Mining Science, vol. 25, 2018, 111–127

www.miningscience.pwr.edu.pl

Mining Science

(Previously Prace Naukowe Instytutu Gornictwa Politechniki Wroclawskiej, ISSN 0370-0798)

ISSN 2300-9586 (print)
ISSN 2353-5423 (online)

Received December 31, 2017; reviewed; accepted August 10, 2018

DETERMINATION OF “m_i” IN THE HOEK–BROWN FAILURE CRITERION OF ROCK

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Abstract: The m_i is an important parameter in the use of the Hoek–Brown failure criterion. It can be estimated using a triaxial compressive strength test but in many projects there is no actual test result for the parameter. An estimation of m_i comes from a reference table giving a constant value. Elsewhere some empirical equations for the value were suggested in the 1990s. Applying these equations is limited use since they are available for a few rock types and the equations are based on just uniaxial compressive strength tests of rock. In this research rocks were divided into three categories (Igneous, Sedimentary and Metamorphic) and three empirical formulas are suggested for the categories based on uniaxial compressive strength (σ_{ci}) and tensile strength (σ_t) of rocks by nonlinear regression. The equations have been obtained by a combination of the two independent parameters and the trial and error method was used to find the equations with the highest correlation coefficient. The data base uses data from many original international research projects and much data from Iranian tunnelling projects. The models have a high level of accuracy and have been used to describe most rock types although the authors know that the technique can be improved using a new and larger collection of data in the future.

Keywords: *Hoek–Brown criterion, m_i, rock strength, regression, empirical model*

1. INTRODUCTION

The uniaxial compressive strength value is a useful criterion in predicting rock failure in a uniaxial field stress. But when there is a triaxial field stress, we need a mathematical function which considers all three principal stresses to find the rock fracturing boundary. This function is a failure envelope which can be obtained using

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doi: 10.5277/msc182509

a mathematical analysis or through an empirical approach. Six recent studies have shown (Sheory 1997) that empirical models using a nonlinear form have a better agreement with experiences.

There are a number of models that describe the compressive failure envelope rock mass but the most famous practical model is that proposed by Hoek and Brown. Most of the famous models are shown in Table 1.

Table 1. Important empirical intact rock failure criterion

No.	Models	References
1	$(\sigma_1-\sigma_3)^2=a+b(\sigma_1+\sigma_3)$	Fairhurst 1964
2	$\sigma_1=\sigma_c+\sigma_3+a\sigma_3^b$	Hobbs 1964
3	$\sigma_1=\sigma_c+a\sigma_3^b$	Murrel 1965
4	$\sigma_1=\sigma_c+\sigma_3+a\sigma_3$	Bodonyi 1970
5	$\sigma_1=\sigma_3+a(\sigma_1+\sigma_3)^b$	Franklin 1971
6	$\frac{\sigma_1}{\sigma_c}=a+b\left[\frac{\sigma_3}{\sigma_c}\right]^\alpha$	Bieniawski 1974 Yudhbir et al. 1983
7	$\sigma_1=\sigma_3+\sigma_c\left(m\frac{\sigma_3}{\sigma_c}+s\right)^{\frac{1}{2}}$	Hoek & Brown 1980
8	$\sigma_1=\sigma_3+B\sigma_c\left(\frac{\sigma_c}{\sigma_3}\right)^\alpha$	Ramamurthy et al. 1985
9	$\frac{\sigma_1}{\sigma_c}=\left[\left(\frac{M}{B}\right)\left(\frac{\sigma_3}{\sigma_c}\right)+1\right]^B$	Johnston 1985
10	$\sigma_1+\sigma_3\left[1+\frac{\sigma_3}{\sigma_t}\right]^b$	Balmer 1952 Sheory et al. 1989
11	$\sigma_1=\sigma_3+A\sigma_c\left[\frac{\sigma_3}{\sigma_c}-S\right]^{\frac{1}{B}}$	Yoshida 1990
12	$\sigma_1=\sigma_3+(\sigma_3+\sigma_t)B\left[\frac{\sigma_c}{\sigma_3+\sigma_t}\right]^\alpha$	Ramamurthy 2001
13	$\sigma_1=\sigma_3+\sigma_c\left(m_i\frac{\sigma_3}{\sigma_c}+1\right)^\alpha$	Hoek et al. 2002
14	$\frac{\sigma_1}{\sigma_c}=1+m\left(\frac{\sigma_3}{\sigma_c}\right)^{0.5}$	Mogi 2007
15	$\sigma_1=\left(\sqrt{\sigma_3}+\sqrt{\sigma_c}\right)^2$	You 2011

2. ROCK CONSTANT IN HOEK-BROWN CRITERION

The currently most famous of these empirical rock failure criteria is the Hoek–Brown criterion, and it is used in many rock mechanical software packages, such as Phase2, FLAC, UDEC, PLAXIS. The criterion has been improved several times in papers given in 1983, 1988, 1992, 1995 and 2002. These improvements have increased the accuracy of rock failure prediction. The final generalized form of the model for rock mass is as follows (Hoek et al. 2002):

$$\sigma_1 = \sigma_3 + \sigma_{ci}\left(m_b \frac{\sigma_3}{\sigma_{ci}} + S\right)^a \tag{1}$$

where, m_b, S and a are rock constants and S is one and a is 0.5 for intact rocks. The basic equation that Hoek & Brown proposed to describe intact rock is as follow (Hoek & Brown 1980):

$$\sigma_1 = \sigma_3 + \sigma_{ci}\left(m_i \frac{\sigma_3}{\sigma_{ci}} + 1\right)^{0.5} \tag{2}$$

where m_b is a rock mass constant which it has been evaluated based on the m_i value. The m_i is determined by a regression analysis based on the principal stresses at failure (Eqs. (11)–(19)). Other simple way to obtain the mi is by using a reference table (Hoek et al. 1995) & (Roclab 2007). The parameter depends upon the mineralogy, composition and grain size of the intact rock (Hoek et al. 2002).

The m_b, S and a values can be calculated by an empirical equation (Hoek et al. 2002) and also they can be determined knowing the peak and residual strength conditions of the rock mass. In fact m_b is a reduced value of the rock constant (m_i) which can be estimated (Hoek et al. 2002) by:

$$m_b = m_i \exp\left(\frac{GSI - 100}{28 - 14D}\right) \tag{3}$$

S and a are also estimate for rock mass as follows:

$$S = \exp\left(\frac{GSI - 100}{9 - 3D}\right) \tag{4}$$

$$a = 0.5 + \frac{1}{\left(e^{-\frac{GSI}{15}} - e^{-\frac{20}{3}}\right)} \tag{5}$$

where *GSI* is a rock mass classification system (Geological Strength Index) and *D* is disturbance factor which depends upon the degree of disturbance of the rock mass as a result of blasting or due to impact damage or stress relaxation. The value of the parameter is between zero for an undisturbed rock mass and one for a completely disturbed rock mass. It is usually applied to the zone of disturbed (damaged) rock mass, around a tunnel, limited to a range between 1 and 3 meters. Equations 3 to 5 evaluate the peak strength of the rock mass. Cai and his colleagues (Cai et al. 2007) introduced some empirical models to evaluate GSI in residual condition (GSI_r) and also m_r, S_r and a_r are as follows:

$$GSI_r = GSIe^{-0.0134GSI} \tag{6}$$

$$m_r = m_i \exp\left(\frac{GSI_r - 100}{28}\right) \tag{7}$$

$$S_r = \exp\left(\frac{GSI_r - 100}{9}\right) \tag{8}$$

$$a_r = 0.5 + \frac{1}{\frac{1}{6}\left(e^{-\frac{GSI_r}{15}} - e^{-\frac{20}{3}}\right)} \tag{9}$$

$$\sigma_1 = \sigma_3 + \sigma_{ci}\left(m_r \frac{\sigma_3}{\sigma_{ci}} + S_r\right)^{a_r} \tag{10}$$

The three main independent parameters are the m_i, *GSI* and *D* values. *GSI* has been updated several times in 1995, 1998 (Hoek et al. 1995; 1998) and 2002 (Sonmez & Ulusay 2002), 2004 (Cai et al. 2004), 2009 (Russo 2009) and finally in 2013 by Hoek and his colleagues (Hoek et al. 2013). The Value of D was introduced by Hoek (Hoek et al. 2002) as a reference table and also more recently has been developed as an equation and a graph (Sonmez & Ulusay 2002). But methods to derive m_i have not been updated into a new form. The authors have tried to introduce three empirical formulae to calculate m_i for igneous, sedimentary and metamorphic rocks. These models are based on more than one hundred real data sets. Figure 1 illustrates the effect of different values of the constant m_i upon the Mohr failure envelope for intact rock (S = 1 and a = 0.5) where σ_{ci} is equal to one.

Hoek and Brown showed two graphs for estimating m_i, first using *GSI vs.* ratio of the cohesive strength and uniaxial strength of intact rock and second using *GSI vs.* internal friction angle of the rock mass but they are not sufficiently accurate (Fig. 2). The value of the m_i can be evaluated using data (five or more tests) from triaxial tests. The tests should be carried out over a confining stress range from zero to one half of the uniaxial compressive strength of the rock sample (Hoek & Brown, 1997). Equation 2 was then changed to allow a linear regression analysis (Eq. (11)).

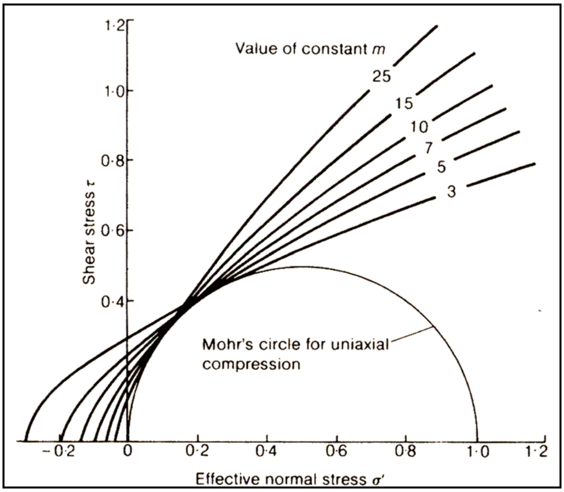

Fig. 1. Effect of m_i value on shape of the Mohr failure envelope (Hoek 1983)

$$(\sigma_1 - \sigma_3)^2 = S\sigma_{ci}^2 + m_i\sigma_{ci}\sigma_3 \tag{11}$$

$$y = A + Bx \tag{12}$$

$$y = \sigma_3 \tag{13}$$

$$y = (\sigma_1 - \sigma_3)^2 \tag{14}$$

$$A = S(\sigma_{ci}^2) \tag{15}$$

S is one for intact rock and then:

$$\sigma_{ci} = \sqrt{A} \tag{16}$$

$$B = m_i\sigma_{ci} \Rightarrow m_i = \frac{B}{\sigma_{ci}} \tag{17}$$

$$a = \frac{\Sigma y}{n} - \left[\frac{\Sigma xy - \left(\frac{\Sigma x \Sigma y}{n} \right)}{\Sigma x^2 - \left(\frac{(\Sigma\Sigma x)^2}{n} \right)} \right] \frac{\Sigma x}{n} \tag{18}$$

$$B = \frac{\Sigma xy - \left(\frac{\Sigma x \Sigma y}{n}\right)}{\Sigma x^2 - \left(\frac{\Sigma x^2}{n}\right)} \tag{19}$$

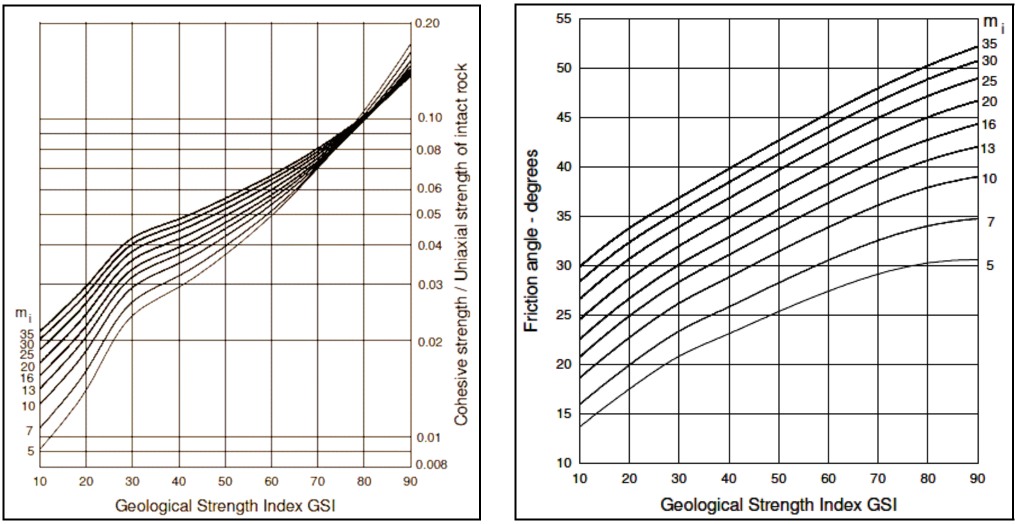

Fig. 2. Relationship between ratio of $\frac{C}{\sigma_{ci}}$ and *GSI* and also between ϕ and *GSI* (Hoek & Brown 1997)

Another method to estimate of m_i is through empirical models, Hosseini illustrated that m_i has a relationship with the rock uniaxial compressive strength (Hosseini 1993). He used empirical data from different references from other scientists to carry out non-linear regression and suggested seven equations describing the mechanical behavior for claystone, coal, granite, granodiorite, limestone, sandstone and shale.

3. THE EFFECT OF "m_i" ON THE SIZE OF THE PLASTIC ZONE IN TUNNELS

Examining Eqs. (1), (2) and (3) it is clear that there is a non-linear relationship between m_i and σ_1. The importance of the m_i can be seen by considering the stress analysis of surface and underground spaces. As a primary example consider rock mass around a typical road tunnel. The depth of the tunnel is 100 meters and its diameter and height are 12.2 and 8 meters, respectively. The tunnel is unsupported which will provide a maximum size for the plastic zone. The rock substance strength is assumed to have a typical average value shown in Tables 2 and 3 and the applied field stress is hydrostatic ($K = 1$).

Table 2. Intact rock and rock mass properties

σ_{ci} (MPa)	E_i (MPa)	GSI	D	γ (kN/m^3)	E_m (MPa)
30	10000	30	0	27	810

Table 3. Constant values of the Hoek-Brown criterion in this research (Peak and residual strength)

Peak strength				Residual strength		
m_i	m_b	S	a	m_r	S_r	a_r
5	0.410	0.0004	0.522	0.228	0.00014	0.544
10	0.821	0.0004	0.522	0.575	0.00014	0.544
15	1.230	0.0004	0.522	0.864	0.00014	0.544
20	1.640	0.0004	0.522	1.150	0.00014	0.544
25	2.052	0.0004	0.522	1.440	0.00014	0.544
30	2.460	0.0004	0.522	1.730	0.00014	0.544

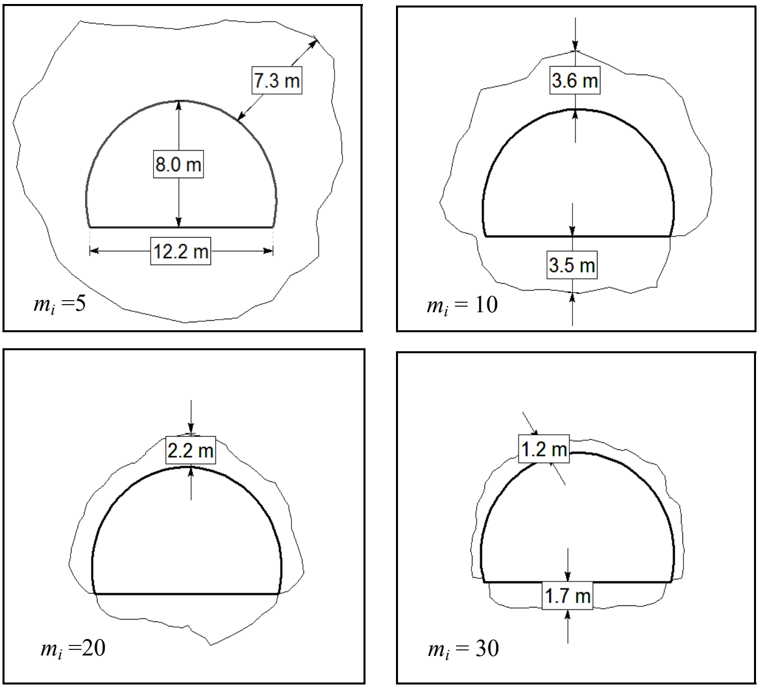

Fig. 3. Influence of the m_i on the size of yielding zone around a tunnel

Residual Strength parameters have been calculated using the Eqs. (6)–(9) which proposed by Cai and his colleagues (Cai et al. 2007). A wide range of values were selected for m_i from 5 to 30 (5, 10, 15, 20, 25, and 30), from which the size of the plastic zone was calculated using a finite element software (Phase[2]) which models the elasto-plastic behavior of rock and soil into the post failure condition, by considering residual strength parameters. Figure 3 illustrates some results of the numerical modelling with different values of the m_i and a graph has been plotted in Figure 4 showing the influence of the m_i on size of yielding zone (plastic zone).

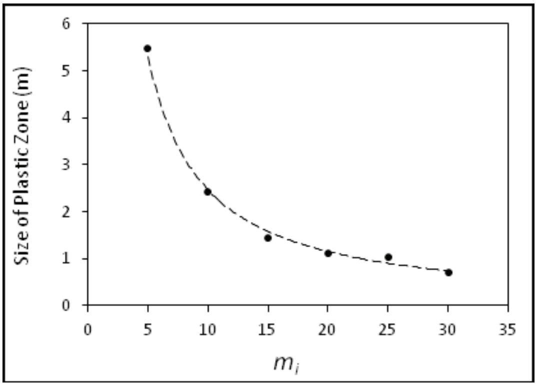

Fig. 4. Variation of m_i versus plastic zone

4. STATISTICAL ANALYSIS TO EVALUATE "m_i"

Collection of accurate data on the triaxial compressive strength of different rocks is difficult and expensive. The author has collected significant volumes of data for three types of rocks from several important references. Some data comes from Iranian tunnel projects (Ettehad Rah Co. 2015; Ettehad Rah Co. 2008; Iranoston Co. 2012; Pars Co. 2014; Rahvar e Iran Co. 2014) and many of them are from several other researchers around the world (Akai et al. 1970; Aldritch 1969; Attewell & Sandford 1974; Barat 1995; Betourney et al. 1991; Borecki et al. 1982; Brace 1964; Broch 1974; Chan et al. 1972; Dayre & Giraud 1986; Donath 1964; Everling 1960; Franklin & Hoek 1970; Gnirk & Cheatham 1965; Hareland et al. 1993; Hobbs 1964; Hobbs 1970; Hoek 1983; Hoshino et al. 1972; Hoskins 1969; Hosseini & Vutukuri 1993; Jaeger 1970; Johnston 1985; Johnson et al. 1987; Kovari & Tisa 1975; Kwasniewski 1983; Misra 1972; Mogi 1965; Mogi 1966; Murrel 1965; Ouyang & Elsworth 1991; Ramamurthy 1989; Ramez 1967; Rao et al. 1983; Shea-Albin et al. 1991; Sheory et al. 1989; Shimada, et al. 1983; Singh 1995; Vutukuri & Farough Hosseini 1993; Wang & Kemeny 1995; Wilhelmi & Somerton 1967). However it should be noted that most of the data has come from an especial reference entitled "Empirical rock failure criteria" (Sheory 1997).

The data includes two independent parameters (uniaxial compressive strength (σ_{ci}) and tensile strength (σ_t)) as well as the dependent parameter (m_i). Scattering of the points were poor quality when the plot was just between m_i and σ_{ci} or m_i and σ_t. An initial attempt to find a regression between the two independent parameters was not enough successful, when it looked at the full data field (Fig. 5). A simple and primary equation for all data as a first attempt suggested as follow:

$$y = 1.8575x^{-0.893} \tag{20}$$

$$\frac{\ln m_i}{\frac{\sigma_{ci}}{\sigma_t}} = 1.8575\left(\frac{\sigma_{ci}}{\sigma_t}\right)^{-0.893} \tag{21}$$

$$\ln m_i = 1.8575\left(\frac{\sigma_{ci}}{\sigma_t}\right)^{0.107} \tag{22}$$

$$m_i = e^{1.858\left(\frac{\sigma_{ci}}{\sigma_t}\right)^{0.107}} \tag{23}$$

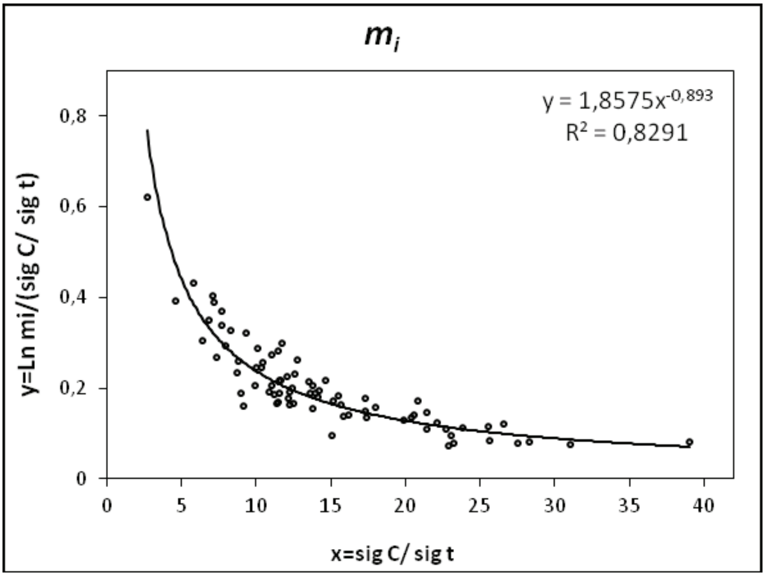

Fig. 5. Regression graph for all type of rocks

The data was then divided into the three categories of rock: sedimentary, metamorphic and igneous. This trial and error approach gave the best combination for the two parameters (σ_{ci}, σ_t) (in more than thirty cases) as an individual independent parameter and, as a result, is suggested with the following equation.

For igneous rocks:

$$m_i = e^{\left[1.2\left(\frac{\sigma_{ci}-2\sigma_t}{\sigma_t}\right)^{0.30}\right]} \tag{24}$$

$$r^2 = 0.961$$

where m_i is the constant of intact rock in Hoek–Brown criterion, σ_{ci} is the uniaxial compressive strength of intact rock in MPa and σ_t is the tensile strength of intact rock in MPa. The number of cases to generate the regression analysis was 27 and the graph of the curve is shown in Figure 6. Figure 7 shows the cross value graph between the measured and estimated data.

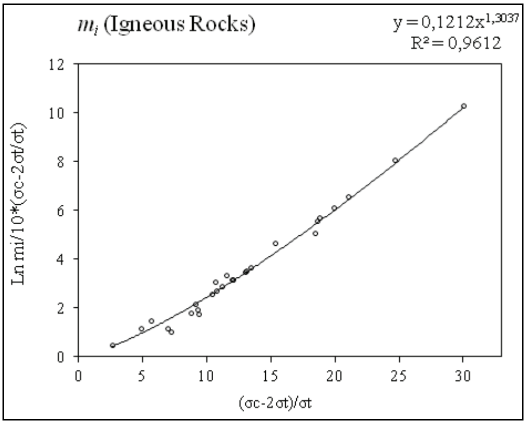

Fig. 6. Regression graph for igneous rocks

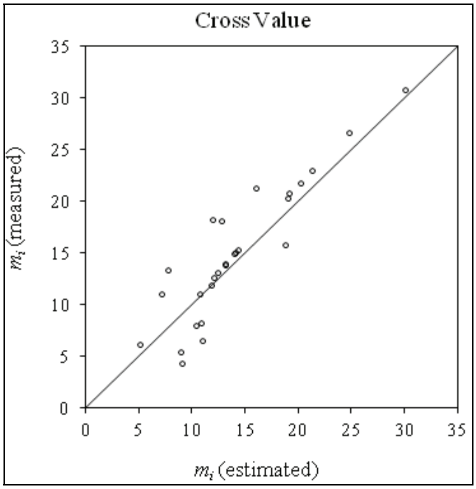

Fig. 7. Cross value graph for igneous rocks

For sedimentary rocks:

$$m_i = e^{\left[1.3\left(\frac{\sigma_{ci}-2.5\sigma_t}{\sigma_t}\right)^{0.26}\right]} \tag{25}$$

$$r^2 = 0.9884$$

The number of cases used for the regression analysis was 59. Figures 8 and 9 show the regression curve and the cross value graph, respectively.

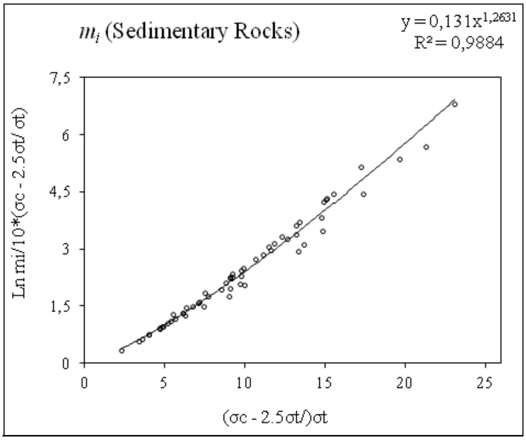

Fig. 8. Regression graph for sedimentary rocks

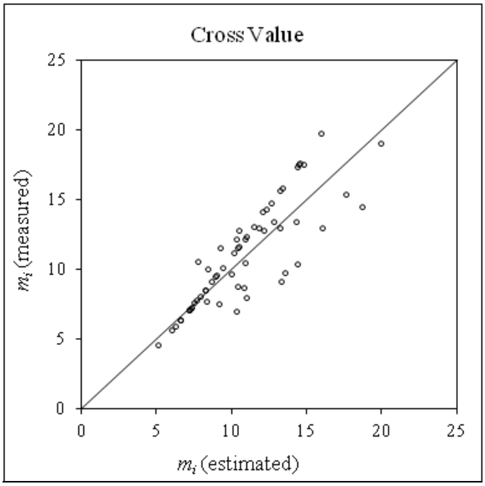

Fig. 9. Cross value graph for sedimentary rocks

For metamorphic rocks:

$$m_i = e^{\left[1.3\left(\frac{\sigma_{ci} - 1.5\sigma_t}{\sigma_t}\right)^{0.28}\right]} \tag{26}$$

$$r^2 = 0.993$$

The number of cases for the regression analysis was 21. Figures 10 and 11 show the regression curve and the cross value graph for these rocks (metamorphic), respectively.

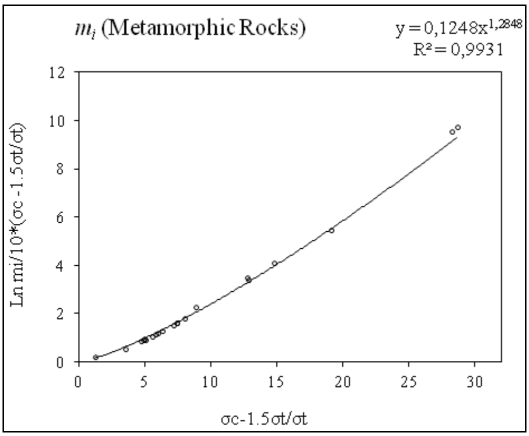

Fig. 10. Regression graph for metamorphic rocks

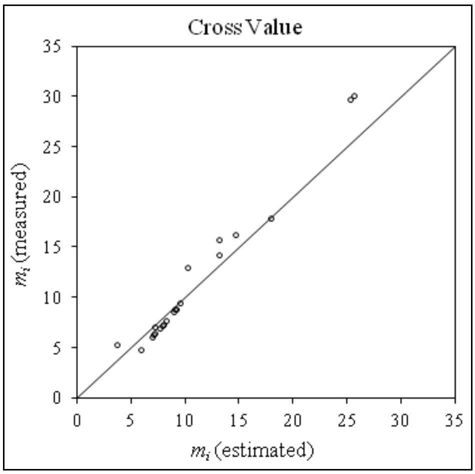

Fig. 11. Cross value graph for metamorphic rocks

The relationship can be written as a generalized formula as shown below or can be found using a table for the constant values of the model such as shown in Table 4. List of rock types used in the analysis is in Table 5.

$$m_i = e^{\left[a\left(\frac{\sigma_{ci} - b\sigma_t}{\sigma_t}\right)^c\right]} \tag{27}$$

Table 4. Constant values for the suggested models

Rock category	*a*	*b*	*c*
Igneous	1.2	2	0.30
Sedimentary	1.3	2.5	0.26
Metamorphic	1.3	1.5	0.28

Table 5. Different types of rock in the analysis

Rock category	Type of rocks
Igneous	Granite, Granodiorite, Granite Breccia, Diorite, Lamprophyre, Agglomerate Tuff, Basalt, Rhyolite, Quartz Diorite, Diabase, Gabbro, Tuff, Andesite.
Sedimentary	Sandstone, Limestone, Dolomite, Coal, Siltstone, Shale, Marl, Travertine, Anhydrite.
Metamorphic	Quartzite, Gneiss, Marble, Schist, Slate.

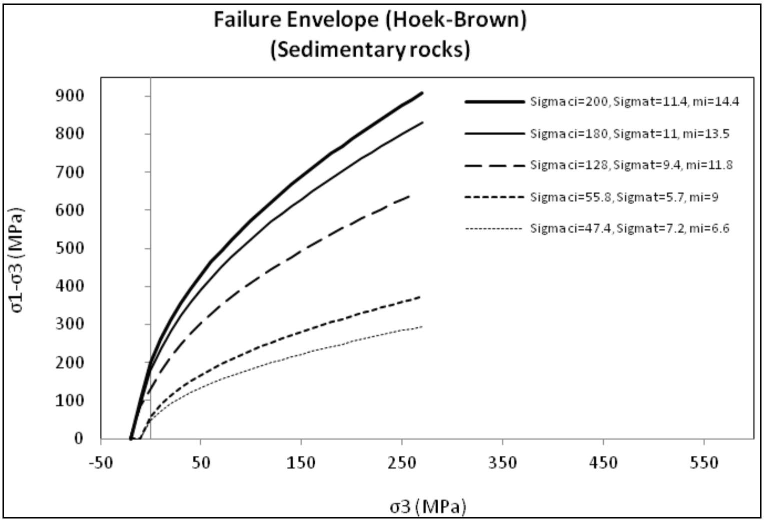

Fig. 12. Failure envelopes for sedimentary rocks

Based on the three suggested models for evaluation of the m_i, failure envelop for each one have been plotted which the graphs for sedimentary rocks has been shown in Figure 12, for igneous rocks has been shown in Figure 13 and finally for metamorphic rocks has been illustrated in Figure 14. The curves are Hoek-Brown failure criteria for intact rock. It means that the parameter *S* equal one and the parameter *a* equal 0.5. But m_i has been calculated by the equations 24, 25 & 26 which they have obtained by uniaxial compressive strength and tensile strength of the intact rock for each category.

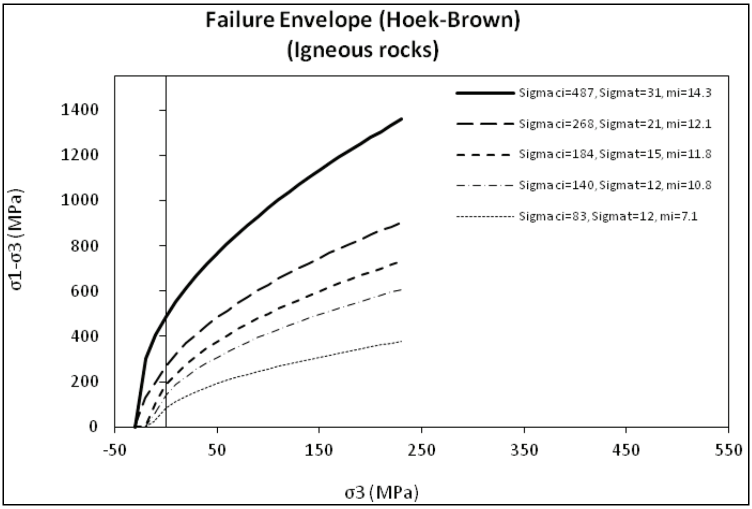

Fig. 13. Failure envelopes for igneous rocks

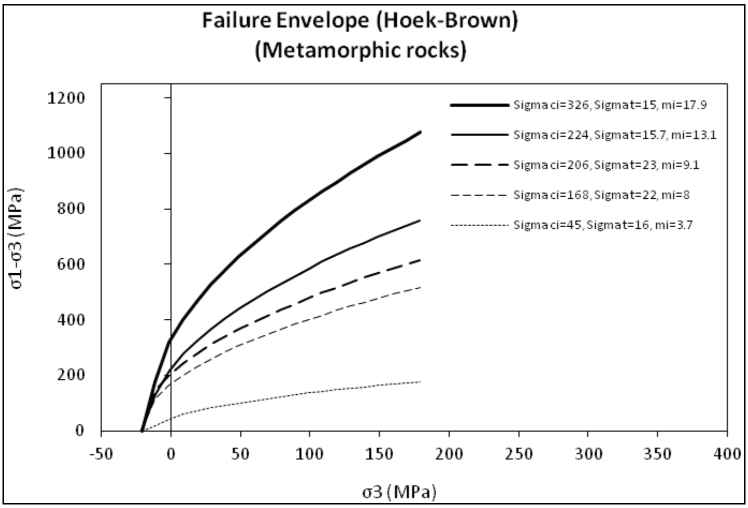

Fig. 14. Failure envelopes for metamorphic rocks

5. CONCLUSIONS

The application of most models (reference tables, graphs and empirical formulae) are limited in their ability to estimate the value of m_i. The author proposes a new model derived from a large database and with rock types divided into three categories (Igneous, Sedimentary and Metamorphic). Three empirical formulae have been suggested for the categories based on a specific combination of uniaxial compressive strength (σ_{ci}) and tensile strength (σ_t) of the intact rock as independent parameters. The equations have been obtained by trial and error to achieve equations found to have a high value for the correlation coefficient (Igneous: 0.961, Sedimentary: 0.988 and Metamorphic: 0.993). The models have thus a high level of accuracy and have been tested to cover most rock types but the author recognizes that this can be improved with larger access to databases in the future.

ACKNOWLEGMENTS

This research was partially supported by Azad University with research code 123a52.

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