MECHANICS AND CONTROL
Vol. 32 No. 3 2013 http://dx.doi.org/7494/mech.2013.32.3.110

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SIMULATION STUDY OF THE NON-COLLOCATED CONTROL OF A CANTILEVER BEAM

ABSTRACT

This article deals with the simulation of the active vibration control of a cantilever beam. For these purposes, a lumped parameter model has been developed and the simplest controller has been designed to ensure the structural stability of the control loop. The controller is of a proportional velocity feedback type. The control loop can also be stable in the case of a very small inherent damping of the cantilever beam. The lumped-parameter model is based on the Euler-Bernoulli beam theory. The developed tools can be used to simulate the collocated and non-collocated active vibration control. Since this article is intended to study the behaviour of a non-collocated control system of the transducer is sensing the vibration of the free end of the beam, while the actuator force acts near the fixed end.

Keywords: *cantilever beam, non-collocated control, simulation, active suppression of vibrations*

BADANIA SYMULACYJNE AKTYWNEGO STEROWANIA DRGANIAMI BELKI WSPORNIKOWEJ

Artykuł dotyczy symulacji aktywnego sterowania drganiami belki wspornikowej. W tym celu zaproponowano model o parametrach skupionych oraz najprostszy regulator, który zapewnia stabilność strukturalną. Zastosowano regulator proporcjonalny, z prędkościowym sprzężeniem zwrotnym. Pętla sterowania może być stabilna nawet w przypadku bardzo małego tłumienia drgań własnych belki. Model o parametrach skupionych został wyprowadzony na podstawie teorii Eulera-Bernoulliego. Zaproponowane podejście może zostać zastosowane do symulacji aktywnego sterowania drganiami w przypadku, gdy czujnik i element wykonawczy są umieszczone w tym samym miejscu bądź w różnych miejscach na belce. Ponieważ artykuł dotyczy badań własności aktywnego sterowania drganiami w drugim z tych przypadków, przetwornik pomiarowy drgań został umieszczony na swobodnym końcu belki, natomiast element wykonawczy działa na utwierdzonym końcu.

Słowa kluczowe: *belka wspornikowa, sterowanie, symulacja, aktywna redukcja drgań*

1. INTRODUCTION

It is possible to find a large number of publications that deal with the properties of cantilevered beams. There are calculations of the static deformation of the beam, the resonant frequencies and modal shapes of vibration and frequency response functions. The simulation models are frequently based on the use of the FE methods (Chudnovsky *et al*. 2006). State-space methods have also been used, but the application of a simplified model, such as has been discussed, e.g., in (Khot *et al*. 2012), for the simulation of active vibration control cannot be found.

The paper deals with the simulation of the active vibration control of a cantilever beam. To analyze the vibration of the cantilever beam a lumped-parameter model is created. This approach is motivated by the methods used in the study of active vibration control systems. Active vibration control is usually collocated, which means that the sensor and actuator are located at the same place (Preumont *et al*. 2008).

On the other hand, this paper is intended to study the behaviour of non-collocated control systems. Any number of sensors and actuators can be generally used for active vibration damping. The most interesting layout is the case when the transducer is sensing the vibration of the free end of the beam, while the actuator force acts on the clamped end.

The dynamic properties of the cantilever beam determine the resonant frequencies and corresponding modal shapes, which can also be determined experimentally by modal analysis. The mathematical model of the mentioned beam is designed to be simple enough and its modal properties are close enough to the results of experimental measurements.

2. LUMPED-PARAMETER MODEL

In the structural analysis of beams a general Timoshenko theory or a simplified Euler-Bernoulli engineering theory are most often used (Shames *et al*. 2006). The difference is that the simplified theory is valid for beams with the length that is at least twenty times bigger than the thickness, which will be assumed in this study. Therefore, shear deformation will no be taken into account, but only the bending deformation of the beam structure.

A cantilever beam of the length L can be divided into discrete elements of the same length ΔL that are modelled using rigid-body dynamics. How to create the lumped-parameter model of the cantilever beam of the rectangular cross section

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and to associate this multibody system with the Cartesians coordinates x, y, z is shown in figure 1, which represents the ways of the modelling of the beam element mass. The cantilever beam is clamped at the origin of the xy-plane and its centerline is parallel to the z-axis. It is assumed that the beam can move only in the yz-plane. Let N be the number of flexible links in the model. The massless link of a pair of adjacent beam elements is modelled in the mentioned plane with a torsion spring.

The individual coordinates of the multibody elements are usually associated with the gravity centers. Such a coordinate system requires an additional set of constrains (algebraic equations) for the continual linking of the individual adjacent elements at one connection point, except in the case where the element mass is concentrated at one point. If the mass of the beam element is spread along the whole length of this element and the coordinate system is chosen in such a way that it describes the motion of the connection points of two adjacent elementary beams then the additional equations are not needed. The two mentioned variants of the mass distribution along the element of the beam are shown in figure 1. The vertical co-ordinates of these points are marked by $y_1, y_2,...y_N$, and can be arranged into a column vector:

$$\mathbf{y} = [y_1, y_2,...y_N]^T. \tag{1}$$

The angle of rotation with respect to the horizontal axis is denoted by $\delta_1, \delta_2,...\delta_N$. The co-ordinates of the beam equidistant points in the Cartesian coordinates and the independent generalized coordinates for the Lagrangian equations of motion are identical. For further derivation only the motion in the y-direction is important. Because small deformations are assumed, the displacements of the meeting points in the direction of the z-axis are neglected. For small angles their measure in radians is given by the formula:

$$\delta_n = (y_n - y_{n-1})/\Delta L, \quad \Delta\delta_n = \delta_{n+1} - \delta_n. \tag{2}$$

The coordinates of the gravity center of the elementary beams are as follows:

$$Y_1 = y_1/2, \quad Y_n = (y_n + y_{n-1})/2. \tag{3}$$

Both of these models, which are shown in figure 1, assume the torsion springs with the same bending stiffness. This bending stiffness $K_\delta = \tau/\Delta\delta$ of the flexible links of the adjacent elementary beams relates the applied bending moment τ to the resulting relative rotation $\Delta\delta$ of the elementary beams. The potential energy V of the cantilever beam with a continuum replaced by its lumped parameter model is as follows:

$$V = \sum_{n=0}^{N-1} \frac{1}{2} K_\delta (\Delta\delta_n)^2. \tag{4}$$

For the beam in the horizontal position, the force of gravity is simply added to the force acting on the element.

The kinetic energy T of the cantilever beam which is replaced by the lumped parameter model according to the variants in figure 1 is as follows:

$$T = \sum_{n=1}^{N} \left[\frac{1}{2} \Delta m \left(\frac{\mathrm{d}Y_n}{\mathrm{d}t} \right)^2 + \frac{1}{2} \Delta J_x \left(\frac{\mathrm{d}\delta_n}{\mathrm{d}t} \right)^2 \right], \tag{5}$$

where ΔJ_x is the moment of inertia in kg m^2 about a horizontal axis perpendicular to the centerline of the elementary beam. The cantilever beam being a conservative system, Lagrange's equations of motion can be written as follows:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{y}_n} \right) - \frac{\partial T}{\partial y_n} + \frac{\partial V}{\partial y_n} = 0, \quad n = 1, 2,..., N. \tag{6}$$

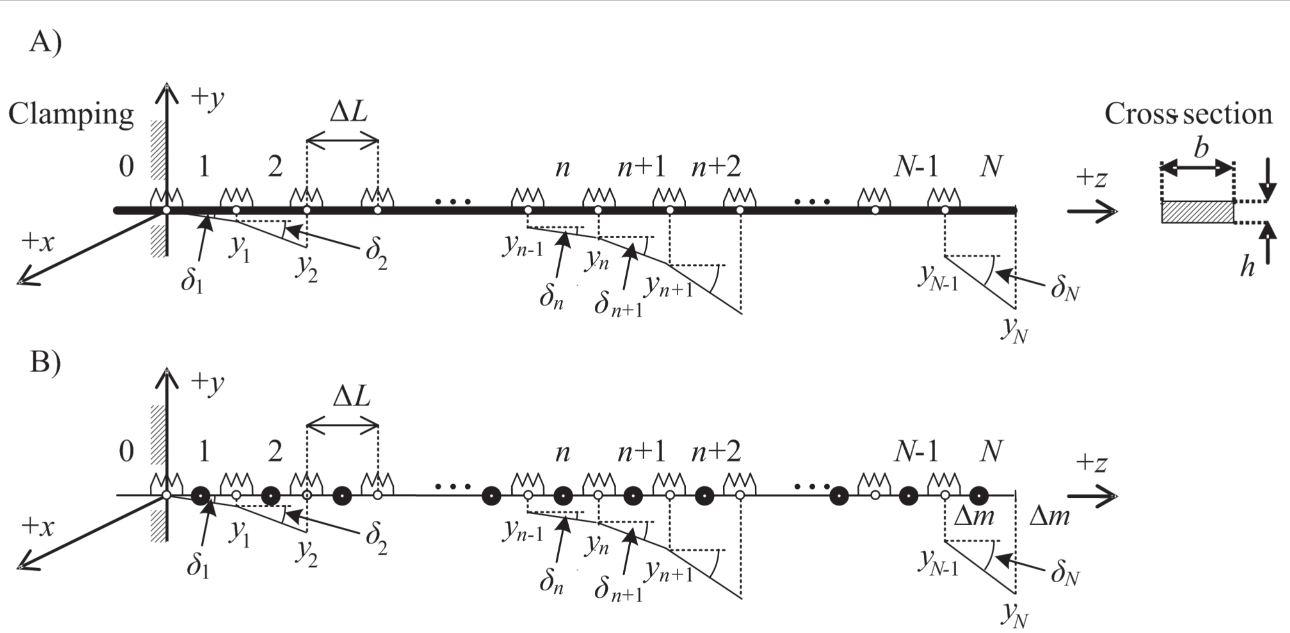

Fig. 1. Coordinates and elements of a cantilever beam for two ways of mass distribution along the element (A – uniform, B –concentrated at the gravity centre)

Introducing symbols **M** for the mass square matrix and **K** for the stiffness square matrix and y for the vertical coordinate column vector, the matrix equation of motion has the standard form

$$\mathbf{M}\ddot{\mathbf{y}} + \mathbf{K}\mathbf{y} = \mathbf{0}. \tag{7}$$

The stiffness- and mass matrices are of the following forms

$$\mathbf{K} = \frac{K_\delta}{\Delta L^2}\begin{bmatrix} 6 & \dots & & & & & \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ & 1 & -4 & 6 & -4 & 1 & \\ & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ & & & & & \dots & 1 \end{bmatrix}, \tag{8}$$

$$\mathbf{M} = \begin{bmatrix} B & A & & & \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ & A & B & A & \\ & \vdots & \vdots & \vdots & \vdots \\ & & & A & B/2 \end{bmatrix},$$

where parameters A and B are defined as follows:

$$A = \frac{\Delta m}{4} - \frac{\Delta J_x}{\Delta L^2}, \quad B = 2\left(\frac{\Delta m}{4} + \frac{\Delta J_x}{\Delta L^2}\right). \tag{9}$$

The sum of the row entries of the mass matrix and the difference between them is given by the following formula:

$$B + 2A = \Delta m \tag{10}$$
$$B - 2A = 4\frac{\Delta J_x}{\Delta L^2}$$

If the mass of the element is concentrated in its center of gravity then the moment of inertia will be replaced by zero ($\Delta J_x = 0$), i.e., the entries on the main diagonal are doubled with respect to the other two diagonals, $B = 2\,A = \Delta m/2$. It is known that for a solid cuboid of height h and length ΔL: $\Delta J_x = \Delta m(\Delta L^2 + h^2)/12$. The parameters A and B can be calculated according to the formulas:

$$A = \frac{\Delta m}{4}\left\{1 - \frac{1}{3}\left[1 + \left(\frac{h}{\Delta L}\right)^2\right]\right\}, \quad B = \frac{\Delta m}{2}\left\{1 + \frac{1}{3}\left[1 + \left(\frac{h}{\Delta L}\right)^2\right]\right\}. \tag{11}$$

The assumption about a uniform mass distribution along the beam element changes the value of the parameters A and B by more than 30%.

2.1. The bending stiffness of flexible links

Handbooks of mechanics (Flugge ed. 1962) provide formulas for the deflection y and slopes δ_a and δ_b of a cantilever beam at the free end. The following formulas refer to figure 2:

$$\delta_a = \frac{pL^2}{3EI_x}, \quad \delta_b = \frac{pL^2}{2EI_x}, \quad y = \frac{pL^3}{3EI_x}. \tag{12}$$

The slope δ_a corresponds to a straight line which approximates the beam according to figure 1. Because it is not clear which slope is best suited to the approximation of the bending stiffness, we define it by the following formula:

$$K_\delta = \frac{\tau}{\Delta\delta} = \eta\frac{EI_x}{\Delta L}, \tag{13}$$

where $E = 2.14\times10^{11}$ N/m^2 is Young's modulus of the beam material, $I_x = bh^3/12$ is the area moment of inertia of the beam cross-section about the horizontal axis and η is a multiplication factor which will be discussed below in this section.

We have prepared a specimen of a cantilever beam to be tested with the following parameters: $L = 0.5$ m, $b = 0.04$ m, $h = 0.005$ m. The value of the multiplicative factor in

Table 1

Multiplication factor for the calculation of bending stiffness

Number N	1	2	5	10	20	50	100	200
Factor η	3	1.8750	1.3200	1.1550	1.0763	1.0302	1.0150	1.0075

Table 2

Resonance frequencies

$N = 5$					
Mode n	1	2	3	4	5
Freq Hz	16.1	103	291	564	839
($\Delta J_x = 0$)	16.2	108	341	857	3089

$N = 10$										
Mode n	1	2	3	4	5	6	7	8	9	10
Freq Hz	16.5	104	292	574	950	1415	1953	2523	3041	3421
($\Delta J_x = 0$)	18	112	322	655	1141	1840	2890	4651	8469	26527
$N \to \infty$										
Mode n	1	2	3	4	5	6	7	8	9	10
Freq Hz	16.9	106	296	580	951	1432	2001	2663	3421	4273

equation (13) was designed so that the beam deflection at the free end of the lumped-parameter model fits the deflection obtained using the formula for the calculation of the deflection y in figure 2. It was found that the value of this factor depends on the number N of the elementary beams. The results are shown in table 1. By increasing the number of elements the factor η tends to one, the bending stiffness tends to $K_\delta = EI_x/\Delta L$, and the discrete cantilever beam approaches a continuous beam. $N = 5$ will be used in the simulations, therefore the correction factor is important.

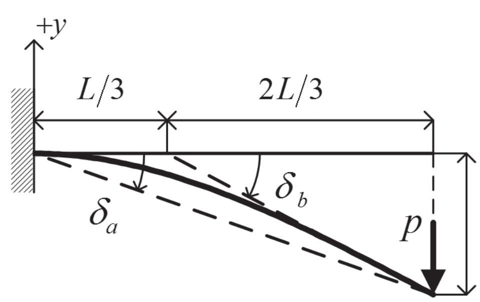

Fig. 2. Estimation of the bending stiffness of a cantilever beam

2.2. Resonant frequencies

The resonant frequencies can be obtained by calculating the eigenvalues and eigenvectors of the square matrix $\mathbf{M}^{-1}\mathbf{K}$. The eigenvalues form a diagonal spectral matrix $\mathbf{\Lambda} = \mathrm{diag}(\lambda_1, \lambda_2,...,\lambda_N)$ and the eigenvectors are arranged in the columns of a square matrix $\mathbf{U} = [\mathbf{u}_1, \mathbf{u}_2,..., \mathbf{u}_N]$. Matrix $\mathbf{U}$ will be used for the calculation of the orthogonal eigenvectors in the chapter discussing the transfer functions. The roots of the diagonal entries λ_n, $n = 1,..., N$ of the spectral matrix determine the frequencies f_n, $n = 1,..., N$ in Hz: $f_n = \sqrt{\lambda_n}/2\pi = \omega_n/2\pi$.

The exact resonant frequencies f_n^* of the clamped-free cantilever beam are given by the formula (Flugge ed. 1962):

$$f_n^* = \frac{\lambda_n^2}{2\pi L^2}\sqrt{\frac{EI_x}{\rho S}}, \quad n = 1, 2,..., \tag{14}$$

where ρS is the mass per unit length, $\rho = 7850$ kg/m^3 is a specific mass, $\lambda_1 = 1.8751$, $\lambda_2 = 4.6941$, $\lambda_3 = 7.8548$, $\lambda_4 = 10.9955$ and $\lambda_n = (2n - 1)\pi/2$ for n large. The results of the calculation of the resonance frequencies f_n for $N = 5$ and 10 and the frequencies f_n^* for an infinite number of elements ($N \to \infty$) are listed in table 2. The calculation of the resonant frequencies for the infinite number of beam elements has been done using formula (14). Except for the highest frequencies of the approximate model these frequencies agree well with the theoretically calculated values. Hence, the calculations that have taken account of the correction factor listed in table 1, show close conformity of the resonant frequencies of the discrete- and continuous model. The resonant frequencies were also verified by experimental modal analysis, with deviations up to 10% (Šuránek *et al*. 2013a, 2013b).

As is evident from table 2, a satisfactory approximation of the frequency response of the beam in the frequency range from 0 to 1 kHz can be obtained using the first 5 vibration modes. The first 10 modes describe the frequency response up to 5 kHz. When the kinetic energy originating from the rotation of the beam elements is neglected ($\Delta J_x = 0$) then this simplification leads to a significant deviation, especially at high resonant frequencies. Neither sensor nor actuator does work in an infinitely wide frequency band, therefore the frequency range of the control system is limited.

2.3. Forced vibrations

The forced vibration of the cantilever beam is described by the equation of motion with the external forces $F_1, F_2,..., F_N$ on the right-hand side. These forces are acting at the gravity centers of the beam elements and can be assembled into a vector $\mathbf{F} = [F_1, F_2,..., F_N]^T$, so the equation of motion reads:

$$\mathbf{M}\ddot{\mathbf{y}} + \mathbf{K}\mathbf{y} = \mathbf{F}. \tag{15}$$

The presence of viscous damping, such as a dissipative force, extends the left side of the equation of motion by an additional term which is proportional to velocity $\dot{\mathbf{y}}$:

$$\mathbf{M}\ddot{\mathbf{y}} + \mathbf{C}\dot{\mathbf{y}} + \mathbf{K}\mathbf{y} = \mathbf{F}, \quad \mathbf{K} = \alpha\mathbf{M} + \beta\mathbf{K}, \tag{16}$$

where the matrix $\mathbf{C}$, assuming Rayleigh's damping, is a linear combination of the mass and stiffness matrices, and α, β are constants of proportionality. For this model, the dependence of the damping ratio ξ on the frequency f_0 in Hz can be obtained from the formula: $\xi = \pi(\alpha/f_0 + \beta f_0)$. In this paper the values of Rayleigh's damping are selected as follows $\alpha = 0.159$ and $\beta = 0{,}0000411$.

2.4. Transfer functions

The vibrations of mechanical structures are very lightly damped. Damping is only a few percent of the critical value. The purpose of active vibration control is to increase the ability of structures to suppress vibrations by adding additional electronic feedback. To analyze the effect of active vibration damping we assume that the system is not damped passively at all.

If $\mathbf{y}(t)$ and $\mathbf{F}(t)$ are complex harmonic functions of time ($\exp(j\omega t)$) and $\mathbf{Y}$ and $\mathbf{F}$ are their complex magnitudes, respectively, the transfer function in the form of a square matrix $\mathbf{H}$, relating the displacement y_r, $r = 1,...,N$ to the forces F_q, $q = 1,...,N$ is given by the formula (Hi *et al*. 2001, Genta 2009):

$$\mathbf{Y} = (\mathbf{K} - \lambda\mathbf{M})^{-1} = \mathbf{H}\mathbf{F}. \tag{17}$$

The modal transform $\mathbf{y} = \mathbf{V}\mathbf{q}$ is the basis for the derivation of the transfer function. The relationship of the transfer function to the modal properties of the structure can be defined when the modal transformation matrix $\mathbf{V}$ has the following property: $\mathbf{V}^T\mathbf{M}\mathbf{V} = \mathbf{I}$. It can be proved that the orthonormal eigenvectors $\mathbf{v}_n$, $n = 1,...,N$ arranged in the matrix $\mathbf{V} = [\mathbf{v}_1, \mathbf{v}_2,...,\mathbf{v}_N]$ may be calculated from a set of the eigenvectors $\mathbf{u}_n$, $n = 1,..., N$ for the coordinates $\mathbf{y}$ by the following formula:

$$\mathbf{v}_n = \mathbf{u}_n / \sqrt{\mathbf{u}_n^T \mathbf{M} \mathbf{u}_n}, \quad n = 1,...,N. \tag{18}$$

The individual elements of matrix **H** are expressed as follows

$$H_{r,q}(j\omega) = \sum_{n=1}^{N} \frac{v_{n,r}\, v_{q,n}}{\omega_n^2 - \omega^2}, \quad r, q = 1, 2,...,N, \tag{19}$$

where $v_{q,r}$, q, $r = 1,...,N$ is the r-th element of the q-th normalized eigenvector. The transfer function $H_{r,q}(j\omega)$ relates the force acting on the q-th lumped mass, to the displacement y_r of the r-th lumped mass. The poles of the transfer function lie on the imaginary axis. The system is on the stability margin, not stable and simultaneously not unstable. The parameter $k_n = v_{n,r}\, v_{q,n}$ is called a modal constant.

3. ACTIVE VIBRATION CONTROL

3.1. Configuration of AVC

The purpose of the active vibration control (AVC) system is to compensate the effect of a disturbing external force. It is desirable to relocate the poles of the transfer function of the controlled system from the imaginary axis to the left half-plane of the complex plane. The cantilever beam is considered as an MIMO system composed of N lumped masses. The vibrations of all these masses can be controlled by forces acting on all of them.

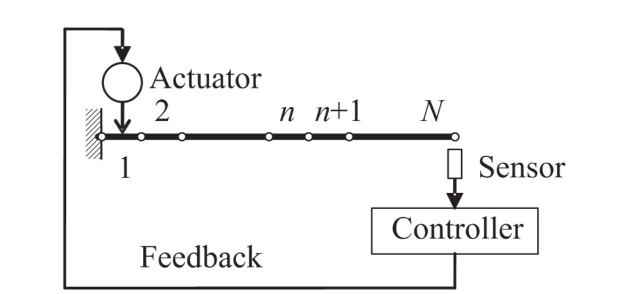

Fig. 3. Non-collocated system of active vibration control

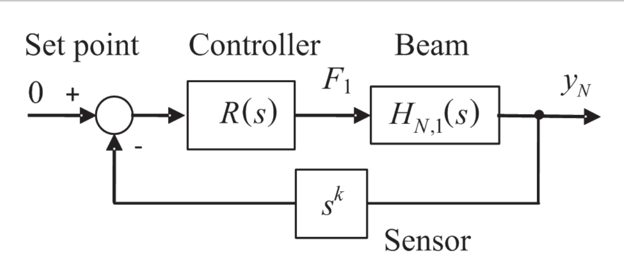

Fig. 4. Blok diagram of the control loop

The location of the sensor and actuator that is shown in figure 3, is a more practical configuration of active vibration control than the MIMO system just mentioned. In this figure, the actuator is the source of the force acting perpendicularly to the beam at the element center of gravity. The system discussed is of a SISO type, with a controller with a transfer function $R(s)$, as is shown in the block diagram of the closed-loop system in figure 4. For the non-collocated system it is assumed that the correcting force F_1 acts on the lumped mass indexed by $q = 1$ that is closest to the clamped end of the beam and the vibrations are sensed at the lumped mass with index $r = N$. The number of the beam elements and the sampling period for digital control also influence the number of poles and the frequency range of active vibration control. The transfer function of the vibration sensor is assumed to be equal to s^k. The output of the controlled system y_N is measured by either a displacement sensor ($k = 0$), or a velocity sensor ($k = 1$) or an acceleration sensor ($k = 2$). Consequently, the physical quantity for the set point of the controller is either position, or velocity, or acceleration.

The type of the sensor will be discussed later. As actuator one can use an electrodynamic exciter or piezoactuator. Especially, piezoactuators exhibit hysteresis. However, the analysis will focus on the ideal source of force.

3.2. Controller types

The transfer function of the closed-loop system $\widetilde{H}_{N,SP}$ relates the displacement y_N of the N-th lumped mass to the set point y_{SP} and the function $H_{N,1}$, relating the displacement y_N to the feedback force F_N acting on the first lumped mass:

$$\widetilde{H}_{N,SP}(s) = \frac{Y_N(s)}{Y_{SP}(s)} = \frac{R(s)H_{N,1}(s)}{1 + s^k R(s) H_{N,1}(s)} \tag{20}$$

$$= \frac{R(s)\sum_{n=1}^{N} \frac{v_{n,1}v_{N,n}}{\omega_n^2 + s^2}}{1 + s^k R(s)\sum_{n=1}^{N} \frac{v_{n,1}v_{N,n}}{\omega_n^2 + s^2}} = \frac{M(s)}{N(s)}.$$

The denominator of the transfer function of the closed-loop system is given as follows:

$$N(s) = \prod_{n=1}^{N} (\omega_n^2 + s^2) + s^k R(s) \sum_{n=1}^{N} v_{n,N} v_{1,n} \prod_{\substack{k=1 \\ k \neq n}}^{N} (\omega_k^2 + s^2). \tag{21}$$

With the exception of the term $R(s)s^k$ both polynomials are functions of complex variable s squared, thus there are no odd powers of s. When odd powers are missing, the system is not asymptotically stable; we can say it is structurally unstable for any values of the coefficients of the polynomials in the denominator of the transfer function. For a structurally stable system, the variable s which is raised to odd powers has to be added:

$$s^k R(s) \sum_{n=1}^{N} v_{n,N} v_{1,n} \prod_{\substack{k=1 \\ k \neq n}}^{N} (\omega_k^2 + s^2) = \sum_{n=1}^{N} T_{D,n} s^{2n-1} \tag{22}$$

$$\Rightarrow R(s) = \frac{\sum_{n=1}^{N} T_{D,n} s^{2n-1-k}}{\sum_{n=1}^{N} v_{n,N} v_{1,n} \prod_{\substack{k=1 \\ k \neq n}}^{N} (\omega_k^2 + s^2)}$$

where $T_{D,n}$, $n = 1,..., N$ are positive coefficients.

The controller designed according to formula (22) contains a total of N unknown parameters. The following choice of the controller with a derivative time constant T_D allows easy tuning:

$$s^k R(s) = T_D s \quad \Rightarrow \quad R(s) = T_D s^{1-k}. \tag{23}$$

This type of controller is termed velocity feedback. The transfer functions of the suitable controllers for all the relevant three types of vibration sensors are given in table 3.

Table 3

Types of the controller ensuring structural stability

Sensor	Controller
Displacement ($k = 0$)	$R(s) = T_D s$
Velocity ($k = 1$)	$R(s) = T_D$
Acceleration ($k = 2$)	$R(s) = T_D/s$

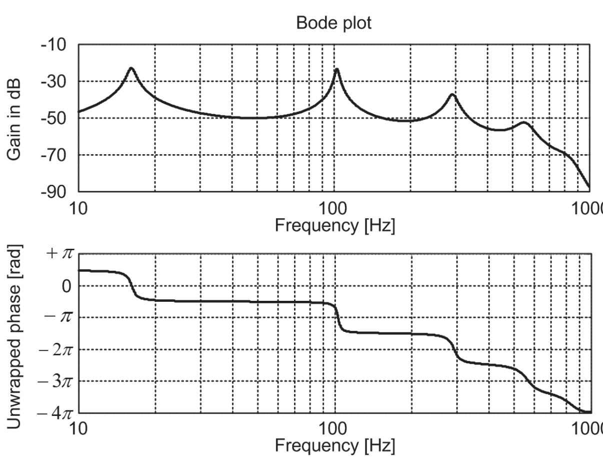

Fig. 5. The Bode plots of the frequency response $j\omega H_{N,1}(j\omega)$

With velocity taken as the controlled variable the time constant T_D becomes a proportional gain in the open-loop transfer function of the damped beam as follows:

$$G_{r,q}(j\omega) = j\omega T_D H_{r,q}(j\omega) \tag{24}$$

$$= \sum_{n=1}^{N} \frac{j\omega T_D v_{n,r} v_{q,n}}{\omega_n^2 + 2\zeta\omega\omega_n - \omega^2}, \quad r, q = 1,2,...,N,$$

With respect to the resonant frequency the low frequency asymptote rises at +20 dB/decade and high frequency asymptote drops off at –20 dB/decade. Frequency bands around each resonance frequency of the beam frequency response are well separated due to the low damping of the metal structures. This means that the magnitude of the transfer function (24) around a resonant frequency is determined almost entirely by the frequency response of the associated second order system and is practically not affected by the contributions from other second-order systems if one focuses on the transfer function that relates the velocity to the force.

The Bode plot of the frequency response $j\omega H_{N,1}(j\omega)$, which relates the velocity at the free end of the damped cantilever beam to the force acting close to the clamped end, is shown in figure 5. This frequency response shows the significance of the individual vibrational modes. The resonance peaks are reduced with increasing frequency as expected from Rayleigh's damping model.

4. SIMULATION RESULTS

The equation of motion (16) is a second-order ordinary differential equation. After the introduction of substitution $\mathbf{x}_1 = \mathbf{y}$ and $\mathbf{x}_2 = \dot{\mathbf{y}}$, the second order equation of motion is divided into two ordinary differential equations of the first order:

$$\dot{\mathbf{x}}_1 = \mathbf{x}_2 \tag{25}$$

$$\dot{\mathbf{x}}_2 = \mathbf{M}^{-1}\mathbf{F} - \mathbf{M}^{-1}\mathbf{C}\mathbf{x}_2 - \mathbf{M}^{-1}\mathbf{K}\,\mathbf{x}_1$$

The corresponding Matlab-Simulink arrangement is shown in figure 6. The simulation also requires the specification of the initial conditions: $\mathbf{x}_1(0) = \mathbf{y}(0)$ and $\mathbf{x}_2(0) = \dot{\mathbf{y}}(0)$. The Simulink model contains standard blocks. All connections are vectors.

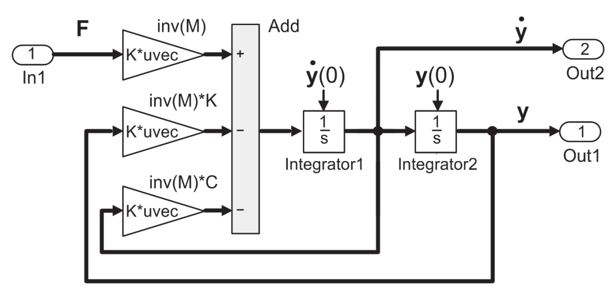

Fig. 6. Matlab-Simulink model of the cantilever beam for five elements

The effect of active vibration control can be demonstrated by the vibration decay of the beam which is bent into a stationary deflected position by the force of 10 N and then is suddenly released:

$$\dot{\mathbf{y}}(0) = \mathbf{0} \tag{26}$$

$$\mathbf{y}(0) = \mathbf{y}_0 = -\mathbf{K}^{-1}[0, 0,...,10].$$

The shape of the static deflection due to a concentrated force acting at the free end of the beam does not match fully its first eigenvector but is composed of other eigenvectors:

$$\mathbf{y}(0) = \alpha_1\mathbf{v}_1 + \alpha_2\mathbf{v}_2 + \alpha_3\mathbf{v}_3 + \alpha_4\mathbf{v}_4 + \alpha_5\mathbf{v}_5 \tag{27}$$

where $\alpha_1,...,\alpha_5$ are coefficients and $\mathbf{v}_1,...,\mathbf{v}_5$ are the normalized eigenvectors, which are calculated according to formula (19). Relative influence factors on the resulting deflection is given by the following table:

$\mathbf{v}_1$	$\mathbf{v}_2$	$\mathbf{v}_3$	$\mathbf{v}_4$	$\mathbf{v}_5$	(27)
97,2%	2,4%	0,3%	0,008%	0,003%.	

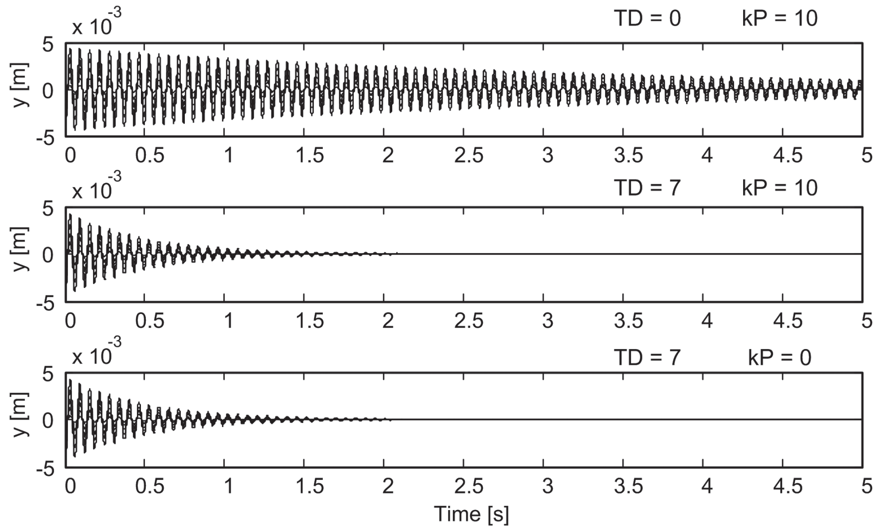

Fig. 7. The effect of ACV on the decay of vibration

In this case, 5 modes of vibration are mainly excited, but important vibrations are excited only for the lowest 2 modes.

The closed loop model of the active vibration control is shown in figure 8. The circuit configuration contains the subsystem which is shown in figure 6, and also an inverter and a PD controller (*Gain P* and *Gain D*).

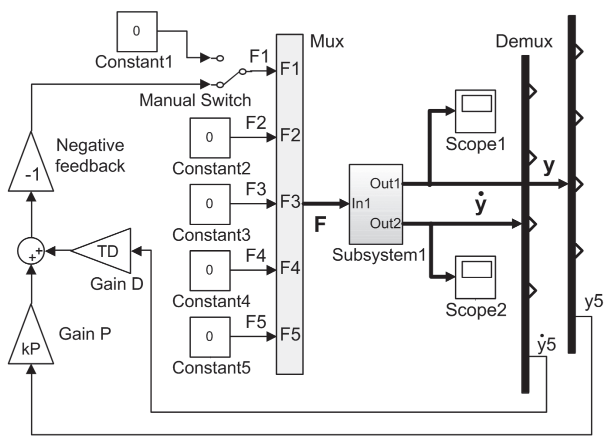

Fig. 8. Matlab-Simulink model of the closed loop control system

The effect of ACV on the decay of vibrations is shown in figure 7. The time constant of the derivative component strongly influences the damping effect. The margin value of the derivative time constant is slightly greater than 7. The slowest decay of the beam vibration is reached when AVC is switched off ($T_D = 0$). The decay of vibration is much faster than in the case when AVC is switched off when the derivative time constant is non-zero ($T_D = 7$). The choice of the proportional gain (kP) is almost without influence.

5. CONCLUSIONS

The lumped-parameter model of a cantilever beam was designed using the method based on modal analysis. It was proved that the cantilever beam can be actively damped by a force which is controlled by velocity feedback. The feedback of the D type is sufficient for the damping of lightly-damped systems. The proportional controller is almost without influence.

Acknowledgments

This research has been supported by the Czech Grant Agency project No P101/12/2520 'Active vibration damping of rotor with the use of parametric excitation of journal bearings' and elaborated in the framework of the project Opportunity for young researchers, reg. no. CZ.1.07/2.3.00/30.0016, supported by Operational Programme Education for Competitiveness and co-financed by the European Social Fund and the state budget of the Czech Republic.

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