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Application of stereology for assessment of two-phase flow structures

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Abstract

This paper describes a method for two-phase flow structures evaluation of gas-liquid mixture based on theoretical assumptions for the stereology in materials science. This assessment is based on the analysis of digital images, obtained with a high-speed camera. The bright field technique was used for process visualization. The images obtained during the visualisation were, in fact, projections of the structures. For the given recording conditions, the stereological analysis applied was based on the linear method and on the method of random and directed secants. The new methods of determining important parameters of two-phase flow were proposed on the basis of collected data. The parameters are as follows: the volume fraction, the interfacial area, the number of objects in one of the phases of the mixture and two other structural parameters of the selected two-phase fluid obtained from the analysis of the two-dimensional image (average length of chords for projected objects, average free distance for convex shapes from the projection).

Keywords: Stereological analysis; Two-phase flow pattern; Pattern recognition

Nomenclature

A – total image area, or channel's normal cross-sectional area, m^2

A_A – total field of flat sections on the individual β-phase of the image per unit area, m^2/m^2

A'_A – total field of flat projections on the individual β-phase of the image per unit area, m^2/m^2

A_β – total field of flat sections on the individual β-phase, m^2

L – total length of the secants marked on the picture, m

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L_β – total length of the chords on the individual β-phase, m
L_L – total length of the β-phase chords per unit length of secant, applied to the cross-sectional image, m/m
L'_L – total length of the β-phase chords per unit length, secant applied to the projection image, m/m
$\bar{I}$ – average chord length of the cross-sectional image, m
$\bar{I}'$ – average chord length of the projection of the image, m
l_k – length of the kth secant, m
l_{ik} – chord length and the ith β-phase object on the kth secant, m
N'_A – number of cross-sections of objects per unit surface projection, pcs/m^2 (pieces per meter)
N'_L – number of objects' chord per unit length of the secant applied to the projection of the image, pcs/m
N_V – number of objects in the volume, pcs/m^3
S_V – relative interfacial surface area, m^2/m^3
t – thickness of the analysed structure, m
V – total volume of the space or the total volume of the mixture, m^3
V_β – total volume of the β-phase, m^3
V_V – total volume of the β-phase objects per unit volume of the mixture, m^3/m^3
w_{Go} – superficial gas-phase velocity, m/s
w_{Lo} – superficial liquid-phase velocity, m/s
λ – average free distance for convex shapes, m/pcs

1 Introduction

The issues of two-phase flow, in particular of gas-liquid two-phase flow, are under laboratory investigations and are important in operations of industrial installations. They result from difficulties in the interpretation of flow phenomena that occur in two-phase flow systems, as well as from imperfections of two-phase flow diagnostic methods. One of the basic questions regarding the two-phase gas-liquid flow is the flow structure. The structure of the flow influences intensity of mass and heat transfer processes. Knowledge about the flow structure can be used for assessing the exploitation conditions of two-phase flow devices. This is why conventional and widely accepted approach to the two-phase mixture flow research strongly indicates the need for determining the type of flow structures and the extent of their occurrence.

2 Presentation of two-phase flow structure evaluation issues

The area in which two-phase flow occurs is three-dimensional space, and its structure should be considered. Each phase of the gas-liquid mixture can take any geometric shape under the action of external forces. The flow itself, as a dynamic phenomeon, changes these shapes. Also thermal processes and the mass transfer between the components of the mixture, can affect the geometric shape formation of individual phase components. This means that the two-phase flow structure must also be considered as a function of time and the nature of flow is thus transient. The solution to this problem was to classify the structures for different cases of two-phase flow, for example: inside differently orientated channels [2,4], and the tube bundle space [33], which takes the two-phase mixture. The object of the assessment here is the concentration distribution of the phases inside the channel.

First classification attempts were based on direct observations by the investigator. The next step was introducing visualisation methods, allowing for further development of the classification methods by obtaining images of the process. For image acquisition, a variety of techniques for both recording and observation have been used. Image registration techniques include, among others: high speed photography [1,7,16], X ray photography (with very short duration of exposure) [17], densitometric radiation methods using: X rays [19,25,35], γ rays [5,11,12], β rays [14], and various types of process tomography: electric capacitance tomography (ECT) [10], electric resistance tomography (ERT) [8], microwave tomography [3], X ray transmission tomography [20], or γ ray transmission tomography [9], ultrasonic tomography [37] and optical tomography [38]. Observational techniques include, for example, a mirror scanner [15] and axial observation of the annular flow [18].

Classification of two-phase flow structures and the possibility of their registration solved the problem of their identification. However, in addition to extensive tomographic systems, it is still a subjective assessment made by the observer. Tracking the process under industrial conditions requires an objective identification, based on the evaluation of measurable features, characteristic for the two-phase flow. Solutions to this problem were attempted in different ways, namely:

- by creating flow maps [17,29,30,32],
- by using a variety of measurement techniques such as measurement probes [6,22,24], trapping methods [26,39],
- by using computer processing and analysis of signals [21,32], or images [13,27,31,34].

3 Stereological methods for quantitative description of the structure

Quantitative stereoscopic and stereological terms were introduced in the 1960s by a group of material-testing scientists, mathematicians and scientists from other disciplines. They became convinced that the images of different types of structures have many common features, if they are analysed with a certain level of generalisation. Progress was made in the area of methods for obtaining a materials' generalised three-dimensional description, on the basis of two-dimensional images [28].

The development of methods for quantitative interpretation of the three-dimensional characteristics of objects based on their two-dimensional images, requires knowledge of geometry, including stereometry. In these pictures, we deal with projections or sections of solids, surfaces and lines (Fig. 1).

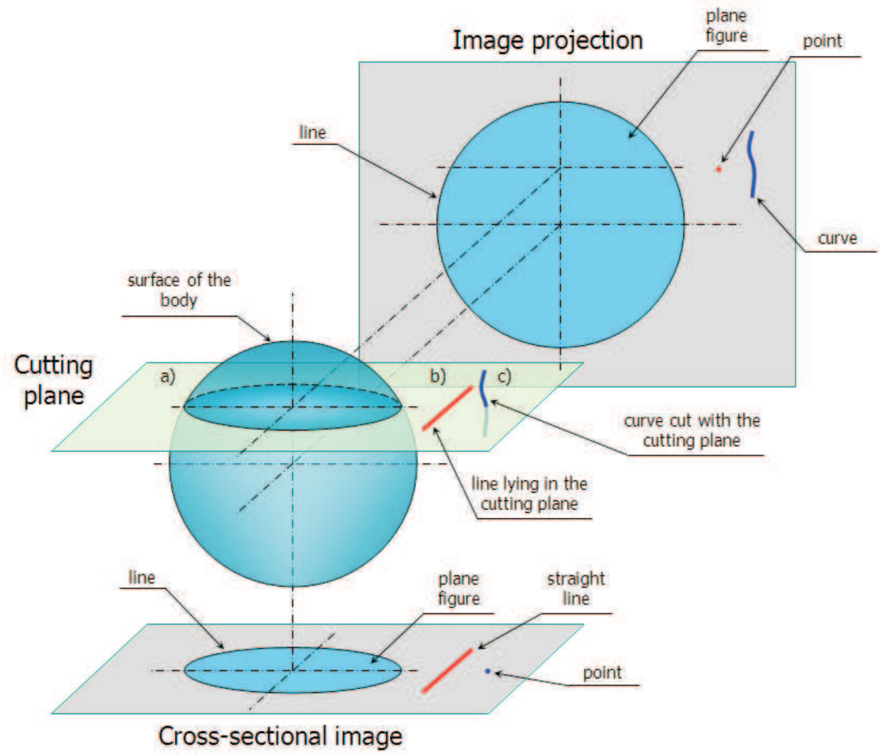

Figure 1. The difference between the projected image and the cross-sectional image of: a) solid (ball), b) line lying in the cutting plane and c) curve cut with the cutting plane.

It turned out that the application of probability theory and mathematical statistics methods are also required. This is due to the fact that the spatial structures are characterised by considerable degrees of randomness and a high size and shape variabilities. The randomness of the imaging and measurements process necessitates the application of mathematical statistics. Obtaining the appropriate image requires the selection of the correct test sample and the localisation of the measurement area. At all these stages there is a high degree of randomness, which justifies the statistical approach.

The interest in stereology led mathematicians from intuitive ways of measurements to its correct formalisation. Further development allowed the development of the precise measurement procedure, which minimizes the impact of potential errors associated with the choices of observation areas [28].

4 The problem of system orientation

Proper stereological analysis of the two-phase fluid flow structure requires the determination of the surface layout. This information is important for the determination of the correct plane of imaging in order to obtain a representative image.

A partially orientated system on the plane surface is a system in which only part of the total surface is orientated in a certain way. Typically, in the case of multiphase mixtures, the surface is orientated parallel to the flow direction. Figure 2 shows the most common interfacial surface setups: A) partially linear surface perpendicular to the flow direction (these objects are usually small and their participation is not significant); B) an isometric interface setup; C–G) partially linear surface parallel to the direction of mixture flow; H) an interface setup, which can be treated as fully linearly orientated (it is characteristic for annular flow, in which the interfacial area is dominant in the gas core and orientated according to the flow direction of the mixture).

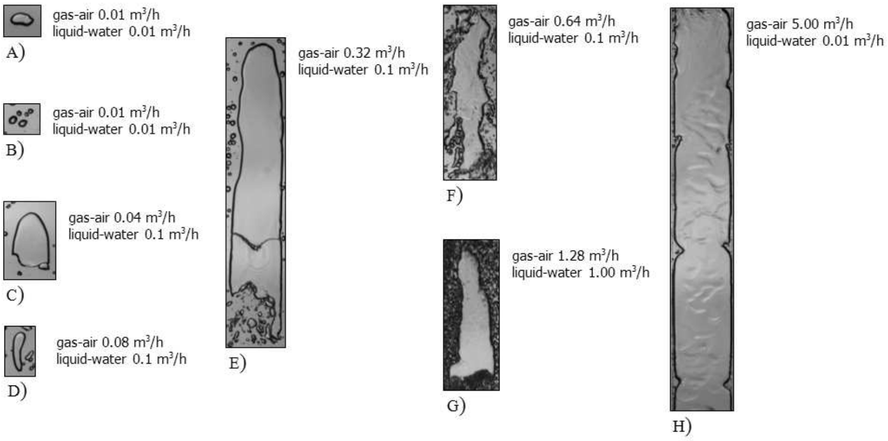

Figure 2. Sample images of the interfaces of the various phases of concurrent flow streams in the ascending vertical channel.

Any partially orientated surface arrangement can be considered as consisting of two planar systems superimposed on one another. One of them is fully orientated, and the second isometric. To determine both surfaces of the system, we need to have the image of the structure parallel to the orientation axis, i.e., longitudinal section of the test channel. In this case the relative surface projection takes the form of linear elements (closed loops and lines).

5 Data acquisition

As mentioned above, an image is required for the stereological analysis which must be taken in a suitable area of the test channel and in an appropriate manner. To do this, an appropriate video track has been set up (Fig. 3).

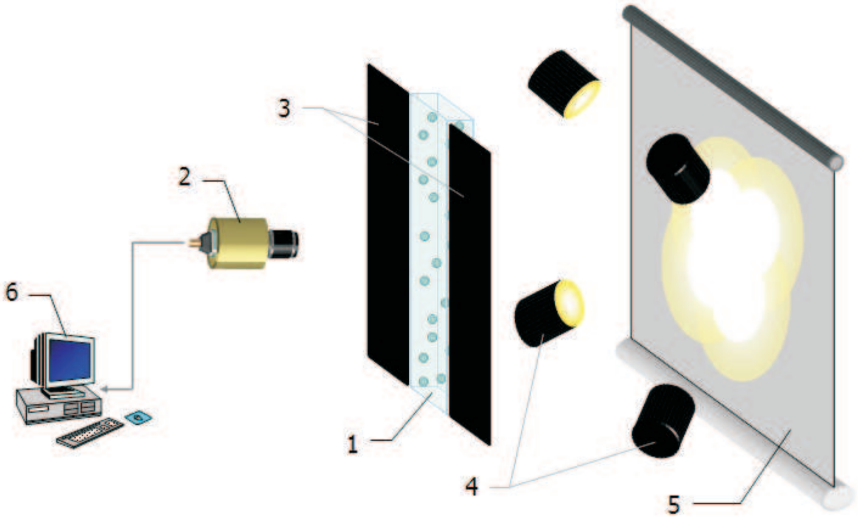

Figure 3. Schematic view of the image acquisition system.

A two-phase mixture flows inside a transparent rectangular channel (1) with dimensions of 1200×50×6 mm. The entire process is recorded by a high-speed video camera (2). To avoid the adverse reflection of light, screens (3) have been installed on both sides of the channel. The lighting system (4) consists of four halogen lamps with a power output of 1 kW each, arranged to form a spot of light on the screen (5), which is an arrangement known as bright-field illumination. In order to obtain uniform illumination, the lamps are connected to a light intensity control unit (not shown) with separate control for each lamp. Images recorded by the video camera, in the form of digital data, are stored in the memory buffer, and then transferred to a computer (6). Due to the rapid changes in the process and the requirement for high-quality images, the following high speed video camera was used: VDS Vosskühler HCC-1000, with a CMOS sensor. The maximum recording rate of this camera is 1850 fps at a resolution of 1024×1024 pixels.
Using the measurement setup described above, video images of gas-liquid mixture

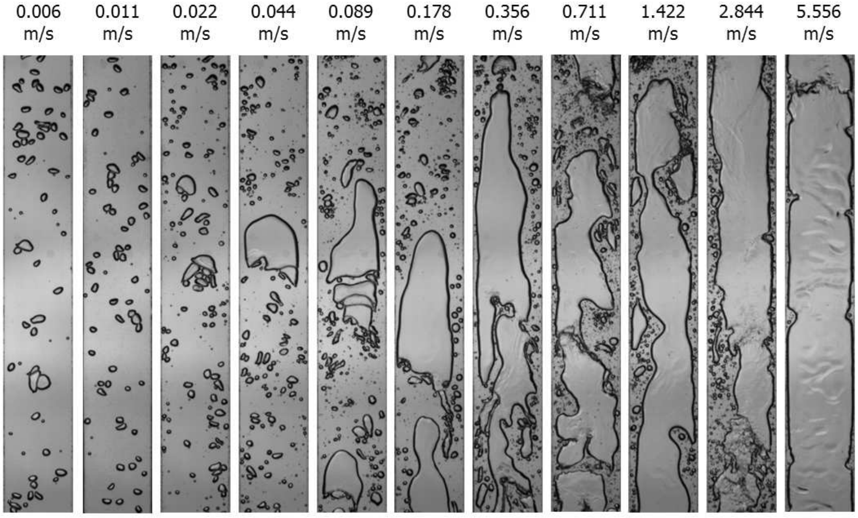

Figure 4. Sample images of the recorded flows, ranked by increasing superficial velocity of the gas phase (w_{Go}), at a constant superficial velocity of the liquid phase $w_{Lo} = 0.011$ m/s.

structures (wherein the gas is air and the liquid is water), have been captured. Those images are projections of objects, which are gas bubbles (Fig. 4). Due to the fact that those images are sections and not cross-sections, determining the selection of stereological parameters used on the image is based on the methods, similar to those used in thin-film analysis used in metallography.

6 Image processing

The resulting images of the multiphase mixture structures require the extraction and highlighting of interesting objects and their attributes. To do this, the following graphical unit operations have been applied: cleaning the objects from noise, separation of the objects (which is important in the case of chains or swarm of bubbles), filling the 'holes' in the objects, as in the case of a water-air flow is very important, because in the image we see only the edges of the interface, and the interior of the bubble is indistinguishable from the surrounding fluid (Fig. 5). After the mentioned image processing operations, all bubbles are filled with a solid colour, which can also be selected to create the best contrast with the background, i.e., the surrounding fluid.

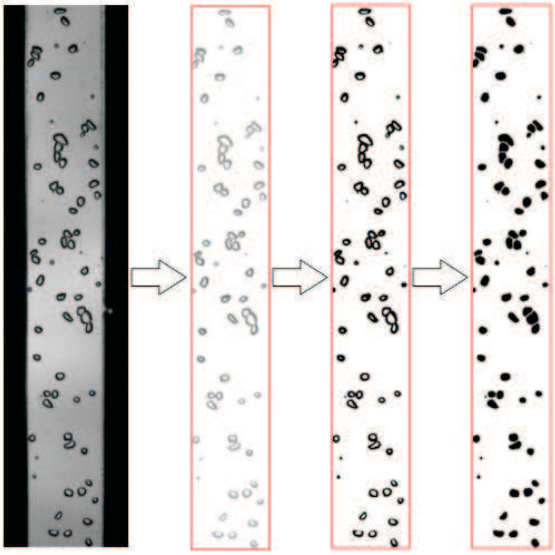

Figure 5. Selected image processing steps.

7 Stereological analysis

Evaluation of the two-phase mixture structures is carried out with the use of stereological methods and is based on the comparison of the stereological parameter groups, set for various two-phase mixture images. The most interesting parameters, from point-of-view of flow phenomena, were: the volume fraction, interfacial area, the number of objects of one phase in the mixture, and two additional structural parameters (average length of chords for projected objects and average free distance for convex shapes from the projection). Determination of desired parameters requires certain expressions, shown in Tab. 1. Due to the measurement set up used, the channel geometry and physical characteristics of the mixture components, those expressions are derived from the principles of the overall projection on the plane and in space (Fig. 6). All small, spherical objects of the gas phase, such as bubbles, take the form of circles on the plane, perpendicular A_T to the flow direction of mixture. For this reason, the mean chord length in cross-section $\bar{l}$ is equal to the average chord length of the objects in projection $\bar{l}'$. However, for large objects, such as bubbles and plugs, when the average chord length is greater than the thickness of the channel, then the mean chord length of the cross-sectional thickness of the channel is equivalent to the thickness of the channel (t).

Table 1. Selected stereological parameters under investigations [28].

Relative volume of convex shapes from the projection	Relative area interfacial of closed body from the projection	Number of closed bodies in the volume from the projection	Average length of chords for projected objects	Average free distance for convex shapes from the projection
$V_V = \frac{A'_A \cdot \overline{I}}{t}$	$S_V = \frac{4A'_A}{t}$	$N_V = \frac{N'_A}{t}$	$\overline{l}' = \frac{L'_L}{N'_L}$	$\lambda = \frac{t - A'_A \cdot \overline{I}}{N'_L \cdot \overline{I}'}$

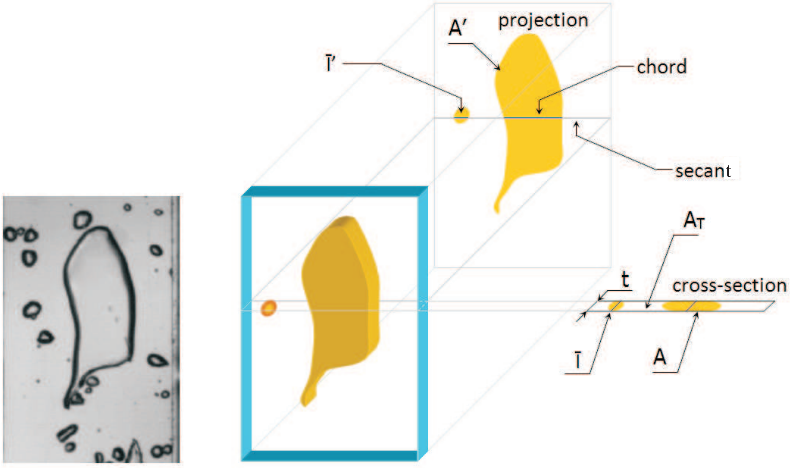

Figure 6. Relationship between the projection and the cross-section of the object in the measuring point.

Finally, the chord and secant lengths, and their quantities were counted on digitized images by application of the random-secant and targeted-secant methods [28,36], which are variations of the linear method, derived from the Cavalieri-Hacquert principle (Fig. 7):

$$\frac{V_\beta}{V} = \frac{A_\beta}{A} = \frac{L_\beta}{L} \Rightarrow V_V = A_A = L_L \,, \tag{1}$$

$$L_L = \frac{\sum\limits_i l_{ik}}{l_k} \,. \tag{2}$$

This rule states that, the volume fraction of the specific phase in the mixture (V_β/V) is equal to the fraction of the specific surface occupied by this phase of the total surface of the mixture on the image (A_β/A), which is also equal to

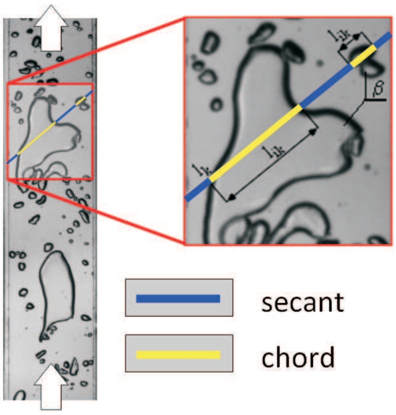

Figure 7. The notion of the linear method.

the specific chord length to the secant (L_β/L). This means that the percentage distributions of the specific phase in the volume of the mixture (V_V) in the section plane (A_A) and in the length of the secant line (L_L) are the same.

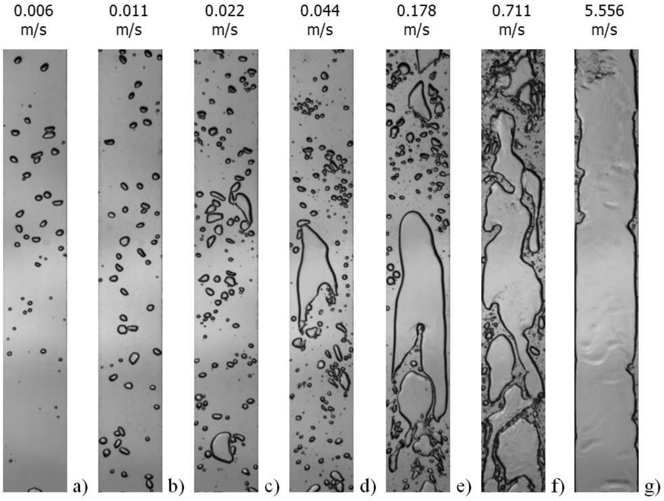

Figure 8. Selected images of recorded flow structures, arranged according to the increasing superficial gas velocity (w_{Go}) at a constant superficial velocity of the liquid phase $w_{Lo} = 0.01$ m/s.

According to general classification, the structure images shown in Fig. 8 are representative examples of two-phase gas-liquid mixture flow in the vertical channel, from the bubbling structures (a–c) through a plug (d–e) and foam structures (f), ending with the annular flow structure (g). The base measurement was made by image scanning by projecting random secants on the image and counting the visible number of chords and their lengths. Image scanning was performed in two directions, along the flow axis and perpendicular to it. The gathered statistical data of the chords, secants and their mutual orientations relative to the flow direction of the two-phase mixture, were used to calculate the five selected stereological parameters, shown in Tab. 2.

Table 2. Example of stereological parameters, calculated for selected flow regimes at a constant superficial velocity of the gas phase $w_{Lo} = 0.011$ m/s (presented in Fig. 8.) [23].

	Unit	a) $w_{Go} = 0.006$ m/s		b) $w_{Go} = 0.011$ m/s		c) $w_{Go} = 0.022$ m/s	
		Average value	Absolute error	Average value	Absolute error	Average value	Absolute error
$\overline{l}'$	mm/pcs	2.98	0.298	4.16	0.246	4.68	0.189
N_V	pcs/dm^3	0.22	–	0.56	–	1.00	–
S_V	mm^2/mm^3	0.04	0.004	0.10	0.006	0.19	0.008
V_V	%	1.11	0.220	7.06	0.570	14.66	0.650
λ	mm/pcs	266.45	80.095	69.85	9.720	31.87	2.706

	Unit	d) $w_{Go} = 0.044$ m/s		e) $w_{Go} = 5.556$ m/s	
		Average value	Absolute error	Average value	Absolute error
$\overline{l}'$	mm/pcs	4.65	0.143	9.60	0.160
N_V	pcs/dm^3	1.45	–	1.72	–
S_V	mm^2/mm^3	0.27	0.008	0.42	0.007
V_V	%	21.00	0.650	53.63	1.130
λ	mm/pcs	20.34	1.253	4.90	0.185

	Unit	f) $w_{Go} = 0.711$ m/s		g) $w_{Go} = 0.178$ m/s	
		Average value	Absolute error	Average value	Absolute error
$\overline{l}'$	mm/pcs	17.08	0.209	62.18	0.255
N_V	pcs/dm^3	1.26	–	0.46	–
S_V	mm^2/mm^3	0.41	0.005	0.39	0.002
V_V	%	70.91	1.180	94.13	0.770
λ	mm/pcs	2.35	0.068	0.36	0.004

To quantify the structure of the gas-liquid mixture, we primarily use the volume fraction (V_V) and interfacial surface (S_V). However, the starting point for further analysis a complete set of all above five stereological parameters, which allows a qualitative assessment of the structure and its proper classification to generally accepted two-phase fluid flow structure characterisation.

8 Interpretation of the results

Stereological methods for gas-liquid mixture structure identification can be performed by observing the correlations between the stereological parameters and the analysed structure (Tab. 3). Those dependencies are the result of the trends observed during the analysis of phase superficial velocity (Fig. 9) [23].

Table 3. Stereological parameter change trends, observed during the research for the structures presented in Fig. 8.

I'	↓	↓	~ (↓)	~ (↑)	↑	↑
N_V	↑	↓	↑	↓	~ (↑)	↓
S_V	~ (↑)	↓	↑	↓	↑	~ (↓)
V_V	↓	↓	↑	↓	↑	↑
λ	↑	↑	↓	↑	↓	↓
$w_{Lo} = 0.011$ m/s						
Increase w_{Go}			c → d		a → b b → c d → e	e → f f → g
Decrease w_{Go}	f → e g → f	b → a c → b e → d		d → c		

'~' – slight change

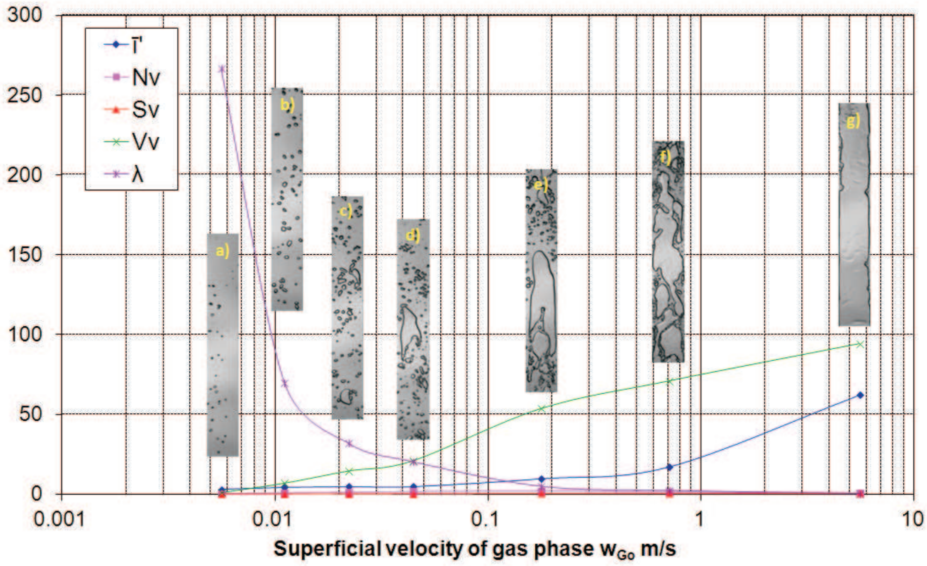

Figure 9. Changes of the stereological parameters of the flow at a constant superficial velocity of gas phase $w_{Lo} = 0.011$ m/s.

For an increasing rate of gas-phase superficial velocity (w_{Go}) and at a constant superficial velocity of the liquid phase (w_{Lo}), a decrease in the average free distance (λ) between objects in dispersed phase is observed. If we compare the stereological parameters for two different images, and the λ value is decreasing when we change the images, then it means that, the superficial gas velocity is increasing for the structure shown in the second image. The opposite situation occurs for the relative volume fraction and the average chord length for dispersed phase objects. This means that if we know the exact hydrodynamic parameters, during proper flow in the apparatus, we can deduce how they have changed as a result of disturbances, such as a sudden increase in temperature or pressure.

9 Conclusions

It has been determined that there is a set of stereological parameters for each structure that enables the quantitative estimation of the structure. Moreover, it has been found that the observation of interrelation of all stereological parameters during the changing of the flow structure, allows control of the operating conditions of the apparatus. Knowledge about the character of changes taking place in the structure may be used for constant adjustment of the structure via an automatic feedback system.

The presented method can be extended with additional parameters, and even allows the experimenter to create his or her own parameters based on the principles developed for stereology. This improves the method's usability and makes it easily adaptable for other identification tasks.

Any flow structure image, independent of the visualisation technique used, can be treated using the presented method. The success of such treatment is, however, dependent on distinguishing the exact phases and knowledge of the place where the images were taken. For this reason, it is one of the noninvasive methods, in which the measurement does not interfere with the analysed flow.

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