Cezary BARTMAŃSKI, Andrzej STANIEK

CENTRAL MINING INSTITUTE, DEPARTMENT OF TECHNICAL ACOUSTICS AND LASER TECHNIQUE 1 Plac Gwarków Sg., 40-166 Katowice

Evaluation of Dynamic Parameters of Mechanical Constructions Based on Wavelet Analysis of Vibration Signals

Abstract

The paper presents the test procedure for determining dynamic parameters of mechanical structures, by using the method of wavelet analysis of vibration signals as a response of the construction to impulse excitation. Basic mathematical foundations have been shown for a mechanical system with one degree of freedom and its generalization for mechanical systems with many degrees of freedom. The correct functioning of the programme, which is the implementation of procedure for the MATLAB platform, has been tested with the help of programmable simulation of vibration signals and sinusoidal mechanical signals. The author has presented the results of the identification of dynamic parameters of two, selected mechanical structures: a free hanging steel plate and an acoustic screen. The results have been verified by comparing them with those obtained by modal analysis.

Keywords: vibroacoustics, digital signal processing, wavelet analysis.

1. Introduction

One of the directions of the undertaken mechanical research of engineering constructions is studying the impact of dynamic loads on their technical condition. On the basis of these studies, the state of degradation of the mechanical properties of a structure is assessed and specific measures are taken, aimed at avoiding a situation threatening the safety of the structure, or even its destruction.

For diagnostic purposes of the effects of dynamic loads which engineering structures are subjected to during their lifetime, vibratory signals are commonly used. These signals, generated by the action of dynamic loads on buildings, are good carriers of information about the dynamic properties of these objects, allowing to infer about their condition, extent of destruction etc.

In the Department of Technical Acoustics and Laser Technology of the Central Mining Institute studies of dynamic properties of engineering structures are conducted, using, among others, experimental methods directly on real objects. Past experience points to the possibility of diagnosing the technical condition of the objects, based on their basic parameters like natural and damped vibrations, i.e. the period of natural vibrations and damping decrement.

In practice, a number of methods for determining dynamic parameters of mechanical structures are used (a method based on a modal analysis of a vibrating object, a natural vibration method and others). There are some difficulties in determining the parameters characterizing the vibration energy dissipation precisely. The test procedure presented in the article, using wavelet analysis method of vibration signals is an attempt to use modern non-stationary signals analysis tools to overcome these difficulties.

2. Wavelet analysis of vibration signals

2.1. Continuous wavelet transform

The continuous wavelet transform is applicable for signals of finite energy (class $L^2(R)$ function). The basis for the wavelet analysis is the wavelet decomposition of signal x(t) based on a set of orthogonal basis functions, called wavelets $\psi_{ab}(t)$. The coefficients of this decomposition, called wavelet coefficients are defined by the wavelet transform of x(t) [1]:

$$(W_{\psi}x)(a,b) = \langle x(t), \psi_{a,b} \rangle = \int_{-\infty}^{\infty} x(t) \cdot \psi_{a,b}(t)^* dt \quad (1)$$

Wavelet family (set of basis functions) is made up of a basic wavelet (i.e. the mother wavelet) $\psi(t)$ with scaling and shifting in time:

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \cdot \psi\left(\frac{t-b}{a}\right) \tag{2}$$

where:

 $(W_{\psi}x)(a,b) - \text{wavelet coefficient,}$ a - the scale coefficient,b - shift in time domain, $a, b \in \mathbb{R}, a > 0,$ $<math>\psi_{a,b}(t)^* - \text{complex conjugate to } \psi_{a,b}(t).$

Primary wavelet must satisfy the condition of admissibility, defined as follows:

$$\int_0^\infty \frac{|\Psi(\omega)|^2}{\omega} < \infty \tag{3}$$

where: $\Psi(\omega)$ – Fourier transform of the function $\psi(t)$.

The average value of the so defined function is zero. The wavelet has a finite energy and non-zero values only in a finite period of time, and as a result, it has a form of short-term oscillations.

The values of wavelet coefficients are a measure of the degree of correlation between the signal and base wavelets.

In diagnostics, one of the most commonly used wavelet is a complex Morlet wavelet described by the formula [2]:

$$g(t) = \sqrt{\pi f_b} e^{j2\pi ft} e^{-t^2/f_b}$$
(4)

where: f_b - parameter specifying a wavelet band. A wavelet time diagram is shown in Fig. 1.



Fig.1. Time diagram of the Morlet wavelet

Wavelet coefficients, being the result of a signal decomposition, are not directly related to the parameters of the time-frequency signal representation. However, in specific cases, e.g. for the Morlet wavelet, you can specify the relationship between scale a and the value of frequency f:

$$a = \frac{f_s}{f} \tag{5}$$

where: f_s – signal sampling frequency.

2.2. The model of a mechanical construction

In general, the mechanical structure (a building or an engineering construction) should be considered as a system with many degrees of freedom (MDOF). For simplicity reasons, it has been assumed that the dynamic model of such a system consists of a set of independent linear system models with a single degree of freedom (SDOF), corresponding to various forms of natural vibrations. Each SDOF system is characterized by the value of natural frequencies and damping coefficient of these vibrations. The model of a mechanical system with a single degree of freedom is shown in Figure 2.



Fig. 2. The model of a mechanical system with a single degree of freedom

Dynamic properties of the model are described by the differential equation:

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = f(t) \tag{6}$$

where:

m – the vibrating weight, kg, c- damping coefficient, N/m s⁻¹, k– coefficient of elasticity N/m.

2.3. Determination of damping

The research of mechanical objects was made by measuring natural vibration, after stimulating the construction to vibrate by a force impulse. Using the following formulae:

$$\omega_d = \sqrt{\frac{k}{m}} \tag{7}$$

$$\zeta = \frac{c}{2\sqrt{k*m}} \tag{8}$$

we can describe the SDOF system response to an impulse excitation with the formula:

$$A(t) = A_0 e^{-\zeta \omega_n t} \sin(\omega_d t + \varphi) \tag{9}$$

where: ω_n – natural vibrations pulsation, $\omega_d = \omega_n \sqrt{1 - \zeta^2}$ – damped natural vibrations pulsation.

The vibration of energy dissipation of the SDOF system is manifested by the disappearance of the vibration amplitude envelope:

$$A(t) = A_0 e^{-\zeta \omega_n t} \tag{10}$$

According to the mathematical allocution conducted in article [2], the Morlet wavelet g(t) given by formula (4), a wavelet transform module for scale value a_0 , may be expressed by the formula:

$$\left| \left(W_g x \right) (a_0, b) \right| \approx A_0 e^{-\zeta \omega_n b} \left| G^* \left(\pm j a_0 \omega_n \sqrt{1 - \zeta^2} \right) \right| \quad (11)$$

where:

- $G(\cdot)$ Fourier transform of the g(t) function (the Morlet wavelet).
- $G^*(\cdot)$ function coupled to $G(\cdot)$.

Hence, after taking the logarithm of both sides of the equation we get:

$$\ln |(W_g x)(a_0, b)| \approx -\zeta \omega_n b + \ln \left(A_0 \left| G^* \left(\pm j a_0 \omega_n \sqrt{1 - \zeta^2} \right) \right| \right)$$
(12)

From the equation it follows that the value of the damping ratio ζ can be determined from the slope of a straight line of the module graph $|(W_g x)(a_0, b)|$ for scale factor a_0 corresponding to pulsation ω_n , drawn on a logarithmic scale.

The value of the ω_n pulsation may be determined in advance, using one of the methods used in practice (e.g. the method of spectral analysis of a vibration signal). Where ω_n pulse is not known, the scale factor a_0 is determined on the basis of the function $|(W_g x)(a, b)|$ (of a scalogram) analysis as the value, for which the wavelet transform module reaches the maximum value. The value of pulsation ω_n can then also be determined, using formula (4).

Given the assumption made at the outset, the response system with N degrees of freedom can be described as a superposition of response systems with a single degree of freedom, describing the given forms of natural vibrations:

$$h(t) = \sum_{i=1}^{N} A_i e^{-\zeta_i \omega_{n_i} t} \sin\left(\sqrt{1 - \zeta_i^2} \omega_{n_i} t + \varphi_i\right)$$
(13)

for i = 1, 2, ... N.

The linearity of the wavelet transform allows to treat the wavelet transform vibration signal as the sum of transforms signals corresponding to various forms of natural vibrations. The procedure for determining the parameters of the MDOF dumping model is reduced to the use of the procedure described above, in relation to SDOF systems, describing the given forms of natural vibrations.

3. Procedure validation

The described procedure is implemented in the form of the mfile into MATLAB. It has been tested if the program is operating correctly by a programmable simulation of vibration signals and by using sinusoidal mechanical signals.

For the purposes of the programmable simulation, files were generated, containing the samples of signals with preset values: frequency f and damping ratio ξ . An example of the time history of the test signal and a graph of wavelet coefficients modules for a preset scale value a_0 are shown in Figs. 3 and 4.



Fig. 3. Time history of a test signal

The result of the program processing was examined, for signals in the range of the expected vibration frequency f and damping ratio ξ during real construction tests. Table 1 shows an example of the tests' result. Errors connected with determining dynamic parameters of signals did not exceed 5%.



Fig. 4. Graph of wavelet coefficients modules for a preset scale value a_0

Tab. 1. Processing errors of test signals

	Test signal	parameters		Processia	ng results	
No.	f	ξ	f	δ	ξ	δ
	Hz	-	Hz	%	-	%
1	10	0.05	9.7177	-2.8	0.0507	1.4
2	20	0.05	19.4123	-2.9	0.0515	3.0
3	30	0.05	29.2571	-2.5	0.0513	2.6
4	40	0.05	38.8246	-2.9	0.0515	3.0
5	60	0.05	58.5143	-2.5	0.0513	2.6
6	80	0.05	76.5607	-4.3	0.0522	4.4
7	100	0.05	96.3765	-3.6	0.0519	3.8

The standard uncertainty of measurement resulting from the selected in the study resolution of wavelet analysis is about 0.5%. The results are satisfactory, taking into account that in the case of wavelet analysis, there is no close link between scale a and analyzed signal frequency f. Processing errors are a component of the uncertainty surrounding the results of measurements of the dynamic mechanical constructions.

In order to confirm (verify) the correctness of a signal processing in the entire measurement circuit, preliminary measurements of a mechanical signal were conducted. In order to do this, a steel plate was stimulated to vibrate by using an exciter. Plate surface vibration was measured with a laser vibrometer. The recorded measurement results in the form of a series of instantaneous values of the vibration signal, then processed, using this procedure. The study was conducted for sinusoidal vibrations of constant amplitude for the three frequencies: 30 Hz, 80 Hz and 120 Hz. It was checked for errors of determination (identification) of the plate vibration frequency. The test results are shown in Table 2.

Fab. 2. Frequency	v identification	errors of steel	plate vibration
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No.	Frequency of the plate vibration	Measured frequency	Error
	Hz	Hz	%
1	30	28.9266	-3.6
2	80	76.9925	-3.8
3	120	115.0562	-4.1

4. Implementation of tests and results

The subject of the study were the selected mechanical vibrations of structures, which were caused by impulse excitation of the structure to vibrate. The study was carried out on the basis of digital recordings of vibration signals.

Figure 5 shows a typical graph of time history of how a tested construction responded to force impulse. It has the form of a vibration of a decreasing, over time, amplitude. The rate of the decrease is the function of the scattering intensity of vibrations' energy, a quantitatively described value of the attenuation factor. In the laboratory, these signals were subjected to a wavelet analysis and then, processed further.



Fig. 5. Time history of the response of a mechanical construction to impulse force

The result of a signal wavelet analysis is a set of wavelet coefficients $(W_g x)(a, b)$, illustrated in the graph as a scalogram (Fig. 6). On its basis the value of scale *a* was determined, for which the largest value of a wavelet coefficient was achieved. For the presented registration it amounted to: a = 13.4.



Fig. 6. Scalogram of the impulse response of the tested object

Further, for the predetermined value a_0 , a graph $|(W_g x)(a_0, b)|$ was created (Fig. 7). It is a sectional view of the scalogram taken along the line parallel to the axis b and intersecting the scale axis at a_0 . This chart allows you to specify a range of values b for which a decline in the value of wavelet coefficients modules can be observed. In the example:

$$b_1 = 1350 < b < 1850 = b_2 \tag{14}$$



Fig. 7. Graph of wavelet coefficients modules for the specified scale value a_0

For a logarithmic measure of the ordinate, the graph section for the specified range of values *b* is approximately straight (Fig. 8).

Using the method of linear regression, for values *b* belonging to a specified interval $\langle b_1; b_2 \rangle$, this section is replaced with a straight segment (straight line on the graph). On the basis of the two values of parameter *b* from the interval $\langle b_1; b_2 \rangle$, and of the two values of wavelet coefficients modules, a damping coefficient is determined, according to the formula:



Fig. 8. Graph of wavelet coefficients modules for the specified scale value *a*₀ for the ordinate presented in logarithmic measure

$$\zeta = \frac{1}{2\pi(b_2 - b_1)f_n} \ln \left| \frac{(W_g x)(a_0, b_2)}{(W_g x)(a_0, b_1)} \right|$$
(15)

The procedure described above has been implemented into the program, performed using MATLAB software platform.

Below are examples of the results of dynamic parameter identification of two selected mechanical constructions, obtained by using the described procedure.

The first object was a simple mechanical construction in the form of a freely hanging steel plate with dimensions of 1000 mm \times 1000 mm \times 30 mm (Fig. 9.). The study was conducted in laboratory conditions.



Fig. 9. Hanging steel plate

The second object was an acoustic barrier located on expressway (Fig. 10).

The studied constructions were excited to vibrate with the help of hammers for modal testing. The smaller hammer type 8202 from the Brüel & Kjær company, weighing 280 g was used in laboratory tests while the larger hammer, type 086D50, from the PCB Piezotronics company, weighing 5.5 kg, in field research. For the measurement and recording of vibration signals was used a laser vibrometer type PSV-400 from POLITEC with an additional piezoelectric transducer of vibration type 393B12 from the PCB Piezotronics company. During the research horizontal components of vibrations were measured. An acceleration sensor was attached to the support structure of the acoustic barrier with the help of a magnetic holder.



Fig. 10. Acoustic barrier which is the subject of research

a) A steel plate test

Using laser vibrometer scanning properties, vibration measurement of the steel plate was performed at the nodes of a regular grid ($7 \times 7 = 49$ measuring points) covering the entire surface of the plate. Tens of vibration signals registration were taken. Preliminary identification of the vibration frequency of the metal plate, made on the basis of the review of the received scalograms, reveals several vibration frequencies associated with different modes of the plate vibrations. These frequencies are shown in the form of local maxima of the scalogram (Fig. 11).



Fig. 11. Scalogram of a vibrating steel plate for the impulse excitation

By properly adjusting the value range of scale a, a portion of the scalogram was chosen for the analysis. The analysis of all registrations were limited to two value ranges of scale a, for the largest local maxima of the scalogram. Table 3 shows the results of processing vibration signals. The modes of vibrations corresponding to the analyzed fragments of the scalogram were marked as Mode 1, Mode 2 and so on.

Tab. 3. Measurement results of for a steel plate

	Мо	de 1	Mode 2		
	f ξ		f	ξ	
	Hz	-	Hz	-	
The average value	32.6	0.0419	120.3	0.00934	
The standard deviation	3.54	0.0147	12.4	0.00383	
Number of measurements	1	8	2	4	
The standard uncertainty	0.834	0.00347	2.53	0.00078	
The relative standard uncertainty, %	2.6	8.3	2.1	8.4	

b) Acoustic barrier test

As in case of a steel plate, the vibration measurements of the acoustic barrier were made at the nodes of a regular grid ($9 \times 9 = 81$ measuring points) consisting of a large part of the surface of an acoustic barrier that is mounted between steel columns of the supporting structure. Tens of vibration signal registrations were taken. Based on the analysis of the scalograms, ranges of scale *a* were identified, involving local maxima of the scalograms (Fig. 12).

Additionally, the vibrations of the barrier steel structure were tested, using a vibration acceleration transducer. Vibration measurement was performed at one measuring point. There were taken 30 registrations of signals of the steel construction responses to impulse excitation. Tables 4 and 5 show the obtained results.



Fig. 12. Scalogram of the vibrating surface of the barrier to impulse excitation

Tab.	4.	Measurement	results	of the	acoustic	barrier si	urface
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	Mo	Mode 1		de 2	Mode 3	
	f ξ		f	٤	f	ک
	Hz	-	Hz	-	Hz	-
The average value	7.547	0.0701	21.997	0.0223	24.384	0.0224
The standard deviation	0.865	0.0062	1.145	0.0040	0.724	0.0057
Number of measurements	23	3	23	3	34	Ļ
The standard uncertainty	0.180	0.0129	0.2387	0.00083	0.124	0.0010
The relative standard uncertainty, %	2.4	18.5	1.1	3.7	0.51	4.4

Tab. 5. Measurement results of the acoustic barrier steel construction

	Moo	de 1	Mo	de 2
	f	٤	f	ξ
	Hz	-	Hz	-
The average value	27.770	0.0289	57.383	0.0265
The standard deviation	1.799	0.00208	1.817	0.00439
Number of measurements	3	4	1	3
The standard uncertainty	0.309	0.00036	0.504	0.00122
The relative standard uncertainty %	1.1	1.2	0.9	4.6
	Moo	de 3	Mo	de 4
	f	ξ	f	ئې
	Hz	-	Hz	-
The average value	75.928	0.0212	98.034	0.0224
The standard deviation	6.459	0.00502	2.60	0.00167
Number of measurements	1	7	4	
The standard uncertainty	1.57	0.001	1.30	0.00084
The relative standard uncertainty, %	2.1	5.7	1.3	3.7

The results of the statistical analysis of a series of the measurements are an important contribution to uncertainty estimation of the measurement results of dynamic parameters of a mechanical construction, made by the method described. The results obtained show, for a small dispersion thereof, in the identification vibration frequency. Accidental impact of various disturbing factors occurring in practice, points to the need to perform a series of measurements and their statistical analysis.

5. Verification of test results

To verify the obtained results, comparative studies were conducted, with the results obtained using a different method. In this study, as the reference method, a method for modal analysis of vibration was used, implemented in the Test.Lab program from the LMS company.

The study was performed on the previously described two objects. The results of measurements obtained with the help of the two methods are summarized in Tables 6 - 9. For each pair of the results compatibility factor was calculated, given by the formula:

$$Z = \frac{|x_1 - x_{REF}|}{\sqrt{u_{x1}^2 + u_{REF}^2}}$$
(16)

where: x_1 -measurement result obtained by using the wavelet analysis, x_{REF} -measurement result obtained by the reference method (modal analysis), u_{x1} -standard uncertainty of the result in the wavelet analysis method, u_{REF} - standard uncertainty of the result in the reference method.

The results are considered to be satisfactory if value $Z \le 2$. If 2 < Z < 3, the result of the comparison is considered to be doubtful.

The estimated, standard combined uncertainty of the measurements for both methods is given in the tables with the results of the comparison.

Tab. 6. The results of the comparison of frequency identification for the steel plate

	Wavelet analysis method			Modal a				
No.	Mode	f_{I}	u_{fI}	u_{fI}	F	u_{REF}	<i>u_{REF}</i>	Ζ
		Hz	%	Hz	Hz	%	Hz	-
1	1	32.6	5.0	1.6	36.6	4.0	1.5	1.8
2	4							
3	5	120.3	5.0	6.0	118.0	4.0	4.7	0.30

Tab. 7. The results of the comparison of the damping ratio identification for the steel plate

	Wavelet analysis method		Modal a					
No.	Mode	ξι	$u_{\xi I}$	$u_{\xi I}$	ξ ref	u_{REF}	u _{REF}	Ζ
		-	%	-	-	%	-	-
1	1	0.0419	20.0	0.0084	0.0130	20.0	0.0026	3.3
2	4							
3	5	0.00934	15.0	0.0014	0.00900	15.0	0.0014	0.17

Tab. 8. The results of the comparison of the frequency identification for the acoustic barrier

		Wavelet	analysis	s method	Modal a	nalysis 1	method	
No.	Mode	f_I	u_{fI}	u _{f1}	F	u_{REF}	u_{REF}	Z
		Hz	%	Hz	Hz	%	Hz	-
1	1	7.547	5.0	0.4	7.042	4.0	0.28	1.1
2	4	21.997	5.0	1.1	21.363	4.0	0.85	0.46
3	5	24.384	5.0	1.2	25.717	4.0	1.0	0.84

	Wavele		elet analysis method		Modal a			
No.	Mode	ξı	$u_{\xi I}$	$u_{\xi I}$	ξ _{REF}	u_{REF}	u_{REF}	Ζ
		-	%	-	-	%	-	-
1	1	0.0701	20.0	0.014	0.0266	15.0	0.0040	2.98
2	4	0.0223	20.0	0.0045	0.0132	15.0	0.0020	1.86
3	5	0.0224	20.0	0.0045	0.00953	15.0	0.0014	2.74

Tab. 9. The results of the comparison of the damping ratio identification of the acoustic barrier

In all cases, comparative studies confirmed the consistency of the results obtained by both identification methods of vibration frequency.

Comparison of the damping ratio identification results appeared to be worse. In two cases, the consistency of results was obtained, in two others the result of the comparison is questionable, and in one case, the results did not meet the consistency criteria.

The research highlighted the difficulties involved in carrying out a research of real objects. Because, in the light of the results of validation procedures to identify the damping ratio, signal processing method does not raise any objections, the reason for the inconsistence of the results of the comparison was, according to the authors, a low quality of the input data given to processing. Poor signal quality is understood as a small signal to noise ratio. In this case, there are some difficulties in accurate determination of the range of a delay value *b*, in which the function $\ln|(W_g x)(a_0, b)|$ must be approximated by a straight line (Fig. 13). The research found out that a change in the value range *b* by 5% can result, for certain registrations, in a change of the damping ratio value of over 10%.



Fig. 13. Diagram for a noisy signal

Possibly there are two reasons for this state of affairs. Vibration measurements were performed at many points of a vibrating surface, covering its large surface. Not for all forms of vibration points were appropriate. The lack of repeatability of the excitation signal should also be taken into account.



Fig. 14. Time history of the disturbed registration of a vibration signal

The research of the acoustic barrier took place in traffic on a busy road. This traffic caused, in some (frequent) cases, the vibration signal, after an impulse excitation, not to fade but to remain at a level which significantly interfered with the recorded signal and the result of its processing (Fig. 14). This effect was particularly evident for vibrations of the lowest, identified frequency.

6. Conclusions

- The research, which utilizes the presented procedure, has a distinctive character with regard to technical problems related to the application of this procedure in practice.
- It has been observed that there is a considerable spread of the damping ratio obtained by repeated dynamic excitations, which indicates a difficulty in meeting the repeatability of the test conditions in real conditions. The results indicate that in order to identify dynamic parameters of the object model, it is necessary to perform a series of measurements and a statistical analysis of the results.
- The results were verified by performing a comparative study with the results obtained by using the method of modal analysis of vibration signals. The results of the comparison are not satisfactory with respect to the estimate of the damping ratio. Difficulties which have been pointed out, associated with the implementation of the method in practice, in some cases may explain the lack of a positive result of the comparison.

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Cezary BARTMAŃSKI, DSc, eng.

Graduated from Warsaw University of Technology. Research associate in the Department of Technical Acoustics and Radiometry of Central Mining Institute in Katowice. Scientific activity - study of mechanical vibrations and noise in the environment and digital signal processing methods. The subject of special scientific interest is wavelet analysis of vibroacoustic signals and its use in technical diagnostics.



e-mail: cbartmanski@gig.eu

Andrzej STANIEK, PhD

Graduated from Silesian Technical University in 1986. PhD at Central Mining Institute in 2004. Fields of research: modal analysis of mechanical structures, NDT of rock bolt support system, buildings damage detection, metrology, environmental testing. Hobby: climbing, skiing, swimming.



e-mail: astaniek@gig.eu