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Optimization of Operation and Safety of Global Baltic Network of Critical Infrastructure Networks (GBNCIN) with Considering Climate-Weather Change Process (C-WCP) influence – Minimizing GBNCIN Operation Cost

Keywords

critical infrastructure, critical infrastructure network, operation process, climate-weather change process, network of critical infrastructure networks

Abstract

The paper presents a method of the GBNCIN operation and safety, with considering the climate-weather change process, safety optimization. Basic characteristics of the critical infrastructure operation process related to climate-weather change process are shown. Then, the GBNCIN operation cost related to climate-weather change is introduced. Furthermore, by analysis of the operation cost of the GBNCIN impacted by the operation process, related to the climate-weather change process, and its conditional safety functions, mean values of the total sojourn times at particular operation states during certain sufficiently large GBNCIN operation time are fixed. Finally, the GBNCIN operation cost related to climate-weather change minimization, and cost analysis of the GBNCIN operation impacted by climate-weather change, are presented.

1. Introduction

It is being predicted by many different entities working on climate changes and critical infrastructure protection, forecasted climate changes will significantly impact on critical infrastructure systems. Thus, intensive works on adapting infrastructures to possible climate fluctuations, have been processed for last couple of years [European Commission, 2013].

The Baltic Sea area is a region showing significant concentration of various systems showing features resulting with classifying them as forming critical infrastructure. Additionally, geographical conditions of the area, cause potential failure of one of the systems, can lead to a massive negative impact on natural environment and societies located within and around. Also, predicted climate changes do have significant meaning for the Baltic Sea and critical infrastructures located within: one of the greatest (within Europe) increases in sea surface temperature; decreasing trend in the Baltic Sea's ice cover; falling level of the Baltic in the northern shores and rising to the south; increased beach erosion due to increased storminess in the eastern Baltic Sea; and increasing eutrophication problems in coastal waters [Dziula, 2015].

Analysis of nature of some critical infrastructures operating within the Baltic Sea area, their interconnections and interdependencies, resulted with distinguishing certain critical infrastructure networks, defined as a set of interconnected and interdependent critical infrastructures, interacting directly and indirectly at various levels of their complexity and operating activity [EU-CIRCLE, 2015]. The networks have been abbreviated as the Baltic Critical Infrastructure Networks (BCIN). Consequently, distinguished networks, operating within the Baltic Sea area, interacting, and being also interconnected and interdependent, were classified as the Global Baltic Network of Critical Infrastructure Networks (GBNCIN).

2. The GBNCIN Operation Process Related to Climate-Weather Change Process

We consider the Global Baltic Network of Critical Infrastructure Networks (*GBNCIN*) impacted by the operation process related to the climate-weather change process $ZC_{GBNCIN}(t)$, $t \in <0,\infty)$, in a various way at this process states zc_{bl}, $b = 1,2,...,\iota$, $l = 1,2,...,w$. We assume that the changes of the states of operation process related to the climate-weather change process $ZC_{GBNCIN}(t)$, $t \in <0,\infty)$, at the *GBNCIN* operating area have an influence on the *GBNCIN* safety structure and on the safety of particular *BCIN* networks E_i^{GBNCIN}, $i = 1,2,...,n$, as well [Kołowrocki, Soszyńska-Budny, 2011].

We assume, the *GBNCIN* during its operation process is taking $\iota, \iota \in N$, different operation states $z_1, z_2, \ldots, z_\iota$. We define the *GBNCIN* operation process $Z_{GBNCIN}(t)$, $t \in <0,+\infty)$, with discrete operation states from the set $\{z_1, z_2, \ldots, z_\iota\}$.

Moreover, we assume the climate-weather change process *C*(*t*), $t \in <0,+\infty)$, at the *GBNCIN* operating area is taking w, $w \in N$, different climate-weather states $c_1, c_2, \ldots, c_w$. Climate-weather conditions can have also influence on *GBNCIN* safety.

Then, the joint process of *GBNCIN* operation process and climate-weather change process called the *GBNCIN* operation process related to climate-weather change is proposed and it is marked by $ZC_{GBNCIN}(t)$, $t \in <0,+\infty)$. Further, we assume that it can take $\iota w, \iota, w \in N$, different operation states related to the climate-weather change $zc_{11}, zc_{12}, \ldots, zc_{\iota w}$.

We assume that the *GBNCIN* operation process related to climate-weather change $ZC_{GBNCIN}(t)$, at the moment $t \in <0,+\infty)$, is at the state zc_{bl}, $b = 1,2,...,\iota$, $l = 1,2,...,w$, if and only if at that moment, the operation process $Z_{GBNCIN}(t)$ is at the operation states z_b, $b = 1,2,...,\iota$, and the climate-weather change process *C*(*t*) is at the climate-weather state c_l, $l = 1,2,...,w$, can be expressed as follows:

$$(ZC_{GBNCIN}(t) = zc_{bl}) \Leftrightarrow (Z_{GBNCIN}(t) = z_b \cap C(t) = c_l),$$
$$t \in <0,+\infty),\ b = 1,2,...,\iota,\ \ l = 1,2,...,w.$$

The transient probabilities of the *GBNCIN* operation process related to climate-weather change $ZC_{GBNCIN}(t)$ at the operation states zc_{bl}, $b = 1,2,...,\iota$, $l = 1,2,...,w$, are defined as below:

$$pq_{bl}^{GBNCIN}(t) = P(ZC_{GBNCIN}(t) = zc_{bl}),\ t \in <0,+\infty),$$
$$b = 1,2,...,\iota,\ \ l = 1,2,...,w.$$

Further, the limit values of the transient probabilities of the *GBNCIN* operation process related to climate-weather change process $ZC_{GBNCIN}(t)$ at the operation states zc_{bl}, $b = 1,2,...,\iota$, $l = 1,2,...,w$, are given by

$$pq_{bl}^{GBNCIN} = \lim_{t\to\infty} pq_{bl}^{GBNCIN}(t),\ b = 1,2,...,\iota,$$
$$l = 1,2,...,w, \qquad (1)$$

and in case when the processes $Z_{GBNCIN}(t)$ and $C(t)$ are independent, they can be found from [12]

$$pq_{bl}^{GBNCIN} = p_b^{GBNCIN} q_l,\ b = 1,2,...,\iota,\ l = 1,2,...,w, \qquad (2)$$

where p_b^{GBNCIN}, $b = 1,2,...,\iota$, are the limit transient probabilities of the operation process $Z_{GBNCIN}(t)$ at the particular operation startes z_b, $b = 1,2,...,\iota$, and q_l, $l = 1,2,...,w$, are the limit transient probabilities of the climate-weather change process *C*(*t*) at the particular climate-weather states c_l, $l = 1,2,...,w$.

Other interesting characteristics of the *GBNCIN* operation process $ZC_{GBNCIN}(t)$ are its total sojourn times $\hat{\theta}\hat{C}_{bl}^{GBNCIN}$ at the particular operation states zc_{bl}, $b = 1,2,...,\iota$, $l = 1,2,...,w$, during the fixed sufficiently large *GBNCIN* operation time θ. They have approximately normal distributions with the expected values given by

$$\hat{M}\hat{N}_{bl} = E[\hat{\theta}\hat{C}_{bl}] = pq_{bl}\theta,\ b = 1,2,...,\iota,$$
$$l = 1,2,...,w, \qquad (3)$$

where pq_{bl}, $b = 1,2,...,\iota$, $l = 1,2,...,w$, are defined by (1) and given by (2) in the case the processes $Z_{GBNCIN}(t)$ and *C*(*t*) are independent.

3. The Global Baltic Network of Critical Infrastructure Networks Operation Cost Related to Climate-Weather Change

We may introduce the instantaneous operation cost of the *GBNCIN* impacted by the operation process $ZC_{GBNCIN}(t)$, $t \in <0,\infty)$, related to the climate-weather change process in the form of vector

$$\boldsymbol{K}_{\boldsymbol{GBNCIN}}^{\boldsymbol{4}}(t,\cdot) = [1,\ \boldsymbol{K}_{\boldsymbol{GBNCIN}}^{\boldsymbol{4}}(t,1),\ ...,\boldsymbol{K}_{\boldsymbol{GBNCIN}}^{\boldsymbol{4}}(t,z)],$$
$$t \in <0,\infty),$$

with the coordinates given by

$$\boldsymbol{K}^{4}_{\boldsymbol{GBNCIN}}(t,u) \cong \sum_{b=1}^{\iota}\sum_{l=1}^{w} pq_{bl}^{GBNCIN}[\boldsymbol{K}^{4}_{\boldsymbol{GBNCIN}}(t,u)]^{(bl)}$$
for $t \geq 0$, $u = 1,2,..., z$, (4)

where pq_{bl}^{GBNCIN}, $b = 1,2,...,\iota$, $l = 1,2,...,w$, are the limit transient probabilities at the states zc_{bl}, $b = 1,2,...,\iota$, $l = 1,2,...,w$, of the operation process $ZC_{GBNCIN}(t)$, $t \in < 0,\infty)$, related to the climate-weather change and

$[\boldsymbol{K}^{4}_{\boldsymbol{GBNCIN}}(t,u)]^{(bl)}$, $u = 1,2,..., z$, $b = 1,2,...,\iota$, $l = 1,2,...,w$,

are the coordinates of the *GBNCIN* conditional instantaneous operation costs in the safety state subsets $\{u, u+1,..., z\}$, $u = 1,2,..., z$, impacted by the operation process $ZC_{GBNCIN}(t)$, $t \in < 0,\infty)$, related to the climate-weather change process at the states zc_{bl}, $b = 1,2,...,\iota$, $l = 1,2,...,w$, defined in the form of the vector

$[\boldsymbol{K}^{4}_{\boldsymbol{GBNCIN}}(t,\cdot)]^{(bl)} = [1, [\boldsymbol{K}^{4}_{\boldsymbol{GBNCIN}}(t,1)]^{(bl)},..., [\boldsymbol{K}^{4}_{\boldsymbol{GBNCIN}}(t,z)]^{(bl)}]$, $t \in < 0,\infty)$, $b = 1,2,...,\iota$, $l = 1,2,...,w$.

The dependency (4) can also be clearly expressed in the linear equation for the mean value of the *GBNCIN* total unconditional operation costs in the safety state subsets $\{u, u+1,..., z\}$, $u = 1,2,..., z$,

$$\overline{\boldsymbol{K}}^{4}_{\boldsymbol{GBNCIN}}(u) \cong \sum_{b=1}^{\iota}\sum_{l=1}^{w} pq_{bl}^{GBNCIN}[\overline{\boldsymbol{K}}^{4}_{\boldsymbol{GBNCIN}}(u)]^{(bl)},$$
$u = 1,2,..., z$, (5)

where pq_{bl}^{GBNCIN}, $b = 1,2,...,\iota$, $l = 1,2,...,w$, are the limit transient probabilities at the states zc_{bl}, $b = 1,2,...,\iota$, $l = 1,2,...,w$, of the operation process $ZC_{GBNCIN}(t)$, $t \in < 0,\infty)$, related to the climate-weather change defined by (3) and

$[\overline{\boldsymbol{K}}^{4}_{\boldsymbol{GBNCIN}}(u)]^{(bl)}$, $u = 1,2,..., z$, $b = 1,2,...,\iota$, $l = 1,2,...,w$,

are the mean values of the *GBNCIN* total conditional instantaneous operation costs in the safety state subsets $\{u, u+1,..., z\}$, $u = 1,2,..., z$, at the operation states zc_{bl}, $b = 1,2,...,\iota$, $l = 1,2,...,w$, defined by

$$[\overline{\boldsymbol{K}}^{4}_{\boldsymbol{GBNCIN}}(u)]^{(bl)} = \int_{\boldsymbol{0}}^{[\mu^{4}_{GBNCIN}(u)]^{(bl)}} [\boldsymbol{K}^{4}_{\boldsymbol{GBNCIN}}(t,u)]^{(bl)} dt,$$
$u = 1,2,..., z$, $b = 1,2,...,\iota$, $l = 1,2,...,w$, (6)

where $[\mu^{4}_{GBNCIN}(r_{GBNCIN})]^{(bl)}$, $u = 1,2,..., z$, $b = 1,2,...,\iota$, $l = 1,2,...,w$, are the mean values of the *GBNCIN* conditional lifetimes $[T^{4}_{GBNCIN}(u)]^{(bl)}$, $u = 1,2,..., z$, $b = 1,2,...,\iota$, $l = 1,2,...,w$, in the safety state subset $\{u, u+1,..., z\}$ at the *GBNCIN* operating process related to the climate-weather change state zc_{bl}, $b = 1,2,...,\iota$, $l = 1,2,...,w$, given by [EU-CIRCLE, 2017].

$$[\mu^{4}_{GBNCIN}(u)]^{(bl)} = \int_{0}^{\infty}[\boldsymbol{S}^{4}_{\boldsymbol{GBNCIN}}(t,u)]^{(bl)} dt,$$
$u = 1,2,..., z$, $b = 1,2,...,\iota$, $l = 1,2,...,w$, (7)

and

$[\boldsymbol{S}^{4}_{\boldsymbol{GBNCIN}}(t,u)]^{(bl)}$, $u = 1,2,..., z$, $b = 1,2,...,\iota$, $l = 1,2,...,w$,

are the coordinates of the *GBNCIN* impacted by the operation process related to the climate-weather change process $ZC_{GBNCIN}(t)$, $t \in < 0,\infty)$, conditional safety functions [12]

$[\boldsymbol{S}^{4}_{\boldsymbol{GBNCIN}}(t,\cdot)]^{(bl)} = [1, [\boldsymbol{S}^{4}_{\boldsymbol{GBNCIN}}(t,1)]^{(bl)},..., [\boldsymbol{S}^{4}_{\boldsymbol{GBNCIN}}(t,z)]^{(bl)}]$, $t \in < 0,\infty)$, $b = 1,2,...,\iota$, $l = 1,2,...,w$.

The mean values of the *GBNCIN* total conditional instantaneous operation costs in the safety state subsets $\{u, u+1,..., z\}$, $u = 1,2,..., z$, at the operation states zc_{bl}, $b = 1,2,...,\iota$, $l = 1,2,...,w$, can be alternatively defined for the *GBNCIN* fixed operation time θ by

$$[\overline{\boldsymbol{K}}^{4}_{\boldsymbol{GBNCIN}}(u)]^{(bl)} = \int_{\boldsymbol{0}}^{\hat{M}\hat{N}_{GBNCIN}} [\boldsymbol{K}^{4}_{\boldsymbol{GBNCIN}}(t,u)]^{(bl)} dt,$$
$u = 1,2,..., z$, $b = 1,2,...,\iota$, $l = 1,2,...,w$, (8)

where $\hat{M}\hat{N}_{GBNCIN\,bl}$, $b = 1,2,...,\iota$, $l = 1,2,...,w$, are the mean values of the total sojourn times $\hat{\theta}\hat{C}_{bl}^{GBNCIN}$, $b = 1,2,...,\iota$, $l = 1,2,..., w$, at the particular operation states zc_{bl}, $b = 1,2,...,\iota$, $l = 1,2,..., w$, during the fixed sufficiently large *GBNCIN* operation time θ.

4. The Global Baltic Network of Critical Infrastructure Networks Operation Cost

Related to Climate-Weather Change Minimization

From the linear equation (5) we can see that the mean value of the *GBNCIN* total unconditional operation cost, $\overline{\boldsymbol{K}}^4_{GBNCIN}(u)$, $u = 1,2,...,z$, is determined by the limit values of transient probabilities pq_{bl}^{GBNCIN}, $b = 1,2,...,\iota$, $l = 1,2,...,w$, of the *GBNCIN* operation process at the operation states given by (1) and the mean values of the *GBNCIN* total conditional operation costs $[\overline{\boldsymbol{K}}^4_{GBNCIN}(u)]^{(bl)}$, $u = 1,2,...,z$, $b = 1,2,...,\iota$, $l = 1,2,...,w$, at the operating process related to the climate-weather change zc_{bl}, $b = 1,2,...,\iota$, $l = 1,2,...,w$, given by (6).

Therefore, the *GBNCIN* total unconditional operation cost optimization approach based on the linear programming [Klabjan, Adelman, 2006] can be proposed. Namely, we may look for the corresponding optimal values $\dot{p}q_{bl}^{GBNCIN}$, $b = 1,2,...,\iota$, $l = 1,2,...,w$, of the limit transient probabilities pq_{bl}^{GBNCIN}, $b = 1,2,...,\iota$, $l = 1,2,...,w$, of the *GBNCIN* operation process at the operation states zc_{bl}, $b = 1,2,...,\iota$, $l = 1,2,...,w$, to minimize the mean value $\overline{\boldsymbol{K}}^4_{GBNCIN}(u)$, $u = 1,2,...,z$, of the GBNCIN total unconditional operation cost in the safety state subsets $\{u, u+1,..., z\}$, $u = 1,2,...,z$, under the assumption that the mean values $[\overline{\boldsymbol{K}}^4_{GBNCIN}(u)]^{(bl)}$, $u = 1,2,...,z$, $b = 1,2,...,\iota$, $l = 1,2,...,w$, of the *GBNCIN* total conditional operation costs in the safety state subsets $\{u, u+1,..., z\}$, $u = 1,2,...,z$, at the operation states zc_{bl}, $b = 1,2,...,\iota$, $l = 1,2,...,w$, are fixed. As a special case of the above described the *GBNCIN* total unconditional operation cost optimization problem, if r_{GBNCIN}, $r_{GBNCIN} = 1,2,...,z$, is a *GBNCIN* critical safety state, we formulate the optimization problem as a linear programming model with the objective function of the following form

$$\overline{\boldsymbol{K}}^4_{GBNCIN}(r_{GBNCIN}) \cong \sum_{b=1}^{\iota}\sum_{l=1}^{w} pq_{bl}^{GBNCIN}[\overline{\boldsymbol{K}}^4_{GBNCIN}(r_{GBNCIN})]^{(bl)}, \quad (9)$$

for a fixed r_{GBNCIN}, $r_{GBNCIN} \in \{1,2,..., z\}$ and with the following bound constraints

$$\breve{p}q_{bl}^{GBNCIN} \le pq_{bl}^{GBNCIN} \le \hat{p}q_{bl}^{GBNCIN}, \ b = 1,2,...,\iota, \ l = 1,2,...,w, \quad (10)$$

$$\sum_{b=1}^{\iota}\sum_{l=1}^{w} pq_{bl}^{GBNCIN} = 1, \quad (11)$$

where

$$[\overline{\boldsymbol{K}}^4_{GBNCIN}(r_{GBNCIN})]^{(bl)}, \ [\overline{\boldsymbol{K}}^4_{GBNCIN}(r_{GBNCIN})]^{(bl)} \ge 0, \ b = 1,2,...,\iota, \ l = 1,2,...,w, \quad (12)$$

are fixed mean values of the *GBNCIN* conditional lifetimes in the safety state subset $\{r_{GBNCIN}, r_{GBNCIN}+1,..., z\}$ and

$$\breve{p}q_{bl}^{GBNCIN}, \ 0 \le \breve{p}q_{bl}^{GBNCIN} \le 1 \text{ and } \hat{p}q_{bl}^{GBNCIN}, \ 0 \le \hat{p}q_{bl}^{GBNCIN} \le 1, \ \breve{p}q_{bl}^{GBNCIN} \le \hat{p}q_{bl}^{GBNCIN}, \quad (13)$$

are lower and upper bounds of the transient probabilities pq_{bl}^{GBNCIN}, $b = 1,2,...,\iota$, $l = 1,2,...,w$, respectively.

Now, we can obtain the optimal solution of the formulated by (9)-(13) the linear programming problem. Namely, we can find the optimal values $\dot{p}q_{bl}^{GBNCIN}$, $b = 1,2,...,\iota$, $l = 1,2,...,w$, of the transient probabilities pq_{bl}^{GBNCIN}, $b = 1,2,...,\iota$, $l = 1,2,...,w$, that minimize the mean value of the *GBNCIN* unconditional operation cost in the safety state subset $\{r_{GBNCIN}, r_{GBNCIN}+1,..., z\}$, defined by the linear form (9), giving its minimum value in the following form

$$\dot{\overline{\boldsymbol{K}}}^4_{GBNCIN}(r_{GBNCIN}) \cong \sum_{b=1}^{\iota}\sum_{l=1}^{w} \dot{p}q_{bl}^{GBNCIN}[\overline{\boldsymbol{K}}^4_{GBNCIN}(r_{GBNCIN})]^{(bl)} \quad (14)$$

for a fixed r_{GBNCIN}, $r_{GBNCIN} \in \{1,2,..., z\}$.

Thus, considering (4), the coordinates of the optimal instantaneous operation cost of the *GBNCIN* in the form of the vector

$$\dot{\boldsymbol{K}}^4_{GBNCIN}(t,\cdot) = [1, \dot{\boldsymbol{K}}^4_{GBNCIN}(t,1), \ldots, \dot{\boldsymbol{K}}^4_{GBNCIN}(t,z)], \ t \in <0,\infty),$$

are given by

$$\dot{\boldsymbol{K}}^4_{GBNCIN}(t,u) \cong \sum_{b=1}^{\iota}\sum_{l=1}^{w} \dot{p}q_{bl}^{GBNCIN}[\boldsymbol{K}^4_{GBNCIN}(t,u)]^{(bl)}$$

$$\text{for } u = 1,2,...,z, \quad (15)$$

where $\dot{p}q_{bl}^{GBNCIN}$, $b=1,2,...,\iota$, $l=1,2,...,w$, are the optimal limit transient probabilities at the states zc_{bl}, $b=1,2,...,\iota$, $l=1,2,...,w$, of the operation process $ZC_{GBNCIN}(t)$, $t\in<0,\infty)$, related to the climate-weather change and

$$[\boldsymbol{K}_{GBNCIN}^{4}(t,u)]^{(bl)}, u=1,2,...,z,\ b=1,2,...,\iota,\ l=1,2,...,w,$$

are the coordinates of the *GBNCIN* conditional instantaneous operation costs in the safety state subsets $\{u,u+1,...,z\}$, $u=1,2,...,z$, impacted by the operation process $ZC_{GBNCIN}(t)$, $t\in<0,\infty)$, related to the climate-weather change process at the states zc_{bl}, $b=1,2,...,\iota$, $l=1,2,...,w$.

Replacing in (14) r_{GBNCIN} by *u*, we get the expressions for the optimal mean values of the *GBNCIN* unconditional operation costs in the safety state subset $\{u,u+1,...,z\}$, $u=1,2,...,z$, giving its minimum value in the following form

$$\dot{\overline{\boldsymbol{K}}}_{GBNCIN}^{4}(u) \cong \sum_{b=1}^{\iota}\sum_{l=1}^{w} \dot{p}q_{bl}^{GBNCIN}[\overline{\boldsymbol{K}}_{GBNCIN}^{4}(u)]^{(bl)},\quad \{u,u+1,...,z\},\ u=1,2,...,z. \tag{16}$$

The optimal solutions for the mean values of the *GBNCIN* unconditional operation costs in the particular safety states are

$$\dot{\overline{\overline{\boldsymbol{K}}}}_{GBNCIN}^{4}(u) = \dot{\overline{\boldsymbol{K}}}_{GBNCIN}^{4}(u) - \dot{\overline{\boldsymbol{K}}}_{GBNCIN}^{4}(u+1),\quad u=,1,...,z-1,\ \dot{\overline{\overline{\boldsymbol{K}}}}_{GBNCIN}^{4}(z) = \dot{\overline{\boldsymbol{K}}}_{GBNCIN}^{4}(z), \tag{17}$$

where $\dot{\overline{\boldsymbol{K}}}_{GBNCIN}^{4}(u)$, $u=1,2,...,z$, are given by (16).

Moreover, if we define the corresponding critical operation cost function by

$$\boldsymbol{K}_{GBNCIN}^{4}(t) = \boldsymbol{K}_{GBNCIN}^{4}(t, r_{GBNCIN})\ t\geq 0, \tag{18}$$

and the moment ζ_{GBNCIN}^{4} when the *GBNCIN* operation cost exceeds a permitted level κ, by

$$\zeta_{GBNCIN}^{4} = \boldsymbol{K}_{GBNCIN}^{4}{}^{-1}(\kappa), \tag{19}$$

where $\boldsymbol{K}_{GBNCIN}^{4}(t, r_{GBNCIN})$ is given by (4) for $u=r_{GBNCIN}$ and $\boldsymbol{K}_{GBNCIN}^{4}{}^{-1}(\kappa)$ is the inverse function of the critical operation cost function $\boldsymbol{K}_{GBNCIN}^{4}(t)$ given by (18), then the corresponding optimal critical operation cost function is given by

$$\dot{\boldsymbol{K}}_{GBNCIN}^{4}(t) = \dot{\boldsymbol{K}}_{GBNCIN}^{4}(t, r_{GBNCIN})\ t\geq 0, \tag{20}$$

then the optimal moment $\dot{\zeta}_{GBNCIN}^{4}$ when the *GBNCIN* operation cost exceeds a permitted level κ, is given by

$$\dot{\zeta}_{GBNCIN}^{4} = \dot{\boldsymbol{K}}_{GBNCIN}^{4}{}^{-1}(\kappa), \tag{21}$$

where $\dot{\boldsymbol{K}}_{GBNCIN}^{4}(t, r_{GBNCIN})$ is given by (15) for $u=r_{GBNCIN}$ and $\dot{\boldsymbol{K}}_{GBNCIN}^{4}{}^{-1}(\kappa)$, if it exists, is the inverse function of the optimal critical operation cost function $\dot{\boldsymbol{K}}_{GBNCIN}^{4}(t)$ given by (20).

5. Cost Analysis of the Global Baltic Network of Critical Infrastructure Networks Operation Impacted by Climate-Weather Change

We consider the *GBNCIN*, consisted of *n BCIN* networks E_i^{GBNCIN}, $i=1,2,...,n$, in its operation process $ZC_{GBNCIN}(t)$, $t\in<0,\infty)$, related to climate-weather change and we assume that the operation cost of its single basic *BCIN* network E_i^{GBNCIN}, $i=1,2,...,n$, at the operation state zc_{bl}, $b=1,2,...,\iota$, $l=1,2,...,w$, during the *GBNCIN* operation time θ, $\theta\geq 0$, amounts

$$[K_{BCIN\,i}(\theta)]^{(bl)},\ b=1,2,...,\iota,\ l=1,2,...,w,\ i=1,2,...,n.$$

First, we suppose that the *GBNCIN* is non-repairable and during the operation time θ, $\theta\geq 0$, it has not exceeded the critical safety state r_{GBNCIN}. In this case, the total cost of the non-repairable *GBNCIN* during the operation time θ, $\theta\geq 0$, is given by

$$K_{GBNCIN}(\theta) \cong \sum_{b=1}^{\iota}\sum_{l=1}^{w} pq_{bl}^{GBNCIN}\sum_{i=1}^{n}[K_{BCIN\,i}(\theta)]^{(bl)},\quad \theta\geq 0, \tag{22}$$

where pq_{bl}^{GBNCIN}, $b=1,2,...,\iota$, $l=1,2,...,w$, are transient probabilities defined by (1).

Further, we consider another case and we additionally assume that the *GBNCIN* is repairable after exceeding the critical safety state r_{GBNCIN} and its renovation time is ignored and the cost of its single renovation is constant and equal to $K_{GBNCIN_{ign}}$.

In this case, the total operation cost of the repairable *GBNCIN* with ignored its renovation time during the operation time θ, $\theta \geq 0$, amounts

$$K_{GBNCIN_{ign}}(\theta) \cong \sum_{b=1}^{\iota}\sum_{l=1}^{w} pq_{bl}^{GBNCIN} \sum_{i=1}^{n}[K_{BCIN\,i}(\theta)]^{(bl)} + $$
$$+ K_{GBNCIN_{ign}} H(\theta, r_{GBNCIN}), \ \ \theta \geq 0, \qquad (23)$$

where pq_{bl}^{GBNCIN}, $b = 1,2,...,\iota$, $l = 1,2,...,w$, are transient probabilities defined by (1) and $H(\theta, r)$ is the mean value of the number of exceeding the critical safety state r_{GBNCIN} by the *GBNCIN* operating at the variable conditions during the operation time θ defined in [Kołowrocki, Soszyńska-Budny, 2011].

Now, we assume that the *GBNCIN* is repairable after exceeding the critical safety state r_{GBNCIN} and its renewal time is non-ignored and have distribution function with the mean value $\mu_0^{GBNCIN}(r_{GBNCIN})$ and the standard deviation $\sigma_0^{GBNCIN}(r_{GBNCIN})$ and the cost of the *GBNCIN* single renovation is $K_{GBNCIN_{n-ign}}$.

In this case, the total operation cost of the repairable *GBNCIN* with not ignored its renovation time during the operation time θ, $\theta \geq 0$, amounts

$$K_{GBNCIN_{n-ign}}(\theta) \cong \sum_{b=1}^{\iota}\sum_{l=1}^{w} pq_{bl} \sum_{i=1}^{n}[K_{BCIN\,i}(\theta)]^{(bl)} + $$
$$+ K_{GBNCIN_{n-ign}} \overline{\overline{H}}(\theta, r_{GBNCIN}), \ \ \theta \geq 0, \qquad (24)$$

where pq_{bl}^{GBNCIN}, $b = 1,2,...,\iota$, $l = 1,2,...,w$, are transient probabilities defined by (1) and $\overline{\overline{H}}(\theta, r_{GBNCIN})$ is the mean value of the number of renovations of the *GBNCIN* operating at the variable conditions during the operation time θ defined by [Kołowrocki, Soszyńska-Budny, 2011].

The particular expressions for the mean values $H(\theta, r_{GBNCIN})$ and $\overline{\overline{H}}(\theta, r_{GBNCIN})$ for the repairable *GBNCIN* with ignored and non-ignored renovation times existing in the formulae (23) and (24), respectively defined in [Kołowrocki, Soszyńska-Budny, 2011], are determined for typical multistate repairable critical infrastructure operating at the variable operation conditions.

After the optimization of the *GBNCIN* operation process related to climate-weather change, the *GBNCIN* operation total costs given by (22)-(24) assume their optimal values.

The total optimal cost of the non-repairable *GBNCIN* during the operation time θ, $\theta \geq 0$, after its operation process related to climate-weather change optimization is given by

$$\dot{K}_{GBNCIN}(\theta) \cong \sum_{b=1}^{\iota}\sum_{l=1}^{w} \dot{p}q_{bl}^{GBNCIN} \sum_{i=1}^{n}[K_{BCIN\,i}(\theta)]^{(bl)},$$
$$\theta \geq 0, \qquad (25)$$

where $\dot{p}q_{bl}^{GBNCIN}$, $b = 1,2,...,\iota$, $l = 1,2,...,w$, are optimal transient probabilities found by the procedure presented before.

The optimal total operation cost of the repairable *GBNCIN* with ignored its renovation time during the operation time θ, $\theta \geq 0$, after its operation process related to climate-weather change optimization amounts

$$\dot{K}_{GBNCIN_{ign}}(\theta) \cong \sum_{b=1}^{\iota}\sum_{l=1}^{w} \dot{p}q_{bl}^{GBNCIN} \sum_{i=1}^{n}[K_{BCIN\,i}(\theta)]^{(bl)} + $$
$$+ K_{GBNCIN_{ign}} \dot{H}(\theta, r_{GBNCIN}), \ \ \theta \geq 0, \qquad (26)$$

where $\dot{p}q_{bl}^{GBNCIN}$, $b = 1,2,...,\iota$, $l = 1,2,...,w$, are optimal transient probabilities and $\dot{H}(\theta, r_{GBNCIN})$ is the mean value of the optimal number of exceeding the critical safety state r_{GBNCIN} by the *GBNCIN* operating at the variable conditions during the operation time θ [Kołowrocki, Soszyńska-Budny, 2011].

The total optimal operation cost of the repairable *GBNCIN* with non-ignored its renovation time during the operation time θ, $\theta \geq 0$, after its operation process related to climate-weather change optimization amounts

$$K_{GBNCIN_{n-ign}}(\theta) \cong \sum_{b=1}^{\iota}\sum_{l=1}^{w} pq_{bl}^{GBNCIN} \sum_{i=1}^{n}[K_{BCIN\,i}(\theta)]^{(bl)} + $$
$$+ K_{GBNCIN_{n-ign}} \overline{\overline{H}}(\theta, r_{GBNCIN}), \ \ \theta \geq 0, \qquad (27)$$

where $\dot{p}q_{bl}^{GBNCIN}$, $b = 1,2,...,\iota$, $l = 1,2,...,w$, are optimal transient probabilities and $\dot{\overline{\overline{H}}}(\theta, r_{GBNCIN})$ is the mean value of the optimal number of renovations of the *GBNCIN* operating at the variable operation conditions during the operation time θ [Kołowrocki, Soszyńska-Budny, 2011].

The particular expressions for the optimal mean values $\dot{H}(\theta, r_{GBNCIN})$ and $\dot{\overline{\overline{H}}}(\theta, r_{GBNCIN})$ for the repairable *GBNCIN* with ignored and non-ignored renovation times, respectively defined by [17], may be obtain by replacing the transient probabilities pq_{bl}^{GBNCIN} by their optimal values $\dot{p}q_{bl}^{GBNCIN}$ in the expressions for $H(\theta, r_{GBNCIN})$ and $\overline{\overline{H}}(\theta, r_{GBNCIN})$ defined in [17], that are determined for typical multistate repaired critical infrastructure operating at the variable operation conditions.

The application of the formulae (22)-(24) and (25)-(27) allow us to compare the costs of the non-repairable and repairable CI networks, in a special case of the *GBNCIN*, with ignored and non-ignored times of renovations operating at the variable operation conditions before and after the optimization of their operation processes.

4. Conclusion

The tools presented in the article are useful for the optimization of operation and safety of the GBNCIN, operating at the varying conditions that have an influence on changing its safety structures and its components safety characteristics. Presented: the GBNCIN operation cost related to climate-weather change, the GBNCIN operation cost related to climate-weather change minimization, and cost analysis of the GBNCIN operation impacted by climate-weather change, can be applied for different critical infrastructures, critical infrastructure networks, and networks of critical infrastructure networks.

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