THEORETICAL PROBLEMS OF THE WIDEBAND SONAR TIME-SPATIAL FILTRATION

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The paper presents the synthesis of the time-spatial matched filtration algorithm. First part specifies base for the algorithm-input data. Successively, the matched filtration for continuous function is depicted. This leads to the canonical form of the time-spatial filter. Consecutively, the decomposition into series connection of the matched and spatial filtration is realized. The next step is the signals approximation by Fourier series due to equivalence between approximation and interpolation on uniformly distributed discrete samples. The equation of the spatial filtration - in frequency domain - results from this mathematical operations. Application of the Bluestein's method to solve this equation provides to the final algorithm version. Advantages of the developed algorithm are presented in conclusions.

INTRODUCTION

The present technology in signals processing field makes possible matched filtration realization for multi-element sonar transducer and different types of the impulse modulation. Application of the effective signal processing algorithms provides to the minimization of the equipment cost and dimensions. Adaptation of the general filtration theory to the sonar signal processing methods requires properly signals sampling and their interpolation by complex Fourier series. Such approach allows derivation of the continuous dependences in limited frequency band in accordance with Nyquist's criterion.

1. FORM OF THE INPUT STREAM

Stream transmitted from transducer elements to the signal processing unit has the form of the snapshot sequences received after I/Q detection. To solve the problem the following assumptions are made: echo signal delivered from the I/Q detectors has a complex form *echo(x,t);* where: x is the distance from transducer central element and t is the time. In this and all beneath expressions relative units are used:

- distance is noticed in quantity d (distance between adjacent transducer elements),
- time is noticed in quantity t_s (the sampling period).

In case of point source without noise the signal expression is defined as follow:

$$echo(x,t) = h(t - t_0 + \tau')e^{2\pi j \frac{fg}{fs}(t - t_0 + \tau')} = h(t - t_1)e^{2\pi j \frac{fg}{fs}(t - t_0 + \tau')} \tag{1}$$

where: t_0 is the time delay at the transducer central element $x=0$; $\tau'=\tau'(r,x)$ is the additional time delay at optional transducer element positioned with the distance x from the transducer center and for r direction of arrival; f_g is the signal central frequency; f_s is the sampling frequency and $h(t)$ is the complex signal envelope.

The detection consists in multiplication of the complex echo signal by reference generator signal of frequency f_g. The signal $SIQ(x,t)$ is obtained at the output of the I/Q detectors:

$$SIQ(x,t) = echo(x,t) \cdot e^{-j\varphi_g} e^{-2\pi j \frac{fg}{fs} t} \tag{2}$$

where: φ_g is the random initial I/Q generator phase. Especially, for point source without noise the expression is as follow:

$$SIQ(x,t) = h(t - t_1) \cdot e^{-j\varphi_g - 2\pi j \frac{fg}{fs} t_0} e^{+2\pi j \frac{fg}{fs} \tau'} = h(t - t_1) \cdot e^{-j\varphi_0} e^{+2\pi j \frac{fg}{fs} \tau'} \tag{3}$$

where: φ_0 is the random initial echo signal phase.

The assumption concerning complex signal envelope $h(t)$ is following: 0 for $t<0$ and $t>M$; where M is the impulse length (in t_s unit). The principle of the assumption is illustrated on the figure 1 below.

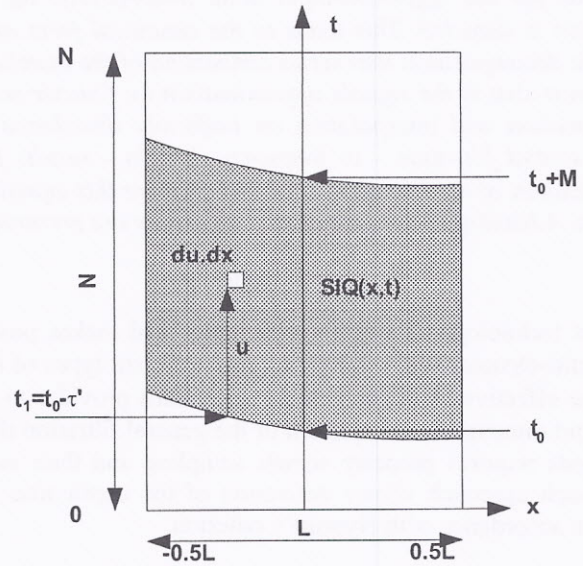

Fig. 1

Additionally, the figure 1 presents the stream segment of N time duration incident at the transducer of L length (in d units). Gray area designates fragment of $SIQ(x,t \neq 0$. Rectangle of dimensions $N \times L$ being the fragment of the input stream is called the segment.

The filtration algorithm consist in matching the pattern for each chosen directions of arrival to the $SIQ(x,t)$ signal for each segment of the received data stream.

2. PROBLEM DEFINITION CANONICAL EQUATION OF THE PROCESSED SEGMENT

Matched filtration requires application of the (4) expression. Each point of the segment with its neighbourhood *dx,dt (where dt=du)* represents complex number of normalized amplitude and phase. They are determined by argument of the complex function envelope *h(t)* and distribution along transducer axis proportionally to the τ' delay. Phase change compensation is realized as the signal multiplication by its replica of parameters (t,τ). Next the summation of obtained results is done:

$$B(r,t) = \int_{-L/2}^{L/2} \int_{0}^{M} \left(SIQ(x,t-\tau-M+u) \cdot h^{*}(u) \cdot e^{-2\pi \cdot j \cdot \frac{fg}{fs} \cdot \tau} \right) dx \cdot du \tag{4}$$

This expression is called the canonical form of the time-spatial matched filtration. When the signal is matched to its pattern (i.e. $t=t_0+M$, $\tau=\tau'$) all phases are fitted and equal φ_0.

$$B(r',t_0+M) = \int_{-L/2}^{L/2} \int_{0}^{M} \left(h(u)h^{*}(u) \cdot e^{-j\varphi_0} \cdot e^{+2\pi \cdot j \cdot \frac{fg}{fs} \cdot \tau'} \cdot e^{-2\pi \cdot j \cdot \frac{fg}{fs} \cdot \tau'} \right) dx \cdot du = M \cdot L \cdot e^{-j\varphi_0} \tag{5}$$

The module of the (5) expression equals signal energy.

3. PROBLEMS DECOMPOSITION - FILTER MATCHED IN TIME DOMAIN AND FILTER MATCHED IN SPATIAL DOMAIN

Canonical equation can be parted into two serially computed algorithms:

- Matched filtration to *t* axis,
- Matched filtration to *x* axis.

The first one is defined by expression:

$$FIR(x,t) = \int_{0}^{M} SIQ(x,t-M+u) \cdot h^{*}(u) du \tag{6}$$

Presently, the (7) expression can be obtained from canonical equation of the time-spatial matched filtration:

$$B(r,t) = \int_{-L/2}^{L/2} \left(FIR(x,t-\tau) \cdot e^{-2\pi \cdot j \cdot \frac{fg}{fs} \cdot \tau} \right) dx \tag{7}$$

This dependence is defined as the canonical form of the spatial filtration. Scheme of the presented hereabove algorithm is shown on the figure 2.

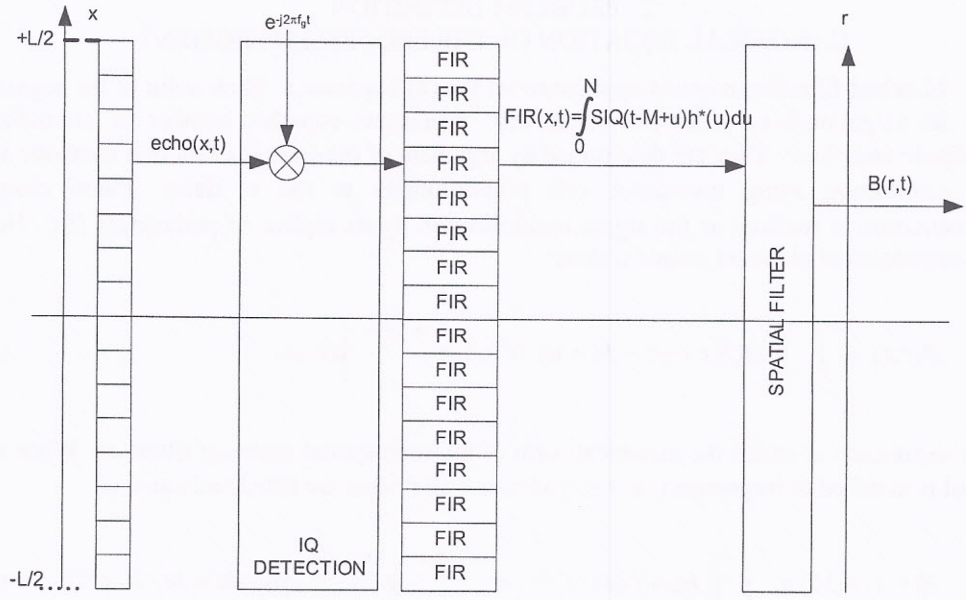

Fig. 2

Both discrete x and t can be treated as continuous variables coming from signal interpolation by Fourier series. Correct reconstruction of the signal can be achieved if Nyquist's criterion is respected. Results of LFM (Linear Frequency Modulation) and HFM (Hyperbolic Frequency Modulation) signals sampling are presented on the figure 3. The frequency band is 75% of the sampling frequency for both illustrated cases.

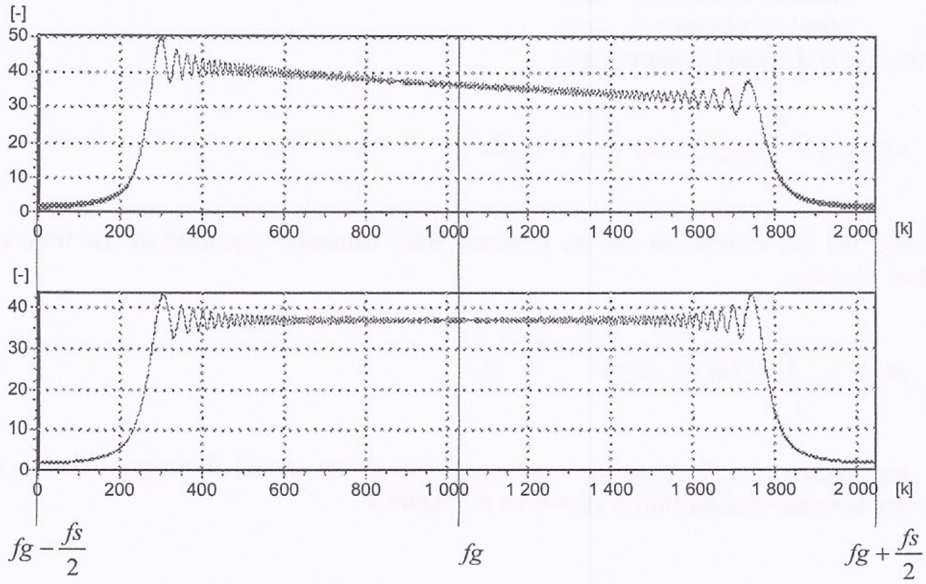

Fig. 3 LFM and HFM signal sampling.

4. SIGNAL APPROXIMATION BY FOURIER SERIES

The input signal can be expanded into Fourier series due to knowledge on *SIQ(x,t)* signal at *t* discrete values in segment of *N* duration and assumption *SIQ(x,t+N)=SIQ(x,t)*. Analogous method is used in case of complex envelope. It provides to following expressions:

$$SIQ(x,t) \approx \frac{1}{N}\sum_{k=0}^{N-1}\left(X(x)[k]\cdot e^{+2\pi\cdot j\cdot\frac{k\cdot t}{N}} \right)$$
$$h^*(t) \approx \frac{1}{N}\sum_{k=0}^{N-1}\left(H^*[k]\cdot e^{-2\pi\cdot j\cdot\frac{k\cdot t}{N}} \right) \tag{8}$$

The approximation sign designates rejection of the high frequency components. The square brackets denote the discrete set and the round brackets denote continuous set. Application of the described hereabove transformations leads to following expression of the matched filtration:

$$FIR(x,t) \approx \frac{1}{N}\sum_{k=0}^{N-1}\left(X(x)[k]H^*[k]\cdot e^{+2\pi\cdot j\cdot\frac{k\cdot t}{N}} \right) \tag{9}$$

Introducing the (9) dependence into canonical equation of the spatial filtration gives expression:

$$B(r,t) = \frac{1}{N}\sum_{k=0}^{N-1}\left(e^{+2\pi\cdot j\cdot\frac{k\cdot t}{N}}\cdot \int_{-L/2}^{L/2}\left(H^*[k]\cdot X(x)[k]\cdot e^{-2\pi\cdot j\cdot\frac{k}{N}\tau}\cdot e^{-2\pi\cdot j\cdot\frac{fg}{fs}\tau} \right)dx \right) \tag{10}$$

This expression is the Fourier series expansion of time-spatial filtration output. The part including integral is the generator of the Fourier series coefficients:

$$Y(r)[k] = \int_{-L/2}^{L/2}\left(H^*[k]\cdot X(x)[k]\cdot e^{-2\pi\cdot j\cdot\left(\frac{k}{N}+\frac{fg}{fs}\right)\cdot\tau} \right)dx \tag{11}$$

This dependence is called as the equation of the spatial filtration in frequency domain. The fundamental numerical problem of the time-spatial filtration is effective calculation of (11) integral.

5. BLUESTEIN METHOD APPLICATION INTO SPATIAL FILTRATION

Bluestein method suggests decomposition of τ delays into three parts:

$$\tau = \tau(x,r) \approx \tau i(x) + \tau c(x+r) + \tau o(r) \tag{12}$$

In this case, the equation of the spatial filtration in frequency domain is defined as follow:

$$Y(r)[k] = e^{-2\pi\cdot j\cdot\left(\frac{k}{N}+\frac{fg}{fs}\right)\cdot\tau o(r)} \int_{-L/2}^{L/2}\left(X(x)[k]H^*[k] e^{-2\pi\cdot j\cdot\left(\frac{k}{N}+\frac{fg}{fs}\right)\cdot\tau i(x)}\cdot e^{-2\pi\cdot j\cdot\left(\frac{k}{N}+\frac{fg}{fs}\right)\cdot\tau c(x+r)} \right)dx \tag{13}$$

The following denotations are introduced:

$$\xi(x)[k] = X(x)[k]H^*[k] e^{-2\pi\cdot j\cdot\left(\frac{k}{N}+\frac{fg}{fs}\right)\cdot\tau i(x)} = X(x)[k]\cdot W1(x)[k]$$
$$W(x)[k] = e^{-2\pi\cdot j\cdot\left(\frac{k}{N}+\frac{fg}{fs}\right)\cdot\tau c(x)} \tag{14}$$
$$W3(x)[k] = e^{-2\pi\cdot j\cdot\left(\frac{k}{N}+\frac{fg}{fs}\right)\cdot\tau o(x)}$$

Presently the integral can be rewritten as:

$$Y(r)[k] = W3[r] \int_{-L/2}^{L/2} \xi(x)[k] W(x+r)[k] dx \tag{15}$$

Next step is approximation of the integrand functions by trigonometric series of period $LE \geq L+K$; where: K is the number of the defined directions of arrival. The following expansions of the integrand function are the results of this operation:

$$\xi(x)[k] = \frac{1}{LE} \sum_{y=-LE/2}^{LE/2-1} \xi E[y,k] e^{-2\pi j \frac{y \cdot x}{LE}}$$
$$W(x)[k] = \frac{1}{LE} \sum_{y=-LE/2}^{LE/2-1} W2[y,k] e^{+2\pi j \frac{y \cdot x}{LE}} \tag{16}$$

Finally the equation of the spatial filtration in frequency domain is defined by formula:

$$Y(r)[k] = \frac{W3(r)[k]}{LE} \sum_{y=-LE/2}^{LE/2-1} \xi E[y,k] W2[y,k] e^{+2\pi j \frac{y \cdot r}{LE}} \tag{17}$$

Expansion limits of integration results from *ξ(x)[k]* function equality to *0* value outside range *(-L/2 , L/2)*.

As mentioned above, the equation of the spatial filtration in frequency domain provides Fourier series coefficients of the output signal. The IDFT should be applied to acquire the signal values in discrete interpolation points. The scheme of the developed algorithm is presented on the figure 4.

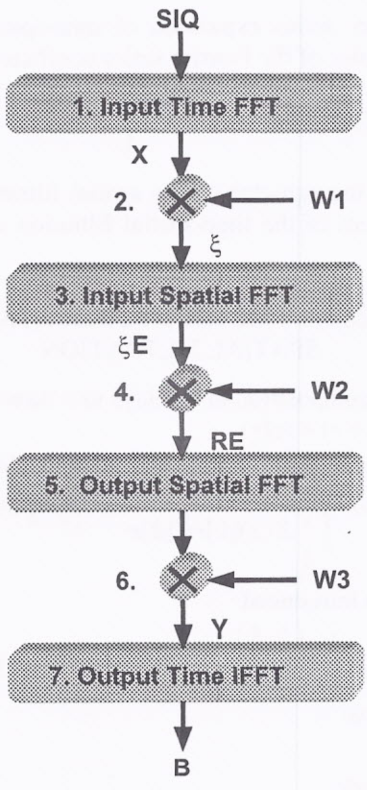

Fig. 4

The time FFT calculate $X[x,k]$ coefficients for each transducer element. The FFT algorithm required N samples being the power of 2. These values are next multiplied by $W1[x,k]$ weighting coefficients. Received ξ segment of $N \times L$ size increases into $N \times LE$ size by zero's padding in order to obtain LE value being the power of 2 as well. On the basis of known $\xi[x,k]$ points $\xi(x)[k]$ function is calculated. Spatial FFT at the input interpolates $\xi E(x)[k]$ function using known $\xi[x,k]$ points. Multiplication by W2 weighting coefficients, spatial FFT and the third multiplication by W3 weighting coefficient deliver the coefficients to output signal approximation by Fourier series. Successively, the application of the time IFFT at the output enables acquisition discrete values of the signal.

6. THE ALGORITHM EXAMPLE FOR FAR FIELD

The method accuracy depends on described hereabove $\tau(x,t)$ function decomposition. In case of far field the following dependence is valid:

$$\tau = fs \cdot x \cdot d \cdot \sin\Theta_r / c \tag{18}$$

where: Θ_r is the direction of arrival, d is the distance between adjacent transducer elements and c is the sound velocity in water medium. To express the delay as a linear function of direction the following assumption is made:

$$\sin\Theta_r = r \cdot \sin\Theta_1 \tag{19}$$

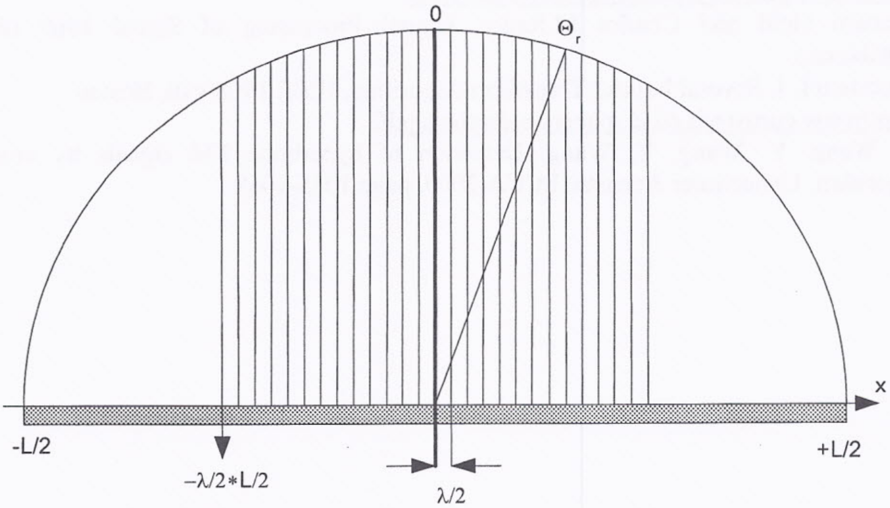

Fig. 5

The value of the elementary Θ_1 angle is defined as:

$$\sin\Theta_1 = \frac{\lambda}{L} = \frac{\left(\frac{c}{fg}\right)/d}{L} \tag{20}$$

This provides to following expression of τ:

$$\tau = \tau(r,x) = \frac{fs}{fg} \cdot \frac{x \cdot r}{L} = \frac{fs}{fg} \cdot \frac{1}{2 \cdot L} \cdot \left(-x^2 + (x+r)^2 - r^2\right) \tag{21}$$

Application of the (21) identity provides to following τ terms in Bluestein method:

$$\tau i(x) = \tau o(x) = -\frac{fs}{2 \cdot fg \cdot L} x^2$$
$$\tau c(x) = +\frac{fs}{2 \cdot fg \cdot L} x^2 \qquad (22)$$

In case of far field the W1, W2 and W3 weighting coefficients are calculated precisely.

CONCLUSIONS

Presented algorithm has various possibilities of matching to any type of signal pattern, both in time and spatial domain. Te filter characteristic change is realized by modification of the W1, W2 and W3 weighting coefficient set. The algorithm operates correctly both in case of near and far field and for different signal modulation types. The time filtration can be matched to the signal and target as well [1]. Algorithm consistently makes use of FFT, what reduces computing power consumption.

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