" defined by von Sydow (1866). This article is a continuation of the solutions contained in publications by Barańska et al. (2021) and Banasik et al. (2022) and used for a complex structure, the so-called polyline band. The conclusions of the tests carried out for such a band are as follows:

1) Linear topographic objects (e.g. roads, rivers, etc.) and closed areas are presented in the form of a band with two edges and their axis.

2) The band axis is subject to generalisation as an unambiguous representation of both its edges. The shape of the simplified axis determines the shape of both edges of the band after its simplification.

3) Contractive self-mapping of any ordered polyline P_O (e.g., band edges and its axis) is an objective transformation, as it depends solely on the scale of generalisation.

4) The generalisation of an ordered polyline P_O in any $s < 1$ scale using contractive self-mapping and a binary tree system in a 1:1 scale has one objective solution.

5) The verification of the simplification of the polyline is carried out by comparing the base and height of the triangles with the measures of Salichtchev's recognition norms.

6) The process of polyline generalisation should be dissociated with the process of transformation between coordinate systems, as the transformation can change the angular-linear measures of the polyline shape.

7) The example of generalisation of the Jałowica River, performed using real data (topographic database BDOT10k), indicates that it is possible to automatically simplify the shape of a polyline by significantly reducing the number of its points. Simplification is achieved by reducing polyline points that are not significant for the recognition of the polyline shape. The applied generalisation method is characterised by the following:

− algorithm automation, requiring no intervention from the cartographer,

− the generalisation process only uses the original, actual points of the polyline, included in databases, and no new points are added to the polyline,

− using the target scale of the map as the only value determining the degree of simplification of the polyline,

− ensuring minimum recognisability of the polyline shape that results from the resolution capabilities of the human eye,

− polylines can be generalised in any scale, including non-standard scales.

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References

Ajdacka, I., & Karsznia, I. (2022). Verifying and improving map specifications of river network selection for automatic generalization of small-scale maps. *Polish Cartographical Review*, *54*(1), 75–91. https://doi.org/10.2478/pcr-2022-0006

Arora, S., Ge, R., Neyshabur, B., & Zhang, Y. (2018). Stronger generalization bounds for deep nets via a compression approach. *Proceedings of the 35th International Conference on Machine Learning, Stockholm, Sweden, 10–15 July*.

Balboa, J. L. G., & Lopez, F. J. A. (2000). Frequency filtering of linear elements by means of wavelets. A method and an example. *The Cartographic Journal, 37*(1), 39–50. https://doi.org/10.1179/caj.2000.37.1.39

Balboa, J. L. G., & López, F. J. A. (2008). Generalization-oriented road line classification by means of an artificial neural network. *Geoinformatica*, *12*(3), 289–312. https://doi.org/10.1007/s10707-007-0026-z

Banasik, P., Chrobak, T., & Biegun, B. (2022). Algorithm of automatic digital cartographic generalisation with the use of contractive self-mapping. *Polish Cartographical Review*, *54*(1), 1–10. https://doi.org/10.2478/pcr-2022-0001

Barańska, A., Bac-Bronowicz, J., Dejniak, D., Lewiński, S., Krawczyk, A., & Chrobak, T. (2021). A unified methodology for the generalisation of the geometry of features. *ISPRS International Journal of Geo-Information, 10*(3), 107. https://doi.org/10.3390/ijgi10030107

Berg, M., Kreveld, M., & Schirra, S. (1995). A new approach to subdivision simplification. *Proceedings of the ACSM/ASPRS Annual Convention and Exposition Technical Papers, Charlotte NC, USA, 27 February – 2 March*, 4, 79–88.

Berg, M., Kreveld, M., & Schirra, S. (1998). Topologically correct subdivision simplification using the bandwidth criterion. *Cartography and Geographic Information Science*, *25*(4), 243–257. https://doi.org/10.1559/152304098782383007

Bjørke, J. T. (1996). Framework for entropy-based map evaluation. *Cartography and Geographic Information Systems*, *23*(2), 78–95. https://doi.org/10.1559/152304096782562136

Bjørke, J. T. (2003). Generalization of road networks for mobile map services: an information theoretic approach. *Proceedings of the 21st International Cartographic Conference (ICA), Durban, RPA, 10–16 August*.

Blana, N., Kavadas, I., & Tsoulos, L. (2023). A constraint-based generalization model incorporating a quality control mechanism. *Geographies*, *3*(2), 321–343. https://doi.org/10.3390/geographies3020017

Borkowski, A., & Keller, W. (2003). Modelling of irregulary sampled surfaces by two-dimensional snakes. *Journal of Geodesy, 77*, 543–553.

Boutoura, C. (1989). Line generalization using spectral techniques. *Cartographica, 26*(3&4), 33–48. https://doi.org/10.3138/G237-67W4-15N4-1R59

Brassel, K. E, & Weibel, R. (1988). A review and conceptual framework of automated map generalization. *International Journal of Geographical Information Systems*, *2*(3), 229–244. https://doi.org/10.1080/02693798808927898

Bronsztejn, I. N., Siemiendiajew, K. A., Musiol, G., & Muhlig, H. (2011). *Nowoczesne kompendium matematyki*. PWN.

Burghardt, D. (2005). Controlled line smoothing by snakes. *GeoInformatica*, *9*(3), 237–252. https://doi.org/10.1007/s10707-005-1283-3

Burghardt, D., & Meier, S. (1997). Cartographic displacement using the snakes concept. In W. Förstner & L. Förstner (Eds.), *Semantic Modeling for the Acquisition of Topographic Information from Images and Maps: SMATI 97* (pp. 59–71). Birkhäuser Verlag.

Burghardt, D., & Schmid, S. (2009). Constraint-based evaluation of automated and manual generalised topographic maps. In G. Gartner & F. Ortag (Eds.), *Cartography in Central and Eastern Europe. Lecture Notes in Geoinformation and Cartography* (pp. 147–162). Springer. https://doi.org/10.1007/978-3-642-03294-3_9

Burghardt, D., Neun, M., & Weibel, R. (2005). Generalization services on the web-classification and an initial prototype implementation. *Cartography and Geographic Information Science*, *32*(4), 257–268. https://doi.org/10.1559/152304005775194665

Burghardt, D., Schmid, S., Duchêne, C., Stoter, J., Baella, B., Regnauld, N., & Touya, G. (2008). Methodologies for the evaluation of generalised data derived with commercial available generalisation systems. *Proceedings of the 11th ICA Workshop on Generalization and Multiple Representation, Montpellier, France, 20–21 June,* 1–16. https://doi.org/10.5167/uzh-6877

Cebrykow, J. (2017). *Generalizacja map statystycznych*. Wydawnictwo UMCS.

Chaudhry, O., & Mackaness, W. (2008). Partitioning techniques to make manageable the generalisation of national spatial datasets. *Proceedings of the 11th ICA Workshop on Generalization and Multiple Representation, Montpellier, France, 20–21 June.*

Chrobak, T. (1999). *Badanie przydatności trójkąta elementarnego w komputerowej generalizacji kartograficznej*. UWND AGH.

Chrobak, T. (2003). Metoda uogólnienia danych w procesie generalizacji obiektów liniowych. The Method of Generalization Data Objects Linear. *Materiały Ogólnopolskiego Sympozjum Geoinformacji „Geoinformacja zintegrowanym narzędziem badań przestrzennych", Wrocław–Polanica Zdrój, Poland, 15–17 September, Archiwum Fotogrametrii, Kartografii i Teledetekcji*, 13 A, 33–38.

Chrobak, T. (2007). *Podstawy cyfrowej generalizacji kartograficznej.* UWND AGH.

Chrobak, T., Szombara, S., Kozioł, K., & Lupa, M. (2017). A method for assessing generalized data accuracy with linear object resolution verification. *Geocarto International*, *32*(3), 238–256. https://doi.org/10.1080/10106049.2015.1133721

Courtial, A., El Ayedi, A., Touya, G., & Zang, X. (2020). Exploring the potential of deep learning segmentation for mountain roads generalisation. *ISPRS International Journal of Geo-Information*, *9*(5), 338. https://doi.org/10.3390/ijgi9050338

Directive 2007/2/EC of the European Parliament and of the Council of 14 March 2007 establishing an Infrastructure for Spatial Information in the European Community (INSPIRE) Document 32007L0002 (2007) (European Union). http://data.europa.eu/eli/dir/2007/2/oj

Douglas, D. H., & Peucker, T. K. (1973). Algorithms for the reduction of the number of points required to represent a digitized line or its caricature. *Cartographica: The International Journal for Geographic Information and Geovisualization*, *10*(2), 112–122 https://doi.org/10.3138/FM57-6770-U75U-7727

Du, J., Wu, F., Yin, J., Liu, C., & Gong, X. (2022). Polyline simplification based on the artificial neural network with constraints of generalization knowledge. *Cartography and Geographic Information Science*, *49*(4), 313-337. https://doi.org/10.1080/15230406.2021.2013944

Dupuis, B., Deligiannidis, G., & Simsekli, U. (2023). Generalization bounds using data-dependent fractal dimensions. *Proceedings of the 40th International Conference on Machine Learning, Honolulu, Hawaii, USA, 23–29 July*, 358, 8922–8968. https://doi.org/10.48550/arXiv.2302.02766

Dziubiński, I., & Świątkowski, T. (Eds.). (1982). *Poradnik matematyczny*. PWN.

EPSG 2180. (2022). https://epsg.io/2180

Jepsen, T. S., Jensen, C. S., & Nielsen, T. D. (2022). Relation fusion networks: graph convolutional networks for road networks. *IEEE Transactions on Intelligent Transportation Systems*, *23*(1), 418–429. https://doi.org/10.1109/TITS.2020.3011799

Jiang, B., & Claramunt, C. (2004). A structural approach to the model generalization of an urban street network. *GeoInformatica*, *8*, 157–171. https://doi.org/10.1023/B:GEIN.0000017746.44824.70

Jiang, B., Liu, X., & Jia, T. (2013). Scaling of geographic space as a universal rule for map generalization. *Annals of the Association of American Geographers*, *103*(4), 844–855. https://doi.org/10.1080/00045608.2013.765773

João, E. (1998). *Causes and Consequences of Map Generalization*. Taylor and Francis.

Karsznia, I., Wereszczyńska, K., & Weibel, R. (2022). Make it simple: effective road selection for small--scale map design using decision-tree-based models. *ISPRS International Journal of Geo-Information*, *11*(8), 457. https://doi.org/10.3390/ijgi11080457

Kass, M., Witkin, A., & Terzopoulos, D. (1987). Snakes: active contour models. *Proceedings of the 1st International Conference on Computer Vision (ICCV'87), London, UK, 8–11 June*, 259–268.

Kilpeläinen, T., & Sarjakoski, T. (1995). Incremental generalization for multiple representations of geographical objects. In J-P. Lagrange, R. Weibel & J-C. Muller (Eds.), *GIS And Generalisation* (pp. 209–218). CRC Press. https://doi.org/10.1201/9781003062646

Kozioł, K. (2013). A line simplification algorithm using interpolation. *Annals of Geomatics*, *11*(3(60)), 45–59.

Kronenfeld, B. J., Stanislawski, L. V., Buttenfield, B. P., & Brockmeyer, T. (2020). Simplification of polylines by segment collapse: minimizing areal displacement while preserving area. *International Journal of Cartography, 6*(1), 22–46. https://doi.org/10.1080/23729333.2019.1631535

Lagrange, F., Landras, B., & Mustiere, S. (2000). Machine learning techniques for determining parameters of cartographic generalisation algorithms. *International Archives of Photogrammetry and Remote Sensing*, *13*(B4), 718–725.

Lang, T. (1969). Rules for robot draughtsman. *The Geographical Magazine*, *42*, 50–51.

Li, Z. (2007). Digital map generalization at the age of enlightenment: a review of the first forty years. *The Cartographic Journal*, *44*(1), 80–93. https://doi.org/10.1179/000870407X173913

Li, Z., & Choi, Y. H. (2002). Topographic map generalization: association of road elimination with thematic attributes. *The Cartographic Journal*, *39*(2), 153–166. https://doi.org/10.1179/caj.2002.39.2.153

Li, Z., & Openshaw, S. (1992). Algorithms for automated line generalization based on a natural principle of objective generalization. *International Journal of Geographical Information Systems*, *6*(5), 373–389. https://doi.org/10.1080/02693799208901921

Li, Z., & Su, B. (1995). From phenomena to essence: envisioning the nature of digital map generalisation. *The Cartographic Journal*, 32(1), 45–47. https://doi.org/10.1179/000870495787073852

Liu, Y., & Li, W. (2019). A new algorithms of stroke generation considering geometric and structural properties of road network. *ISPRS International Journal of Geo-Information*, *8*(7), 304. https://doi.org/10.3390/ijgi8070304

Mackaness, M., Ruas, A., & Sarjakoski, L. T. (2007). *Generalisation of geographic information: cartographic modelling and applications*. Elsevier.

Maudet, A., Touya, G., Duchêne, C., & Picault, S. (2017). DIOGEN, a multi-level oriented model for cartographic generalization. *International Journal of Cartography*, *3*(1), 121–133. https://doi.org/10.1080/23729333.2017.1300997

McMaster, R. B. (1986). A statistical analysis of mathematical measures for linear simplification. *The American Cartographer*, *13*(2), 103–116. https://doi.org/10.1559/152304086783900059

McMaster, R. B. (1987). Automated line generalization. *Cartographica: The International Journal for Geographic Information and Geovisualization*, *24*(2), 74–111, https://doi.org/10.3138/3535-7609-781G-4L20

McMaster, R. B., & Shea, K. S. (1992). *Generalization in digital cartography*. Association of American Geographers.

Muller, J. C., & Mouwes, P. J. (1990). Knowledge acquisition and representation for rule based map generalization: an example from the Netherlands. *Proceedings of the GIS/LIS '90, Anaheim, California, 7–10 November*, 1, 58–67.

Neun, M., Burghardt, D., & Weibel, R. (2009). Automated processing for map generalization with web services. *GeoInformatica*, *13*(4), 425–452. https://doi.org/10.1007/s10707-008-0054-3

Opheim, H. (1982). Fast data reduction of a digitized curve. *Geo-Processing, 2*, 33-40.

Pannekoek, A. J. (1962). Generalization of coastlines and contours. *International Yearbook of Cartography, 2*, 55–75.

Perkal, J. (1958). Próba obiektywnej generalizacji. *Geodezja i Kartografia*, *6*(2), 130–142.

Perkal, J. (1966). On the length of empirical curves, Discussion paper number 10. Michigan Inter-University Community of Mathematical Geographers.

Plazanet, C., Affholder, J. G., & Fritsch, E. (1995). The importance of geometric modeling in linear feature generalization. *Cartography and Geographic Information Systems, 22*(4), 291–305. https://doi.org/10.1559/152304095782540276

Regnauld, N. (2015). Generalisation and data quality. *The International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences*, XL-3/W3, 91–94. https://doi.org/10.5194/isprsarchives-XL-3-W3-91-2015

Robinson, A., Sale, R., & Morrison, J. (1988). *Podstawy kartografii*. PWN.

Rozporządzenie Ministra Rozwoju, Pracy i Technologii z dnia 27 lipca 2021 r. w sprawie bazy danych obiektów topograficznych oraz bazy danych obiektów ogólnogeograficznych, a także standardowych opracowań kartograficznych Dz.U. 2021 poz. 1412 (2021) (Polska). https://isap.sejm.gov.pl/isap.nsf/DocDetails.xsp?id=WDU20210001412

Saalfeld, A. (1999). Topologically consistent line simplification with the Douglas–Peucker algorithm. *Cartography and Geographic Information Science*, *26*(1), 7–18. https://doi.org/10.1559/152304099782424901

Salichtchev, K. A. (1967). Задачи картограбй и абтоматизацьѣ [Cartographic tasks and automation]. Izw. Wysszych Zawied. Gieod. i Aerofot., *4*, 7–10.

Salichtchev, K. A. (2003). *Kartografia ogólna*. PWN.

Sester, M., Anders K-H., & Walter, V. (1998). Linking objects of different spatial data sets by integration and aggregation. *GeoInformatica*, *2*, 335–358. https://doi.org/10.1023/A:1009705404707

Sester, M., Feng, Y., & Thiemann, F. (2018). Building generalization using deep learning. *International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences*, XLII-4, 565–572. https://doi.org/10.5194/isprs-archives-XLII-4-565-2018

Srnka, E. (1970). The analytical solution of regular generalization in cartography. *International Yearbook of Cartography*, *10*, 48–62.

Stoter, J., Burghardt, D., Duchêne, C., Baella, B., Bakke, N., Blok, C., Pla, M., Regnauld, N., Touya, G., & Schmid, S. (2009b). Methodology for evaluating automated map generalization in commercial software. *Computers, Environment and Urban Systems*, *33*(5), 311–324. https://doi.org/10.1016/j.compenvurbsys.2009.06.002

Stoter, J., van Altena, V., Post, M., Burghardt, D., & Duchêne, C. (2016). Automated generalisation within NMAs in 2016. *The International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences*, XLI-B4, 647–652. https://doi.org/10.5194/isprs-archives-XLI-B4-647-2016

Stoter, J., van Smaalen, J., Bakker, N., & Hardy, P. (2009a). Specifying map requirements for automated generalization of topographic data. *The Cartographic Journal*, *46*(3), 214–227. https://doi.org/10.1179/174327709X446637

von Sydow, E. (1866). Drei Karten-Klippen. Geo--Kartographische Betrachtung. *Geographisches Jahrbuch*, 348–361.

Tobler, W. R. (1964). Technical report No. 1, „An experiment in the computer generalization of maps". Department of geography University of Michigan. https://deepblue.lib.umich.edu/bitstream/handle/2027.42/7972/aja5779.0001.001.pdf?sequence=5&isAllowed=y

Touya, G. (2007). A road network selection process based on data enrichment and structure detection.

Proceedings of the 10th ICA Workshop on Generalisation and Multiple Representation, Moscow, Russia, 2–3 August, 595–614.

Touya, G., Zhang, X., & Lokhat, I. (2019). Is deep learning the new agent for map generalization? *International Journal of Cartography*, *5*(2–3), 142–157. https://doi.org/10.1080/23729333.2019.1613071

Trinder, J. C., & Li, H. (1995). Semi-automatic feature extraction by snakes. In A. Gruen, O. Kuebler & P. Agouris (Eds.), *Monte Verità: Automatic extraction of man-made objects from aerial and space images* (pp. 95–104). Birkhäuser Verlag. https://doi.org/10.1007/978-3-0348-9242-1_10

Visvalingam, M., & Whyatt J. D. (1991). Cartographic algorithms: problems of implementation and evaluation and the impact of digitising errors. *Computer Graphics Forum*, *10*(3), 225–235. https://doi.org/10.1111/1467-8659.1030225

Visvalingam, M., & Whyatt, D. (1990). The Douglas–Peucker algorithm for line simplification: re-evaluation through visualization. *Computer Graphics Forum*, *9*(3), 213–225. https://doi.org/10.1111/j.1467-8659. 1990.tb00398.x

Visvalingam, M., & Whyatt, J. (1993). Line generalisation by repeated elimination of points. *Cartographic Journal*, *30*(1), 46–51.

Wang, Z., Muller, J-C. (1998). Line generalization based on analysis of shape characteristics. *Cartography and Geographic Information Systems*, *25*(1), 3–15. https://doi.org/10.1559/152304098782441750

Weibel, R. (1995). Three essential building blocks for automated generalization. In J-P, Lagrange, R. Weibel, & J-C, Muller (Eds.), *GIS and Generalization: Methodology and Practice* (pp. 56–69). Taylor & Francis.

Weibel, R. (1996). A typology of constraints to line simplification. In M. J. Kraak & M. Molenaar (Eds.), *Advances in GIS Research II (7th International Symposium on Spatial Data Handling)* (pp. 533–546). Taylor & Francis.

Weibel, R., & Dutton, G. H. (1998). Constraints-based automated map generalization. *Proceedings of the 8th International Symposium on Spatial Data Handling, Vancouver, BC, Canada,* 214–224.

Weiss, R., & Weibel, R. (2014). Road network selection for small-scale maps using an improved centrality-based algorithm. *Journal of Spatial Information Science*, *9*, 71–99. https://doi.org/10.5311/JOSIS.2014.9.166

Werschlein, T., & Weibel, R. (1994). Use of neural networks in line generalization. *Proceedings of the Fifth European Conference and Exhibition on Geographical Information Systems, Paris, France, 30 March–1 April*, 76–85.

White, E. R. (1985). Assessment of line-generalization algorithms using characteristic points. *The American Cartographer*, *12*(1), 17–28. https://doi.org/10.1559/152304085783914703

Wu, H. (1997). Structured approach to implementing automatic cartographic generalization. *Proceedings of the 18th ICA International Cartographic Conference, Stockholm, Sweden, 23–27 June*, 1, 349–356.

Xiao, T., Ai, T., Yu, H., Yang, M., & Liu, P. (2023). A point selection method in map generalization using graph convolutional network model. *Cartography and Geographic Information Science*, 1–21. https://doi.org/10.1080/15230406.2023.2187886

Yan, H. (2019). *Description Approaches and Automated Generalization Algorithms for Groups of Map Objects*. Springer.

Yan, H., Yang, W., Lu, X., & Li, P. (2023). Quantitative expressions of spatial similarity between road networks in multiscale map spaces. *International Journal of Cartog*raphy, *9*(3), 554–570. https://doi.org/10.1080/23729333.2023.2236266

Yan, H., Zhang, X., Lu, X., & Li, P. (2022). Approach to automating the DP algorithm – taking river simplification as an example. *Geomatics and Information Science of Wuhan University*. https://doi.org/10.13203/j.whugis20210412

Yan, X., & Yang, M. (2023). A deep learning approach for polyline and building simplification based on graph autoencoder with flexible constraints. *Cartography and Geographic Information Science*, 1–18. https://doi.org/10.1080/15230406.2023.2218106

Zasoby Głównego Urzędu Geodezji i Kartografii. Geoportal. https://mapy.geoportal.gov.pl/

Zhang, Q. (2004). Road network generalization based on connection analysis. *Proceedings of the 11th International Symposium on Spatial Data Handling, Leicester, UK, 23–25 August*, 343–353.

Zheng, J., Gao, Z., Ma, J., Shen, J., & Zhang, K. (2021). Deep graph convolutional networks for accurate automatic road network selection. *ISPRS International Journal of Geo-Information*, *10*(11), 768. https://doi.org/10.3390/ijgi10110768

Zheng, L., & Tian, Z. (1997). Refinement of Douglas–Peucker algorithm to move the segments toward only one side. *Proceedings of 18th International Cartographic Conference, Stockholm, Sweden, 23–27 June*, 830–835.

Zhou, J., Shen, J., Yang, S., Yu, Z., Stanek, K., & Stampach, R. (2018). Method of constructing point generalization constraints based on the cloud platform. *ISPRS International Journal of Geo-Information*, *7*(7), 235. https://doi.org/10.3390/ijgi7070235

Zhou, Q., & Li, Z. (2014). Use of artificial neural networks for selective omission in updating road networks. *The Cartographic Journal*, *51*(1), 38–51.