Influence of Machining Accuracy of Links on Dynamic Response of Planar Mechanisms

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Abstract

In the paper the influence of machining accuracy of mechanism links on their dynamic response is under investigation. As an example a of planar slider-crank mechanism is studied. The influence of different cross-section area within the assumed tolerance on the dynamic behaviour of mechanism's connection rod is investigated. For vibration analysis of links the finite element method is used with Bernoulli-Euler beam elements. The calculation were conducted for nominal dimensions of the cross section of the crank and for the two cases for connecting rod: 1) for the maximal stiffness and 2) for the minimal stiffness obtained within the given tolerance. The results of analysis show that the changes in dynamic response for different cross-sectional area of mechanism links, within the assumed tolerance of machining, can be quite significant in the case of high-speed precise mechanisms and manipulators.

Keywords: FEM, Slider-crank mechanism, Machining tolerance

1. Introduction

The influence of machining accuracy of mechanism links on their dynamic response is rarely a subject of research activities however its importance is very clear. The computer simulations conducted in the case of gear shafts [3] have shown that machining tolerance of pins affected the work gearing. It appeared that in the case of toothed gears much larger significance has the preciseness of machining of shaft pins than taking into account other factors such as non-linear phenomena.

In the last years the demand for modelling high-speed, lightweight mechanisms was emphasized. On the other hand these new constructions are more flexible and the accuracy of machining of mechanism links can be significant and its influence on dynamic response should be investigated.

The research works on the modelling and analysis of planar mechanisms can be divided in two categories: the first group of researchers take into account the flexibility of links; the second group of works assume that the links of mechanism are rigid, but considering other factors such as friction and clearances in joints, dynamics of engine and transmission system, etc. The most popular and efficient method for analysing dynamic behaviour of manipulators with taking into account flexibility of links is the finite element method. In the present paper the FEM is applied to dynamic analysis of a slider-crank mechanism. It is assumed that the mechanism links are cylindrical tubes (i.e. with ring-shape cross-section) and the influence of machining tolerances on the dynamic response is under investigation.

2. Modelling of flexible elements by FEM

The finite element method is commonly used for vibration analysis of flexible mechanisms since it allows taking into account flexibility of all links. In recent years considerable attention has been given to the analysis of flexible mechanisms. The need of considering flexibility of members in high-speed mechanisms has arisen due to restrictions on weight and power requirements. A number of investigators have conducted analyses of flexible mechanisms applying special purpose finite element method. In most of these studies, the response of a mechanism was obtained through the superposition of rigid-body and flexible motion components [4, 5]. In these papers it was assumed that the rigid-body motion is known and the elastic displacements are separated from the rigid-body displacement and solved as the unknowns of the system.

Equations of motion are usually derived by using the Lagrangian formulation. In the derivation of this matrix equation the planar linear beam finite elements are used.

For modelling the slider-crank mechanism with taking into account flexibility of links the beam finite elements are used (Fig. 1).

Figure 1. Displacement of nodes of a beam finite element

The beam element consists of 2 nodes and three general displacements at each node, so components of nodal displacements vector in global *xy* and local $\xi\eta$ coordinate frames are as follows:

$$\{s\}^T = [u_1, w_1, \Theta_1, u_2, w_2, \Theta_2], \tag{1}$$

where: u_1, w_1, u_2, w_2 are displacements of nodes 1 and 2 in *x* and *y* direction, respectively; Θ_1, Θ_2 are angular deformations of nodes 1 and 2,

$$\{\delta\}^T = [p_1, v_1, \Theta_1, p_2, v_2, \Theta_2], \tag{2}$$

where p_1, p_2, v_1, v_2 – displacements of nodes 1 and 2 in ξ and η direction, respectively.

The vector $\{\delta\}$ is transformed into $\{s\}$ by

$$\{\delta\} = [T]\{s\}, \tag{3}$$

where $[T]$ is a transformation matrix

$$[T] = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 & 0 & 0 & 0 \\ -\sin\alpha & \cos\alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\alpha & \sin\alpha & 0 \\ 0 & 0 & 0 & -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \tag{4}$$

where α is the angle between local and global coordinate systems.

The shape function $[N_e]$ for elastic displacement of Euler-Bernoulli beam type finite element is as follows

$$[N_e] = \begin{bmatrix} 1-\xi & 0 & 0 & \xi & 0 & 0 \\ 0 & 1-3\xi^2+2\xi^3 & L(\xi-2\xi^2+\xi^3) & 0 & 3\xi^2-2\xi^3 & L(\xi^3-\xi^2) \end{bmatrix}, \quad (5)$$

where $0 \le \xi \le 1$.

In the derivation procedure it is assumed that the shape function for rigid-body motion is the same as for the elastic motion [7]. As a result, the system inertia matrix for elastic displacement vector appears together with the rigid-body acceleration vector on the right hand side of equations of motions:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F\} - [M]\{\ddot{x}_0\}, \quad (6)$$

where $[M]$ is the global inertia matrix, $[C]$ is the global damping matrix, $[K]$ is the stiffness matrix, $\{F\}$ represents generalized forces, and $\{\ddot{x}\}$, $\{\dot{x}\}$ and $\{x\}$ represent acceleration, velocity and displacement vectors (in nodal points), and $\{\ddot{x}_0\}$ is the rigid-body acceleration.

In the above equation two elements are unknown and should be defined: the formulation of a damping matrix and rigid-body acceleration.

In the analysis of flexible mechanisms the classical Rayleigh damping is usually adopted. In these studies the damping, stiffness and inertia matrices are based on the same displacement vectors connected with elastic deformation. In the Rayleigh damping model there are two factors: α – for mass proportional damping and β – for stiffness proportional damping. It is assumed that mass proportional damping dominates when the frequency is low and stiffness proportional damping dominates when the frequency is high. In most cases of multibody dynamics the stiffness proportional damping is only considered:

$$[C] = \beta[K]. \quad (7)$$

The coefficient β is determined based on the first few natural frequencies of the system. The procedure of obtaining damping matrix in finite element analysis of flexible mechanism is discussed in details in [4].

The way of obtaining rigid-body acceleration is presented in the Author's paper [5]. The model of a shape function for the planar rigid-body motion is proposed and the shape function for elastic motion is not used to describe an arbitrary large rigid-body translation. The use of different shape functions for elastic and rigid motions implies that the inertia matrix, standing by rigid body acceleration vector in the equations of motion of flexible mechanisms, depends on both shape functions of elastic and rigid elements.

The components of nodal displacement vector can be expressed in the relative coordinate system *xy* by the following vector:

$$\{s_0\}^T = [u_{01}, w_{01}, u_{02}, w_{02}] \quad (8)$$

or in the local coordinate system ξη by

$$\{\delta_0\}^T = [p_{01}, v_{01}, p_{02}, v_{02}], \quad (9)$$

where $u_{01}, w_{02}, u_{01}, w_{02}$ are displacements of nodes 1 and 2 in x and y direction respectively, $p_{01}, p_{02}, v_{01}, v_{02}$ - displacements of nodes 1 and 2 in ξ and η direction respectively.

A transformation matrix $[T_0]$ for this case is as follows (α is the angle between $\xi\eta$ and global coordinate systems):

$$[T_0] = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 & 0 \\ -\sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & \cos\alpha & \sin\alpha \\ 0 & 0 & -\sin\alpha & \cos\alpha \end{bmatrix}. \quad (9)$$

The shape function for the rigid body motion $[N_{e0}]$ can be expressed as:

$$[N_{e0}] = \begin{bmatrix} 0.5 & 0 & 0.5 & 0 \\ 0 & 1-\xi/L & 0 & \xi/L \end{bmatrix}, \quad (10)$$

where $0 \le \xi \le L$, L is the length of a finite element

The obtained equations of motion for the system are as follows [7]

$$[M_0]\{\ddot{x}_0\} + [K_0]\{x_0\} = \{F_0\}, \quad (11)$$

where $[M_0]$, $[K_0]$ are the system matrices obtained from element matrices; $\{x_0\}$ is the nodal displacement vector, and $\{F_0\}$ is the external system force vector. The presence of stiffness matrix in equations (11) is only necessary due to proper modeling of displacements of the rigid element and has practically no influence on rigid body motion (the kinetic energy of an element and consequently element inertia matrix were taken for rigid elements). If the stiffness matrix was omitted, the nodes could displace in any direction – also in longitudinal direction of the element which is impossible due to the rigidity of elements.

3. Numerical example

The influence of accuracy of link machining on vibration mechanism members was investigated for the example of a planar slider-crank mechanism presented in Fig. 2. The dynamic analysis of this type of mechanisms is studied by many researchers, e.g. Akbari et al. [1] investigated effects of various mechanisms' parameters including crank length, flexibility of the connecting rod and the slider's mass on its dynamic behaviour. Cheng and Liu [2] studied the influence of the crack in the rod on a slider-crank dynamics.

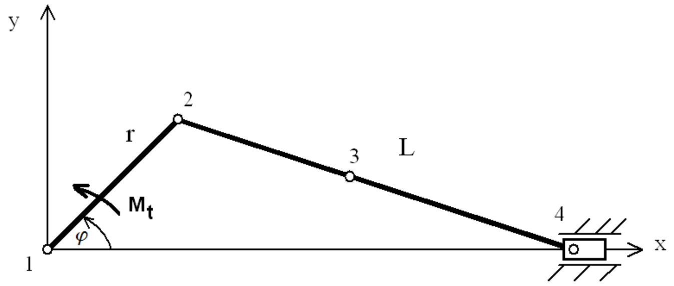

Figure 2. Slider crank mechanism

It was assumed that the crank was manufactured with no machining errors (i.e. it possess nominal dimensions) but the connecting rod is made of tube shape with the nominal cross-sectional dimensions: the inner diameter of 10 mm, the outer diameter of 16 mm. Assuming the IT12 class of tolerance for the inside diameters and IT11 for the outside diameters and applying the possible wall thickness of the tube shape connecting rod are as follows: maximal thickness is equal to (16-10)/2=3 [mm], and minimal thickness (15.89-10.15)/2=2.87 [mm]. Its characteristics are given in Table 1. At point 4 there is attached mass m = 0.2 kg and gravity and friction in rotary pairs and slider are not taken into account.

The coefficient β appearing in the damping matrix formulation (7), based on the natural frequencies of the system, was assumed to be $\beta = 9.03 \cdot 10^{-7}$.

Table 1. Slider-crank parameters

Parameter	Crank	Connecting Rod	
		Minimal value	Maximal value
Length, m	0.1	0.4	
Cross-section area, m^2	1.107×10^{-3}	1.169×10^{-4}	1.225×10^{-4}
Moment of inertia, m^4	1.775×10^{-7}	2.584×10^{-9}	2.726×10^{-9}
Young modulus, N/m^2	0.71×10^{11}		
Density, kg/m^3	2710		

The mechanism is divided into three elements, i.e. crank is represented by one beam finite element while the connecting rod by two finite elements. Taking into account the elastic vibration of links the vector of the unknown functions (nodal displacement vector $\{x\}$) in global coordinates consists of 9 elements: displacements u, and w in the X, and Y direction, respectively, of moving nodes 2, 3, and 4, and nodal deformation angles Θ:

$$\{x\}^T = [u_2, w_2, \Theta_{21}, \Theta_{22}, u_3, w_3, \Theta_3, u_4, \Theta_4]. \tag{12}$$

The input torque M_t is applied to the crank of the mechanism and is assumed to be

$$M_t = M_{t0} + M_{t1} \sin \varphi, \tag{13}$$

where φ is the crank angle.

The data for input torque are as follows: M_{t0} = 0.1 N·m, M_{t1} =0.1 N·m. The zero initial conditions for crank angle and crank rotational speed are assumed.

The governing equations of motion for dynamic analysis of flexible mechanisms are presented in Chapter 2. In numerical analysis midspan displacements of the connecting rod (node 3) were calculated and the results are presented in Fig. 3. It can be seen that it is visible difference in dynamic response for the two cases considered i.e. one – for the maximum stiffness of the connecting rod within the given machining tolerance, and second – minimum stiffness within the tolerance of the cross-section of the connecting rod. From the figure it can be seen that for the lowest link stiffness the amplitude of vibration is greater.

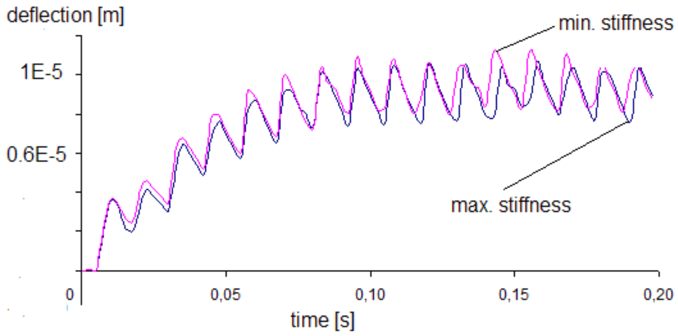

Figure 3. Midspan displacements of connecting rod (node 3).

4. Conclusions

The conducted comparative analysis shows that within tolerance of links the difference in stiffness can be significant. The subject is not sufficiently studied in literature since for the case of high-speed lightweight mechanisms or manipulators the changes in dynamic response may cause additional errors. In the case of rotatable elements (e.g. shafts) the not perfectly axial cross-sectional area (but machining within the given tolerance) causes additional vibrations and influences the working conditions of gears. The conducted transient analysis shows that starting torque has a great influence on the vibration.

The model presented in the paper did not take into account the phenomena that always occur in this type of mechanisms such as clearances and friction in rotary and sliding pairs. The future investigation should be directed at building an extensive model in order to describe possible effect of flexible links on mechanism behaviour including jamming, blocking and rattling.

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