

RESPONSE DUE TO MECHANICAL SOURCE IN SECOND AXISYMMETRIC PROBLEM OF MICROPOLAR ELASTIC MEDIUM

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The second axisymmetric problem in a micropolar elastic medium has been investigated by employing an eigen value approach after applying the Laplace and the Hankel transforms. An example of infinite space with concentrated force at the origin has been presented to illustrate the application of the approach. The integral transforms have been inverted by using a numerical technique to obtain the components of microrotation, displacement, force stress and couple stress in the physical domain. The results for these quantities are given and illustrated graphically.

Key words: eigen value, micropolar elastic medium, Laplace transforms, Hankel transforms, concentrated force, microrotation, Rhomberg's integration.

1. Introduction

Eringen and Suhubi (1964) introduced the theory of microelastic solids in which the microdeformation and microrotation of the material particles contained in a microvolume element with respect to its centroid are taken into account in an average sense. Materials affected by micromotions and microdeformations are known as micromorphic materials. Later Eringen (1964) developed a theory for a subclass of micromorphic materials which are called micropolar solids and these materials show microrotation effect and microrotational inertia. Here, the material's particle in a volume element can undergo only rigid rotational motions about its centre of mass. Micropolar solids may represent the materials that are made up of dipole atoms or dumbbell type molecules and are subjected to surface and body couples. Solid propellant grains, rocks, polymeric materials, wood and fibre glass are few examples of such materials. The deformation in these materials is characterized not only by classical translational degree of freedom represented by the displacement vector field $\mathbf{u}(\mathbf{x},t)$, but also by the rotation vector $\boldsymbol{\phi}(\mathbf{x},t)$.

Das *et al.* (1983) discussed a one-dimensional problem in coupled thermoelasticity using an eigen value approach. Mahalanabis and Manna (1989) discussed the eigen value approach to linear micropolar elasticity by arranging basic equations of linear micropolar elasticity form of matrix differential equation in the Hankel transform domain. Saxena and Dhaliwal (1990) discussed a two-dimensional problem in axisymmetric and plane strain cases in the context of coupled thermoelasticity employing the eigen value approach. The two-dimensional axisymmetric and plane strain problems in homogeneous and isotropic media are investigated by Sharma and Chand (1992) using the eigen value approach. Sharma and Kumar (1996) discussed the axisymmetric problem of generalized an isotropic thermoelasticity by using the eigen value approach after employing the integral transform technique. By using the eigen value approach Das *et al.* (1997) investigated a one-dimensional problem with heat sources distributed over a plane area in an infinite isotropic elastic solids and a two-dimensional problem with the instantaneous heat sources in an infinite transversely isotropic elastic medium. Recently Mahalanabis and Manna (1997) discussed the problem of linear micropolarthermoelasticity by using eigen value approach.

In the paper we consider a two-dimensional second axisymmetric problem in a homogeneous isotropic micropolar elastic medium. The solutions are obtained using the eigen value approach after employing the integral transform technique. The integral transforms are inverted using a numerical approach.

2. Basic equations

Following Eringen (1966) the constitutive relations and the field equations in a micropolar elastic solid without body forces and body couples can be written as

$$t_{kl} = u_{r,r} \delta_{kl} + \mu(u_{k,l} + u_{l,k}) + k(u_{l,k} - \varepsilon_{krl} \phi_r), \quad (2.1)$$

$$m_{kl} = \alpha \phi_{r,r} \delta_{kl} + \beta \phi_{k,l} + \gamma \phi_{l,k}, \quad (2.2)$$

$$(\lambda + 2\mu + k) \nabla \cdot \mathbf{u} - (\mu + k) \nabla \times \nabla \times \mathbf{u} + k \nabla \times \boldsymbol{\phi} = \rho \frac{\partial^2 \mathbf{u}}{\partial t^2}, \quad (2.3)$$

$$(\alpha + \beta + \gamma) \nabla \cdot \boldsymbol{\phi} - \gamma \nabla \times \nabla \times \boldsymbol{\phi} + k \nabla \times \mathbf{u} - 2K \boldsymbol{\phi} = \rho j \frac{\partial^2 \boldsymbol{\phi}}{\partial t^2} \quad (2.4)$$

where, $\lambda, \mu, \alpha, \beta, \gamma, K$ are material constants, ρ is the density, j – the micro inertia, \mathbf{u} - the displacement vector, $\boldsymbol{\phi}$ - the rotation vector, t_{kl} - the force stress tensor and m_{kl} - the couple stress tensor.

3. Formulation and solution

We consider a homogeneous, isotropic micropolar elastic solid. We take a cylindrical polar coordinates system (r, θ, z) and the z -axis is pointing into the medium. Due to the symmetry about the z -axis, all quantities are independent of θ . Since we are discussing the second axisymmetric problem, we have

$$\mathbf{u} = (\theta, u_\theta, 0), \quad \boldsymbol{\phi} = (\phi_r, \theta, \phi_z). \quad (3.1)$$

Using Eq.(3.1) and introducing dimensionless quantities as

$$\begin{aligned} r' &= \frac{r}{h}, & z' &= \frac{z}{h}, & u'_\theta &= \frac{\rho h \omega^{*2}}{\mu}, & \phi'_r &= \frac{\rho h^2 \omega^{*2}}{\mu} \phi_r, \\ \phi'_z &= \frac{\rho h^2 \omega^{*2}}{\mu} \phi_z, & t' &= \frac{\mu}{\rho h^2 \omega^{*2}} t, & t'_{z\theta} &= \frac{l}{K} t_{z\theta}, \\ m'_{zz} &= \frac{l}{Kh} m_{zz}, & m_{zr} &= \frac{l}{Kh} m_{zr}, & \omega^{*2} &= \frac{K}{\rho j}, \end{aligned} \quad (3.2)$$

the set of Eqs (2.3)-(2.4) reduce to (on suppressing the dashes)

$$\left\{ \frac{\partial^2 \phi_r}{\partial r^2} + \frac{1}{r} \frac{\partial \phi_r}{\partial r} - \frac{1}{r^2} \phi_r \right\} + (1-d^2) \frac{\partial^2 \phi_r}{\partial r \partial z} + d^2 \frac{\partial^2 \phi_r}{\partial z^2} - 2n_1 \phi_r - n_1 \frac{\partial u_\theta}{\partial z} = n_2 \frac{\partial^2 \phi_r}{\partial t^2}, \tag{3.3}$$

$$\frac{\partial^2 \phi_z}{\partial z^2} + (1-d^2) \left\{ \frac{\partial^2 \phi_r}{\partial z \partial r} + \frac{1}{r} \frac{\partial \phi_r}{\partial z} \right\} + d^2 \left\{ \frac{\partial^2 \phi_z}{\partial r^2} + \frac{1}{r} \frac{\partial \phi_z}{\partial r} \right\} + n_1 \frac{1}{r} \frac{\partial}{\partial r} (ru_\theta) - 2n_1 \phi_z = n_2 \frac{\partial^2 \phi_z}{\partial t^2}, \tag{3.4}$$

$$\left\{ \frac{\partial^2 u_\theta}{\partial r^2} + \frac{1}{r} \frac{\partial u_\theta}{\partial r} + \frac{\partial^2 u_\theta}{\partial z^2} - \frac{u_\theta}{r^2} \right\} + n_3 \left\{ \frac{\partial \phi_r}{\partial z} - \frac{\partial \phi_z}{\partial r} \right\} = n_4 \frac{\partial^2 u_\theta}{\partial t^2} \tag{3.5}$$

where

$$n_1 = \frac{Kh}{\alpha + \beta + \gamma}, \quad n_2 = \frac{j\mu^2}{\rho h^2 \omega^{*2} (\alpha + \beta + \gamma)}, \quad n_3 = \frac{K}{\mu + K}, \tag{3.6}$$

$$n_4 = \frac{\mu^2}{\rho h^2 \omega^{*2} (\mu + K)}, \quad d^2 = \frac{\gamma}{\alpha + \beta + \gamma}.$$

Applying the Laplace transform with respect to time ‘t’ defined by

$$\{\tilde{\phi}_r(r, z, p), \tilde{\phi}_z(r, z, p), \tilde{u}_\theta(r, z, p)\} = \int_0^\infty \{\phi_r(r, z, t), \phi_z(r, z, t), u_\theta(r, z, t)\} e^{-pt} dt, \tag{3.7}$$

and then the Hankel transform with respect to ‘r’ defined by

$$\tilde{\phi}_z(\xi, z, p) = \int_0^\infty \tilde{\phi}_z(r, z, p) r J_0(\xi r) dr, \tag{3.8}$$

$$\{\tilde{\phi}_r(\xi, z, p), \tilde{u}_\theta(\xi, z, p)\} = \int_0^\infty \{\tilde{\phi}_r(r, z, p), \tilde{u}_\theta(r, z, p)\} r J_1(\xi r) dr,$$

to Eqs (3.3)-(3.5), we obtain

$$\tilde{\phi}_r'' = \frac{1}{d^2} (\xi^2 + n_2 p^2 + 2n_1) \tilde{\phi}_r + \frac{(1-d^2)\xi}{d^2} \tilde{\phi}_z' + \frac{n_1}{d^2} \tilde{u}_\theta', \tag{3.9}$$

$$\tilde{\phi}_z'' = (p^2 n_2 + \xi^2 d^2 + 2n_1) \tilde{\phi}_z - (1-d^2) \xi \tilde{\phi}_r' - n_1 \xi \tilde{u}_\theta, \tag{3.10}$$

$$\tilde{u}_\theta'' = -n_3 \xi \tilde{\phi}_z - n_3 \tilde{\phi}_r' + (n_4 p^2 + \xi^2) \tilde{u}_\theta. \tag{3.11}$$

The system of Eqs (3.9)-(3.11) can be written as

$$\frac{d}{dz}W(\xi, z, p) = A(\xi, p)W(\xi, z, p) \quad (3.12)$$

where

$$W = \begin{bmatrix} U \\ U' \end{bmatrix}, \quad A = \begin{bmatrix} 0 & I \\ A_2 & A_1 \end{bmatrix}, \quad U = \begin{bmatrix} \tilde{\Phi}_r \\ \tilde{\Phi}_z \\ \tilde{u}_\theta \end{bmatrix}, \quad (3.13)$$

$$A_1 = \begin{bmatrix} 0 & \frac{(1-d^2)\xi}{d^2} & \frac{n_1}{d^2} \\ -(1-d^2) & 0 & 0 \\ -n_3 & 0 & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} \frac{1}{d^2}(\xi^2 + n_2 p^2 + 2n_1) & 0 & 0 \\ 0 & p^2 n_2 + \xi^2 d^2 + 2n_1 & -n_1 \xi \\ 0 & -n_3 \xi & n_4 p^2 + \xi^2 \end{bmatrix}$$

0 is the null matrix and I the unit matrix of order 3×3 . To solve Eq.(3.12), we take

$$W(\xi, z, p) = X(\xi, p)e^{lz}, \quad (3.14)$$

so that

$$A(\xi, p)W(\xi, z, p) = lW(\xi, z, p), \quad (3.15)$$

which leads to the eigen value problem. The characteristic equation corresponding to the matrix A is given by

$$\det(A - lI) = 0, \quad (3.16)$$

which on expansion provides

$$l^6 - \lambda_1 l^4 + \lambda_2 l^2 - \lambda_3 = 0 \quad (3.17)$$

where the coefficient $\lambda_1, \lambda_2, \lambda_3$ can be easily evaluated in terms of p, ξ and constants Eq.(3.6).

The eigen values of the matrix A are characteristics roots of Eq.(3.17). We assume that real parts of l_i are positive. The vector $\bar{X}(\xi, p)$ corresponding to the eigen value l_i can be determined by solving the homogeneous equation

$$[A - lI]X(\xi, p) = 0. \quad (3.18)$$

The set of eigenvectors $\bar{X}_i(\xi, p)$, ($i=1,2,3,4,5,6$) may be obtained as

$$X_i(\xi, p) = \begin{bmatrix} X_{i1}(\xi, p) \\ X_{i2}(\xi, p) \end{bmatrix} \tag{3.19}$$

where

$$X_{i1}(\xi, p) = \begin{bmatrix} a_i l_i \\ b_i \\ -\xi \end{bmatrix}, \quad X_{i2}(\xi, p) = \begin{bmatrix} a_i l_i^2 \\ b_i l_i \\ -\xi l_i \end{bmatrix}, \quad l = l_i, \quad i = 1, 2, 3, \tag{3.20}$$

$$X_{j1}(\xi, p) = \begin{bmatrix} -a_i l_i \\ b_i \\ -\xi \end{bmatrix}, \quad X_{j2}(\xi, p) = \begin{bmatrix} a_i l_i^2 \\ -b_i l_i \\ \xi l_i \end{bmatrix}, \quad j = i + 3, \quad l = -l_i, \quad i = 1, 2, 3, \tag{3.21}$$

$$a_i = \xi \left[(d^2 - 1)(n_4 p^2 + \xi^2 - l_i^2) - n_1 n_3 \right] / \Delta_i, \tag{3.22}$$

$$b_i = \left[(n_4 p^2 + \xi^2)(n_2 p^2 + \xi^2 + 2n_1) - l_i^2 (n_2 p^2 + \xi^2 + 2n_1 - n_1 n_3) + d^2 l_i^2 \{ l_i^2 - (n_4 p^2 + \xi^2) \} \right] / \Delta_i, \tag{3.23}$$

$$\Delta_i = n_3 \left[l_i^2 - (n_2 p^2 + \xi^2 + 2n_1) \right]; \quad i = 1, 2, 3. \tag{3.24}$$

The solution of Eq.(3.12) is given by Sharma and Chand (1992)

$$W(\xi, z, p) = \sum_{i=1}^3 \left[B_i X_i(\xi, p) \exp(l_i z) + B_{i+3} X_{i+3}(\xi, p) \exp(-l_i z) \right] \tag{3.25}$$

where B_i ($i = 1, 2, 3, 4, 5, 6$) are arbitrary constants.

Equation (3.25) represents the solution of the general problem in the axisymmetric case of homogeneous isotropic, micropolar elasticity by employing the eigen value approach and therefore can be applied to a broad class of problem in the domains of the Laplace and Hankel transforms.

4. Application

We consider an infinite micropolar elastic space in which a concentrated force of magnitude $F = \frac{-F_0 \delta(r) \delta(t)}{2\pi r}$ acting in the direction of the z -axis at the origin of a cylindrical coordinate system as shown in Fig.1.

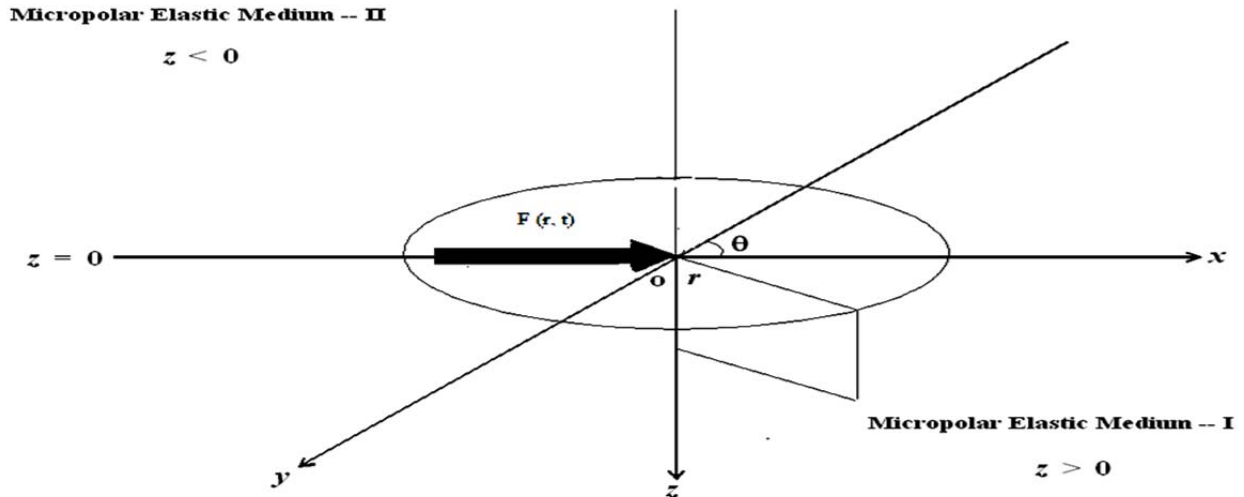


Fig.1. Geometry of the problem.

The problem is axisymmetric with respect to the z -axis. The boundary conditions on the plane $z=0$ are given by

$$\phi_r(r, 0^+, t) - \phi_r(r, 0^-, t) = 0, \quad \phi_z(r, 0^+, t) - \phi_z(r, 0^-, t) = 0, \quad (4.1)$$

$$u_\theta(r, 0^+, t) - u_\theta(r, 0^-, t) = 0, \quad (4.2)$$

$$t_{z\theta}(r, 0^+, t) - t_{z\theta}(r, 0^-, t) = \frac{-F_0 \delta(r) \delta(t)}{2\pi r}, \quad (4.3)$$

$$m_{zr}(r, 0^+, t) - m_{zr}(r, 0^-, t) = 0, \quad m_{zz}(r, 0^+, t) - m_{zz}(r, 0^-, t) = 0. \quad (4.4)$$

Applying the Laplace and Hankel transform to Eqs (4.1)-(4.4), we get

$$\tilde{\phi}_r(\xi, 0^+, p) - \tilde{\phi}_r(\xi, 0^-, p) = 0, \quad \tilde{\phi}_z(\xi, 0^+, p) - \tilde{\phi}_z(\xi, 0^-, p) = 0, \quad (4.5)$$

$$\tilde{u}_\theta(\xi, 0^+, p) - \tilde{u}_\theta(\xi, 0^-, p) = 0, \quad (4.6)$$

$$\tilde{t}_{z\theta}(\xi, 0^+, p) - \tilde{t}_{z\theta}(\xi, 0^-, p) = \frac{-F_0}{2\pi}, \quad (4.7)$$

$$\tilde{m}_{zr}(\xi, 0^+, p) - \tilde{m}_{zr}(\xi, 0^-, p) = 0, \quad \tilde{m}_{zz}(\xi, 0^+, p) - \tilde{m}_{zz}(\xi, 0^-, p) = 0. \quad (4.8)$$

The transformed microrotations, displacement and stresses are given for $z \geq 0$ by

$$\tilde{\phi}_r(\xi, z, p) = \{a_1 l_1 B_4 \exp(-l_1 z) + a_2 l_2 B_5 \exp(-l_2 z) + a_3 l_3 B_6 \exp(-l_3 z)\}, \quad (4.9)$$

$$\tilde{\phi}_z(\xi, z, p) = \{b_1 B_4 \exp(-l_1 z) + b_2 B_5 \exp(-l_2 z) + b_3 B_6 \exp(-l_3 z)\}, \tag{4.10}$$

$$\tilde{u}_\theta(\xi, z, p) = -\xi \{B_4 \exp(-l_1 z) + B_5 \exp(-l_2 z) + B_6 \exp(-l_3 z)\}, \tag{4.11}$$

$$\tilde{u}'_0(\xi, z, p) = \xi \{l_1 B_4 \exp(-l_1 z) + l_2 B_5 \exp(-l_2 z) + l_3 B_6 \exp(-l_3 z)\}, \tag{4.12}$$

$$\begin{aligned} \tilde{t}_{z0}(\xi, z, p) = & l_1 (\xi n_5 - a_1 n_6) B_4 \exp(-l_1 z) + l_2 (\xi n_5 - a_2 n_6) B_5 \exp(-l_2 z) + \\ & + l_3 (\xi n_5 - a_3 n_6) B_6 \exp(-l_3 z), \end{aligned} \tag{4.13}$$

$$\begin{aligned} \tilde{m}_{zr}(\xi, z, p) = & (a_1 l_1^2 n_8 - \xi b_1 n_7) B_4 \exp(-l_1 z) + (a_2 l_2^2 n_8 - \xi b_2 n_7) B_5 \exp(-l_2 z) + \\ & + (a_3 l_3^2 n_8 - \xi b_3 n_7) B_6 \exp(-l_3 z), \end{aligned} \tag{4.14}$$

$$\begin{aligned} \tilde{m}_{zz}(\xi, z, p) = & -[l_1 (\xi a_1 n_9 + b_1 n_{10}) B_4 \exp(-l_1 z) + l_2 (\xi a_2 n_9 + b_2 n_{10}) B_5 \exp(-l_2 z) + \\ & + l_3 (\xi a_3 n_9 + b_3 n_{10}) B_6 \exp(-l_3 z)]. \end{aligned} \tag{4.15}$$

For $z \leq 0$ the above expressions get suitably modified

$$\tilde{\phi}_r(\xi, z, p) = a_1 l_1 B_1 \exp(l_1 z) + a_2 l_2 B_2 \exp(l_2 z) + a_3 l_3 B_3 \exp(l_3 z) \tag{4.16}$$

where $n_5 = \frac{(\mu + K)\mu}{K\rho h^2 \omega^2}, \quad n_6 = \frac{\mu}{\rho h^2 \omega^2}, \quad n_7 = \frac{\beta\mu}{K\rho h^4 \omega^2},$

$$n_8 = \frac{\gamma\mu}{K\rho h^4 \omega^2}, \quad n_9 = \frac{\alpha\mu}{K\rho h^4 \omega^2}, \quad n_{10} = \frac{(\alpha + \beta + \gamma)\mu}{K\rho h^4 \omega^2}, \tag{4.17}$$

$$B_1 = B_4 = F_0 (a_3 b_2 - a_2 b_3) / 4\pi l_1 \Delta^*, \tag{4.18}$$

$$B_2 = B_5 = F_0 (a_1 b_3 - a_3 b_1) / 4\pi l_2 \Delta^*, \tag{4.19}$$

$$B_3 = B_6 = F_0 (a_2 b_1 - a_1 b_2) / 4\pi l_3 \Delta^* \tag{4.20}$$

where $\Delta^* = n_5 [(a_2 b_3 - a_3 b_2) + (a_3 b_1 - a_1 b_3) + (a_1 b_2 - a_2 b_1)].$ (4.21)

Thus the functions $\tilde{\phi}_r, \tilde{\phi}_z, \tilde{u}_\theta, \tilde{t}_{z0}, \tilde{m}_{zr},$ and \tilde{m}_{zz} have been determined in the transformed domain and these enable us to find the micro rotations, displacement, force stress and couple stress.

5. Inversion of the transforms

The solution is obtained by inverting the transforms in Eqs (4.9)-(4.16). These expressions can be formally expressed as functions of z , the parameters of the Laplace and Hankel transforms p and ξ respectively, and hence are of the forms $\bar{f}(\xi, z, p)$. To get the function $f(r, z, t)$ in the physical domain, first we invert the Hankel transform using

$$\bar{f}(r, z, p) = \int_0^{\infty} \xi \bar{f}(\xi, z, p) J_n(\xi r) d\xi. \quad (5.1)$$

Thus, expression Eq.(5.1) gives us the Laplace transform $\tilde{f}(r, z, p)$ of the function $f(r, z, t)$.

Now for the fixed values of ξ , r and z , the function $\tilde{f}(r, z, p)$ in the expression Eq.(5.1) can be considered as the Laplace transform $\bar{g}(p)$ of some function $g(t)$. Following Honig and Hirdes (1992), the Laplace transform of the function $\bar{g}(p)$ can be inverted as given below:

The function $g(t)$ can be obtained by using

$$g(t) = \frac{1}{2\pi i} \int_{C-i\infty}^{C+i\infty} e^{pt} \bar{g}(p) dp \quad (5.2)$$

where C is an arbitrary real number greater than all the real parts of the singularities of $g(p)$. Taking $p = C + iy$, we get

$$g(t) = \frac{e^{Ct}}{2\pi} \int_{-\infty}^{\infty} e^{iyt} \bar{g}(C + iy) dy. \quad (5.3)$$

Now taking $e^{-Ct} g(t)$ as $h(t)$ and expanding it as a Fourier series in $[0, 2L]$, we obtain approximately the formula

$$g(t) = g_{\infty}(t) + E_D \quad (5.4)$$

where

$$g_{\infty}(t) = \frac{C_0}{2} + \sum_{k=1}^{\infty} C_k, \quad 0 \leq t \leq 2L, \quad (5.5)$$

$$C_k = \frac{e^{Ct}}{L} \operatorname{Re} \left\{ e^{\frac{ik\pi t}{L}} \bar{g} \left(C + \frac{ik\pi}{L} \right) \right\}.$$

E_D is the discretization error and can be made arbitrarily small by choosing C large enough.

Since the infinite series in Eq.(5.5) can be summed up only to a finite number of N terms, so the approximate value of $g(t)$ becomes

$$g_N(t) = \frac{C_0}{2} + \sum_{k=1}^N C_k, \quad 0 \leq t \leq 2L. \quad (5.6)$$

Now, we introduce an error E_r that must be added to the discretization error to produce the total approximation error in evaluating $g(t)$ using the formula. The discretization error is reduced by using the

'Korrektur method' and then ' ε - algorithm' is used to reduce the truncation error and hence to accelerate the convergence. The Korrektur method formula to evaluate the function $g(t)$ is

$$g(t) = g_{\infty}(t) - e^{-2CL} g_{\infty}(2L+t) + E'_D$$

where $|E'_D| \ll |E_D|$.

Thus, the approximate value of $g(t)$ becomes

$$g_{N_k}(t) = g_N(t) - e^{-2CL} g_{N'}(2L+t) \quad (5.7)$$

where N' is an integer such that $N' < n$.

We shall now describe the ε - algorithm which is used to accelerate the convergence of the series in Eq.(5.6). Let N be an odd natural number and $S_m = \sum_{k=1}^m C_K$ be the sequence of partial sums of Eqs (5.6).

We define the ε - sequence by

$$\begin{aligned} \varepsilon_{0,m} &= 0, & \varepsilon_{1,m} &= S_m, \\ \varepsilon_{n+1,m} &= \varepsilon_{n-1,m+1} + \frac{1}{\varepsilon_{n,m-1} - \varepsilon_{n,m}}, & n, m &= 1, 2, 3, \dots \end{aligned}$$

The sequence $\varepsilon_{1,1}, \varepsilon_{3,1}, \dots, \varepsilon_{N,1}$ converges to $g(t) + E_D - C_0/2$ faster than the sequence $S_m, m = 1, 2, 3, \dots$. The actual procedure to invert the Laplace transform consists of Eq.(5.7) together with the ε - algorithm. The values of C and L are chosen according to the criteria outlined by Honig and Hirdes (1992).

The last step in the inversion process is to evaluate the integral in Eq.(5.1). This was done using Romberg's integration with an adaptive step size. This method uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limits when the step size tends to zero. The details can be found in Press *et al.* (1986).

6. Numerical results and discussion

Following Gauthier (1982) we take the following values of relevant parameters for the case of aluminium epoxy composite as

$$\begin{aligned} \rho &= 2.19 \times 10^3 \text{ kg/m}^3, & \lambda &= 7.59 \times 10^9 \text{ N/m}^2, & \mu &= 1.89 \times 10^9 \text{ N/m}^2, \\ K &= 0.0149 \times 10^9 \text{ N/m}^2, & \alpha = \beta = \gamma &= 0.0268 \times 10^5 \text{ N}, & j &= 19.6 \times 10^{-8} \text{ m}^2. \end{aligned}$$

Gauthier (1982) considered $\varepsilon = \frac{K}{\mu}$ as the coupling coefficient. The computations were carried out for three values of time, namely: $t = 0.025, 0.075, 0.125$ for fixed $\varepsilon = 0.0078$ and for three values of coupling coefficients, namely: $\varepsilon = 0.0078, 0.01, 0.0125$ for the fixed time $t = 0.075$ at $z = 1.0$ in the range $0 \leq r \leq 6$.

Figure 2 shows the variation of normal microrotations which decreases in the range $0 \leq r \leq 1$ and increases in the range $1.5 \leq r \leq 6$ as time increases from 0.025 to 0.0125 for the fixed value of $\varepsilon = 0.0078$.

Figure 3 shows the variation of normal microrotations which decreases in the range $0 \leq r \leq 1$ and increases in the range $1.5 \leq r \leq 6$ as ε increases from 0.0078 to 0.0125 for the fixed value of time 0.075 .

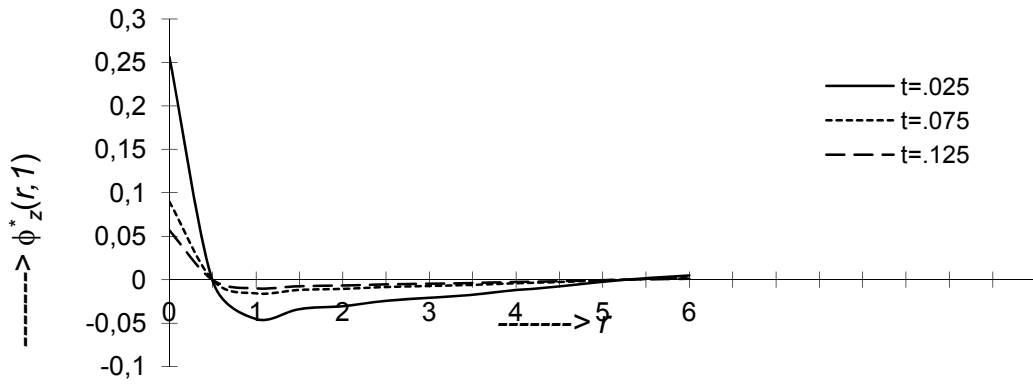


Fig.2. Variation of normal microrotation $\phi_z^*(r, t)$.

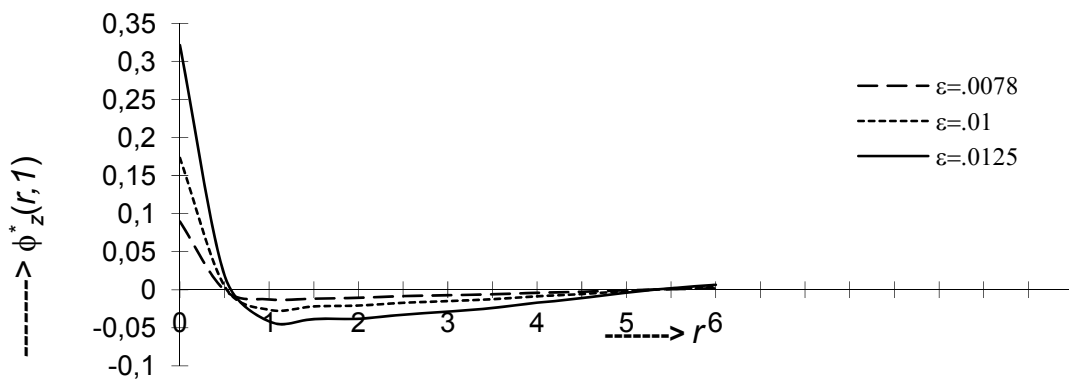


Fig.3. Variation of normal microrotation $\phi_z^*(r, t)$.

The variation of the tangential force stress is shown in Fig.4 which increases in the range $0 \leq r \leq 1$, decreases in the range $1.5 \leq r \leq 2, 5.5 \leq r \leq 6$ and oscillates in the range $2.5 \leq r \leq 5$ as time increases from 0.025 to 0.125 for the fixed value of $\epsilon = 0.0078$.

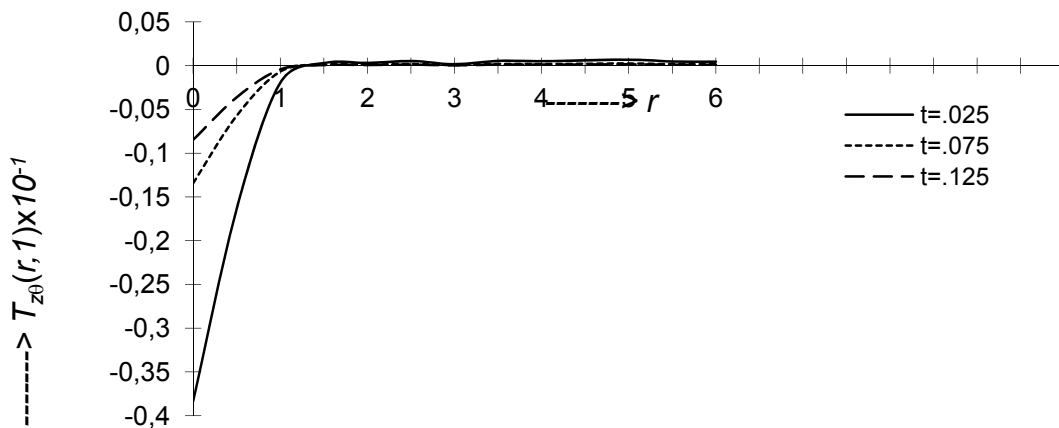


Fig.4. Variation of tangential force stress $T_{z_0}(r, t)$.

Figure 5 shows the variation of the tangential force stress which decreases in the range $0 \leq r \leq 1$, and increases in the range $1.5 \leq r \leq 6$ as ϵ increases from 0.0078 to 0.0125 for the fixed value of time 0.075.

Figure 6 shows the variation of normal couple stress which increases in the range $0 \leq r \leq 1.5$, oscillates in modulus value in the range $2 \leq r \leq 6$ as time increases from 0.025 to 0.125 for the fixed value of $\varepsilon = 0.0078$.

Figure 7 shows the variation of normal couple stress which decreases in the range $0 \leq r \leq 2.5$ and increases in the range $3 \leq r \leq 6$ as ε increases from 0.0078 to 0.0125 for the fixed value of time 0.075.

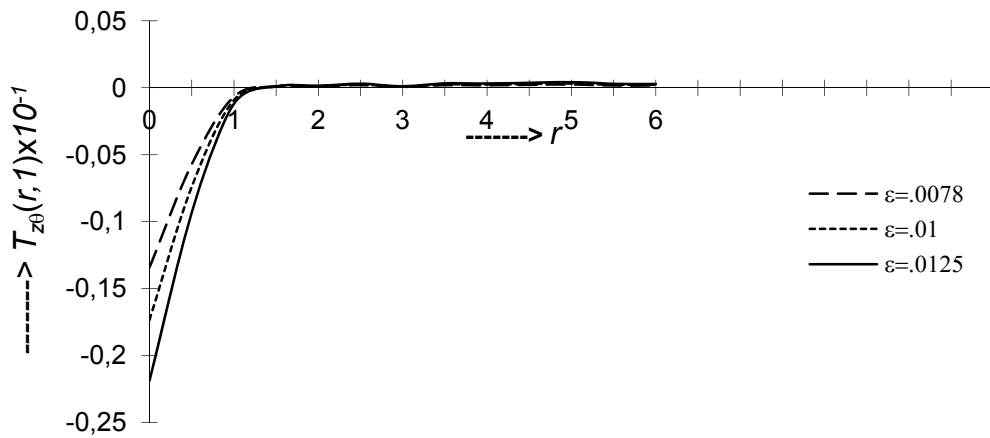


Fig.5. Variation of tangential force stress $T_{z_0}(r, t)$.

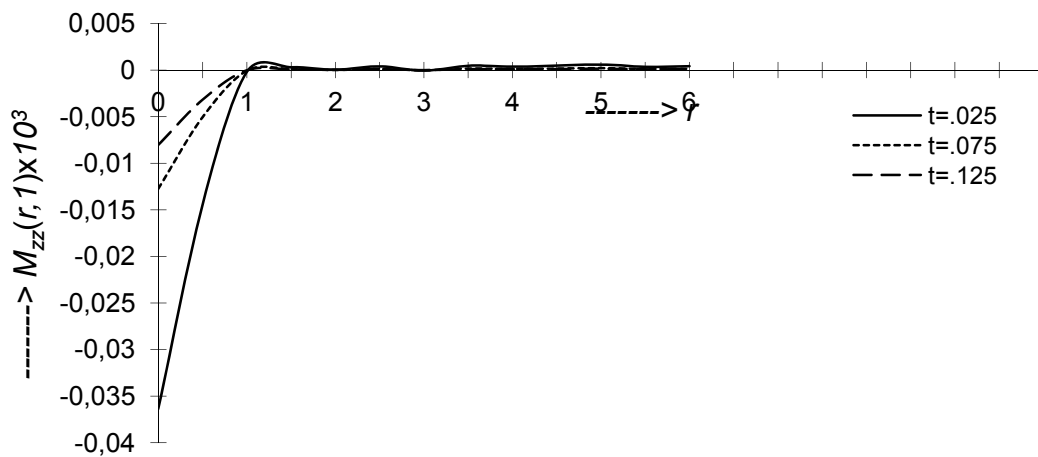


Fig.6. Variation of normal couple stress $M_{zz}(r, t)$.

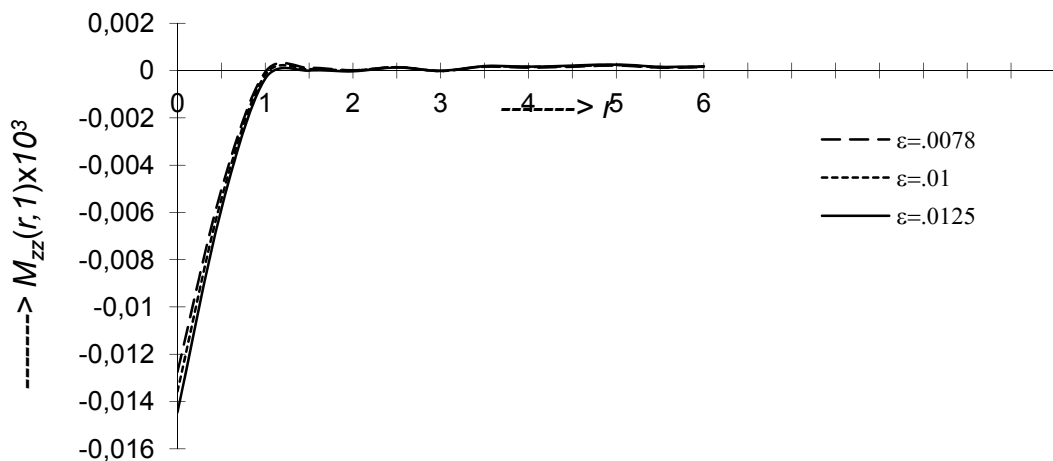


Fig.7. Variation of normal couple stress $M_{zz}(r, t)$.

Conclusion

We observed that all the six quantities showed more variation in their magnitude at all small times and small coupling coefficients and decreased with increase of time and ε .

Nomenclature

- j – micro-inertia
- m_{ij} – couple stress tensor
- t_{ij} – force stress tensor
- \mathbf{u} – displacement vector
- α, β, γ, K – micropolar material constants
- Δ – gradient operator
- δ_{ij} – Kronecker delta
- ε_{ijr} – alternating tensor
- λ, μ – Lamé's constants
- ρ – density
- ϕ – microrotation vector

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