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INFLUENCE OF THE NUMERICAL MODELLING METHODS ON DISPERSIONS AND ERRORS OF ANALYSIS RESULTS OF SHIP HULL AND DECKHOUSE VIBRATIONS

Wpływ metod modelowania numerycznego na rozrzuty i błędy wyników analiz drgań kadłubów i nadbudówek statków

Abstract: *Ships' (especially containers) vibrations significantly impact navigation safety. The presented analyses aims to identify the main forces exciting the ship's superstructure and hull vibrations, test their influence on vibration levels, and verify the assumptions of the computational methodology. Two container ships were analysed. The influence of different modeling methods on the obtained calculation results was investigated. The impact of various operating parameters on the vibration level was also analysed. The numerical analyses results are compared with some empirical formulas. As a result, the calculation confidence level was estimated. The calculation results have been verified by comparison with measurement tests carried out on the real ship.*
Keywords: ship vibrations, finite element method, calculation errors and dispersions

Streszczenie: *Drgania statków (zwłaszcza kontenerowców) mają duży wpływ na bezpieczeństwo żeglugi. Celem przedstawionych analiz jest identyfikacja głównych sił wymuszających drgania nadbudówki i kadłuba statku, zbadanie ich wpływu na poziom drgań oraz weryfikacja założeń metodyki obliczeniowej. Przeanalizowano dwa kontenerowce. Zbadano wpływ różnych metod modelowania na wyniki obliczeń. Przeanalizowano również wpływ różnych parametrów eksploatacyjnych na poziom drgań. Wyniki analiz numerycznych porównano z niektórymi wzorami empirycznymi. W rezultacie oszacowano poziom ufności obliczeń. Wyniki obliczeń zostały zweryfikowane poprzez porównanie z badaniami pomiarowymi wykonanymi na rzeczywistym statku.*
Słowa kluczowe: drgania statków, MES, rozrzuty i błędy obliczeniowe

1. Introduction

Modelling processes and physical phenomena is so common in engineering practice that it is often unnoticeable. Fully realizing that the most accurate model is not a physical reality - it has its limitations - is extremely important when analyzing various engineering issues. In many cases, the complete computation process, from modelling a physical object through a mathematical model to numerical calculations, is not fully understood [5]. Analytical engineers and designers need knowledge of modelling methods (including their scope of application) and the ability to apply optimal methods for assessing a given phenomenon [9]. Seagoing ships, particularly those equipped with slow-speed diesel engines, are exposed to excessive vibration of the ship's superstructure and hull. Ship (especially containers) vibrations impact navigation safety. In most cases, two main systems of the ship are distinguished: the ship's hull (with superstructure and main engine body) and the power transmission system (crankshaft, shaft line and propeller). The ship's adverse dynamic characteristics are very difficult to change once the ship is built. Therefore, it is very important to correctly determine the expected vibration levels when designing units. Mainly, the power transmission system induces forces that excite significant vibrations. Environmental impact functions are less important because they do not generate continuous vibration. The analyses presented in this paper aims to identify the main forces which excite the ship's superstructure and hull vibrations, test their effects on vibration levels, and verify the assumptions of the computational methodology. The numerical calculation results have been compared with the measurement tests. Two container ships were analysed in the manuscript: a medium-sized, with a capacity of approximately 2700 TEU (standard containers - 200 m length) and a large one, with a capacity of 11400 TEU (360 m length). In the study, a finite element model representing the entire ship hulls, including the deckhouse and machinery propulsion system, has been developed.

The ship's hull is a specific object for modelling. One important element in the Finite Element Method (FEM) calculation procedure is the boundary conditions problem. After all, like an aircraft, the ship is not permanently connected to any other structural element — there are no scleronomous constraints. From the point of view of analytical mechanics, there are non-holonomic constraints (constraints depending on the derivative coordinates defining the system positions) and even rheonomous (time-dependent) constraints. However, it is difficult to capture such an approach to water interaction in a mathematical model. Usually, static calculations are based on the equilibrium of two effects: the distribution of gravity forces (ship structure and cargo) and the buoyancy forces (dependent on the ship waterline, i.e. on the volume of the ship's underwater part). Contemporary FEM software allows for the absence of boundary conditions for dynamic calculations. In such a case, we reject the first six, zero (for numerical reasons, this is usually a very small number, e.g. about 10-6) frequencies of free vibrations. For this reason, the problem of ship modelling breaks down into two partially independent issues: the calculation of the overall hull strength [10] and the analysis of the hull and the ship's superstructure vibrations [1]. Specific modelling of some of the following ship's elements is partially common for static and dynamic calculations: the ship's cargo and operating fluids, sea water impact, the stiffeners of the ship hull, the propulsion system, non-structural masses, loads and excitations.

Seagoing ships, particularly those equipped with slow-speed diesel engines, are exposed to excessive vibration of the ship’s superstructure and hull [14]. Ship’s hull vibrations have a major impact on navigation safety. They impact the marine structures and equipment reliability and on the comfort of maritime crews, which is also connected with navigation safety. Thus, shipping safety requires that ship structure systems are free from excessive stress and vibration [14]. In most cases, two main systems of the ship are distinguished: the ship’s hull (with superstructure and main engine body) and the power transmission system (crankshaft, main engine shafting and propeller). The ship’s adverse dynamic characteristics are very difficult and rather expensive to change once it is built. Therefore, it is very important to correctly determine the expected vibration levels when designing units.

The power transmission system [1, 4, 6] induces forces that excite significant vibrations. Environmental impact functions are less important because they do not generate continuous vibration. The analyses presented in this paper aims to identify the main forces that excite the ship’s superstructure and hull vibrations, test their effects on vibration levels and verify the assumptions of the computational methodology. The numerical calculation results have been verified by comparison with the measurement tests.

Basic forces exciting vibrations of the hull and superstructure [4, 7, 8] of a ship equipped with a low-speed engine directly driving the propeller are indicated in Fig. 1. Drive dynamics analysis is necessary to determine hull vibration correctly; nevertheless, the power transmission system vibration analysis requires knowledge of the boundary conditions, i.e. the hull’s stiffness dynamic characteristics. The couplings between the hull and the drive shall be taken into account in any detailed analysis of the ship’s vibrations.

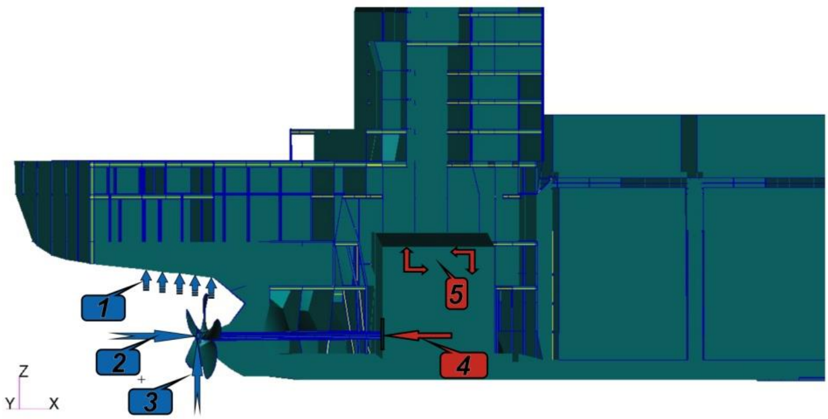

Fig. 1. Forces exciting the ship’s hull and superstructure vibration

The dynamic forces indicated schematically in Fig. 1 have the following meanings:

1. pressure pulses induced on the ship's deck transom by the propeller;
2. longitudinal hydrodynamic forces exciting uncoupled longitudinal vibrations of the power transmission system and, consequently, variable reactions of the thrust bearing;
3. transversal hydrodynamic forces and moments causing flexural vibrations of the main engine shafting and, consequently, variable reactions of the transversal radial bearings (stern bearings and intermediate bearings);
4. dynamic reactions of the thrust bearing from coupled longitudinal-lateral-torsional vibrations [13] of the power transmission system;
5. unbalanced moments (and, possibly, forces) of the main engine coming from the radial gas and mass forces of the piston-crank system.

The easiest way to reduce excessive vibration is to avoid resonance: the free vibration frequency should not be close to the exciting frequency. The problem is that the ship's structure is so complex that a typical ship presents a wide variety of free vibration modes (and frequencies); they concern the global vibrations of the hull, as well as the substructure vibrations or local vibrations of the decks. There are also a lot of frequencies of exciting forces. From a practical point of view, at least three forcing frequencies associated with the propeller (three harmonic components of the hydrodynamic forces) and several to a dozen forcing frequencies associated with the main drive engine should be considered. For example, a medium-sized container ship, which will be analyzed later, is equipped with an eight-cylinder main engine and a five-blade propeller. Therefore, as regards the propeller, we should consider the 5th, 10th, and 15th order of vibrations (multiplicity of the rotational speed) and at least the 3rd, 4th, 5th, 8th, and 16th order of vibrations of the main engine. We should note that half harmonic components appear in four-stroke engines (one forcing event for two engine rotations). An additional difficulty is the impact of environmental conditions (wind, waves, etc.) on the ship's [2] dynamics, as well as the variability of the ship's hull dynamic characteristics associated with cargo condition [6]. For this reason, avoiding all local vibration resonances is very difficult. The designer is usually limited to avoiding the main vibration resonance (with the main harmonic components for the propeller and the main engine), for the global ship's hull and superstructure vibration and the main engine body vibrations. For this reason, it is necessary to first calculate the free vibrations of the structure to be analysed.

This manuscript addressed two container ships: a medium-sized, with a capacity of approximately 2700 TEU (standard containers) and a large one, with a capacity of 11400 TEU. The FEM models for both ships are shown in Fig. 2 and 3. The 2700 TEU container ship model contains 7050 nodes and 26207 elements, while the 11400 TEU container ship model contains 25679 nodes and 84182 elements. Most of the items are 4-nodes plate elements. The mass distribution of the models was compared with real ship mass distribution and tuned by changes in the curb weight of the elements. Containers are modelled by 3-D elements for smaller ships, and for bigger ships they are modelled by constraints with concentrated masses. A typical linear material model of steel was used (with constant Young modulus and Poisson ratio). A problem with constraints and added water masses was discussed in the introduction.

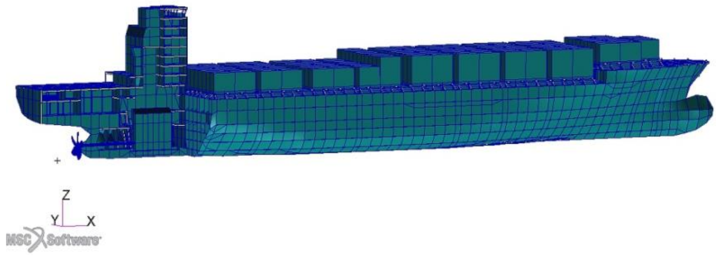

Fig. 2. FEM model of a 2700 TEU container ship

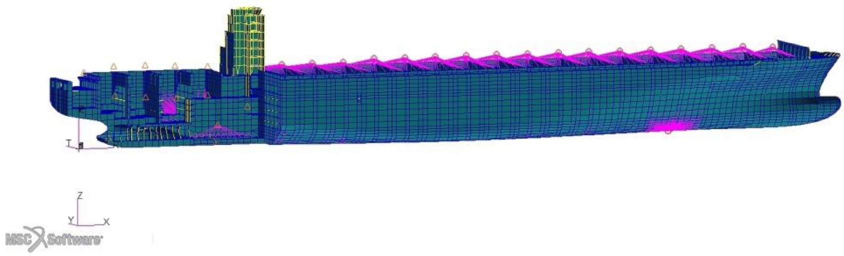

Fig. 3. FEM model of an 11400 TEU container ship

2. Natural Vibrations

Free vibrations allow a qualitative assessment of the ship's excessive vibration risk [15]. Figure 4 shows examples of global hull beam natural vibrations for the ships in question. From the basics of the vibrations theory, it is known that modes (shapes) of eigenvectors are dimensionless. These are vertical and torsional vibration forms with frequencies from 1.6 to 4.6 Hz for a 200-m long container ship and from 0.6 to 1.2 Hz for a ship over 360-m long. Most often, the "beam-type" forms of the ship's hull-free vibrations are benign due to their very low frequencies (except in the extreme cases of very large ships encountering sea state resonance and requiring hydroelastic numerical models). They are usually outside the range of the forcing frequency generated by the drive. At a rated speed of 91 rpm (for the 2700 TEU container ship), the basic forcing frequency of the 8-cylinder engine is 12.1 Hz. However, for this ship, the basic forcing force frequency of the 5-blade propeller is 7.6 Hz. The ship's hull is an example of typical super-resonant vibrations with an important difference. Of course, the forcing frequency may be similar to the higher forms of the ship's hull beam free vibrations. For this reason, at least the first dozen free vibration forms should be considered when analyzing the global vibration of ships. For large ships, such as the 11400 TEU container ship, it may be necessary to define even a few hundred of free vibration forms, to cover the full spectrum of excitation frequencies (in this case, up to 20 Hz). In a smaller ship, the superstructure is situated on the stern to maximize the cargo space. Its potential vibration amplitudes are high for each form. For navigation reasons, large container ships' superstructures are placed closer to the midship. This additionally allows controlling the vibrations of the high superstructures (standing above the container level) more easily.

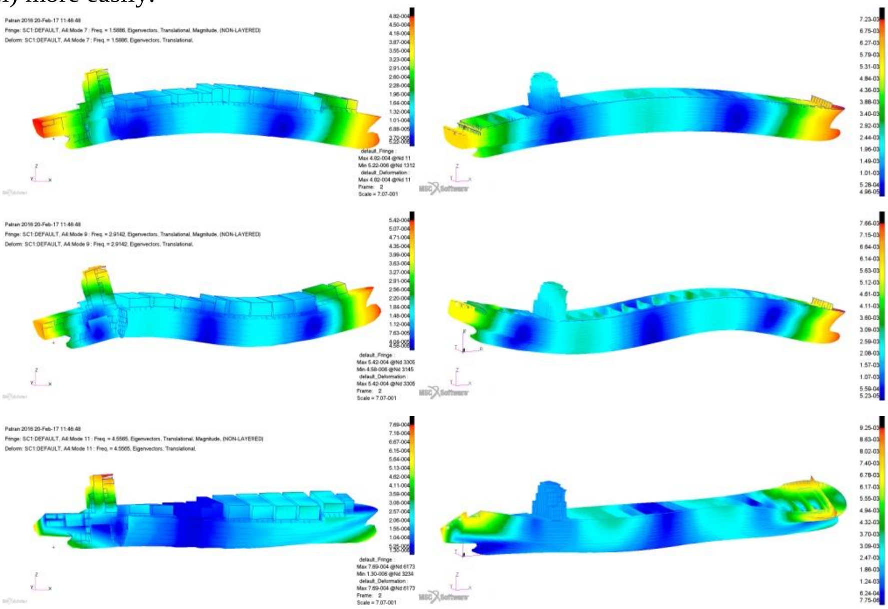

Fig. 4. Global vibrations of 2700 TEU (left) and 11400 TEU (right) container ships

Ship's superstructure vibrations are the most important. They are often combined with the vibrations of the ship's hull and main engine. Figure 5 shows the forms of the discussed ships' superstructure free vibrations. The smaller container ship's deckhouse vibrates relatively independently of the other ship's structures (except for the deck transoms). On the other hand, there are several forms of free vibrations involving the superstructure of the large, very flexible ship. All are strongly combined with various forms of hull vibrations. The methods of analyzing the superstructure isolated from the ship's hull are obsolete. The expected vibration level should be assessed by performing several calculations of the induced vibrations as a function of the forcing frequency (drive rotational speed). The analysis of the alignment of the free vibration frequency difference with the forcing frequency gives too little information if the free vibration forms are numerous and combined with other hull structures.

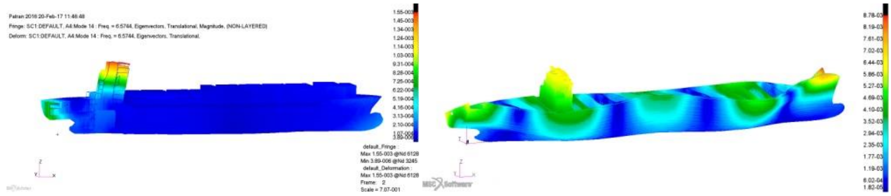

Fig. 5. Superstructure vibrations of 2700 TEU and 11400 TEU container ships

Good consistency between vibration measurements and calculation results is particularly difficult to obtain in marine conditions [3, 4]. Many important design decisions depend on the results of the computational analyses. The reliability and distribution of the vibration calculation results are fundamental in numerical analyses of complex structures. In the first stage of the dynamic calculation, the frequencies and forms of free vibrations need to be specified, as the location of the resonant structure areas depends on them. Free vibrations depend only on the stiffness and mass distribution (the damping effect is negligible). Therefore, the confidence level of the free vibration calculation is relatively high. When correctly calculated, the resonance position should be well defined (with a relatively small error). However, the actual level of the forced amplitude may significantly differ from that resulting from the computational analyses. This is due to poor knowledge of the size of the structural damping in marine conditions. There are no good methods for calculating damping. One can rely on experimentally determined values only.

The first assumption that should be verified during the FEM dynamic calculation is how to create the mass matrix of a mathematical model. A simplified 'lumped'-type matrix may be used, with non-zero elements lying only in the matrix diagonal, or an exact 'coupled'-type matrix for mass coupling, with non-zero elements beyond the diagonal. The effects of using a "lumped" mass matrix are an overestimation error of vibration frequency calculations not exceeding 4%, and the omission of certain free vibration forms. Nevertheless, the omitted vibration modes are of no practical significance (mainly isolated vibrations of separate hull plates). The error may cause the actual resonant rotational speed of the drive to shift downwards by no more than 3 rpm.

The mass matrix is also affected by the ship's performance characteristics. The effect of the ship's loading state on its frequencies and the free vibration forms was examined. The importance of considering the masses of associated water accompanying the hull vibrations was also assessed. The addition of the associated water mass also changes the order of the free vibration form occurrence. If the associated water phenomenon is taken into account, the free vibration frequencies are reduced. This reduction is highest for the lowest ship's hull vibration forms. Regardless of the ship's size, the free vibration frequency decreases by approximately 25% when the hull wetness is taken into account. This applies mainly to vertical forms with a small number of vibration nodes. The more vibration nodes, the less the impact of the associated water. For example, the frequency drop for the 6-node vibration form is 16%. The hull wetting effect on horizontal vibration is also slightly less, at most 20%. The effect of the associated water on the vibrations of the superstructure and main engine is much lower, and does not exceed 6%. This is because neither the superstructure nor the engine body is directly wetted. This impact is so small that it is acceptable not to include the associated water in analyses focused on these ship's elements. The ship's cargo (ballast state -> design cargo state) reduces the hull free vibration frequency. In this case, the ship's size also influences the reduction of the vibration frequency. For the 11400 TEU container ship, the free vibration frequency decrease is even 50%. This is since a large container ship is more flexible (than a small one), and the cargo percentage in its total mass is significantly higher. The ship's vibration image may vary considerably, depending on the cargo state. The measured vibration amplitude levels, which are usually checked once a ship is built during marine tests (ballast condition), may significantly differ from the operational ones (cargo condition).

The defined free vibration frequencies can be roughly pre-verified by comparison with existing empirical dependencies. Of course, verification on the basis of well-conducted experimental tests would be better, but this is not possible at the ship's design stage for obvious reasons. There are many empirical dependencies used to define the vertical and horizontal ship's hull free vibration frequencies. The most known are: the formula of Kumai [1] (see equation 1) and the formula of Yumei et al. [15] (see equation 2).

$$f_n = 1.61 \times 10^6 \sqrt{\frac{I}{\Delta \cdot L^3}}, \tag{1}$$

where f_n is natural frequency of first order vibration, Δ is the displacement of ship, L is length of ship, I is midship section moment of inertia for horizontal axis.

$$f_n = A_n \sqrt{\frac{I}{\Delta \cdot L^3 (1+\alpha)}}, \tag{2}$$

where A_n is natural frequency coefficient of vertical vibration of ship, α is shear and rotational inertia influence coefficient.

Regarding the above formulas, the free vibration frequency is defined by the moment of inertia, the ship displacement, and its length. The ship displacement must take into account the mass of the associated water. Sometimes, in empirical formulas, hull shear corrections are applied. Another interesting aspect is the dependency which links successive forms of ship's vertical free vibrations. The above-mentioned dependencies should only be used for the initial assessment of the ship's hull dynamics. The cause is a large discrepancy between the numerical calculations and the results obtained from empirical formulas. The calculation errors reported in the literature [1] vary from 2 to 17% depending on the ship's type. The differences between the FEM calculations and the dependencies proposed by Yumei et al. [15] for both container ships analyzed are shown in Fig. 6. The estimates of both vertical and horizontal free vibrations are presented there. The results obtained with the dependencies proposed by other authors have similar error rates. The results obtained do not allow the use of the empirical dependencies in ships' dynamics, although the results of vertical free vibrations for the smaller container ship are satisfactory.

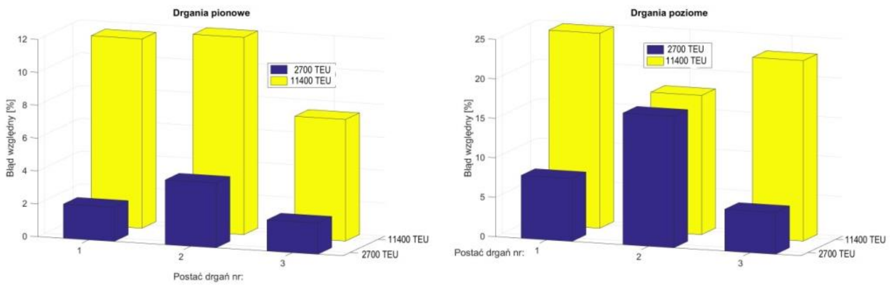

Fig. 6. Errors of calculation made using empirical dependencies

3. Forced Vibrations

Before calculating forced vibrations, a calculation method should be selected. There are two main methods: direct integration and modal superposition. The most common method is the modal superposition of free vibration. In this method, the free vibration forms are combined appropriately. In this case, we should verify the assumption of the number of free vibration forms included in the summation to forced vibrations. If there are not enough free vibration forms to be taken into account, even forced vibration forms might be wrong.

A different problem occurs when analyzing the 11400 TEU container ship's free vibrations. It is a ship with approximately five times the displacement and capacity. The hull design is much more flexible, with much more low free vibration frequencies. Most vibration forms of a given structure (e.g. superstructure) are associated with the vibration forms of the other structures (e.g. bulkhead). Therefore, at least five forms (gathered in two groups) of the superstructure longitudinal free vibrations can be distinguished. All major free vibration forms have frequencies below 5 Hz. To calculate them, 75 normal modes (for the presented analysis) should be calculated. In the range of up to 12 Hz, almost 400 free vibration forms exist. After all, the basic frequency of the engine forces is 20 Hz at rated velocity. In view of the above, the forced vibrations of a large container ship will be analyzed by the direct integration method, not by modal superposition.

The classic approach to calculating forced hull and superstructure vibrations is analyzing them without knowing the dynamic characteristics of the power transfer system (crankshaft, main engine shafting, and propeller). If power transfer system vibrations are not yet calculated and their dynamic characteristics are unknown, the drive bearings reaction is unknown. Therefore, it is impossible to take into account precisely all the excitations shown in Fig. 1. Where such calculations still need to be carried out, they can be easily estimated. In literature [1, 11], the values of dynamic hydrodynamic forces are determined by the percentage of the drive total pressure head or torque. In the same literature [1, 12], empirical dependencies can be found, allowing us to define the values of pressures above the propeller. In summary, the conventional calculation considers mainly forces No. 1 and No. 5 (see Fig. 1) and partly (without considering the main engine shafting vibrations) the forces No. 2 and No. 3.

When analyzing the hull and superstructure dynamics, two main sources of forcing are discussed: the propeller and the main engine [1, 6]. The vibration amplitudes (forcing forces) summation must take into account the time history phase angles between the propeller' blades and the engine's cranks. In the case of excitations induced by the propeller, the dominant force is the harmonic component associated with the number of propeller blades (for the concerned ships, the 5th and 6th component). Engine excitations are represented by a whole spectrum of harmonic components, the most dangerous of which is related to the number of engine cylinders. However, different harmonic orders could be dangerous. The angular mutual alignment of the propeller and crankshaft may significantly affect the superstructure vibrations level [6]. An example of the amplitude distribution of the 2700 TEU container ship's forced vibrations by main engine unbalanced forces, for the 3rd harmonic (X type forcing), for the nominal drive rotational speed (91 rpm), is shown in Fig. 7. The 11400 TEU entire container ship forced vibration forms are not as "clean" as for a smaller container ship. Figure 8 shows the main vibration forms for the considered ship, i.e. those forced by the 3rd and 4th harmonic components of the non-balanced engine moments. Figures 7-10 are from MSC-Software's commercial program Patran. The author has limited influence on the size of descriptions of numerical scales. For this reason, the maximum magnitude of the determined vibrations is given under the descriptions of the drawings.

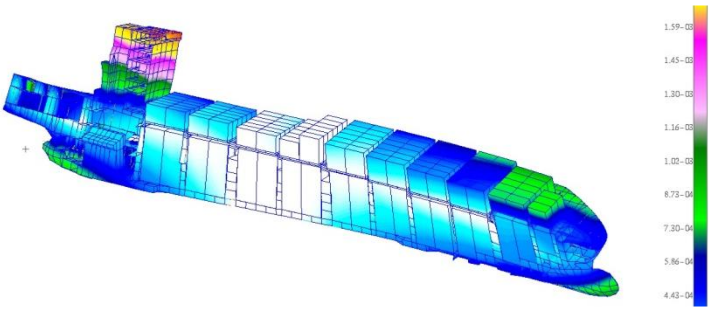

Fig. 7. Vibration velocity amplitudes for the 2700 TEU container ship, forced by the third harmonic component at 91 rpm (max amp. 1.59 mm/s)

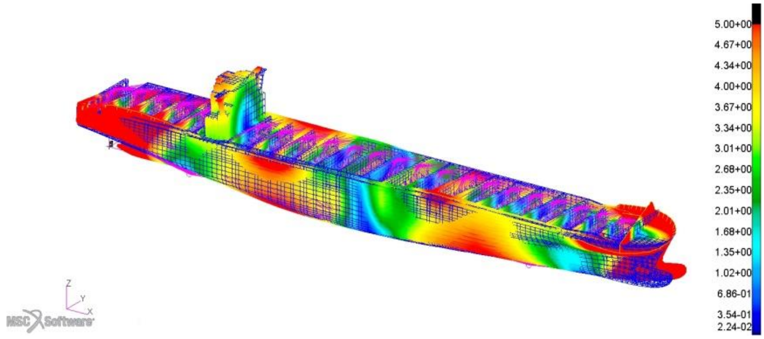

Fig. 8. The 11400 TEU container ship vibrations of 3.2 Hz (48 and 64 rpm) frequency, forced by the 3rd and 4th engine harmonic components (max amp. 5.00 mm/s)

The calculations for the 2700 TEU container ship were done for the same operating velocities as for engine forcing analysis. The most important are resonant rotational speed (83.3 rpm) and rated rotations (91.0 rpm). As shown in Fig. 9, the velocity amplitude fields of vibrations forced by the propeller have similar distributions for both velocities. The 11400 TEU container ship vibrations are more complex compared to a smaller ship vibration. An image of the ship vibration velocity amplitude distribution for drive rotations of 51 rpm is shown in Fig. 10. The higher the engine rotational speeds, the more local vibrations are revealed.

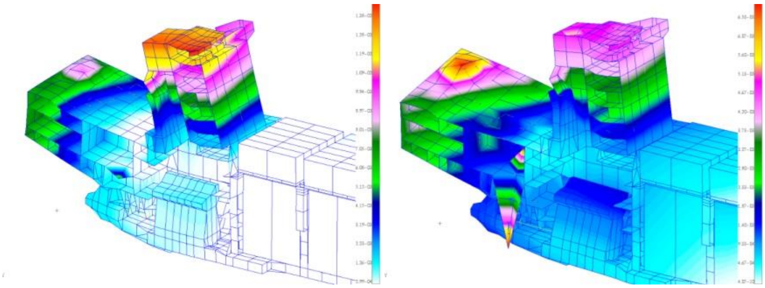

Fig. 9. Vibration velocity amplitudes for the 2700 TEU container ship, forced by the pressure field on the deck transom, for 83.3 rpm (max amp. 13.8 mm/s) and 91.0 rpm (max amp. 6.53 mm/s)

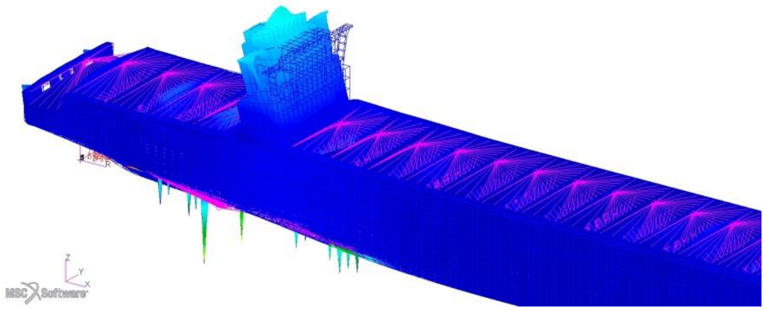

Fig. 10. Vibration velocity amplitudes for the 11400 TEU container ship, forced by the pressure field on deck transom, for 51 rpm (max amp. 2.34 mm/s)

When minimizing vibrations, the selected point vibration amplitude and phase must be determined first, for both excitations sources. For the main engine excitations, the vibrations caused by the coupled longitudinal vibrations and the vibrations caused by the main engine unbalanced moments are to be added together. Propeller excitations consist of water pressure pulses on the deck transom and forces induced by drive uncoupled longitudinal vibrations and of the main engine shafting flexural vibrations. The following analysis will be carried out for a smaller container ship due to wider range of available documentations. Only the 5th harmonic component will be considered in this analysis, as the principal propeller excitations are of the fifth order, and only the same order vibrations can be added as vectors. Figures 11 and 12 summarize the longitudinal vibrations levels of the superstructure bridge wing, produced by the propeller and main engine. Summary results by excitations type are presented.

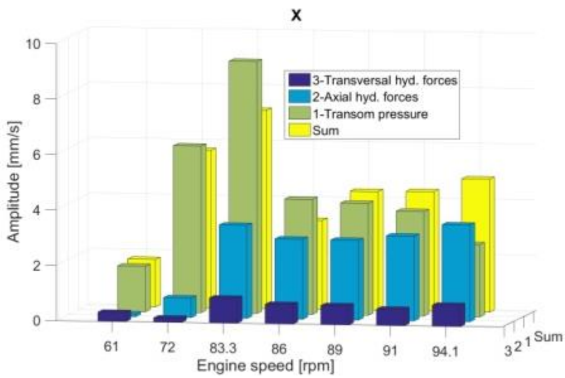

Fig. 11. Vibration velocity amplitudes for the container ship in the longitudinal direction on the bridge wing, excited by the propeller

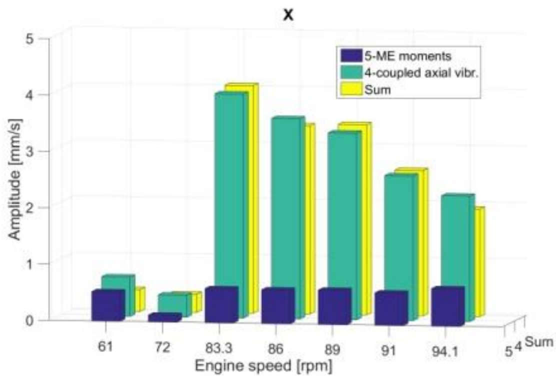

Fig. 12. Vibration velocity amplitudes for the container ship in the longitudinal direction on the bridge wing, excited by the main engine

Since the total vibrations from the propeller and main engine have similar amplitude levels, it is possible to minimize the vibration by changing the propeller and crankshaft crank angular position. Each vibration direction and each drive rotational speed can be minimized. The admitted optimum angle (the propeller blade optimal translation angle in relation to the first crankshaft crank) is α=-20.3°, it gives a minimum level of superstructure longitudinal vibrations at the nominal engine rotational speed. In reality, the propeller and the crankshaft relative angular position are very often random. Therefore, the distribution of calculation results compared to the measurement tests can be significant. Figure 13 shows the size of distribution, i.e. errors which may result from the accidental propeller and crankshaft phasing for the 5th harmonic component. The reference level is the average of the expected vibration amplitudes. During measurements, total vibrations from all harmonic components are measured and standardized. Vibration measurements of the ship's hull and superstructure were carried out using standard methods. A 4-channel measuring system by Brüel & Kjær was used, equipped with sensors - piezoelectric accelerometers. At rated rotations, the expected 5th harmonic component results distribution is about ±60%. However, the total vibration amplitudes distribution will be smaller. The difference in the 5th harmonic results is multiplied by its percentage in the total vibration. It was assumed that the propeller has one dominant harmonic component, i.e. the first blade (5th harmonic).

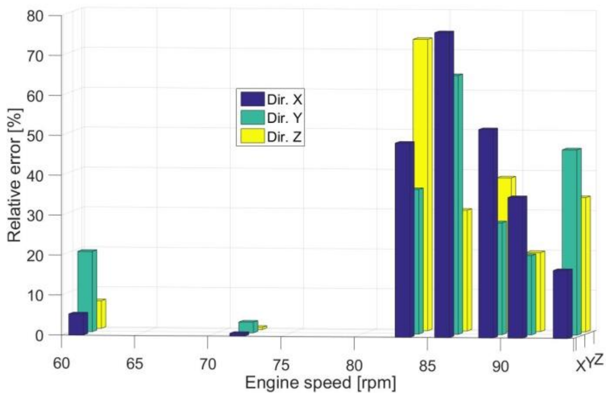

Fig. 13. Expected vibration amplitude result distribution of the 5th harmonic component, for the 2700 TEU container ship's variable propeller and engine phasing

The measurement tests of the 2700 TEU container have been performed. A comparison of the results of calculations with the results of measuring tests is shown in Fig. 14. As the measurements results usually give the values of vibration velocity effective amplitude, the calculation analyses results have been recalculated accordingly. The comparison was presented for the most important hull reference points: the superstructure bridge wing.

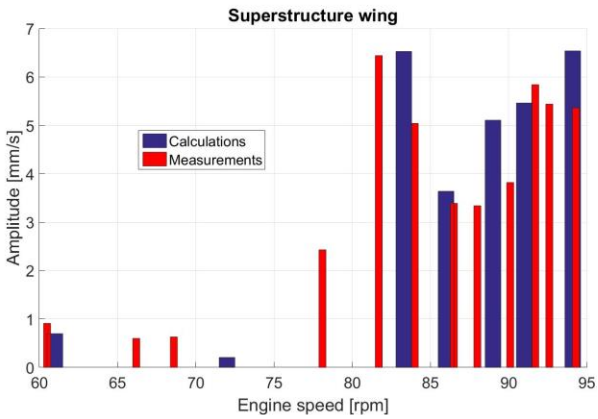

Fig. 14. Measurement and calculation verification of the superstructure bridge wing vibrations, for the 2700 TEU container ship, in ballast condition

4. Conclusions

In comparison to the propeller, the main engine varies in terms of its rotational speed, ship cargo state and the direction of the vibrations. Therefore, it is impossible to eliminate ship vibration to levels not noticeable by the crew and passengers. Due to the complexity of the ship vibrations, it is also difficult to reduce it effectively, especially after the ship has been built. For this reason, in-depth calculation analyses of the ship dynamics are extremely important at the design stage. However, it is important to be aware of the inevitable dispersion of calculation results (and measurement tests).

The presented measurement and calculation verification fully confirmed the validity of the assumptions made and of the used calculation methods. The consistency between the measurement tests and the numerical calculations is highly satisfactory in light of the analyses carried out for the expected dispersions and errors in the numerical modelling of a physical object, the hull. However, different vessel sizes may require different methods of calculation analysis. For this reason, the level of calculation errors may also vary. Each type of ship requires an in-depth analysis of the calculation methodology.

Based on the performed analyses, resonant frequencies should be provided for calculations with good accuracy. However, even correctly performed calculations of vibration amplitudes may significantly differ from the amplitudes obtained by measurement tests. It should be stressed that measurements, especially those carried out in marine conditions, are also subject to several significant errors and dispersions.

In the future, the author, together with the Gdynia Maritime University research intends, intends to conduct similar measurement studies on smaller ships (university school ships). Test results will be compared with appropriate numerical calculations. Contact was established with Croatian researchers (Zagreb University), who cooperate with DNV and Korean shipyards. With this cooperation, research will be possible for the largest sea-going ships. The planned research will allow the conclusions presented in this paper to be generalized.

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