SILESIAN UNIVERSITY OF TECHNOLOGYPUBLISHING HOUSE

SCIENTIFIC PAPERS OF SILESIAN UNIVERSITY OF TECHNOLOGY 2023
ORGANIZATION AND MANAGEMENT SERIESNO. 172

ASSORTMENT ABC CLASSIFICATION PROBLEM AT THE WAREHOUSE MODELLED AS A MULTI-KNAPSACK

Kateryna CZERNIACHOWSKA[1*], Marcin HERNES[2], Sergey SUBBOTIN[3]

[1] Wroclaw University of Economics and Business; Kateryna.Czerniachowska@ue.wroc.pl, ORCID: 0000-0002-1808-6020
[2] Wroclaw University of Economics and Business; Marcin.Hernes@ue.wroc.pl, ORCID: 0000-0002-3832-8154
[3] National University "Zaporizhzhia Polytechnic", Ukraine; Subbotin@zntu.edu.ua, ORCID: 0000-0001-5814-8268
* Correspondence author

Purpose: Appropriate product categorisation in warehouses is an issue facing most warehouses and distribution centres around the globe today. The ABC classification scheme aids in determining the most vital values at the warehouse. ABC classification reduces the effects of excess, end-of-life, or huge volumes of phase-out products.
Design/methodology/approach: ABC classification is a method for classifying products according to their relevance. Products are divided into three categories using the ABC analysis, with category "A" items being the most important, category "B" as medium important and category "C" items being the least important for the distributor. This research model the ABC classification problem as a multi-knapsack and provides an example of commercial and non-commercial solver usage that enables distributors to categorise assortment according to the ABC classification.
Findings: Proposed approach enables the distributor to arrive at the best possible outcomes.
Research limitations/implications: The main limitation of this research is that it does not take into consideration constraints that show that some products cannot be quickly categorised or placed on the shelves because of the availability of storage equipment or warehouse personnel at the time of classification should be considered. Further research may be done on these issues.
Practical implications: Performance comparisons between the proposed approach and the traditional ABC classification method provided by the distributors are conducted.
Originality/value: The main contribution is the improvement of the classification method used in warehouses these days. The proposed approach allows assigning an optimal product mix to ABC categories.

Keywords: ABC classification; multi-knapsack; distribution centre; warehouse.

Category of the paper: Research paper.

1. Introduction

Assortment planning models in the literature and decision selection problems like category management concentrate on the selection of goods in a single category. In other words, variety within a category influences demand contingent upon store visits but has no relation to store choice. However, if the shop cuts diversity across the board based on single category analysis, the business loses appeal, and some consumers may migrate to other merchants, which would decrease customer foot traffic. This concern is especially pertinent to basket consumers since they may buy their entire basket from another vendor if they cannot locate the item they are looking for in one category (Cachon, Kök, 2007).

Each store offers a unique selection. All variations in demand can be taken into account in this situation, but maintaining tailored assortments is expensive.

Since small businesses have close relationships with their consumers, they are able to understand their requirements, purchasing patterns, and seasonal income patterns, which are reflected in the decisions they make regarding their stores (Koul, Jasrotia, 2019).

Cachon and Kok (2007) claimed that each consumer type in their model shop choice is based on the diversity of a particular category, which they refer to as the lead category (Cachon, Kok, 2007).

Rodríguez et al. (2017) considered the following issues given by consumer preferences for the attribute values that might be assigned to the product (i.e., its potential features), the attribute values of goods sold by their rivals, and the attribute values that each producer can use depending on, for example, technological constraints, legal considerations, or the availability of resources.

Three problems that are fundamental to any marketing management process are formalised, and their computational complexity is examined by Rodríguez et al. (2017):

- maximise the number of clients by choosing the product’s attributes,
- determine whether there is a workable strategy that ensures the reach to a specific average number of customers before a deadline,
- determine whether there is a workable strategy that ensures the reach of the maximum customer traffic before the cutoff date of the product’s characteristics (Rodríguez et al., 2017).

Firms can no longer rely on the old sales performance techniques as technology and organisational structures get more complicated, the competition for product differentiation increases and customers become product masters. Organisations must evaluate existing approaches, retest presumptions, and apply innovative solutions to meet today’s expectations (Forum Europe Ltd, 1997).

As an alternative, a shop can use a centralised regime to decide on categories for the whole store. Cachon and Kök (2007) demonstrated that category management never comes up with the best answer and offers both less selection and more expensive rates than ideal (Cachon, Kök, 2007).

The Pareto Principle is the foundation of the well-known classification method known as ABC analysis, which helps businesses decide which goods should be managed as priorities in their inventory. Modern operations management and supply chain literature that covers this subject concentrate on using cost value as the only criterion for categorisation. Jemelka et al. (2016) contended that in order to survive, organisations and supply chains today must be able to offer the right items quickly to relatively niche consumers. This concentration on a single criterion is necessary since there are global suppliers, intermediaries, and buyers, and product lives are shortening quickly (Jemelka et al., 2016).

There are several relatively easy and inexpensive strategies to manage inventory effectively, but some of them are underutilised. It includes the assortment ABC analysis. According to such analysis, the inventory is classified into three categories – Category A, Category B, and Category C. A method that assigns products based on how often they are purchased is used in this class-based storage assignment policy. The method's goal is to store rapidly moving goods close to the delivery zone. The main limitation of the existing approach is that the solution is not optimal.

In this research, we modelled the ABC classification problem as a multi-knapsack problem which differs from the basic multi-knapsack problem having additional bin cost variables in the objective function. The multi-knapsack problem is the problem of assigning a subset of items to the number of distinct knapsacks in a way that the total profit sum of the selected items is maximised without exceeding the capacity of each of the knapsacks (Pisinger, 1999).

The main contribution is the possibility to assign an optimal product mix to ABC categories which improves the solution used in the warehouses at the moment. The next contribution is the demonstration of solver usage as an optimisation engine for solving warehouse tasks. Therefore, we use commercial and non-commercial solvers to solve the warehouse product classification problem optimally.

The work is presented as follows. In Section 2, the relevant literature review. Section 3 presents the definition, notation and mathematical formulation for the ABC assortment classification problem. In Section 4, we briefly describe the input data for the experiment. Section 5 presents the computational experiment. Final comments and possible research directions are given in Section 6.

2. Related literature

2.1. Assortment problem

The measurements of the assortment — assortment width and assortment depth — are used to categorise the products offered in a warehouse and the retail sector. The assortment depth represents the diversity of certain product groupings, whereas the assortment breadth describes the range of various goods.

A wide selection can add value for customers by facilitating shopping. While going too far too soon can be perilous, sticking too closely to the current assortment and image may unfairly restrict the retailer's ability to innovate (Danneels, 2003). The capacity of a retailer's selection policy to successfully develop its significance and appeal to customers over time depends on the logic and sequencing of that policy (Ailawadi, Keller, 2004).

Profitable retail assortment optimisation necessitates the capacity to acquire up-to-date information about product performance across the company, as well as the ability to assess current and historical revenues and profits with trends and execution versus target. Only then can merchants use benchmarking to optimise their selection.

Cachon et al. (2005) created two search models. In general, the independent assortment search model assumes that each retailer's selection is distinct; therefore, if the search option is selected, the consumer expects to see several varieties when searching. Therefore, the consumer's anticipated value from search is unrelated to the retailer's selection (because search yields different variants no matter which variants the retailer carries) (Cachon et al., 2005).

The overlapping assortment model is the name given to the second search model created by Cachon et al. (2005). There are only a few goods on the market in this instance. Because there are fewer potential new versions for the consumer to see if the search is chosen, growing the retailer's selection now decreases the value of the search. A deeper assortment enhances the chance a buyer purchases some variety because it decreases the value of search, which is a significant impact that is not reflected by the no-search model (Cachon et al., 2005).

Researchers have documented the impacts of cross-selling products. It is well-accepted that there is a correlation between buying consumer goods from different categories that complement one another or act as substitutes for one another (Urban, 1998; Seetharaman et al., 2005; Raeder, Chawla, 2011; Oestreicher-Singer et al., 2013).

When there are several things to exhibit but little available shelf space, the issue gets more challenging. Retailers are therefore being forced to create effective decision support systems to control product availability due to low profits, the increasing demand for operational efficiency, and an ever-increasing focus on the needs of the consumer (Hübner, 2017).

Hariga et al. (2007) assert that inventory control, shelf space allocation, product selection, and item display area selection are essential procedures that significantly affect the financial operation of business owners. In order to avoid sub-optimality, the decision-making process for various operations should be specifically connected. However, this integration, along with the inherent non-linearity of relevant mix models, will raise the complexity notably for real-world applications (Lotfi, Torabi, 2011).

Several recent works examine assortment selection and stocking decisions for a group of interchangeable items in a single category (e.g., van Ryzin, Mahajan, 1999; Mahajan, van Ryzin, 2001; Smith, Agrawal, 2000; Kok, Fisher, 2007; Hübner, Schaal, 2017), while other publications solely examine stocking decisions (Netessine, Rudi, 2003; Yücel et al., 2009).

Retailers may improve store groups and drive appropriate assortments by form, shop, department, and category by starting product assortment optimisation process flow using current behavioural-based shopping clustering data. Retailers may get rid of duplicates and underperforming. Stock Keeping Units (SKUs) with different retail assortment planning strategies. They prevent out-of-stocks and overstock, boost supply chain efficiency and forecast accuracy, and, most importantly, raise customer happiness and profitable revenue growth.

Simonson (1999) focused on the ways in which retailers can influence purchase decisions with the help of product assortment:

1. Customers typically evaluate only a group of the available products, with no regard to the entire assortment. Therefore retailers can influence which product groups should be evaluated together and create such groups to achieve their goals.
2. If the assortment is constant, the retailer can change the manner in which the products are displayed.
3. The assortment preferences could be manipulated by the marketing mix variables such as sales promotions. Therefore the retailer could continue achieving the sales goals (Simonson, 1999).

Sales companies must be prepared to review their fundamental strategies, reevaluate generally held presumptions, and incorporate fresh approaches to the ever-evolving problem of selling in order to meet the needs of the modern marketplace. Forum Europe Ltd (1997) presented the results of the research into sales productivity. Their goal was to create a foundation for improving a business' sales efforts.

Modern retail faces constant issues of managing space and selection in diverse shop layouts. Supported by information and communication technology, retail sectors are progressively resorting to e-retailing (and its variants such as retail through social networks and mobile shopping), which permits considerable overcoming of the restrictions of space and selection. Nonetheless, the vast majority of retail sales continue to take place in stationary retail formats, where suppliers (manufacturers and distributors/wholesalers) try to tackle the persistent challenge of a trade-off between limited space for product demonstration/sale and a diverse range of items (Dujak et al., 2016).

2.2. Assortment substitution

Substitution is a crucial factor in assortment planning. Customers can choose an alternative product if one they'd prefer isn't available (or choose not to buy any other product). Typically, there is only one round of replacement allowed, so if a client cannot find both her original pick and her alternative choice, the sale is lost (Corsten et al., 2018).

Subject to a maximum number of distinct assortments, assortments can range from store to store. When the customer's initial preference is not known, Fisher and Vaidyanathan (2009) introduced a model of substitution behaviour and took into account the impact of substitution while selecting assortments for the retail chain. To estimate the demand for attribute levels and replacement probabilities, they used the sales history of the SKUs that the retailer carried. They then extrapolate this information to estimate demand for any potential SKU, even those that the shop does not presently carry. The researchers provided a number of different criteria for selecting SKUs (Fisher, Vaidyanathan, 2009).

Fisher and Vaidyanathan (2014) developed a method for determining the best assortments, which includes a demand model, an estimation strategy, and assortment selection heuristics. Substitution is quantifiable, varies greatly, and significantly affects the ideal assortment (Fisher, Vaidyanathan, 2014).

The assortment planning problem for a single store using the exogenous demand model was modelled by Hübner et al. (2016) and Kök and Fisher (2007). Furthermore, it's frequently believed that substitution only occurs between items belonging to the same category.

On the one hand, Hübner et al. (2016) suggested using a stochastic news vendor model for perishable and non-perishable goods that simultaneously optimises assortment and order sizes under storage constraints, and that makes it possible to solve retail-specific specific problems using an optimal procedure or a near-optimal solution heuristic procedure for large problem cases. They consider product categories that are collectively delivered just once throughout a sales period, such as daily, every other day, just once during a season, etc.

On the other hand, Huh et al. (2016) discovered that buyers are likely to stay inside a category even though they believe the substitute product to be inferior to the original. According to their hypothesis, cross-category substitutes are paradoxically more rewarding than within-category substitutes due to their greater likeness to desired stimuli. Sloot et al. (2005) analysed switching categories and concluded that this tendency was minor. They examined the effect of product-, store-, situation-, and consumer-related variables in their study.

Campo et al. (2004) examined how consumers respond to unexpected, transient out-of-stocks (OOS), as contrasted to permanent assortment reductions (PAR). They compared and contrasted OOS and PAR reactions, as well as the underlying causes of each, and then empirically tested our theories in two different product categories. Their findings suggested that store losses in the event of a PAR could be significantly greater than those in the event of a stock-out for the same item (Campo et al., 2004).

If a customer's first choice product isn't offered by a retailer, they might choose to buy nothing or another item that is close enough to their top choice to make them eager to buy it. Both assortment optimisation and estimate must take into account the probability of substitution. In order to estimate substitution probabilities, it is necessary to separate sales of an SKU to customers who most favoured that SKU from sales to customers who preferred a different SKU but made a substitution when their favoured SKU was not available in the selection (Fisher, Vaidyanathan, 2014).

2.3. ABC classification

The majority of organisations today struggle with properly managing their inventory. When distributors are juggling a variety of consumer demands, it can be challenging to keep the optimum level of inventory at the warehouse. Nevertheless, the distributor will be able to manage assortment effectively if he has a strong inventory control system.

With the economy's recent rapid growth, citizens' way of life and level of consumption has quickened, and demand for retail services is rising as well. People have progressively started to do their grocery shopping at chain stores. Activity-based classification (ABC) is used in some supermarkets' warehouses. The primary method for classifying commodities in storage is the ABC method. Lin and Ma (2021) provided a novel method based on ABC classification, specifically for a niche market like chain supermarkets, in order to address the drawbacks of conventional classification techniques. (Lin, Ma, 2021).

Customers' perceptions of the variety of goods and services that a retailer provides in one place have a big impact on the image of the business. The advantages of a large selection are obvious. First, the retailer's salience increases with the breadth of the product selection since there are more scenarios in which the consumer may remember and consider the merchant. Salience is the most fundamental component of a brand. Second, the ease of one-stop shopping made possible by a wide product selection is more crucial than ever for today's time-pressed consumer (Messinger, Narasimhan, 1997; Ailawadi, Keller, 2004), which puts pressure on merchants to expand their selection. Third, consumers frequently browse at multiple stores; they might choose to buy a certain category in the store they are now visiting rather than another one depending on the in-store selection and marketing mix activities. Retailers with a wider selection benefit from the fact that unplanned purchases make up a sizeable amount of consumers' overall shopping baskets (Ailawadi, Keller, 2004).

Retailers have the option to differentiate their product lines by offering things that are exclusive to the market, but doing so can make pricing comparisons more difficult or, in the worst-case scenario, impossible. Where the potential financial rewards from discovering a lower price far outweigh the expenses of search, the consequences of a one-time search for an item (for example, the purchase of a big appliance) on retailer decisions regarding price and assortment are particularly applicable (Stassen et al., 1999).

The design of the warehousing and handling system goes through a number of steps, starting with the definition of the system limitations and requirements, and concluding with the evaluation of the preferred design. The goal of data analysis is to provide the basis for the designer's suggestions for appropriate operational systems and techniques, layouts, furnishings, technology, workforce levels, and pricing. There are numerous ways to exhibit and analyse data, including graphs, charts, statistical analysis, tables, drawings, and networks (De Koster et al., 2017).

Based on the Pareto principle, which claims that 20% of people are responsible for 80% of the impacts, ABC analysis is a straightforward method of classifying materials in terms of value and quantity. The Pareto principle for inventory management states that 20% of SKUs use 80% of the total annual cost. SKUs are categorised based on their Annual Usage Value, as determined by the ABC analysis.

- A class: the highest annual consumption value.
- B class: the medium consumption value.
- C class: the lowest consumption value (Mor et al., 2021).

The XYZ analysis is a method for allocating products according to changes in demand. The goal of the XYZ analysis is to organise the products according to their consumption in order to determine the best inventory plan. The following are the classification procedures:

- X class: the minimal variance.
- Y class: some variance.
- Z class: the most variation (Mor et al., 2021).

FSN analysis is a method of classifying inventory items as groups according to the rate of movement in the warehouse. Such classification is given as:

- F class: fast-moving.
- S class: slow-moving.
- N class: non-moving (Mor et al., 2021).

Presenting sales amounts, picking accessions, and inventory levels throughout the entire spectrum of SKUs in descending order of magnitude is one really helpful analytical tool. Pareto analysis, ABC analysis, or the 80/20 rule are other names for this technique. The results of 80/20 analysis, more frequently than not, show that around 20% of the stock range accounts for roughly 80% of the total inventory, 80% of the picking efforts and 80% of the sales. In order to create solutions that are appropriate for the material being held and handled, the designer is able to categorise and identify the considerably relevant SKUs in the range of items as well as identify various characteristics for various areas of the item range (De Koster et al., 2017).

The methodology presented in the study by Mor et al. (2021) can be used to identify the parts that will become obsolete in an automobile spare parts warehouse. The authors proposed a framework based on ABC-XYZ and FSN analysis to prioritise the spare parts based on their criticality, and inventory management techniques are used to reduce costs (Mor et al., 2021).

An effort was made to utilise lean concepts to increase the effectiveness of a manufacturing industry warehouse's operations as part of a continuous improvement approach. By using appropriate inventory models, such as ABC analysis, inventory can be managed more effectively. Baby et al. (2018) showed the sales warehouse as a representation of a number of problems with product delivery to clients. The authors aimed to improve warehouse operations by offering lean alternatives. Wastes are classified into transportation, inventory, waiting, delay, space, movement, overproduction, and defect waste groups. Wastes are discovered using lean technologies. Value stream mapping was utilised to track waste, and by switching to a U-shaped flow pattern and completing ABC inventory analysis, the majority of operational wastes, such as long order picking times, delayed vehicle loading, and incorrect storage, are avoided. Depending on ABC inventory analysis, the new inventory arrangement places products in order of their contribution to sales. Most warehouse operations have at least a 40% improvement (Baby et al., 2018).

Truly benefit is mostly dependent on quick order fulfilment; hence slow order picking deters customers. By creating a dynamic class-based storage assignment algorithm that puts quickly moving goods on lower shelves and closer to the shipping area, the goal of this project by McInerney and Yadavalli (2022) was to boost warehouse throughput. An ABC class-based storage policy was used in place to achieve this. The preferred storage locations are given to the A-class products, the second-best storage locations to the B-class products, and the remaining storage spaces to the C-class products (McInerney, Yadavalli, 2022).

Because sometimes nearby containers are not available, similar products are frequently stored in separate locations. Additionally, when the default container is empty, the computer system does not direct pickers to alternative containers. Pickers then report inaccurately that certain items are out of stock. There would be fewer needless restocks and order delays if the products were distributed according to a storage policy that placed them in close proximity to default and secondary containers (McInerney, Yadavalli, 2022).

The relocation of stock sections and the reduction of forklift handling distances were studied by Jemelka et al. (2016). Forklift tracks can be shortened by analysing the stock in accordance with the turnover of the raw materials and dispersing the storage areas in accordance with the ABC technique based on the turnover. The authors also included a comparison of the original 12-section plan based on the actual experience of warehouse operators and the computed solution based on the raw material cycle using the ABC approach (Jemelka et al., 2016).

Li et al. (2016) developed an integrated mechanism for optimisation purposes based on the ABC categorisation and the mutual compatibility of products and presented a novel dynamic storage assignment problem. A data mining method known as the "product affinity-based heuristic" was created for computing pairwise associations between products.

Incoming SKUs are distributed among the available locations according to random storage procedures. Class-based storage assignment methods place SKUs in regions designated for their corresponding classes based on their demand rates and ABC classification. By reducing the necessary travel lengths, the suggested storage assignment approach aims to optimise the order pickup procedure (Li et al., 2016).

Designing a warehouse and handling system involves more than just producing a simple drawing that specifies the dimensions and locations of racks or other storage spaces, handling areas, aisle runs, and vehicle charging points. In addition to that, it specifies things like the unit loads (like pallets), operating systems, equipment types and quantities, service and ancillary activities, communication and information systems, organisational structure, number of employees, and associated operating and capital costs. The external layout and the necessary room for vehicle access, parking, and manoeuvring, as well as for site security, parking, and any other operations, should also be indicated (De Koster et al., 2017).

Among the present issues at the warehouse are the poorly planned warehouse layout and the close proximity of the shelves, which made it impossible for the warehouseman to control the inventory of the goods. Due to overproduction, the overstocked supply typically arises at the end of the year (Hanafi et al., 2019). Hanafi et al. (2019) rearranged the facility layout depending on demand using the ABC classification approach.

The system that may reduce the total costs to the supplier meets one of the best requirements for a trustworthy supply management system. The entire cost is determined by the cost of storage, the cost of orders, and the cost of inventory shortages. On the other hand, only quantitative and qualitative factors are taken into account while making decisions in traditional supply control management (Hanafi et al., 2019).

2.4. A traditional method of ABC classification

The ABC classification method is a useful tool for identifying key products within the warehouse. From the standpoints of manufacturing, inventory control, revenue-generating, and sales generation, it is very important to categorise the products.ABC categorisation adds the most value to the business.

Companies shouldn’t allot the same amount of resources to each product in a warehouse because not all of them generate the same revenue and profitability. The ABC analysis is a method of inventory classification that divides the products into three categories: A, B, and C, according to their revenue.

Since there is no perfect universal classification, the appropriate one must be selected based on the specifics of each organisation. In the ABC system, there are four primary classification categories for product references:

- Classification of ABC products by rotation.
- Classification of ABC products by unit cost.
- Classification of ABC products by total inventory value.

- Classification of ABC references by use and value (AR Racking, ABC Inventory Method in the Warehouse: Origin, Characteristics and Advantages, https://www.ar-racking.com/en/news-and-blog/storage-solutions/quality-and-security/abc-inventory-method-in-the-warehouse-origin-characteristics-and-advantages).

Category A: Goods falling under category A are the most valuable and important items. About 20% of all products make up segment A products, which generate 80% of the company's revenue. It is regarded as a niche market with few products but high sales (Deskera; https://www.deskera.com/...).

Being the priority references category, the corporation must commit more resources to it to carry out, periodically and frequently, more extensive and complicated stock controls. A company will incur significant losses if there is an inventory issue with category A products, such as a stock shortage or depletion. To expedite the order preparation process, category A products in the ABC model must be placed in easily accessible locations adjacent to the dispatch area. This must be taken into account in order to create the warehouse plan and properly organise all of the stock. Products in this category can be kept in storage facilities with easy access to unit loads or, when appropriate, in automated storage facilities to reduce the time it takes to load and unload cargo (AR Racking, ABC Inventory Method in the Warehouse: Origin, Characteristics and Advantages, https://www.ar-racking.com/en/news-and-blog/storage-solutions/quality-and-security/abc-inventory-method-in-the-warehouse-origin-characteristics-and-advantages)

Category B: Products in category B are slightly more expensive than those in segment B. It controls 30% of the market for goods and generates 15% of the revenue. In addition, although there are more items in this category, they are less useful (Deskera; https://www.deskera.com/...).

Any reference that falls into the category that lies between A and C should have its status frequently examined in case it needs to be changed to either A or C in the future. After organising and reserving the finest spots for the A references, they will be placed in the warehouse's most direct and accessible locations. Typically, category B products are kept in intermediate levels with convenient access to some unit loads but not always (AR Racking; https://www.ar-racking.com/...).

Category C: Products falling under category C are more numerous but less effective at bringing in revenue. Comparatively to categories A and B, category C has the highest stock ownership at 50% but only produces 5% of the revenue (Deskera; https://www.deskera.com/...).

Due to their low demand and limited turnover in the warehouse, these products should only receive the barest minimum of resources. Inventory control may be occasional, employ basic techniques, be situated in the warehouse remote from the dispatch location, at greater or lower accessibility levels, and be sufficient to prevent obsolescence or expiration issues. Category C references must be evaluated to determine whether investing resources in storage and stock is desirable for the organisation, as storage costs may end up being more than the income realised through marketing (AR Racking; https://www.ar-racking.com/...).

To sum it up, A represents the most vital items, B indicates moderately necessary commodities, and C indicates the least critical inventories. Any organisation determines how the ABC categories are divided, but the percentages should be roughly 80%, 15%, and 5%. In the ABC analysis, "A" stands for the most significant inventory, "B" for moderately necessary inventory, and "C" for the least important inventory (AR Racking; https://www.ar-racking.com/...; Cips; https://www.cips.org/...; Deskera; https://www.deskera.com/...; EazyStock, Drakeley; https://www.eazystock.com/...; Katana; https://katanamrp.com/...).

ABC classification is performed according to the following steps:

1. Calculate sales/profit/consumption of each product during the investigated period.
2. List the products in descending order based on their sales/profit/consumption value.
3. Sum up the number of items sold and the sales/profit/consumption value.
4. Calculate the cumulative percentage of items sold and the cumulative percentage of the sales/profit/consumption values.
5. Determine the thresholds for splitting the data into categories A, B and C. The threshold for determining the ABC split groups will be unique to the company and the offered assortment. In this research, we set 80%/15%/5% for categories A, B and C (EazyStock, Drakeley; https://www.eazystock.com/...)

3. Problem definition and formulation

P - the total number of products.

B - the total number of categories.

i - product index, $i = 1,...,P$.

j - category index, $j = 1,...,B$.

v_i - value of the product i.

c_j - cost of the category j.

W_j - capacity of the category j.

Decision variable:

$$x_{ij} = \begin{Bmatrix} 1, \text{ if product } i \text{ is assigned to category } j \\ 0, \text{ otherwise} \end{Bmatrix}$$

There is a number of products P at the warehouse. The products must be classified into B categories (such as A, B, C) according to their rotation in the warehouse. The most demanded products and, therefore, those products that generate the most movements in the warehouse should be assigned to category A, and those with hardly any rotation to the last category (e.g. C). The demand or value of the product is characterised by v_i. Each category j has its capacity according to the Pareto principle W_j. The costs resulting from product storage and moving from the warehouse are characterised by c_j. Generally, the storage costs for the lowest category (e.g. C) can be even higher than the profitability obtained from their marketing. The lowest costs are in the first category A.

The goal is to assign products to the categories so that distributor's profit is as much as possible.

The model can then be formulated as follows:

$$\max \sum_{i=1}^{P} \sum_{j=1}^{B} x_{ij} v_i (1 - c_j) \tag{1}$$

subject to:

The number of products in each category cannot exceed the available category capacity:

$$\forall (j) [\sum_{i=1}^{P} x_{ij} \le W_j] \tag{2}$$

(category capacity)

Each product must be assigned to one category.

$$\forall (i) [\sum_{j=1}^{B} x_{ij} = 1] \tag{3}$$

(product to category assignment)

4. Experimental data

For the experiment, we used the "Online Retail II UCI", which is a real online retail transaction data set of two years provided by Kaggle (Kaggle; https://www.kaggle.com/...).

This "Online Retail II" dataset includes every transaction made by a UK-based, registered, and non-store online retailer between December 1, 2009, and December 9, 2011. The company primarily offers distinctive gifts for all occasions. The company has a large number of wholesalers as clients.

Attribute information:

- Invoice number. Nominal, which is uniquely assigned to each transaction.
- Product (item) code. Nominal, which is uniquely assigned to each distinct product.
- Product (item) name. Nominal.
- Quantity. Numeric. The quantities of each product (item) per transaction.
- Invoice date and time. Numeric. The day and time when a transaction was generated.
- Unit price. Numeric. Product price per unit in sterling.
- Customer number. Nominal, which is the number uniquely assigned to each customer.
- Country name. Nominal. The name of the country where a customer resides.

The first six attributes are essential for the current research.

We transformed the data into a structure presented in Figure 1. The information about the products and invoices is needed for the research.

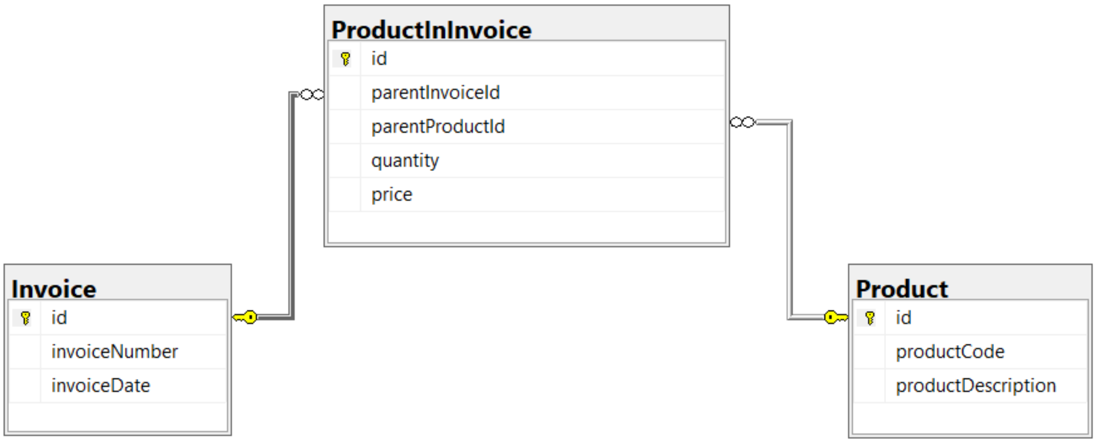

Figure 1. Database diagram based on the downloaded data.

Source: own elaboration.

The dataset could be explained as follows. There is a number of products that the company offers in the assortment. There is a number of invoices with the dates on which the products were sold or returned. In one invoice, there could be many different products. Products can be sold or returned within the invoices.

Table 1 **Błąd! Nie można odnaleźć źródła odwołania.**explains the input data. There were 8196 products and 46 373 invoices between 01.01.2010 and 31.12.2011. The average number of products in an invoice was 43, with minimum and maximum values of 1 and 2544, accordingly. Invoices reported the number of sold (8158) and returned (5090) products which gave the 22 053 839 sold product quantities and 846 360 product quantities. For the analysis, we chose the 2-year timeframe from 01.01.2010 to 31.12.2011.

Table 1.
Statistics on the input data

Parameter	Value
The total number of products	8196
The total number of invoices	46 373
The minimum number of products in invoices	1
The average number of products in invoices	43
The maximum number of products in invoices	2544
The total number of products in invoices which were sold	8158
The total number of products in invoices which were returned	5090
The sum of quantities of products in invoices which were sold	22 053 839
The sum of quantities of products in invoices which were returned	846 360
Timeframe of analysed sales data in invoices	01.01.2010 – 31.12.2011

Source: own elaboration.

5. Experiment

The goal of the computational experiment was to test if the optimal solution could be found by commercial and non-commercial solvers and compare it to the solution made by the distributor evaluation using a real-size database.

For the experiments, the database was built in MS SQL using:

- SQL Server 2019.0150.2000.05.
- SQL Server Management Studio v18.12.1.
- SQL Server Management Studio 15.0.18424.0.

The main program, which used libraries for solvers, was written in C# using:

- Microsoft Visual Studio Community 2019
- Version 16.8.2.
- VisualStudio.16.Release/16.8.2+30717.126.
- Microsoft .NET Framework.
- Version 4.8.03752.

The computer parameters are:

- Processor: AMD Ryzen 5 1600 Six-Core Processor 3.20 GHz.
- System type: 64-bit Operation System, x64-based processor.
- RAM: 16GB.
- Operation system: Windows 10.

We modelled the assortment categorisation problem as a multiple knapsack problem, and we used both the free-of-charge MIP solver and the CP-SAT solvers to solve it. Next, we used the commercial CPLEX solver. Later provided the comparison of the optimal solutions found by the solvers with the practical distributor's solution.

Google OR-Tools is an open-source software suite for optimisation used for tackling the world's known problems in vehicle routing, flows, integer and linear programming, and constraint programming (https://developers.google.com/optimization). Using such libraries, optimisation problems could be modelled and solved in C++, Python, C#, or Java. We used C# for dealing with all solvers.

Table 2 reports the time required for product categorisation by solvers and by the distributor. The table also provides us with the information to determine which solver will earn the best results for the warehouse. We had 39 test instances. Among them, there were 24 instances which represented 1 month, 8 instances which represented 3 months, 4 instances which represented 6 months, 2 instances for 1 year and 1 instance for the whole 2-year period. The average number of products for 1 instance was 5153 with its minimal and maximal values 3797 and 8196 accordingly. We analysed only the transactions with sales during the selected period, i.e. the products that were not returned or sold were not taken into account. This is done only for the experiment in different instances. Otherwise, the products without sales will be assigned to category C. We didn't want to disturb our results with such cases. In practice, the widest period (e.g. a year) is selected, and all products are categorised, even the products with a lack of sales.

CPLEX solver was the fastest solver, which found the solution on average in 0.11 seconds, with its fastest and longest time at 0.07 seconds and 0.22 seconds, accordingly. MIP solver and the CP-SAT solvers executed comparably, founding the solution on average in 1.98 seconds for MIP and 2.32 seconds for CP-SAT. The minimum and maximum solution time vary from 1.18 seconds to 4.39 seconds for MIP. The minimum and maximum solution time vary from 1.51 seconds to 3.85 seconds for CP-SAT. The solution found by the distributor in SQL was the slowest and, on average 3.28 seconds. As it could be observed, for SQL, the time step was 1 second, while for solvers, it was more precise.

Table 2.
Product categorisation solution time

Dates	Start date	End date	Number of products	MIP time [s]	CP-SAT time [s]	CPLEX time [s]	SQL time[s]
1 month	01.01.10	31.01.10	5 019	1.89	2.19	0.14	3.00
	01.02.10	28.02.10	4 752	1.80	2.06	0.09	4.00
	01.03.10	31.03.10	5 284	2.09	2.30	0.11	3.00
	01.04.10	30.04.10	4 776	1.95	1.99	0.09	4.00
	01.05.10	31.05.10	4 770	1.80	1.97	0.09	4.00
	01.06.10	30.06.10	4 887	1.80	2.34	0.14	3.00
	01.07.10	31.07.10	4 749	1.92	2.46	0.08	3.00
	01.08.10	31.08.10	4 831	1.84	2.12	0.10	3.00
	01.09.10	30.09.10	4 904	1.72	2.28	0.11	3.00
	01.10.10	31.10.10	5 080	1.86	2.15	0.10	4.00
	01.11.10	30.11.10	5 312	1.97	2.40	0.12	4.00
	01.12.10	31.12.10	4 861	1.58	2.29	0.09	3.00
	01.01.11	31.01.11	4 445	1.56	1.88	0.09	4.00
	01.02.11	28.02.11	4 125	1.45	2.42	0.09	3.00
	01.03.11	31.03.11	4 315	1.44	2.69	0.09	3.00
	01.04.11	30.04.11	4 267	1.56	1.85	0.09	3.00
	01.05.11	31.05.11	4 186	1.43	1.80	0.08	4.00
	01.06.11	30.06.11	4 386	1.51	2.12	0.10	3.00
	01.07.11	31.07.11	4 463	1.57	2.02	0.10	3.00
	01.08.11	31.08.11	4 334	1.41	1.81	0.08	4.00
	01.09.11	30.09.11	4 470	1.45	1.92	0.09	3.00
	01.10.11	31.10.11	4 532	1.53	1.93	0.10	3.00
	01.11.11	30.11.11	4 631	1.62	2.01	0.10	4.00
	01.12.11	31.12.11	3 797	1.18	1.51	0.07	3.00
3 months	01.01.10	31.03.10	6 074	2.69	2.71	0.12	3.00
	01.04.10	30.06.10	5 737	2.41	2.69	0.12	3.00
	01.07.10	30.09.10	5 758	2.47	2.56	0.13	3.00
	01.10.10	31.12.10	5 807	2.28	2.68	0.13	3.00
	01.01.11	31.03.11	5 125	1.94	2.26	0.11	4.00
	01.04.11	30.06.11	5 247	1.97	2.31	0.10	3.00
	01.07.11	30.09.11	5 084	1.92	2.28	0.11	3.00
	01.10.11	31.12.11	5 008	1.48	2.22	0.11	3.00
6 months	01.01.10	30.06.10	6 517	2.90	2.76	0.15	3.00
	01.07.10	31.12.10	6 348	2.54	2.78	0.14	3.00
	01.01.11	30.06.11	5 759	2.38	2.52	0.14	3.00
	01.07.11	31.12.11	5 486	1.58	2.42	0.13	3.00

Cont. table 2.

1 year	01.01.10	31.12.10	7 317	3.50	3.32	0.20	3.00
	01.01.11	31.12.11	6 329	2.73	2.65	0.14	3.00
2 years	01.01.10	31.12.11	8 196	4.39	3.85	0.22	4.00
Min			3 797	1.18	1.51	0.07	3.00
Avg			5 153	1.98	2.32	0.11	3.28
Max			8 196	4.39	3.85	0.22	4.00

Source: own elaboration.

All solvers found optimal solutions for all instances. Comparing solvers' performance, it could be noted that the value of the objective function was the same for all solvers, the number of products in each category was the same for all solvers, while the value for different categories for one selected instance differed for solvers. All products were assigned to one of the categories by all solvers.

Table 3 **Błąd! Nie można odnaleźć źródła odwołania.**reports the comparison of the solution found by the distributor to the optimal solution found by the solvers. The profit ratio calculated by the total profit found by the distributor divided by the total found by the solver was, on average, 99.9994%. For 17 instances of 39 ones, the solution found by the solver was better than the distributor's solution. This difference is, on average, 0.00128% and varies from 0.00002% to 0.00650%. Not so much, but in large warehouses or distribution centres, even such profit ratio difference provides for savings. So solver usage is advisable for large problem instances.

Table 3.

Performance of the distributor's method

Dates	Start date	End date	Profit ratio	Solvers were better
1 month	01.01.10	31.01.10	100.0000%	
	01.02.10	28.02.10	99.9940%	0.00599%
	01.03.10	31.03.10	99.9977%	0.00227%
	01.04.10	30.04.10	100.0000%	
	01.05.10	31.05.10	100.0000%	
	01.06.10	30.06.10	100.0000%	
	01.07.10	31.07.10	99.9992%	0.00075%
	01.08.10	31.08.10	100.0000%	
	01.09.10	30.09.10	99.9985%	0.00147%
	01.10.10	31.10.10	99.9999%	0.00006%
	01.11.10	30.11.10	100.0000%	
	01.12.10	31.12.10	99.9998%	0.00015%
	01.01.11	31.01.11	99.9997%	0.00033%
	01.02.11	28.02.11	99.9999%	0.00013%
	01.03.11	31.03.11	100.0000%	
	01.04.11	30.04.11	99.9935%	0.00650%
	01.05.11	31.05.11	100.0000%	
	01.06.11	30.06.11	100.0000%	
	01.07.11	31.07.11	100.0000%	
	01.08.11	31.08.11	100.0000%	
	01.09.11	30.09.11	100.0000%	
	01.10.11	31.10.11	100.0000%	
	01.11.11	30.11.11	99.9998%	0.00024%
	01.12.11	31.12.11	99.9967%	0.00332%

Cont. table 3.

3 months	01.01.10	31.03.10	99.9999%	0.00007%
	01.04.10	30.06.10	100.0000%	
	01.07.10	30.09.10	99.9998%	0.00017%
	01.10.10	31.12.10	100.0000%	0.00002%
	01.01.11	31.03.11	100.0000%	
	01.04.11	30.06.11	99.9999%	0.00006%
	01.07.11	30.09.11	100.0000%	
	01.10.11	31.12.11	99.9999%	0.00014%
6 months	01.01.10	30.06.10	99.9998%	0.00018%
	01.07.10	31.12.10	100.0000%	
	01.01.11	30.06.11	100.0000%	
	01.07.11	31.12.11	100.0000%	
1 year	01.01.10	31.12.10	100.0000%	
	01.01.11	31.12.11	100.0000%	
2 years	01.01.10	31.12.11	100.0000%	
Min			99.9935%	0.00002%
Avg			99.9994%	0.00128%
Max			100.0000%	0.00650%

Source: own elaboration.

6. Conclusion

Distribution expectations on the profitability of the warehouse or distribution centre have increased; this means that they are now more educated and tend to increase savings and increase the gained profit. Therefore they are interested in methods of assortment categorisation, which allow them to achieve their goals and to be better than their competitors.

According to the industry, the kind of items, or the particular requirements of the warehouse, there are different ways to categorise or define the importance of a product for a corporation. Wrong product categorisation could lead to low profits because of increased costs of picking. By changing the category of the product, the distributor's revenue might potentially increase, and the product picking costs may be decreased. In other words, changing product categories at the warehouse and distribution centre may help a company to achieve higher results.

This research shows how commercial and non-commercial solvers could be used for assortment categorisation and what profitability could be achieved compared to the distributor's categorisation method. The proposed approach to model assortment categorisation problem as a multi-knapsack one and usage of solvers provides businesses with the right tools to make appropriate decisions and increase their sales.

The results showed that for 17 instances from 39 ones, the solution achieved found by the solver was better than the wholesaler's solution on an average of 0.00128%. This difference varied from 0.00002% to 0.00650% for the rest instances. There is no big difference with the optimal solution on paper, but in large warehouses or distribution centres, even a small percentage improvement provides wholesalers with savings. In larger warehouses the performance of the proposed ABC classification method can be increasingly significant.

A complete assortment optimisation tool should integrate assortment optimisation, connection visibility, and quality measurement capabilities. Distributors who use such technology in their business expect to gain a competitive edge. Distributors can increase the profitability of the assortment and clearly define its category of it by integrating automation as proposed in this research.

Each classification has a precise area. Keeping track of the ABC classification is crucial since the demand for certain products can vary. ABC analysis could be counter-productive. Not all enterprises might choose to categorise their product line using this method. Any company that fails to plan and put in a classification system will not have efficient management over its inventory management, and hence its gained profits will be reduced. The ABC classification allows for efficient stock management and a focus on important stock lines.

The main limitation of this research is not taking into account constraints which indicate that some products cannot be placed on the shelves immediately or some of them cannot be classified immediately. Moreover, the availability of storage equipment or warehouse staff in the classification moment should be taken into account. These issues may be the subject of further research.

Acknowledgements

The project is financed by the Ministry of Science and Higher Education in Poland under the programme "Regional Initiative of Excellence" 2019 - 2022 project number 015/RID/2018/19 total funding amount 10 721 040,00 PLN.

References

1. Ailawadi, K.L., Keller, K.L. (2004). Understanding retail branding: Conceptual insights and research priorities. *Journal of Retailing, Vol. 80, Iss. 4*, pp. 331-342, https://doi.org/10.1016/j.jretai.2004.10.008.
2. AR Racking, *ABC Inventory Method in the Warehouse: Origin, Characteristics and Advantages.* Retrieved from: https://www.ar-racking.com/en/news-and-blog/storage-solutions/quality-and-security/abc-inventory-method-in-the-warehouse-origin-characteristics-and-advantages.

3. Baby, B., Prasanth, N., Jebadurai, S.S. (2018). Implementation of lean principles to improve the operations of a sales warehouse in the manufacturing industry. *International Journal of Technology*, *Vol. 9*, *Iss. 1,* pp. 46-54, https://doi.org/10.14716/ijtech.v9i1.1161.
4. Cachon, G.P., Kök, A.G. (2007). Category management and coordination in retail assortment planning in the presence of basket shopping consumers. *Management Science*, *Vol. 53, No. 6*, pp. 934-951, https://doi.org/10.1287/mnsc.1060.0661.
5. Cachon, G.P., Terwiesch, C., Xu, Y. (2005). Retail assortment planning in the presence of consumer search. *Manufacturing and Service Operations Management*, *Vol. 7, No. 4*, pp. 330-346, https://doi.org/10.1287/msom.1050.0088.
6. Campo, K., Gijsbrechts, E., Nisol, P. (2004). Dynamics in consumer response to product unavailability: Do stock-out reactions signal response to permanent assortment reductions? *Journal of Business Research*, *Vol. 57, No. 8*, pp. 834-843, https://doi.org/10.1016/S0148-2963(02)00486-1.
7. Cips, *Getting to grips with ABC classification.* Retrieved from: https://www.cips.org/intelligence-hub/operations-management/abc-classification.
8. Corsten, H., Hopf, M., Kasper, B., Thielen, C. (2018). Assortment planning for multiple chain stores. *OR Spectrum*, *Vol. 40, No. 4*, pp. 875-912, https://doi.org/10.1007/s00291-017-0496-9.
9. Danneels, E. (2003). Tight-loose coupling with customers: The enactment of customer orientation. *Strategic Management Journal, Vol. 24, Iss. 6*, pp. 559-576, https://doi.org/10.1002/smj.319.
10. De Koster, R.B.M., Johnson, A.L., Roy, D. (2017). Warehouse design and management. *International Journal of Production Research, Vol. 55, Iss. 21*, pp. 6327-6330, https://doi.org/10.1080/00207543.2017.1371856.
11. Deskera, R. *What is ABC Analysis in Inventory Control?* Retrieved from: https://www.deskera.com/blog/abc-analysis/.
12. Dujak, D., Kresoja, M., Franjkovic, J. (2016). Space Management in Category Management - a Comparative Analysis of Retailers. *Strategic Management*, *May*, pp. 409-424. Retrieved from: https://www.researchgate.net/publication/311311064_Space_Management_in_Category_Management_-_a_Comparative_Analysis_of_Retailers.
13. EazyStock, Drakeley, P., *How to calculate and apply ABC classification of inventory.* Retrieved from: https://www.eazystock.com/blog/calculate-apply-abc-classification-inventory/.
14. Fisher, M.L., Vaidyanathan, R. (2009). An Algorithm and Demand Estimation Procedure for Retail Assortment Optimisation. *Management*, *Vol. 2007*, pp. 1-36.
15. Fisher, M., Vaidyanathan, R. (2014). A demand estimation procedure for retail assortment optimisation with results from implementations. *Management Science, Vol. 60, No. 10*, pp. 2401-2415, https://doi.org/10.1287/mnsc.2014.1904.

16. Forum Europe Ltd. (1997). Super sales - how top performers manage the seller/client interface. *Managing Service Quality: An International Journal*, *Vol. 7, No. 2*, pp. 95-101, https://doi.org/10.1108/09604529710796591.
17. Hanafi, R., Mardin, F., Asmal, S., Setiawan, I., Wijaya, S. (2019). Toward a green inventory controlling using the ABC classification analysis: A case of motorcycle spares parts shop. *IOP Conference Series: Earth and Environmental Science*, *Vol. 343, No. 1*, pp. 012012, https://doi.org/10.1088/1755-1315/343/1/012012.
18. Hariga, M.A., Al-Ahmari, A., Mohamed, A.R.A. (2007). A joint optimisation model for inventory replenishment, product assortment, shelf space and display area allocation decisions. *European Journal of Operational Research*, *Vol. 181, No. 1*, pp. 239-251, https://doi.org/10.1016/j.ejor.2006.06.025.
19. Hübner, A. (2017). A decision support system for retail assortment planning. *International Journal of Retail and Distribution Management*, *Vol. 45, No. 7-8*, pp. 808-825, https://doi.org/10.1108/IJRDM-09-2016-0166.
20. Hübner, A., Kuhn, H., Kühn, S. (2016). An efficient algorithm for capacitated assortment planning with stochastic demand and substitution. *European Journal of Operational Research*, *Vol. 250, No. 2*, pp. 505-520, https://doi.org/10.1016/j.ejor.2015.11.007.
21. Hübner, A., Schaal, K. (2017). An integrated assortment and shelf-space optimisation model with demand substitution and space-elasticity effects. *European Journal of Operational Research*, *Vol. 261, No. 1*, pp. 302-316, https://doi.org/10.1016/j.ejor.2017.01.039.
22. Huh, Y.E., Vosgerau, J., Morewedge, C.K. (2016). More Similar but Less Satisfying: Comparing Preferences for and the Efficacy of Within- and Cross-Category Substitutes for Food. *Psychological Science*, *Vol. 27, No. 6*, https://doi.org/10.1177/0956797616640705.
23. Jemelka, M., Chramcov, B., Kříž, P. (2016). Redistribution of stock sections according to the materials turnover and the abcmethod. *Annals of DAAAM and Proceedings of the International DAAAM Symposium*, *Vol. 27, No. 1*, pp. 816-821, https://doi.org/10.2507/27th.daaam.proceedings.118.
24. Katana, *ABC inventory: get the most out of your best-selling products.* Retrieved from: https://katanamrp.com/abc-inventory/.
25. Kök, A.G., Fisher, M.L. (2007). Demand estimation and assortment optimisation under substitution: Methodology and application. *Operations Research*, *Vol. 55, No. 6*, pp. 1001-1021, https://doi.org/10.1287/opre.1070.0409.
26. Koul, S., Jasrotia, S.S. (2019). Product adoption by small retailers in India. *International Journal of Retail and Distribution Management*, *Vol. 47, No. 11*, pp. 1163-1180, https://doi.org/10.1108/IJRDM-03-2018-0053.
27. Li, J., Moghaddam, M., Nof, S.Y. (2016). Dynamic storage assignment with product affinity and ABC classification—a case study. *International Journal of Advanced*

Manufacturing Technology, *Vol. 84, No. 9-12*, pp. 2179-2194, https://doi.org/10.1007/s00170-015-7806-7.
28. Lin, H.L., Ma, Y.Y. (2021). A New Method of Storage Management Based on ABC Classification: A Case Study in Chinese Supermarkets' Distribution Center. *SAGE Open*, *Vol. 11, No. 2*, https://doi.org/10.1177/21582440211023193.
29. Lotfi, M.M., Torabi, S.A. (2011). A fuzzy goal programming approach for mid-term assortment planning in supermarkets. *European Journal of Operational Research*, *Vol. 213, No. 2*, pp. 430-441, https://doi.org/10.1016/j.ejor.2011.04.001.
30. Mahajan, S., Van Ryzin, G. (2001). Stocking retail assortments under dynamic consumer substitution. *Operations Research*, *Vol. 49, No. 3*, pp. 334-351, https://doi.org/10.1287/opre.49.3.334.11210.
31. McInerney, S., Yadavalli, S. (2022). Increasing warehouse throughput through the development of a dynamic class-based storage assignment algorithm. *South African Journal of Industrial Engineering*, *Vol. 32*, pp. 157-167, https://doi.org/10.7166/33-2-2651.
32. Messinger, P.R., Narasimhan, C. (1997). A model of retail formats based on consumers' economising on shopping time. *Marketing Science*, *Vol. 16, No. 1*, pp. 1-23, https://doi.org/10.1287/mksc.16.1.1.
33. Mor, R.S., Bhardwaj, A., Kharka, V., Kharub, M. (2021). Spare Parts Inventory Management in the Warehouse: A Lean Approach. *International Journal of Industrial Engineering and Production Research*, *Vol. 32, No. 2*, pp. 1-11, https://doi.org/10.22068/ijiepr.1110.
34. Netessine, S., Rudi, N. (2003). Centralised and competitive inventory models with demand substitution. *Operations Research*, *Vol. 51, No. 2*, pp. 329-335, https://doi.org/10.1287/opre.51.2.329.12788.
35. Oestreicher-Singer, G., Libai, B., Sivan, L., Carmi, E., Yassin, O. (2013). The network value of products. *Journal of Marketing*, *Vol. 77, No. 3*, pp. 1-14, https://doi.org/10.1509/jm.11.0400.
36. Pisinger, D. (1999). An Exact Algorithm for Large Multiple Knapsack Problems. *European Journal of Operational Research*, *Vol. 114, No. 3*, pp. 528-541, https://doi.org/10.1016/S0377-2217(98)00120-9.
37. Raeder, T., Chawla, N.V. (2011). Market basket analysis with networks. *Social Network Analysis and Mining*, *Vol. 1, No. 2*, 97-113, https://doi.org/10.1007/s13278-010-0003-7.
38. Rodríguez, I., Rabanal, P., Rubio, F. (2017). How to make a best-seller: Optimal product design problems. *Applied Soft Computing Journal*, *Vol. 55*, pp. 178-196, https://doi.org/10.1016/j.asoc.2017.01.036.
39. Seetharaman, P.B., Chib, S., Ainslie, A., Boatwright, P., Chan, T., Gupta, S., Mehta, N., Rao, V., Strijnev, A. (2005). Models of multi-category choice behavior. *Marketing Letters*, *16, No. 3-4*, pp. 239–254, https://doi.org/10.1007/s11002-005-5888-y.

40. Simonson, I. (1999). The effect of product assortment on buyer preferences. *Journal of Retailing*, *Vol. 75, No. 3*, pp. 347-370, https://doi.org/10.1016/S0022-4359(99)00012-3.
41. Sloot, L.M., Verhoef, P.C., Franses, P.H. (2005). The impact of brand equity and the hedonic level of products on consumer stock-out reactions. *Journal of Retailing*, *81, No. 1*, pp. 15-34, https://doi.org/10.1016/j.jretai.2005.01.001.
42. Smith, S.A., Agrawal, N. (2000). Management of multi-item retail inventory systems with demand substitution. *Operations Research*, *48, No. 1*, pp. 50-64, https://doi.org/10.1287/opre.48.1.50.12443.
43. Stassen, R.E., Mittelstaedt, J.D., Mittelstaedt, R.A. (1999). Assortment overlap: Its effect on shopping patterns in a retail market when the distributions of prices and goods are known. *Journal of Retailing*, *Vol. 75, No. 3*, pp. 371-386, https://doi.org/10.1016/S0022-4359(99)00013-5.
44. Urban, T.L. (1998). An inventory-theoretic approach to product assortment and shelf-space allocation. *Journal of Retailing*, *Vol. 74, No. 1*, pp. 15-35, https://doi.org/10.1016/S0022-4359(99)80086-4.
45. Van Ryzin, G., Mahajan, S. (1999). On the relationship between inventory costs and variety benefits in retail assortments. *Management Science*, *Vol. 45, No. 11*, pp. 1496-1509, https://doi.org/10.1287/mnsc.45.11.1496.
46. Yücel, E., Karaesmen, F., Salman, F.S., Türkay, M. (2009). Optimizing product assortment under customer-driven demand substitution. *European Journal of Operational Research*, *Vol. 199, No. 3*, pp. 759-768, https://doi.org/10.1016/j.ejor.2008.08.004.