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# Minimization of Operation Cost of Critical Infrastructure Network with Cascading Effects Considering Climate-Weather Change Influence

**Keywords**

critical infrastructure, operation process, climate-weather change process, critical infrastructure operation cost minimization

**Abstract**

In the paper optimization of operation process and minimization of operation cost for interconnected and interdependent critical infrastructure (CI) networks with cascading effects at variable operation conditions related to the climate-weather change are proposed. A multistate series network with assets dependent according to local load sharing (LLS) rule is analyzed and optimization of operation and safety of CI network with the LLS rule is introduced. For such CI network, the optimal transient probabilities that minimize the mean value of the total operation costs are found. Finally, cost analysis of CI network operation impacted by climate-weather change is presented in case the CI network is non-repairable and in case it is repairable after exceeding its critical safety state.

## 1. Introduction

Considering interdependent Critical Infrastructures (CI) at variable operation states and changing in time climate-weather conditions we use the general safety model of the multistate critical infrastructure constructed in [Kołowrocki, et al., 2017a-b]. To build this general model, first the critical infrastructure operation process related to the climate-weather change [Klabjan, Adelman, 2006], [Kołowrocki, 2014], [Kołowrocki, Soszyńska-Budny, 2011, 2012a-b, 2014] has to be described. In this paper we present the multistate approach [Xue, 1985], [Xue, Yang, 1995] to CI networks safety analysis. Basic theory of multistate critical infrastructure safety related to the climate-weather change process in its operating area is given in [Kołowrocki, et al., 2017a-c]. We assume that the critical infrastructure can change its safety structure and its components safety parameters during variable operation process [Kołowrocki, Soszyńska-Budny, 2011, 2012a-b] and at different climate-weather states of the critical infrastructure operating area [Kołowrocki, et al., 2017b-c].

Taking into account changing in time critical infrastructure operation and climate-weather conditions, that can have significant influence on the CI safety, we assume that these changes can also affect the model of dependency between CI assets. Further, applying the linear programming [Klabjan, Adelman, 2006], [Kołowrocki, Soszyńska-Budny, 2011] to the optimization of operation and safety of interdependent CI networks, it is possible to find optimal values of the limit transient probabilities of the critical infrastructure operation process that minimize the mean value of the CI network operation cost. Then, the optimal critical operation cost function and the optimal moment when the CI network operation cost exceeds a permitted level can be determined.

## 2. Critical Infrastructure Operation Process Related to Climate-Weather Change Process

We consider, similarly as in [Kołowrocki, et al., 2017c], the critical infrastructure impacted by the operation process related to the climate-weather change process $ZC(t),\ t \in <0,\infty)$, in a various way at this process states $zc_{bl},\ b = 1,2,\dots,v,\ l = 1,2,\dots,w.$ We assume that the changes of the states of operation process related to the climate-weather change process $ZC(t)$, $t \in <0,\infty)$, at the critical infrastructure operating area have an influence on the critical infrastructure safety structure and on the safety of the critical infrastructure assets $A_i$, $i = 1,2,...,n$.

We assume, as in [Kołowrocki, et al., 2017b-c], that the critical infrastructure during its operation process is taking $v, v \in N$, different operation states $z_1, z_2, \ldots, z_v$. We define the critical infrastructure operation process $Z(t)$, $t \in <0,+\infty)$, with discrete operation states from the set $\{z_1, z_2, \ldots, z_v\}$. These operation states may take into account actual load or demand and can influence on lifetimes of CI assets in the safety state subsets and way they decrease through various models of dependency. The vector of limit values of transient probabilities of the critical infrastructure operation process $Z(t)$ at the particular operation states $z_b$, $b = 1,2,...,v$, is defined in [Kołowrocki, et al., 2017c].

Moreover, as in [Kołowrocki, et al., 2017b-c], we assume that the climate-weather change process $C(t)$, $t \in <0,+\infty)$, at the critical infrastructure operating area is taking $w$, $w \in N$, different climate-weather states $c_1$, $c_2$,…, $c_w$. Climate-weather conditions can have also influence on CI network safety including cascading effect and models of failure dependency between assets and subnetworks of CI network. The vector of limit values of transient probabilities of the climate-weather change process $C(t)$ at the particular climate-weather states $c_l$, $l = 1,2,...,w$, is defined in [Kołowrocki, et al., 2017c].

Then, the joint process of critical infrastructure operation process and climate-weather change process called the critical infrastructure operation process related to climate-weather change is proposed and it is marked by $ZC(t)$, $t \in <0,+\infty)$. Further, we assume that it can take $vw, v, w \in N$, different operation states related to the climate-weather change $zc_{11}, zc_{12}, \ldots, zc_{vw}$. We assume that the critical infrastructure operation process related to climate-weather change $ZC(t)$, at the moment $t \in <0,+\infty)$, is at the state $zc_{bl}$, $b = 1,2,...,v$, $l = 1,2,...,w$, if and only if at that moment, the operation process $Z(t)$ is at the operation states $z_b$, $b = 1,2,...,v$, and the climate-weather change process $C(t)$ is at the climate-weather state $c_l$, $l = 1,2,...,w$, can be expressed as follows:

$$(ZC(t) = zc_{bl}) \Leftrightarrow (Z(t) = z_b \cap C(t) = c_l),$$
$$t \in <0,+\infty). \quad (1)$$

The transient probabilities of the critical infrastructure operation process related to climate-weather change $ZC(t)$ at the operation states $zc_{bl}$, $b = 1,2,...,v$, $l = 1,2,...,w$, are defined in [Kołowrocki, et al., 2017c]:

$$pq_{bl}(t) = P(ZC(t) = zc_{bl}),\ t \in <0,+\infty). \quad (2)$$

Further, the limit values of the transient probabilities of the critical infrastructure operation process related to climate-weather change process $ZC(t)$ at the operation states $zc_{bl}$, $b = 1,2,...,v$, $l = 1,2,...,w$, are given by

$$pq_{bl} = \lim_{t \to \infty} pq_{bl}(t), \quad (3)$$

and in case when the processes $Z(t)$ and $C(t)$ are independent, they can be found from [Kołowrocki, et al., 2017b-c]

$$pq_{bl} = p_b q_l,\ b = 1,2,...,v,\ l = 1,2,...,w, \quad (4)$$

where $p_b, b = 1,2,...,v$, are the limit transient probabilities of the operation process $Z(t)$ at the particular operation startes $z_b$, $b = 1,2,...,v$, and $q_l$, $l = 1,2,...,w$, are the limit transient probabilities of the climate-weather change process $C(t)$ at the particular climate-weather states $c_l$, $l = 1,2,...,w$.

Other interesting characteristics of the critical infrastructure operation process $ZC_{bl}(t)$ are its total sojourn times $\hat{\theta}\hat{C}_{bl}$ at the particular operation states $zc_{bl}$, $b = 1,2,...,v$, $l = 1,2,...,w$, during the fixed sufficiently large critical infrastructure operation time $\theta$. They have approximately normal distributions with the expected values given by

$$\hat{M}\hat{N}_{bl} = E[\hat{\theta}\hat{C}_{bl}] = pq_{bl}\theta, \quad (5)$$

where $pq_{bl}$, $b = 1,2,...,v$, $l = 1,2,...,w$, are defined by (3) and given by (4) in the case the processes $Z(t)$ and $C(t)$ are independent.

## 3. LLS Model of Dependency Related to CI Operation and Climate-Weather Change Processes

In local load sharing (LLS) model of dependency for a multistate CI series network, described in [Blokus-Roszkowska, Kołowrocki, 2017a-c] and [Kołowrocki, et al., 2017a], the coefficients of the network load growth may be defined differently in various operation and climate-weather condions. In LLS rule, we assume that after the departure of asset $A_j$, $j$ = 1,…,$n$, in the network from the safety state subset {$u$,$u$+1,…,$z$}, $u$ = 1,2,…,$z$, the lifetimes of remaining assets $A_i$, $i$ = 1,…,$n$, $i \neq j$, in the safety state subsets decrease dependently on the coefficients of the network load growth concerned with the distance from the asset $A_j$. Thus, we assume that these coefficients of the network load growth in LLS rule can take different values or formulae at various operation states related to the climate-weather change $zc_{bl}$, $b=1,2,...,v$, $l=1,2,...,w$. For example, if we denote the load on CI at the state $zc_{bl}$, $b=1,2,...,v$, $l=1,2,...,w$, by $L_{bl}$, and maximal load on CI by $L_{MAX}$, the coefficients of the network load growth in LLS rule can be determined from following formula

$$q^{(bl)}(\upsilon, d_{ij}) = [1 - \frac{L_{bl}}{L_{MAX}}] \cdot q(\upsilon, d_{ij}), i = 1,\dots,n,$$
$$j = 1,\dots,n,\ \upsilon = u, u-1,\dots,1,\ u = 1,2,\dots,z-1, \quad (6)$$

where $L_{bl} \leq L_{MAX}$, $b=1,2,...,v$, $l=1,2,...,w$, and the coefficients of the network load growth $q^{(bl)}(\upsilon, d_{ij})$, $0 < q^{(bl)}(\upsilon, d_{ij}) \leq 1$, $i$ = 1,…,$n$, $j$ = 1,…,$n$, and $q^{(bl)}(\upsilon, 0) = 1$ for $\upsilon$ = $u$,$u$-1,…,1, $u$ = 1,2,…,$z$-1, are non-increasing functions of assets' distance $d_{ij} = |i - j|$ from the asset that has got out of the safety state subset {$u$,$u$+1,…,$z$}, $u$ = 1,2,…,$z$. The distance between network assets can be interpreted in the metric sense as well as in the sense of relationships in the functioning of the network assets.

We denote by $E[T_i^{(bl)}(u)]$ and $E[T_{i/j}^{(bl)}(u)]$, $i$ = 1,2,...,$n$, $j$ = 1,2,...,$n$, $u$ = 1,2,…,$z$, the mean values of assets' lifetimes $T_i^{(bl)}(u)$ and $T_{i/j}^{(bl)}(u)$, respectively, before and after departure of one fixed asset $A_j$, $j$ = 1,…,$n$, from the safety state subset {$u$,$u$+1,…,$z$}, $u$ = 1,2,…,$z$, at the operation state related to the climate-weather change $zc_{bl}$, $b=1,2,...,v$, $l=1,2,...,w$. With this notation, in considered local load sharing rule, the mean values of assets lifetimes in the safety state subset {$\upsilon$,$\upsilon$+1,…,$z$}, $\upsilon$ = $u$,$u$-1,…,1, $u$ = 1,2,…,$z$, at particular state $zc_{bl}$, $b=1,2,...,v$, $l=1,2,...,w$, are decreasing according to the following formula:

$$E[T_{i/j}^{(bl)}(\upsilon)] = q^{(bl)}(\upsilon, d_{ij}) \cdot E[T_i^{(bl)}(\upsilon)],$$
$$i = 1,\dots,n, j = 1,\dots,n. \quad (7)$$

In different states not only different values of coefficients of the network load growth can be assumed, but in special cases the different models of dependency between assets and subnetworks can be adopted. We can also assume that in some CI operation states related to the climate-weather change cascading effect can be observed, while in others no dependency between assets or subnetworks are assumed. Then, $q^{(bl)}(\upsilon, d_{ij}) = 1$, for some states $b \in \{1,2,...,v\}$, $l \in \{1,2,...,w\}$, and for the other coefficients are given by (6).

For assumed LLS model of dependency, we get the conditional safety function of a multistate series CI network impacted by the operation process related to the climate-weather change process $ZC(t)$, $t \in <0,\infty)$, in the form of the vector

$$[\boldsymbol{S}_{LLS}^{4}(t,\cdot)]^{(bl)} = [1, [\boldsymbol{S}_{LLS}^{4}(t,1)]^{(bl)}, \dots, [\boldsymbol{S}_{LLS}^{4}(t,z)]^{(bl)}],$$
$$t \geq 0,\ b=1,2,...,v,\ l=1,2,...,w, \quad (8)$$

with the coordinates

$$[\boldsymbol{S}_{LLS}^{4}(t,u)]^{(bl)} = \prod_{i=1}^{n} [S_i\ (t, u+1)]^{(bl)}$$
$$+ \int_0^t \sum_{j=1}^{n} [[\tilde{f}_j\ (a, u+1)]]^{(bl)} \cdot \prod_{\substack{i=1 \\ i \neq j}}^{n} [S_i\ (a, u+1)]^{(bl)}$$
$$\cdot [S_j\ (a,u)]^{(bl)} \cdot \prod_{i=1}^{n} [S_{i/j}\ (t-a, u)]^{(bl)}]da, \quad (9)$$

for $u = 1,2,\dots,z-1$,

$$[\boldsymbol{S}_{LLS}^{4}(t,z)]^{(bl)} = \prod_{i=1}^{n} [S_i\ (t,z)]^{(bl)}, \quad (10)$$

where:

$[S_i\ (t, u+1)]^{(bl)}$ – the conditional safety function coordinate of an asset $A_i$, $i$ = 1,…,$n$, at the state $zc_{bl}$, $b=1,2,...,v$, $l=1,2,...,w$,

$[\tilde{f}_j\ (t, u+1)]^{(bl)}$ – the conditional density function coordinate of an asset $A_j$, $j$ = 1,…,$n$, at the state $zc_{bl}$, $b=1,2,...,v$, $l=1,2,...,w$, corresponding to the distribution function $[\tilde{F}_j\ (t, u+1)]^{(bl)}$, given by

$$[\tilde{F}_j\ (t, u+1)]^{(bl)} = 1 - \frac{[S_j\ (t, u+1)]^{(bl)}}{[S_j\ (t,u)]^{(bl)}},$$
$$u = 1,2,\dots,z-1,\ t \geq 0, \quad (11)$$

$[S_j(t,u)]^{(bl)}$ – the conditional safety function coordinate of an asset $A_j$, $j = 1,\dots,n$, at the state $zc_{bl}$, $b=1,2,\dots,v,\ l=1,2,\dots,w,$

$[S_{i/j}(t,u)]^{(bl)}$ – the conditional safety function coordinate of an asset $A_i$, $i = 1,\dots,n$, at the state $zc_{bl}$, $b=1,2,\dots,v,\ l=1,2,\dots,w,$ after departure from the safety state subset $\{u+1,\dots,z\}$, $u = 1,2,\dots,z-1$, by the asset $A_j$, $j = 1,\dots,n$, such that

$$[S_{i/j}(t-a,u)]^{(bl)} = \frac{[S_{i/j}(t,u)]^{(bl)}}{[S_i\ (a,u)]^{(bl)}},$$
$$u=1,2,\dots,z-1,\ 0<a<t,\ t\geq 0. \tag{12}$$

Similarly, the conditional safety function of CIs and CI networks with other safety structures, presented in [Blokus-Roszkowska, Kołowrocki, 2017a-c] and [Kołowrocki, et al., 2017a], and impacted by the operation process related to the climate-weather change process, can be found. Namely, the conditional safety function of the multistate series-parallel and series-"*m* out of *k*" CI network with assets dependent according to LLS rule, impacted by the operation process related to the climate-weather change process, can be determined.

## 4. Optimization of Operation and Safety of CI Network with the LLS Rule

### 4.1. Operation Cost of CI Network Related to Climate-Weather Change

We may introduce the instantaneous operation cost of the CI network with LLS rule impacted by the operation process $ZC_{bl}(t)$, $t\in<0,\infty)$, related to the climate-weather change process in the form of vector

$$\boldsymbol{K}_{LLS}^4(t,\cdot)=[1,\boldsymbol{K}_{LLS}^4(t,1),\dots,\boldsymbol{K}_{LLS}^4(t,z)], \tag{13}$$

for $t\in<0,\infty)$, with the coordinates given by

$$\boldsymbol{K}_{LLS}^4(t,u)\cong\sum_{b=1}^{v}\sum_{l=1}^{w}pq_{bl}[\boldsymbol{K}_{LLS}^4(t,u)]^{(bl)},$$
$$u=1,2,\dots,z, \tag{14}$$

where $pq_{bl}$, $b=1,2,\dots,v,\ l=1,2,\dots,w,$ are the limit transient probabilities at the states $zc_{bl}$ of the operation process $ZC(t)$, $t\in<0,\infty)$, related to the climate-weather change defined by (3) and

$[\boldsymbol{K}_{LLS}^4(t,u)]^{(bl)}$, $u=1,2,\dots,z,\ b=1,2,\dots,v,\ l=1,2,\dots,w,$

are the coordinates of the CI network conditional instantaneous operation costs in the safety state subsets $\{u,u+1,\dots,z\}$, $u=1,2,\dots,z,$ assuming LLS model of assets' dependency, impacted by the operation process $ZC(t)$, $t\in<0,\infty)$, related to the climate-weather change process at the states $zc_{bl}$, $b=1,2,\dots,v,\ l=1,2,\dots,w,$ defined in the form of the vector

$$[\boldsymbol{K}_{LLS}^4(t,\cdot)]^{(bl)}=[1,[\boldsymbol{K}_{LLS}^4(t,1)]^{(bl)},\dots,[\boldsymbol{K}_{LLS}^4(t,z)]^{(bl)}],$$

for $t\in<0,\infty)$, $b=1,2,\dots,v,\ l=1,2,\dots,w.$

The dependency expressed in (14) can also be clearly shown in the linear equation for the mean value of the CI network with LLS rule total unconditional operation costs in the safety state subsets $\{u,u+1,\dots,z\}$, $u=1,2,\dots,z,$

$$\overline{\boldsymbol{K}}_{LLS}^4(u)\cong\sum_{b=1}^{v}\sum_{l=1}^{w}pq_{bl}[\overline{\boldsymbol{K}}_{LLS}^4(u)]^{(bl)},\ u=1,2,\dots,z, \tag{15}$$

where $pq_{bl}$, $b=1,2,\dots,v,\ l=1,2,\dots,w,$ are the limit transient probabilities at the states $zc_{bl}$, $b=1,2,\dots,v,$ $l=1,2,\dots,w,$ of the operation process $ZC(t)$, $t\in<0,\infty)$, related to the climate-weather change defined by (3) and

$[\overline{\boldsymbol{K}}_{LLS}^4(u)]^{(bl)}$, $u=1,2,\dots,z,\ b=1,2,\dots,v,\ l=1,2,\dots,w,$
are the mean values of the total conditional instantaneous operation costs of CI network with LLS rule in the safety state subsets $\{u,u+1,\dots,z\}$, $u=1,2,\dots,z,$ at the operation states $zc_{bl}$, $b=1,2,\dots,v,\ l=1,2,\dots,w,$ defined by

$$[\overline{\boldsymbol{K}}_{LLS}^4(u)]^{(bl)}=\int_0^{[\mu_{LLS}^4(u)]^{(bl)}}[\boldsymbol{K}_{LLS}^4(t,u)]^{(bl)}dt,$$
$$u=1,2,\dots,z,\ b=1,2,\dots,v,\ l=1,2,\dots,w, \tag{16}$$

where $[\mu_{LLS}^4(u)]^{(bl)}$ are the mean values of the CI network conditional lifetimes $[T_{LLS}^4(u)]^{(bl)}$, $u=1,2,\dots,z,\ b=1,2,\dots,v,\ l=1,2,\dots,w,$ in the safety state subset $\{u,u+1,\dots,z\}$, assuming LLS dependency model, at the critical infrastructure operating process related to the climate-weather change state $zc_{bl}$, $b=1,2,\dots,v,\ l=1,2,\dots,w,$ given in [Kołowrocki, et al., 2017b]

$$[\mu_{LLS}^4(u)]^{(bl)} = \int_0^\infty [\boldsymbol{S}_{LLS}^4(t,u)]^{(bl)} dt, \; u=1,2,...,z, \quad (17)$$

and

$$[\boldsymbol{S}_{LLS}^4(t,u)]^{(bl)} \;\; u=1,2,...,z, \; b=1,2,...,v, \; l=1,2,...,w,$$

are the coordinates of the CI network with LLS rule impacted by the operation process related to the climate-weather change process $ZC(t)$, $t \in <0,\infty)$, conditional safety functions [Kołowrocki, et al., 2017b], given by (8)-(10).

The mean values of the CI network with LLS rule total conditional instantaneous operation costs in the safety state subsets $\{u,u+1,...,z\}$, $u=1,2,...,z$, at the operation states $zc_{bl}$, $b=1,2,...,v$, $l=1,2,...,w$, can be alternatively defined for the critical infrastructure fixed operation time $\theta$ by

$$[\overline{\boldsymbol{K}}_{LLS}^4(u)]^{(bl)} = \int_{\boldsymbol{0}}^{\hat{M}\hat{N}_{bl}} [\boldsymbol{K}_{LLS}^4(t,u)]^{(bl)} dt,$$
$$u=1,2,...,z, \; b=1,2,...,v, \; l=1,2,...,w, \quad (18)$$

where $\hat{M}\hat{N}_{bl}$, $b=1,2,...,v$, $l=1,2,...,w$, are the mean values of the total sojourn times $\hat{\theta}\hat{C}_{bl}$ at the particular operation states $zc_{bl}$, $b=1,2,...,v$, $l=1,2,...,w$, during the fixed sufficiently large critical infrastructure operation time $\theta$, determined by (5).

## 4.2. Minimization of Operation Cost for CI Network Related to Climate-Weather Change

From the linear equation (15) we can see that the mean value of the CI network total unconditional operation cost, assuming LLS dependency model, $\overline{\boldsymbol{K}}_{LLS}^4(u)$, $u=1,2,...,z$, is determined by the limit values of transient probabilities $pq_{bl}$, $b=1,2,...,v$, $l=1,2,...,w$, of the critical infrastructure operation process at the operation states given by (3) and the mean values of the CI network with LLS rule total conditional operation costs $[\overline{\boldsymbol{K}}_{LLS}^4(u)]^{(bl)}$, $u=1,2,...,z$, $b=1,2,...,v$, $l=1,2,...,w$, at the operating process related to the climate-weather change $zc_{bl}$ given by (16).

Therefore, the critical infrastructure total unconditional operation cost optimization approach based on the linear programming [Klabjan, Adelman, 2006], [Kołowrocki, Soszyńska-Budny, 2011] can be proposed. Namely, we may look for the corresponding optimal values $\dot{p}q_{bl}$, $b=1,2,...,v$, $l=1,2,...,w$, of the limit transient probabilities $pq_{bl}$ of the critical infrastructure operation process at the operation states $zc_{bl}$, $b=1,2,...,v$, $l=1,2,...,w$, to minimize the mean value $\overline{\boldsymbol{K}}_{LLS}^4(u)$ of the CI network total unconditional operation cost in the safety state subsets $\{u,u+1,...,z\}$, $u=1,2,...,z$, assuming LLS dependency model, under the assumption that the mean values $[\overline{\boldsymbol{K}}_{LLS}^4(u)]^{(bl)}$ of the CI network total conditional operation costs in the safety state subsets $\{u,u+1,...,z\}$, $u=1,2,...,z$, at the operation states $zc_{bl}$, $b=1,2,...,v$, $l=1,2,...,w$, are fixed. As a special case of the above described the critical infrastructure total unconditional operation cost optimization problem, if $r$, $r=1,2,...,z$, is a CI network critical safety state, we formulate the optimization problem as a linear programming model with the objective function of the following form

$$\overline{\boldsymbol{K}}_{LLS}^4(r) \cong \sum_{b=1}^{v}\sum_{l=1}^{w} pq_{bl}[\overline{\boldsymbol{K}}_{LLS}^4(r)]^{(bl)},$$

for a fixed $r$, $r \in \{1,2,...,z\}$ and with the following bound constraints

$$\breve{p}q_{bl} \le pq_{bl} \le \hat{p}q_{bl}, \; b=1,2,...,v, \; l=1,2,...,w,$$

$$\sum_{b=1}^{v}\sum_{l=1}^{w} pq_{bl} = 1,$$

where

$$[\overline{\boldsymbol{K}}_{LLS}^4(r)]^{(bl)}, \; [\overline{\boldsymbol{K}}_{LLS}^4(r)]^{(bl)} \ge 0,$$
$$b=1,2,...,v, \; l=1,2,...,w,$$

are fixed mean values of the critical infrastructure conditional lifetimes in the safety state subset $\{r,r+1,...,z\}$ and

$$\breve{p}q_{bl}, \; 0 \le \breve{p}q_{bl} \le 1 \text{ and } \hat{p}q_{bl}, \; 0 \le \hat{p}q_{bl} \le 1,$$
$$\breve{p}q_{bl} \le \hat{p}q_{bl}, \; b=1,2,...,v, \; l=1,2,...,w,$$

are lower and upper bounds of the transient probabilities $pq_{bl}$, $b=1,2,...,v$, $l=1,2,...,w$, respectively.

Now, we can obtain the optimal solution of the formulated by (19)-(23) the linear programming problem using the procedure presented in [Kołowrocki, et al., 2017c]. Namely, we can find the optimal values $\dot{p}q_{bl}$, $b=1,2,...,v$, $l=1,2,...,w$, of the transient probabilities $pq_{bl}$ that minimize the mean value of the CI network unconditional operation cost in the safety state subset $\{r, r+1, ..., z\}$, assuming LLS dependency model, defined by the linear form (19), giving its minimum value in the following form

$$\dot{\bar{\boldsymbol{K}}}^4_{LLS}(r) \cong \sum_{b=1}^{v}\sum_{l=1}^{w} \dot{p}q_{bl} [\bar{\boldsymbol{K}}^4_{LLS}(r)]^{(bl)} \qquad (24)$$

for a fixed $r$, $r \in \{1,2,...,z\}$.

Thus, considering (14), the coordinates of the optimal instantaneous operation cost of the CI network with LLS rule in the form of the vector

$$\dot{\boldsymbol{K}}^4_{LLS}(t,\cdot) = [1, \dot{\boldsymbol{K}}^4_{LLS}(t,1), \ldots, \dot{\boldsymbol{K}}^4_{LLS}(t,z)], \ t \in <0,\infty),$$

are given by

$$\dot{\boldsymbol{K}}^4_{LLS}(t,u) \cong \sum_{b=1}^{v}\sum_{l=1}^{w} \dot{p}q_{bl} [\boldsymbol{K}^4_{LLS}(t,u)]^{(bl)}, \quad u = 1,2,...,z, \qquad (25)$$

where $\dot{p}q_{bl}$ are the optimal limit transient probabilities at the states $zc_{bl}$, $b = 1,2,...,v$, $l = 1,2,...,w$, of the operation process $ZC(t)$, $t \in <0,\infty)$, related to the climate-weather change and

$$[\boldsymbol{K}^4_{LLS}(t,u)]^{(bl)}, \ u = 1,2,...,z, \ b = 1,2,...,v, \ l = 1,2,...,w,$$

are the coordinates of the critical infrastructure conditional instantaneous operation costs in the safety state subsets $\{u, u+1, ..., z\}$, $u = 1,2,...,z$, impacted by the operation process $ZC(t)$, $t \in <0,\infty)$, related to the climate-weather change process at the states $zc_{bl}$, $b = 1,2,...,v$, $l = 1,2,...,w$.

Replacing in (24) $r$ by $u$, we get the expressions for the optimal mean values of the CI network with LLS rule unconditional operation costs in the safety state subset $\{u, u+1, ..., z\}$, $u = 1,2,...,z$, giving its minimum value in the following form

$$\dot{\bar{\boldsymbol{K}}}^4_{LLS}(u) \cong \sum_{b=1}^{v}\sum_{l=1}^{w} \dot{p}q_{bl} [\bar{\boldsymbol{K}}^4_{LLS}(u)]^{(bl)}, \ u = 1,2,...,z. \qquad (26)$$

The optimal solutions for the mean values of the CI network with LLS rule unconditional operation costs in the particular safety states are

$$\dot{\bar{\bar{\boldsymbol{K}}}}^4_{LLS}(u) = \dot{\bar{\boldsymbol{K}}}^4_{LLS}(u) - \dot{\bar{\boldsymbol{K}}}^4_{LLS}(u+1), \ u = 1, \ldots, z-1,$$
$$\dot{\bar{\bar{\boldsymbol{K}}}}^4_{LLS}(z) = \dot{\bar{\boldsymbol{K}}}^4_{LLS}(z), \qquad (27)$$

where $\dot{\bar{\boldsymbol{K}}}^4_{LLS}(u)$, $u = 1,2,...,z$, are given by (26).

Moreover, if we define the corresponding critical operation cost function by

$$\boldsymbol{K}^4_{LLS}(t) = \boldsymbol{K}^4_{LLS}(t,r), \ t \geq 0, \qquad (28)$$

and the moment $\zeta^4_{LLS}$ when the critical infrastructure operation cost exceeds a permitted level $\kappa$, by

$$\zeta^4_{LLS} = \boldsymbol{K}^4_{LLS}{}^{-1}(\kappa), \qquad (29)$$

where $\boldsymbol{K}^4_{LLS}(t,r)$ is given by (14) for $u = r$ and $\boldsymbol{K}^4_{LLS}{}^{-1}(\kappa)$ is the inverse function of the critical operation cost function $\boldsymbol{K}^4_{LLS}(t)$ given by (28), then the corresponding optimal critical operation cost function is given by

$$\dot{\boldsymbol{K}}^4_{LLS}(t) = \dot{\boldsymbol{K}}^4_{LLS}(t,r), \ t \geq 0, \qquad (30)$$

then the optimal moment $\dot{\zeta}^4_{LLS}$ when the CI network operation cost exceeds a permitted level $\kappa$, is given by

$$\dot{\zeta}^4_{LLS} = \dot{\boldsymbol{K}}^4_{LLS}{}^{-1}(\kappa), \qquad (31)$$

where $\dot{\boldsymbol{K}}^4_{LLS}(t,r)$ is given by (25) for $u = r$ and $\dot{\boldsymbol{K}}^4_{LLS}{}^{-1}(\kappa)$, if it exists, is the inverse function of the optimal critical operation cost function $\dot{\boldsymbol{K}}^4_{LLS}(t)$ given by (30).

### 4.3. Cost Analysis of CI Network Operation Impacted by Climate-Weather Change

We consider the multistate series CI network, consisted of $n$ assets dependent according to the LLS rule, in its operation process $ZC(t)$, $t \in <0,\infty)$, related to climate-weather change and we assume that the operation cost of its single basic asset at the operation state $zc_{bl}$, $b = 1,2,...,v$, $l = 1,2,...,w$, during the critical infrastructure operation time $\theta$, $\theta \geq 0$, amounts

$$[K_i(\theta)]^{(bl)}, \ b = 1,2,...,v, \ l = 1,2,...,w, \ i = 1,2,...,n.$$

First, we suppose that the CI network is non-repairable and during the operation time $\theta$, $\theta \geq 0$, it has not exceeded the critical safety state $r$. In this case, the total cost of the non-repairable CI network with LLS rule during the operation time $\theta,\ \theta \geq 0,$ is given by

$$K_{LLS}(\theta) \cong \sum_{b=1}^{v}\sum_{l=1}^{w} pq_{bl} \sum_{i=1}^{n}[K_i\ (\theta)]^{(bl)},\ \theta \geq 0, \quad (32)$$

where $pq_{bl},\ b=1,2,...,v,\ l=1,2,...,w,$ are transient probabilities defined by (3).

Further, we consider another case and we additionally assume that the CI network is repairable after exceeding the critical safety state $r$ and its renovation time is ignored and the cost of its single renovation is constant and equal to $K_{ign}$.

In this case, the total operation cost of the repairable CI network assuming LLS dependency model with ignored its renovation time during the operation time $\theta,\ \theta \geq 0,$ amounts

$$K_{LLS\_ign}(\theta) \cong \sum_{b=1}^{v}\sum_{l=1}^{w} pq_{bl} \sum_{i=1}^{n}[K_i\ (\theta)]^{(bl)} + K_{ign}H(\theta,r),\ \theta \geq 0, \quad (33)$$

where $pq_{bl},\ b=1,2,...,v,\ l=1,2,...,w,$ are transient probabilities defined by (3) and $H(\theta,r)$ is the mean value of the number of exceeding the critical safety state $r$ by the critical infrastructure operating at the variable conditions during the operation time $\theta$ defined by (3.58) in [Kołowrocki, Soszyńska-Budny, 2011] and in [Guze, Kołowrocki, 2017].

Now, we assume that the CI network is repairable after exceeding the critical safety state $r$ and its renewal time is non-ignored and have distribution function with the mean value $\mu_0(r)$ and the standard deviation $\sigma_0(r)$ and the cost of the critical infrastructure single renovation is $K_{n-ign}$.

In this case, the total operation cost of the repairable CI network assuming LLS dependency model with not ignored its renovation time during the operation time $\theta,\ \theta \geq 0,$ amounts

$$K_{LLS\_n-ign}(\theta) \cong \sum_{b=1}^{v}\sum_{l=1}^{w} pq_{bl} \sum_{i=1}^{n}[K_i\ (\theta)]^{(bl)} + K_{n-ign}\overline{\overline{H}}(\theta,r),\ \theta \geq 0, \quad (34)$$

where $pq_{bl},\ b=1,2,...,v,\ l=1,2,...,w,$ are transient probabilities defined by (3) and $\overline{\overline{H}}(\theta,r)$ is the mean value of the number of renovations of the critical infrastructure operating at the variable conditions during the operation time $\theta$ defined by (3.92) [Kołowrocki, Soszyńska-Budny, 2011] and in [Guze, Kołowrocki, 2017].

The particular expressions for the mean values $H(\theta,r)$ and $\overline{\overline{H}}(\theta,r)$ for the repairable CI network with ignored and non-ignored renovation times existing in the formulae (33) and (34), are determined in Chapter 3 [Kołowrocki, Soszyńska-Budny, 2011] for typical multistate repairable critical infrastructure operating at the variable operation conditions and in [Guze, Kołowrocki, 2017].

After the optimization of the critical infrastructure operation process related to climate-weather change, the CI network operation total costs given by (32)-(34) assume their optimal values.

The total optimal cost of the non-repairable CI network with LLS rule during the operation time $\theta,$ $\theta \geq 0,$ after its operation process related to climate-weather change optimization is given by

$$\dot{K}_{LLS}(\theta) \cong \sum_{b=1}^{v}\sum_{l=1}^{w} \dot{p}q_{bl} \sum_{i=1}^{n}[K_i\ (\theta)]^{(bl)},\ \theta \geq 0,$$

where $\dot{p}q_{bl},\ b=1,2,...,v,\ l=1,2,...,w,$ are optimal transient probabilities found by the procedure presented in Section 7.3 in [Kołowrocki, et al., 2017c].

The optimal total operation cost of the repairable CI network assuming LLS dependency model with ignored its renovation time during the operation time $\theta,\ \theta \geq 0,$ after its operation process related to climate-weather change optimization amounts

$$\dot{K}_{LLS\_ign}(\theta) \cong \sum_{b=1}^{v}\sum_{l=1}^{w} \dot{p}q_{bl} \sum_{i=1}^{n}[K_i\ (\theta)]^{(bl)} + K_{ign}\dot{H}(\theta,r),\ \theta \geq 0,$$

where $\dot{p}q_{bl},\ b=1,2,...,v,\ l=1,2,...,w,$ are optimal transient probabilities and $\dot{H}(\theta,r)$ is the mean value of the optimal number of exceeding the critical safety state $r$ by the critical infrastructure operating at the variable conditions during the operation time $\theta$ [Kołowrocki, Soszyńska-Budny, 2011].

The total optimal operation cost of the repairable critical infrastructure with non-ignored its renovation time during the operation time $\theta,\ \theta \geq 0,$ after its operation process related to climate-weather change optimization amounts

$$K_{LLS\_n-ign}(\theta) \cong \sum_{b=1}^{v}\sum_{l=1}^{w} pq_{bl} \sum_{i=1}^{n}[K_i\ (\theta)]^{(bl)} + K_{n-ign}\overline{\overline{H}}(\theta,r),\ \theta \geq 0,$$

where $\dot{p}q_{bl}$, $b = 1,2,...,v$, $l = 1,2,...,w$, are optimal transient probabilities and $\dot{\bar{\bar{H}}}(\theta, r)$ is the mean value of the optimal number of renovations of the critical infrastructure operating at the variable operation conditions during the operation time $\theta$ [Kołowrocki, Soszyńska-Budny, 2011].

The particular expressions for the optimal mean values $\dot{H}(\theta,r)$ and $\dot{\bar{\bar{H}}}(\theta,r)$ for the repairable critical infrastructure with ignored and non-ignored renovation times existing in the formulae (36) and (37), may be obtain by replacing the transient probabilities $pq_{bl}$ by their optimal values $\dot{p}q_{bl}$ in the expressions for $H(\theta,r)$ and $\bar{\bar{H}}(\theta,r)$, that are determined in Chapter 3 [Kołowrocki, Soszyńska-Budny, 2011], for typical multistate repaired critical infrastructure operating at the variable operation conditions.

The application of the formulae (32)-(34) and (35)-(37) allow us to compare the costs of the non-repairable and repairable CI networks with ignored and non-ignored times of renovations operating at the variable operation conditions before and after the optimization of their operation processes.

These operation total costs of CI networks and their optimal values can allow to compare the costs of the non-repairable and repairable CI networks with ignored and non-ignored times of renovations operating at the variable operation conditions before and after the optimization of their operation processes.

## 5. Conclusion

In this paper optimization of operation and safety of CI networks with cascading effects is presented. More exactly, a multistate series network with assets dependent according to local load sharing (LLS) rule is described and analysis of operation cost related to climate-weather change of such CI network with LLS rule is given. Similarly, CIs and CI networks with other safety structures and models of dependencies can be considered [Kołowrocki, et al., 2017a, c].

Namely, the conditional safety function of the multistate parallel, "*m* out of *n*", parallel-series and "*m* out of *l*"-series CI network with assets dependent according to equal load sharing (ELS) rule, impacted by the operation process related to the climate-weather change process, can be determined. In ELS model of dependency, described in [Blokus-Roszkowska, Kołowrocki, 2017a-b], [Kołowrocki, et al., 2017a], we assume that after decreasing the safety state by some of assets the increased load can be shared equally among the remaining assets. Considering the CI network impacted by the operation process related to the climate-weather change, we assume that asset stress proportionality correction coefficient can take different values at particular CI network operation states [Kołowrocki, et al., 2017c].

Considering cascading effects in networks with more complex structures we can link the results of safety analysis for CI networks with the LLS and ELS rule. Then, apart from the dependency of subnetworks' departures from the safety states subsets we can take into account the dependencies between assets in subnetworks. This way we can proceed with parallel-series and "*m* out of *l*"-series CI networks assuming the dependence between their parallel, respectively "*m* out of *l*", subnetworks according to the LLS rule and the dependence between their assets in subnetworks according to the ELS rule, by constructing a mixed load sharing (MLS) model of dependency [Blokus-Roszkowska, Kołowrocki, 2017a-b], [Kołowrocki, et al., 2017a].

Then, similarly as in this paper for the CI network with LLS rule, we can determine the optimal instantaneous operation cost of CI networks assuming other models of dependency [Kołowrocki, et al., 2017c]. The operation total costs of the CI network with the ELS rule and MLS rule can be automatically optimized after the optimization of the critical infrastructure operation process related to climate-weather change. Then, the total optimal cost of the non-repairable and repairable CI network during the operation time can be estimated and the results can be compared.

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