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Preliminary Numerical Estimation of Selected Terminal Ballistic Effects Produced by Explosion of Mortar Projectile

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Abstract. Results of theoretical modelling of mortar projectile's fragments propulsion were shown. Taking into account universality of application of the considered ammunition, it seems to be reasonable to conduct simulations of projectile's fragments propulsion and interaction with the environment. In the conducted investigations, due to dynamic character of the whole phenomena, characterized by extremely high values of strains and strain's rate, the meshless explicit approach was used (Smoothed Particle Hydrodynamics method implemented in AUTODYN software). This approach minimalized the negative effects of deformation of "classical" Lagrangian mesh. In order to validate a numerical model, the results were compared with the simplified Gurney's formula, which provides high accuracy of fragment's velocity for regular shapes of casing. Comparison of the results showed low value of relative discrepancy (lower than 10%) for the cylindrical part of the casing in which detonation was fully developed and resulted in higher values of relative discrepancy of initial velocity for the non – cylindrical region, especially where the detonation was not developed.
Keywords: mechanics, Gurney's formula, mortar projectile, explosion, terminal ballistics, meshless methods

1. INTRODUCTION

The mechanisms of influence of different types of ammunition on enemy is strongly dependent on projectile's construction and conditions of its use. The most popular type of considered devices are fragmentation projectiles, in which the fragments are launched by products of detonation of high explosive. One of the crucial parameters for such projectiles is fragments' initial velocity, which significantly determines destructive potential of the applied shell. Commonly, during estimation of lethal abilities of various systems, the Gurney's formula is used [1]. This simplified approach is based on one-dimensional model of the explosive system and works correctly especially in cases of regular geometries (cylinders, plates – Fig. 1), ensuring approximately 10% accuracy [2].

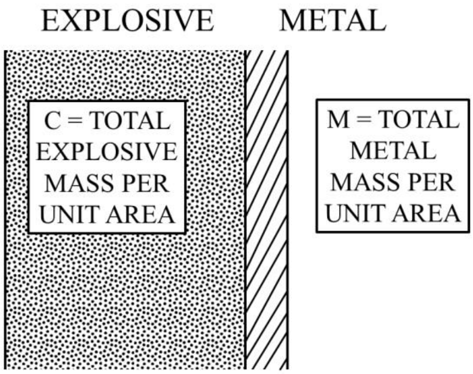

Fig. 1. Open-faced sandwich [3]

For the cylindrical symmetry, Gurney assumed that the gases move radially outward from the central axis of the explosive charge. Moreover, he imposed that the radial velocity of gases varies directly with the distance from the axis to the casing [4]. In accordance with the Gurney's model, the fragment's initial velocity is approximately equal to:

$$v_c = \sqrt{2E}\sqrt{\frac{\frac{C}{M}}{1+\frac{C}{2M}}} \tag{1}$$

where E denotes kinetic energy per unit mass, C – mass of the explosive per unit length, M – mass of metal casing per unit length.

The constants $\sqrt{2E}$ for various explosives is commonly available and its value depends on confinement conditions – it depends on a thickness of casing and strength properties of casing material. For example, for the TNT, the value of $\sqrt{2E}$ varies in the range between 2.039 km/s (for thin case) and 2.505 km/s (for thick one). The relationship between initial velocity of fragments and the ratio *C*/*M* for two presented values of $\sqrt{2E}$ have been shown in Fig. 2.

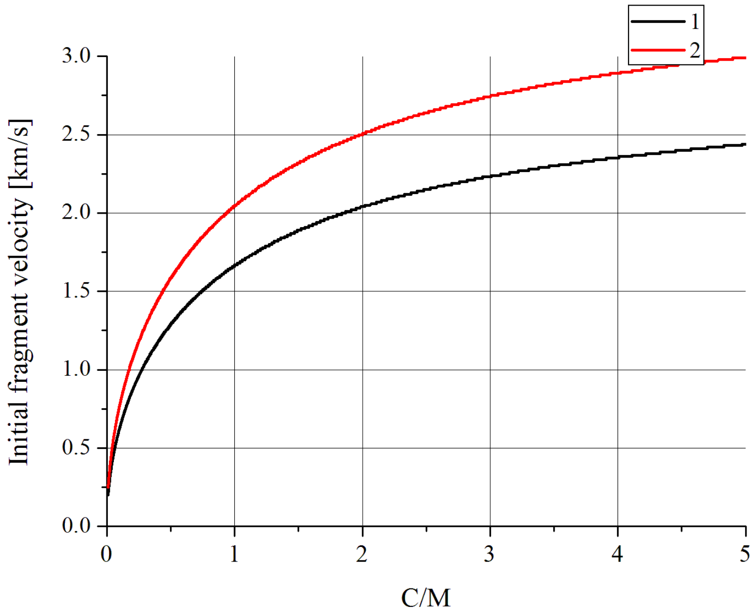

Fig. 2. Comparison of initial velocities, predicted making use of Eq. (1) for two values of $\sqrt{2E}$ (1 - 2.039 km/s; 2 - 2.505 km/s), as the function of C/M

Taking into account, that in real conditions the Gurney's assumptions are not satisfied (especially for conditions of projectile's shapes different from cylinder and spheres), in the presented work, the results of numerical estimation of fragments velocity, produced by explosion of mortar projectile, have been shown and compared with the results obtained making use of Gurney's formula. These results are treated as the introduction to further works in the consideration area.

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1. NUMERICAL MODEL

1.1. Numerical approach

Numerical modelling of extremely dynamic phenomena, like materials launching by detonation products needs a special approach.

The reason of difficulties, during the calculations, is the presence of large deformations (strain) of material, which results in impossibility of application of classical Lagrangian approach, well-known in finite elements analysis (method fails due to large distortion of elements).

One of the alternative ways for investigations of similar problems is application of meshless methods. Their efficiency was confirmed repeatedly [5]. During the performed analyses, the commercial AUTODYN 3D explicit code was applied. In the used software, the Smoothed Particle Hydrodynamics (SPH) method was implemented. This approach is the specific case of Lagrangian method, in which classical elements were replaced by particles. Particles, represented by points, are characterized by so-called "smoothing length" and each neighbouring particle, which is included into "area of interaction", defined by doubled smoothing length, affects the parameters at the considered point. The influence of parameters of neighbouring particles on the parameters at the point x_I has been shown in Fig. 3. The kernel function W has non – zero values for distance between "neighboring" particle and the considered point is less than $2h$ (where h is the smoothing length).

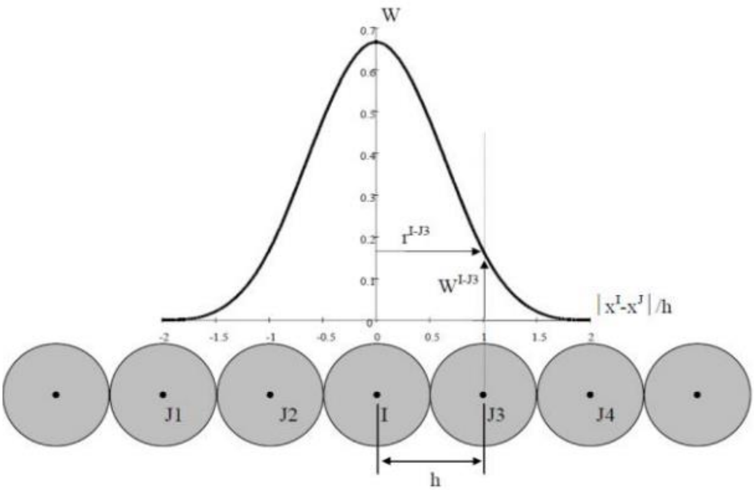


Fig. 3. Geometrical representation of smoothing length [6]

The value of each "smoothed" field variable (density, velocity, energy) at the point x_I can be found using the following expression:

$$f_I = \sum_{J=1}^{N} m_J \frac{f_J}{\rho_J} W(x_J - x_I, h) \tag{2}$$

Using the above presented approximation, the equations expressing conservation laws can be evaluated, as done in [7].

Unfortunately, this approach suffers from several disadvantages. The most important are particle inconsistency, inaccuracy at domain boundaries, and instabilities at tensile stress state. As it can be found in literature, the stability and accuracy of algorithm depends on particles density and sensitivity on this parameter should be analysed in each case.

1.2. Geometry and material models

In this work, the projectile of 98-mm mortar was investigated. To ensure low computational cost of solution, the quarter of the full 3D model was considered, fulfilled by symmetry conditions. The geometry of under consideration object was shown in Fig. 4. The whole projectile consists of four main parts: steel casing with fuse, high explosive (TNT), and fins made of aluminum alloy.

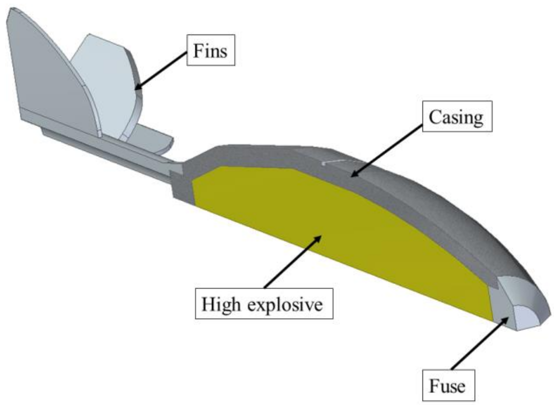


Fig. 4. Geometry of investigated system

In order to model the behaviour of high explosive, the Jones-Wilkens-Lee (JWL) equation of state was applied. For other materials, the linear equation of state and the elastic – plastic material model was used, completed by Johnson-Cook strength and failure models (Johnson-Cook failure model for steel and maximum effective plastic strain failure model for aluminum alloy). Flow criterion was assumed to be in compliance with von Mises hypothesis.

In case of JWL equation of state, the pressure of detonation products is expressed by the following relation [8]:

$$p_{JWL} = A_{JWL}\left(1-\frac{\omega\eta}{R_1}\right)\exp\left(-\frac{R_1}{\eta}\right)+B_{JWL}\left(1-\frac{\omega\eta}{R_2}\right)\exp\left(-\frac{R_2}{\eta}\right)+\omega\rho_{DP}e \quad (3)$$

where A_{JWL}, B_{JWL}, R_1, R_2 and ω are the constants obtained in dynamic conditions, $\eta = \rho_{\mathrm{DP}}/\rho_{\mathrm{HE}}$ (ρ_{DP} – density of detonation products, ρ_{HE} – initial density of HE), e – internal energy of gases.

The linear equation of state, applied for metals, has the following form:

$$p_{Lin} \approx p_0 + K\left(\frac{\rho}{\rho_0}-1\right) \quad (4)$$

where K is the bulk modulus, ρ – material density and ρ_0 – initial material density referring to the initial pressure p_0.

Johnson - Cook expression, describing dynamic yield stress, was assumed as the following:

$$Y = \left[A_{JC} + B_{JC}\varepsilon_p{}^n\right]\left[1 + C_{JC}\ln\dot{\varepsilon}_p{}^*\right]\left[1 - T_H{}^m\right] \quad (5)$$

In above expressions ε_p is the effective plastic strain and $\dot{\varepsilon}_p{}^*$ is the normalized value of strain rate (divided by the reference value $\dot{\varepsilon}_{p\,ref}$). Moreover, $T_{\mathrm{H}} = (T - T_{\mathrm{room}}) / (T_{\mathrm{melt}} - T_{\mathrm{room}})$, where T denotes material temperature, T_{melt} is its melting temperature, T_{room} – room temperature. The constants in Eq. (5): A_{JC}, B_{JC}, C_{JC}, n, and m, are evaluated using experimental data.

The failure model introduces quantity, which defines a level of failure of material:

$$D = \sum \frac{\Delta\varepsilon}{\varepsilon^f} \quad (6)$$

where

$$\varepsilon^f = \left[D_1 + D_2\exp(D_3\sigma^*)\right]\left[1 + D_4\ln\left|\dot{\varepsilon}^*\right|\right]\left[1 + D_5T^*\right]$$

and D_1, D_2, D_3, D_4 are the constants.

For the $D = 1$, material completely fails and, as assumed in calculation, is eroded. In Tables 1 and 2, material properties applied for calculations were summarized.

Table 1. Parameters of models describing high explosive [9].

	Parameter	TNT
1	A_{JWL} [GPa]	373.769989
2	B_{JWL} [GPa]	3.747100
3	R_1	4.15
4	R_2	0.9
5	ω	0.35
6	Density of high explosive ρ_{HE} [kg/m^3]	1630
7	C-J pressure p_{CJ} [GPa]	21
8	C-J detonation velocity D_{CJ} [m/s]	6930
9	Energy of explosion e_0 [MJ/m^3]	6000

Table 2. Parameters of models describing metals [10,11].

	Parameter	**Steel**	**Aluminum alloy**
1	Density ρ_0 [kg/m^3]	7830	2770
2	Bulk modulus K [GPa]	159	70
3	Specific heat λ [J/(kg K)]	477	875
4	Shear modulus μ [GPa]	77	27.6
5	Quasi – static yield stress A_{JC} [MPa]	792	337
6	Hardening constant B_{JC} [MPa]	510	0.343
7	Hardening exponent n	0.26	0.41
8	Strain rate constant C_{JC}	0.014	0.01
9	Thermal softening exponent m	1.03	1
10	Melting temperature T_m [K]	1793	877
11	Reference strain rate $\dot{\varepsilon}_{pref}$	1	1
12	Damage constant D_1	0.05	not applied
13	Damage constant D_2	3.44	not applied
14	Damage constant D_3	-2.12	not applied
15	Damage constant D_4	0.002	not applied
16	Damage constant D_5	0.61	not applied
17	Effective plastic strain at failure ε^p_{eff}	not applied	0.3

2. RESULTS

As the results of calculations, the pressure distributions and velocity of casing fragments, as the function of time, was obtained. In Fig. 5, the spatial pressure distribution for several moments was presented.

As it can be seen, the maximum pressure at the front of a detonation wave differs from Chapman-Jouguete pressure about 15-20 % (Table 1). The reason of this situation is difference between SPH "particles' density" and thickness of reaction zone in detonation wave – in the presented considerations, the structure of a detonation wave was not investigated. Basing on the conducted calculations, it can be concluded that acceptable distance between particles, for the presented problem, is approximately equal to 1 mm.

a b

c d

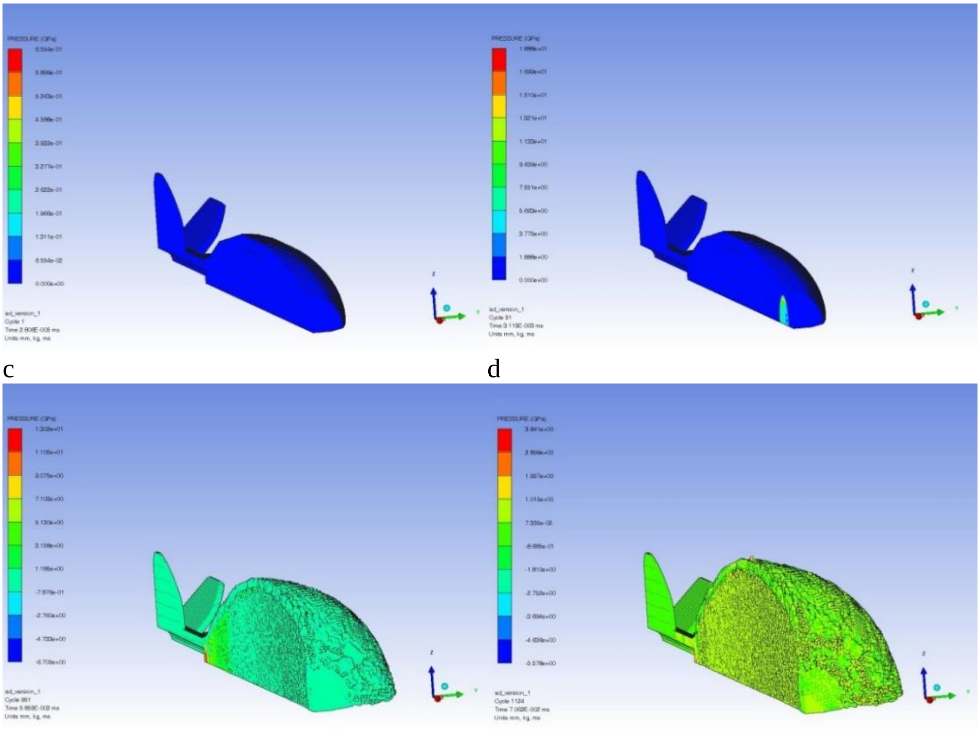


Fig. 5. Pressure distribution for several moments of a launching process

Moreover, the fragmentation process differs from real situation due to omission of stochastic inhomogeneity of material properties.

In Fig. 6, the distribution of gauge points for velocity measurements was presented. In Fig. 7, the velocity components as a function of time were shown. In Fig. 8, the resultant velocity as the function of time was sketched.

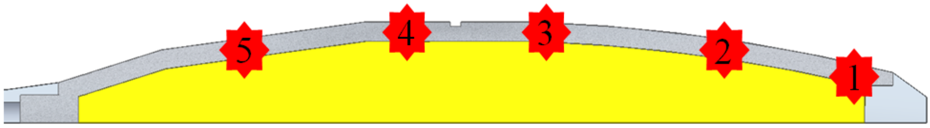
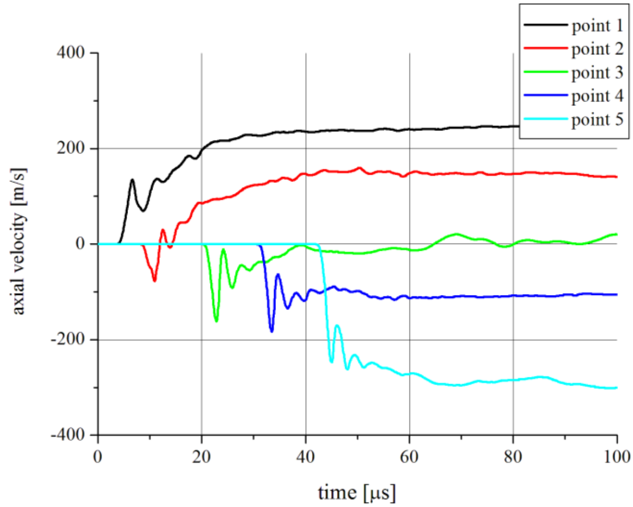


Fig. 6. The distribution of gauge points on the projectile casing

a



b

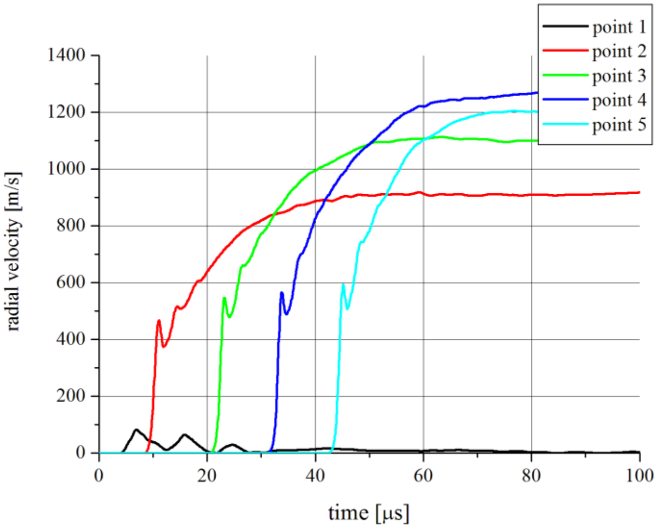


Fig. 7. Values of velocity components as a function of time for gauge points

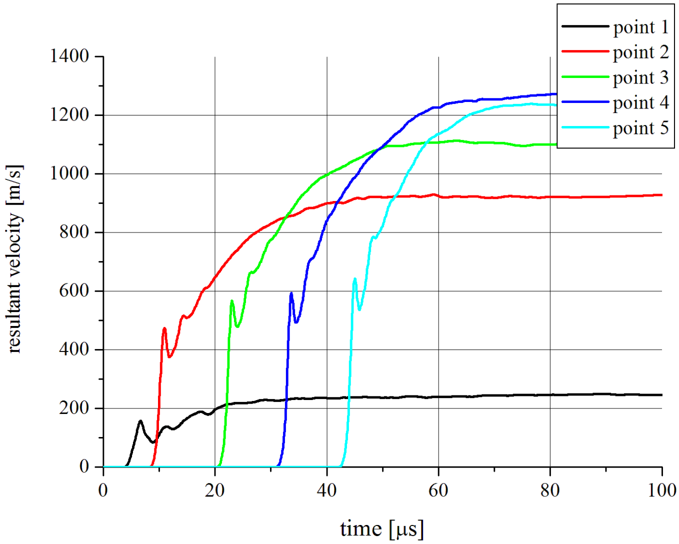


Fig. 8. Values of resultant velocity as a function of time for gauge points

In order to compare the results of numerical calculations with the results of analytical considerations, the obtained values of velocity were summarized in Table 3.

Table 3. Comparison of the Gurney's velocity with the numerical results

Point No.	**Estimated value of C/M**	**Gurney's lower value of velocity [m/s]**	**Gurney's higher value of velocity [m/s]**	**Resultant velocity from numerical model [m/s]**	**Relative discrepancy for lower Gurney's velocity [%]**	**Relative discrepancy for higher Gurney's velocity [%]**
1	0.14	737	906	246	199	268
2	0.23	926	1137	918	1	24
3	0.385	1159	1423	1106	5	29
4	0.385	1159	1423	1265	8	12
5	0.23	926	1137	1199	23	5

Due to small value of casing thickness (approximately 10 mm), the lower value of Gurney's velocity should be taken as the correct value. As it can be seen, the largest discrepancy is noticed for the region where the detonation process is not fully developed. The second region of higher values of relative error is the part of the projectile, where the diameter of the HE decreases (concentration of the parameters).

3. CONCLUSIONS

The presented results of calculations showed the correctness of applied algorithm and compliance of the obtained values with analytical investigations. More serious divergence between these values was observed for the region of detonation development (in the vicinity of fuse) and in the part where the diameter of HE changes. At the cylindrical fragment of the casing, the relative error of analytical results was less than 10%. The conducted verification of a numerical model allows for further analyses of more complex terminal ballistics phenomena.

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Wstępne oszacowania numeryczne wybranych efektów balistyki końcowej powstałych na skutek wybuchu pocisku moździerzowego

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Streszczenie. W pracy przedstawiono wyniki teoretycznego modelowania napędzania odłamków pocisku moździerzowego. Biorąc pod uwagę powszechność stosowania rozważanego typu amunicji, zasadnym wydaje się przeprowadzenie symulacji numerycznych napędzania odłamków oraz ich oddziaływania z otoczeniem. Mając na uwadze dynamiczny charakter badanego zjawiska, charakteryzującego się dużymi odkształceniami oraz szybkościami odkształceń rozważanych materiałów, symulacje przeprowadzono z wykorzystaniem bezsiatkowej metody SPH, bazującej na jawnym schemacie numerycznym. Obliczenia przeprowadzono z wykorzystaniem środowiska AUTODYN. Wykorzystana metoda wyeliminowała negatywny wpływ deformacji elementów w klasycznym Lagrange'owskim sformułowaniu modelowania ruchu fazy stałej. Walidacja modelu teoretycznego została przeprowadzona w oparciu wyniki uzyskane przy użyciu wzorów Gurney'a dla rozpatrywanego układu. W obszarze „rozwiniętej detonacji", stwierdzono satysfakcjonującą dla celów inżynierskich rozbieżność pomiędzy wynikami numerycznymi oraz referencyjnymi (na poziomie mniejszym niż 10 %). Większą rozbieżność pomiędzy wynikami uzyskanymi z zastosowaniem obu podejść uzyskano dla obszarów, których geometria charakteryzowała się stożkowym kształtem ładunku wybuchowego oraz w obszarach, w których detonacja nie rozwinęła się w pełni.
Słowa kluczowe: wzory Gurney'a, pocisk moździerzowy, wybuch, balistyka końcowa, metody bezsiatkowe.

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