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Semi-Markov reliability model of system composed of main subsystem, cold backup component and switch

Keywords
Semi-Markov model, reliability characteristics

Abstract
Probabilistic model of a system composed of a main component, an emergency backup component and the automatic switch are discussed in this paper. The reliability model is semi-Markov process describing evolution of the system. Conditional time to failure of the system is represented by a random variable denoting the first passage time of the process from the given state to the subset of states. The appropriate theorems of the Semi-Markov processes theory allow us to evaluate the reliability function and some reliability characteristics. To calculate the reliability function and mean time to failure of the system we apply theorems of the Semi-Markov processes theory concerning the conditional reliability functions.

1. Introduction

A model presented here is an essential modification of the models that have been considered by Barlow and Proshan [1], Brodi and Pogosian [2], Koroluk and Turbin [5] and Grabski [3], [4]. As a model of the two different cold standby system we construct so *Semi-Markov process* by defining the renewal kernel of that one. Construction of the renewal kernel is an important first step in solving the problem. The conditional time to failure of the system is described by a random variable that means the first passage time from the given state to the subset of states. To obtain the conditional reliability functions of the system we use appropriate system of integral equations. Passing to the Laplace transforms we get system of linear equations for transforms. The solution are Laplace transforms of the conditional reliability functions. Applying theorem on the conditional expectation of the first passage time to given subset of states we compute the mean time to failure of the system.

2. Description and assumptions

A system consists of a main component, an emergency backup component and the automatic switch. An operating unit A denotes main component of the system, the backup unit B means emergency component. In case of the unit A failure or repair of the unit B, takes place automatically switching over an operation process. The time to failure of the main subsystem is a random variable ζ_A with distribution is determined by the PDF $f_A(x),\ x \geq 0$. When the main subsystem A fails, the backup subsystem B is immediately put in motion by the switch, (switching time is omitted). The failed system is repaired (renewed). A repair time of the basic power system A is a random variable γ_A having distribution given by the PDF $h_A(x),\ x \geq 0$. At once after repairing the main subsystem A is put in motion by the switch and the not damaged component B is renewed in a negligibly short time. A time to failure of the component B is a random variable ζ_B with distribution defined by the PDF $f_B(x),\ x \geq 0$. If the backup subsystem B fails during repair period of the main subsystem A, then follows damage to of the whole system. The failure of the system takes place when the main subsystem A fails and the emergency subsystem fails before repairing the basic subsystem A or when the subsystem A fails and the switch fails. Let U be a random variable having a binary distribution

$P(U = k) = a^k(1-a)^{1-k}, \qquad k = 0,1,$
$0 < a < 1,$

where $U = 0$ if a switch is "down" at the moment of the subsystem A failure or renewal and $U = 1$ otherwise.
A restoring time of the whole system is the random variable γ with the distribution determined by the PDF $h(x), \ x \geq 0$. Moreover we assume that all random variables mentioned above are independent. A main goal of this paper is computing a reliability function and a mean time to failure. To solve the problem, 4 state semi-Markov reliability model is constructed. Moreover we assume that all random variables mentioned above are independent.

3. Model construction

To describe the reliability evolution of the system, we have to define the states and the renewal kernel. We introduce the following states:
0 – failure of the entire system due to a failure of a switch,
1 – failure of the entire system due to the failure of the backup subsystem B during repair period of subsystem A,
2 – failure of the main subsystem A, the backup subsystem B is working,
3 – both the main system and backup subsystem are "up" and system A is working.We assume that 3 is the initial state. Let us construct a process in the following way.
Let $0 = \tau_0$ and $\tau_1, \tau_2, \ldots$ denote instants of the subsystems failures or instants of the subsystem B or whole system repair. Let $\{X(t);\ t \geq 0\}$ be a stochastic process, with the right continuous trajectories, keeping constant values on half-intervals $[\tau_n, \tau_{n+1}),\ n = 0,1,\ldots$ defined by the rule

$$X(0) = 3, \quad X(t) = i \quad \text{for} \quad t \in [\tau_n, \tau_{n+1}),$$

$$n = 0,1.2 \ldots, \qquad i \in S = \{0,1,2,3\}$$

It is a semi-Markov process.

The possible one step state changes of the process are shown in *Figure 1*

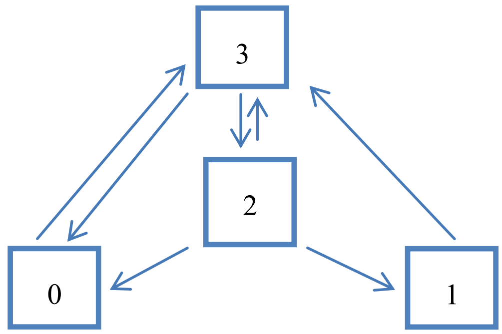

Figure 1. The possible state changes of the process

To determine the semi-Markov process as a model we have to define its initial distribution and all elements of its kernel [3], [4], [5]. Recall that the semi-Markov kernel is the matrix of transition probabilities of the Markov renewal process [4]

$$Q(t) = [Q_{ij}(t):\ i,j \in S], \tag{1}$$

$$Q_{ij}(t) = P(\tau_{n+1} - \tau_n \leq t, X(\tau_{n+1}) = j \mid X(\tau_n) = i), \quad t \geq 0. \tag{2}$$

The sequence $\{X(\tau_n):\ n = 0,1,\ldots\}$ is homogeneous Markov chain with transition probabilities

$$p_{ij} = P(X(\tau_{n+1}) = j \mid X(\tau_n) = i) = \lim_{t\to\infty} Q_{ij}(t) \tag{3}$$

The function

$$G_i(t) = P(T_i \leq t) = P(\tau_{n+1} - \tau_n \leq t \mid X(\tau_n) = i) = \textstyle\sum_{j \in S} Q_{ij}(t) \tag{4}$$

is the CDF of the random variable T_i denoting time spent in state i when the successor state is unknown. This random variable is called a waiting time. The function

$$F_{ij}(t) = P(\tau_{n+1} - \tau_n \leq t \mid X(\tau_n) = i, X(\tau_{n+1}) = j) = \frac{Q_{ij}(t)}{p_{ij}} \tag{5}$$

is CDF of a random variable T_{ij} that is called a *holding time* of state i, if the next state will be j. From (5) we have

$$Q_{ij}(t) = p_{ij} F_{ij}(t). \tag{6}$$

The kernel (1) of the SM process $\{X(t):\ t \geq 0$ takes the form

$$Q(t)=\begin{bmatrix}0 & 0 & 0 & Q_{03}(t)\\ 0 & 0 & 0 & Q_{13}(t)\\ Q_{20}(t) & Q_{21}(t) & 0 & Q_{23}(t)\\ Q_{30}(t) & 0 & Q_{32}(t) & 0\end{bmatrix}. \quad (7)$$

From (2) and from the assumptions we can calculate all elements of the semi-Markov kernel $Q(t),\ t\geq 0$. The elements $Q_{03}(t)$ and $Q_{13}(t)$ are CDF of the system renewal time.

$$Q_{03}(t)=Q_{13}(t)=H(t)=\int_0^t h(x)dx. \quad (8)$$

From the system description and assumptions we get the following equalities:

$$Q_{20}(t)=P(U=0,\gamma_A\leq t,\zeta_B>\gamma_A)= \quad (9)$$
$$=(1-a)\int_0^t h_A(x)[1-F_B(x)]\,dx,$$

$$Q_{21}(t)=P(\zeta_B\leq t,\ \zeta_B<\gamma_A)= \quad (10)$$
$$=\int_0^t f_B(x)[1-H_A(x)]\,dx,$$

$$Q_{23}(t)=P(U=1,\gamma_A\leq t,\zeta_B>\gamma_A)= \quad (11)$$
$$=\ a\int_0^t h_A(x)[1-F_B(x)]dx$$

In similar way we obtain

$$Q_{30}(t)=P(U=0,\zeta_A\leq t)=(1-a)\,F_A(t), \quad (12)$$

$$Q_{32}(t)=P(U=1,\zeta_A\leq t)=a\,F_A(t). \quad (13)$$

All elements of $Q(t)$ have been defined, hence the semi-Markov model is constructed. It is necessary to calculate the transition probabilities of the embedded Markov chain. From (1)- (13) we obtain transition probabilities matrix of the embedded Markov chain $\{X(\tau_n):\ n=0,1,\ldots\}$

$$P=\begin{bmatrix}0 & 0 & 0 & 1\\ 0 & 0 & 0 & 1\\ p_{20} & p_{21} & 0 & p_{23}\\ 1-a & 0 & a & 0\end{bmatrix}, \quad (14)$$

where

$$p_{20}=(1-a)\int_0^\infty h_A(x)[1-F_B(x)]dx, \quad (15)$$

$$p_{21}=\int_0^\infty f_B(x)[1-H_A(x)]dx, \quad (16)$$

$$p_{23}=a\int_0^\infty h_A(x)[1-F_B(x)]dx. \quad (17)$$

According to (4) and (8) - (13) we get CDF's of the times

$$G_0(t)=G_1(t)=H(t), \quad (18)$$

$$G_2(t)=\int_0^t h_A(x)[1-F_B(x)]dx+ \quad (19)$$

$$+\int_0^t f_B(x)[1-H_A(x)]\,dx\ ,$$

$$G_3(t)=F_A(t). \quad (20)$$

Expectation of the waiting times are

$$E(T_0)=E(T_1)=\int_0^\infty xh(x)dx=E(\gamma), \quad (21)$$

$$E(T_2)=\int_0^\infty x\,h_A(x)[1-F_B(x)]dx+ \quad (22)$$

$$+\int_0^\infty x\,f_B(x)[1-H_A(x)]dx,$$

$$E(T_3)=\int_0^\infty x\,f_A(x)dx=E(\,\zeta_A). \quad (23)$$

4. Reliability characteristics

Assume that evolution of a system reliability is describe by a finite state space S semi-Markov process $\{X(t): t\geq 0\}$. Elements of a set S represent the reliability states of the system. Let S_+ consists of the functioning states (up states) and S_- contains all the failed states (down states). The subset S_+ and S_- form a partition of S, i.e., $S=S_+\cup S_-$ and $S_+\cap S_-=\emptyset$.

Let $\{X(t):\ t\geq 0\}$ be the continuous-time semi-Markov process with a discrete state space S and a kernel $Q(t),\ t\geq 0$. A value of random variable

$$\Delta_D=\min\{n\in:\ X(\tau_n)\in D\}$$

denotes a discrete time (a number of state changes) of *a first arrival* at the set of states $D\subset S$ of the embedded Markov chain $\{X(\tau_n):\ n\in \mathrm{N}_0\}$. A value of a random variable $\Theta_D=\tau_{\Delta_D}$ denotes a first passage time to the subset D or the time of *a first arrival* at the set of states $D\subset S$ of semi-Markov process $\{X(t):\ t\geq 0\}$. A function

$$\Phi_{iD}(t)=P(\Theta_D\leq t|\,X(0)=i), \qquad t\geq 0$$

is Cumulative Distribution Function (CDF) of a random variable Θ_{iD} denoting the first passage time from the state $i\in D'$ to the subset D.
If $D=S_-$ and $i\in S_+=D'$ then $\Phi_{iD}(t),\ t\geq 0$, is CDF of a time to failure of an object if initial state is $i\in S_+$. For the regular semi-Markov processes such that,

$$f_{iD} = P(\Delta_D < \infty | X(0) = i) = 1, \quad i \in D',$$

the distributions $\Phi_{iA}(t), \quad i \in A'$ are proper and they are the only solutions of the equations system

$$\Phi_{iD}(t) = \sum_{j \in D} Q_{ij}(t) + \sum_{k \in S} \int_0^t \Phi_{kD}(t-x) dQ_{ik}(x), \quad i \in A'.$$

Applying a Laplace-Stieltjes L-S transformation for this system of integral equations we obtain the linear system of equations for (L-S) transforms
$\tilde{\varphi}_{iD}(s) = \sum_{j \in D} \tilde{q}_{ij}(s) + \sum_{k \in D'} \tilde{q}_{ik}(s) \tilde{\varphi}_{kD}(s)$,
where $\tilde{\varphi}_{iD}(s) = \int_0^\infty e^{-st} d\Phi_{iD}(t)$
are L-S transforms of the unknown CDF of the random variables Θ_{iA}, $i \in A'$ and

$$\tilde{q}_{ij}(s) = \int_0^\infty e^{-st} dQ_{ij}(t).$$

are L-S transforms of the given functions $Q_{ij}(t)$, $i,j \in S$. That linear system of equations is equivalent to the matrix equation

$$(I - q_{D'}(s))\varphi_{D'}(s) = b(s), \tag{24}$$

where $I = [\delta_{ij}: \ i,j \in D']$

$$q_{D'}(s) = [\tilde{q}_{ij}(s): \ i,j \in D'] \tag{25}$$

is the square sub-matrix of the L-S transforms of the matrix $q(s)$, while

$$\varphi_{D'}(s) = [\tilde{\varphi}_{iD}(s): \ i \in A']^T,$$
$$b(s) = [\textstyle\sum_{j \in D} \tilde{q}_{ij}(s): \ i \in D']$$

are one column matrices of the corresponding L-S transforms. The linear system of equations for the L-S transforms allows us to obtain the linear system of equations for the moments of the random variables Θ_{iD}, $i \in D'$. The expectations $E(\Theta_{iA})$, $i \in A'$ we obtain by solving the equation

$$(I - P_{D'})\overline{\Theta}_{D'} = \overline{T}_{A'}, \tag{26}$$

where

$$P_{D'} = [p_{ij}: \ i,j \in D'], \quad \overline{\Theta}_{D'} = [E(\Theta_{iD}): \ i \in D']^T,$$

$$\overline{T}_{A'} = [E(T_i): \ i \in D']$$

and I is the unit matrix. To find the second moments $E(\Theta_{iD}^2)$, $i \in D'$ we have to solve the matrix equation

$$(I - P_{D'})\overline{\Theta}_{D'}^2 = B_D, \tag{27}$$

where

$$P_{A'} = [p_{ij}: i,j \in D'], \quad \overline{\Theta}_{A'} = [E(\Theta_{iA}^2): i \in D']^T,$$

$$B_D = [b_{iD}: i \in A']^T,$$

$$b_{iD} = E(T_i^2) + 2 \sum_{k \in D'} p_{ik} E(T_{ik}) E(\Theta_{kD}).$$

We have two way of calculating reliability function. The first one consists in computing PDF's of the time of *a first arrival* at the set of states $D \subset S$ from set $i \in D'$, where $D = S_-$ and $D' = S_+$. To get the reliability function we have to solve the matrix equation (24). In this case a set of "down" states is $D = \{0,1\}$ and the set of "up" states is $D' = \{2,3\}$.

The matrix (25) is

$$q_{D'}(s) = \begin{bmatrix} 0 & \tilde{q}_{23}(s) \\ \tilde{q}_{32}(s) & 0 \end{bmatrix}$$

The equation (24) takes the form

$$\begin{bmatrix} 1 & -\tilde{q}_{23}(s) \\ -\tilde{q}_{32}(s) & 1 \end{bmatrix} \begin{bmatrix} \tilde{\varphi}_{2D}(s) \\ \tilde{\varphi}_{3D}(s) \end{bmatrix} = \begin{bmatrix} \tilde{q}_{20}(s) + \tilde{q}_{21}(s) \\ \tilde{q}_{30}(s) \end{bmatrix}$$

The Laplace transform of the system reliability function is given by the rule

$$\tilde{R}(s) = \frac{1 - \tilde{\varphi}_{3D}(s)}{s}.$$

The second way of computing the reliability function consists in solving system o integral equation for conditional reliability functions.
Suppose that $i \in S_+ = D'$ is an initial state of the process. The conditional reliability function is defined by

$$R_i(t) = P(\forall u \in [0,t], \ X(u) \in D' | X(0) = i), \quad i \in D'.$$

Notice that

$$R_i(t) = P(\Theta_{S_-} > t | X(0) = i) =$$
$$= 1 - P(\Theta_{S_-} > | X(0) = i) = 1 - \Phi_{iS_-}(t)$$
$$i \in D'.$$

From the Chapman-Kolmogorov property of a two dimensional Markov chain $\{(X(\tau_n), \tau_n): \ n = 0,1,2,\ldots\}$, we can obtain the integral system of eqations

$$R_i(t) = 1 - G_i(t) +$$

$+\sum_{j\in S+}\int_0^t R_j(t-u)dQ_{ij}(u), \quad i \in S_+.$

Passing to the Laplace transform we get

$$\tilde{R}_i(s) = \frac{1}{s} - \tilde{G}_i(s) + \sum_{j\in D'} \tilde{q}_{ij}(s)\tilde{R}_j(s), \qquad i \in D'$$

where $\tilde{R}_j(s) = \int_0^\infty e^{-st} R_j(t)dt.$

The matrix form of the equation system is

$$(I - q_{D'}(s))R(s) = W_{D'}(s), \tag{28}$$

where

$$R(s) = [\tilde{R}_i(s):\ i \in D']^T, \tag{29}$$
$$W_{D'}(s) = [\tfrac{1}{s} - \tilde{G}_i(s):\ i \in D']^T \tag{30}$$

are one column matrices, and

$$q_{D'}(s) = [\tilde{q}_{ij}(s):\ i,j \in D'], \qquad I = [\delta_{ij}:\ i,j \in D']$$

are square matrices. Note that

$$\tilde{G}_i(s) = \frac{1}{s}\sum_{j\in D} \tilde{q}_{ij}(s).$$

Elements of the matrix $\tilde{R}(s)$ are the Laplace transforms of the conditional reliability functions.We obtain the reliability functions $R_i(t),\ i \in D'$ by inverting the Laplace transforms $\tilde{R}_i(s),\ i \in D'$.
Taking under cosideration (28)- (30) we obtain the matrix equation

$$\begin{bmatrix} 1 & -\tilde{q}_{23}(s) \\ -\tilde{q}_{32}(s) & 1 \end{bmatrix}\begin{bmatrix} \tilde{R}_2(s) \\ \tilde{R}_3(s) \end{bmatrix} =$$

$$\frac{1}{s}\begin{bmatrix} 1 - \tilde{q}_{20}(s) - \tilde{q}_{21}(s) \\ 1 - \tilde{q}_{30}(s) \end{bmatrix}.$$

The solution of the equivalent linear equations system is

$$\tilde{R}_2(s) = \frac{1}{s} - \frac{\tilde{q}_{30}(s)+\tilde{q}_{32}(s)\tilde{q}_{20}(s)+\tilde{q}_{32}(s)\tilde{q}_{21}(s)}{s\,(1-\tilde{q}_{23}(s)\tilde{q}_{32}(s))},$$

$$\tilde{R}_3(s) = \frac{1}{s} - \frac{\tilde{q}_{30}(s)+\tilde{q}_{32}(s)\tilde{q}_{20}(s)+\tilde{q}_{32}(s)\tilde{q}_{21}(s)}{s\,(1-\tilde{q}_{23}(s)\tilde{q}_{32}(s))}.$$

To calculate the conditional mean times to failure we apply equation (26). I our case we have

$$\begin{bmatrix} 1 & -p_{23} \\ -p_{32} & 1 \end{bmatrix}\begin{bmatrix} E(\Theta_{2D}) \\ E(\Theta_{3D}) \end{bmatrix} = \begin{bmatrix} E(T_2) \\ E(T_3) \end{bmatrix},$$

where known elelements of the above matrices are given by (17), (22) and (23). The solution of the equivalent linear equations system is

$$E(\Theta_{2D}) = \frac{E(T_2)+p_{23}E(T_3)}{1-p_{23}p_{32}}, \tag{31}$$

$$E(\Theta_{3D}) = \frac{E(T_3)+p_{32}E(T_2)}{1-p_{23}p_{32}}, \tag{32}$$

where

$$E(T_2) = \int_0^\infty x\, h_A(x)[1 - F_B(x)]dx +$$

$$+\int_0^\infty x\, f_B(x)[1 - H_A(x)]dx = E(\min[\zeta_B, \gamma_A)]$$

$$E(T_3) = \int_0^\infty x\, f_A(x)dx = E(\zeta_A),$$

$$p_{23} = a\int_0^\infty h_A(x)[1 - F_B(x)]dx,$$

$$p_{32} = a.$$

5. Example

We assume that time to failure of the main subsystem denoted ζ_A, is exponentialy distributed

$$f_A(x) = \alpha_A\, e^{-\alpha_A x}, x \ge 0,\ \alpha_A > 0\ ;$$

a repair time of the basic subsystem A is a random variable γ_A having distribution given by the PDF

$$h_A(x) = \mu_A^2\, x\, e^{-\mu_A x},\ x \ge 0\ ;$$

a time to failure of the emergency subsystem B is a random variable ζ_B, with an exponential PDF

$$f_B(x) = \alpha_B\, e^{-\alpha_B x},\ x \ge 0;$$

reliability of a switch is

$$P(U = 1) = a\ ;$$

a restoring time of the whole system is the random variable γ having distribution given by the PDF

$$h(x) = \mu^2\, x\, e^{-\mu x},\ x \ge 0.$$

We suppose
$E(\zeta_A) = 16529$ [h], $E(\gamma_A) = 96[h]$,

$E(\zeta_B) = 12690$ [h], $E(\gamma) = 496[h]$,

$a = 0{,}996.$

Therefore

$\alpha_A = 0{,}0000605, \quad \mu_A = 0{,}020833$

$\alpha_B = 0{,}0000788, \quad \mu = 0{,}004032$

Substituting the numerical data to (8)-(13) we obtain for $t \geq 0$ elements of the kernel (7).

$Q_{03}(t) = 1 - e^{-0.004032t}(1 + 0.004032t);$

$Q_{13}(\mathrm{t}) = 1 - e^{-0.004032\,t}(1 + 0.004032t);$

$Q_{20}(\mathrm{t}) = 0.004(1 - e^{-0.020911t}(1 + 0.020911t));$

$\mathrm{Q}_{21}(\mathrm{t}) = 0.1801952(0.0417448 - e^{-0.0209118t}(0.0000788 + 0.020833(2 + 0.020833t) + 0.0000016\,t))$

$Q_{23}(t) = 0.988508(1 - e^{-0.020911t}(1 + +0.020911t))$;

$Q_{30}(t) = 0.004(1 - e^{-0.0000605t})$:

$Q_{32}(t) = 0.996(1 - e^{-0.0000605t})$

The Laplace- Stielties transforms of these functions are

$\tilde{q}_{03}(\mathrm{s}) = \frac{0.00001626}{(0.00403225+s)^2}$;

$\tilde{q}_{13}(\mathrm{s}) = \frac{0.00001626}{(0.00403225+s)^2}$;

$\tilde{q}_{20}(\mathrm{s}) = \frac{0.000001736}{(0.02084008+s)^2}$;

$\tilde{q}_{21}(\mathrm{s}) = \frac{0.00000328+0.0000788\,s}{(0.020911+s)^2}$;

$\tilde{q}_{23}(\mathrm{s}) = \frac{0.00043225}{(0.0.020911+s)^2}$;

$\tilde{q}_{30}(\mathrm{s}) = \frac{0.000000242}{0.0000605+s}$;

$\tilde{q}_{32}(\mathrm{s}) = \frac{0.0000060258}{0.0000605+s}$.

Using (18), (19) and (20) we get the CDF's of wating times.

$G_0(\mathrm{t}) = 1 - e^{-0.00403225t}(1 + 0.00403225\,t);$

$G_1(\mathrm{t}) = 1 - e^{-0.00403225t}(1 + 0.00403225\,t);$

$G_2(\mathrm{t}) = 1 - e^{-0.020911\,t}(1 + 0.02084t)$;

$G_3(\mathrm{t}) = 1 - e^{-0.0000605t}$.

The corresponding expectation are

$E(T_0)$=496; $E(T_1)$=496;

$E(T_2) = 95{,}4596;\ \ E(T_3) = 16529.$

Applaing the rules (15), (16) and (17) we obtain the transition probabilities of the matrix (14).

$p_{20} = 0.00397;\ p_{21} = 0.00752;\ p_{23} = 0.98850;$

$\mathrm{p}_{32} = 0.996$

Substituting Laplace- Stielties transforms of kernel elemnts to (29) we obtain the Laplace transform of the reliability function.

$\tilde{R}(s) = \tilde{R}_3(s)$.

The reliability function of the system we obtain using the procedure InverseLaplaceTransform[$\tilde{R}(s)$,s,t] in the MATHEMATICA computer program.

Finally we have

$R_3(t) = 1.00005468\,e^{-9.29198722\times10^{-7}t} - 2e^{-0.0209415854\,t}(0.00002734\ \cos\ 0.00111570\,t - 0.0002052140\sin\ 0.00111570\,t)$

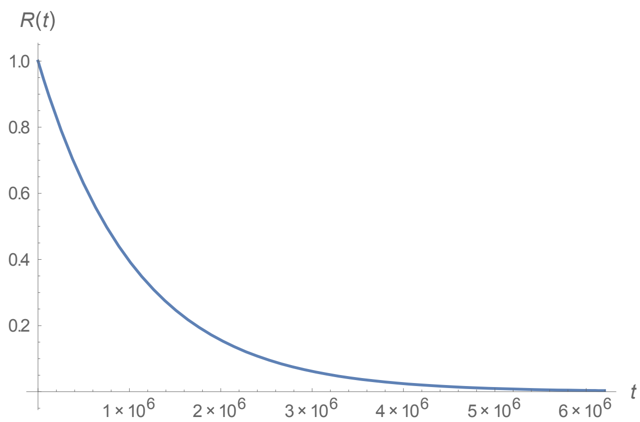

Figure 1.The reliability function of the system.

Using solution (31) we obtain the mean time to failure of the system

$\mathrm{MTTF} = E(\Theta) = 1076254.90\ \ [h]$

6. Conclusions

Semi-Markov reliability model composed of main and cold backup subsystem is presented in the paper.

- Results of semi-Markov process theory allowed us to compute reliability characteristics of the system.
- The Laplace transform of unconditional reliability function of the system is

  $\tilde{R}(s) = \tilde{R}_3(s),$

  where the Laplace transform of conditional reliability functions $\tilde{R}_3(s)$, is given by (29).
- The mean time to failure of the considered cold standby system depend on lifetimes probability distribution of the main and cold backup subsystem and the renewal mean time of the main component and also on the switch reliability:

$$E(\Theta) = \frac{E(\zeta_A) + a\, E(T_2)}{1 - a^2 \int_0^\infty h_A(x)[1 - F_B(x)]dx}.$$

From (11) and (17) it is equivalent to the equality

$E(\Theta) = \frac{E(\zeta_A) + a\, E(T_2)}{1 - a^2 \mathrm{P}(\zeta_B > \gamma_A)}$, where $\mathrm{P}(\zeta_B > \gamma_A)$ denotes

probability that the time to failure of the backup subsystem B is grater than the renewal time of main component. Substituting $a = x$, $\mathrm{P}(\zeta_B > \gamma_A) = y$ and $E(\Theta) = f(x, y)$ we obtain a function of the real variables x and y :

$$f(x, y) = \frac{E(\zeta_A) + E(T_2)\, x}{1 - x^2\, y}$$

that allows to analyse that function mathematically.

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