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AN ESTIMATION METHOD OF TRAFFIC SAFETY LEVEL OF VEHICLES ON A LEVEL CROSSING

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Abstract

Let us consider a level crossing as a technical object with a certain level of safety. In the paper we propose a new method based on the theory of geometric programming. It allows to solve the problem of minimizing common risk of the object safety violation in a simple analytic form due to the choice of object protection parameter set. There is given a numerical example illustrating the calculated scheme of the method.

1. Introduction

It is known that a large proportion of traffic incidents is committed on level crossing. Thus a task of ensuring safety on level crossing is relevant. Here we consider a simplified formulation and solution of the task, since the total volume of its solution is a problem and it is beyond the scope of this study.

2. Main results

In this paper the traffic on the level crossing is considered as a complex object with a certain level of safety. Consider the following safety threats:
U_1 – drive over level crossing at red traffic light by drivers of a group I;
U_2 – drive over level crossing at red traffic light by drivers of a group II;
U_3 – a vehicles collision on the level crossing that does not stop on the tracks;
U_3 – collision with a train and other traffic incidents, which lead to a stop of transport on the tracks.

The group II consists of a car thieves, a drunk drivers, pursued criminals and other persons whose contact with the police is tantamount to arrest them. The group I consists of violators which do not belong to the group

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II. Common threat U of object safety consists of at least one of the threat U_1, U_2, U_3, U_4: $U = \bigcup U_i$.

In fact, the number of threats is much more than $n = 4$. But to keep things simple we'll assume $n = 4$.This is sufficient to illustrate the proposed method of estimating and minimizing the common risk of the object safety violation.

Suppose that the events U_i are independent, $i = 1, 2, 3, 4$, and the probability $y = P(U)$ (common risk of the object safety violation) is expressed as a sum of particular risks u_i :

$$y = u_1 + u_2 + u_3 + u_4, \tag{1}$$

where $u_1 = P(U_1), u_2 = P(U_2)(1 - u_1), u_3 = P(U_3)(1 - u_1)(1 - u_2), u_4 = P(U_4)(1 - u_1)(1 - u_2)(1 - u_3)$.

In addition,

$$u_1 = P(U_1) \approx \frac{M_1}{N_1}.$$

The fraction $\frac{M_1}{N_1}$ is an estimate of the particular risk u_1, where N_1 is total number of vehicles which passed through the crossing for time T (let us say $T = 1$ day) and M_1 is number of drivers of the group I which passed through the crossing for time T at red traffic lights.

Similarly, we estimate other particular risks, for example,

$$u_2 = P(U_2)(1 - u_1) \approx \frac{M_2}{N_2}.$$

The fraction $\frac{M_2}{N_2}$ is an estimate of the particular risk u_2, where N_2 is total number of vehicles which passed through the level crossing for time T without violators of the group I. Number M_2 is number of drivers of the group II which passed through the level crossing for time T at red traffic lights.

The object in question has safety protection system. This system includes signal operator, road inspectors, technical means of preventing violations such as barriers, remote control system barriers etc.

Protection system gives the following parameters:
x_1 – time of duty by road inspectors;
x_2 — time between duty;
x_3 – average time between the opening and closing of the barriers.

In fact, the number of parameters x_i is much more than $m = 3$. But this is enough to illustrate the estimation method of minimizing the common risk of the object safety violation.

A Table below is a fragment of empirical data for 10 observations, which we used for the calculations. Each row in the Table corresponds to time T=1 day for situation on the level crossing.

Probability of threats U_i (multiplied by 10^3)				Protection parameters (in hours)		
u_1	u_2	u_3	u_4	x_1	x_2	x_3
2.1	0.20	5.8	0.51	2	4	0.25
0.42	0.53	1.09	0.20	2.5	3	0.20
0.75	0.29	3.00	0.63	3	5	0.25
2.0	0.52	0.17	0.20	1.5	2	0.10
5.9	0.16	3.80	0.43	1.3	3	0.20
1.8	0.35	1.12	0.12	1.3	2	0.20
0.30	0.53	3.08	0.52	4	5	0.25
2.8	0.23	1.28	0.40	1.7	3	0.15
4.9	0.13	7.50	1.22	2	5	0.20
3.3	0.20	1.44	2.8	3	6	0.10

These data are mainly expert evaluation of road inspectors, ambulance workers and staff which services the technical means of preventing violations. Following our paper [1], we assume that the vector $x = (x_1, x_2, x_3)$ of protection parameters is positive.

Let $u_i = u_i(x)$ be a polynomial

(2) $$u_i = u_i(x) = C_i \cdot \prod_{j=1}^{3} x_j^{a_{ij}}, \quad C_i > 0, \quad i = 1, 2, 3.$$

A matrix $A = (a_{ij})$ is called an exponent matrix.

Taking the logarithm of both side of (2), we obtain

(3) $$\ln u_i(x) = a_{i0} + a_{i1} \ln x_1 + a_{i2} \ln x_2 + a_{i3} \ln x_3, \quad i = 1, 2, 3, 4, \quad C_i = e^{a_{i0}}$$

Thus, we can get coefficients a_{ij} by methods of linear regression analysis [2]. Writing the equation (3) for the first row of the Table, we obtain for the risk u_1:

(4) $$\ln 2,1 = a_{10} + a_{11} \ln 2 + a_{12} \ln 4 + a_{13} \ln 0,25,$$

or

$$a_{10} + 0,693 a_{11} + 1,386 a_{12} - 1,386 a_{13} = 0,742$$

Similarly, writing the equation (3) for next tows of the Table we obtain an algebraic system

(5) $$F a^1 = w^1,$$

where $a^1 = (a_{10} \quad a_{11} \quad a_{12} \quad a_{13})^T$ is a column of vector of required coefficients in the polynomial $u_1(x) = C_1 \cdot \prod_{j=1}^{3} x_j^{a_{1j}}, \quad C_1 = e^{a_{10}}$ and the matrix F is expressed as $F = (\mathbf{1}, \ln x^1, \ln x^2, \ln x^3)$.

Moreover, $\mathbf{1}$ is a column of ones; $\ln x^j$ j=1,2,3, is a column for values $\ln x_{js}$ of the $\ln x_j$ for the parameter x_j and 10 observations $s = 1, 2, \ldots, 10$; w^1 is a column for values $\ln u_{is}(x)$ of the $\ln u_i(x)$;

$$w^1 = (\ln 2.1 \quad \ln 0.42 \ldots \quad \ln 3.3)^T.$$

According to the method of least squares (MLS) the solution $a^i = \dot{a}^i$ of the equation (5) is given in the form [2]:

$$\dot{a}^1 = F^+ w^1 \tag{6}$$

Here F^+ is so called pseudo-inverse matrix of the matrix F. Calculation method for the matrix F^+ is given in the paper [4]. Recall that the pseudo-inverse is defined and unique for all matrices whose entries are real or complex numbers. Vector $\dot{a}^i$ is a solution of equation (5) under the condition that the equation is compatibility. In the converse case, $\dot{a}^i$ is the best approximation solution (according to the MLS).

Thus,

$$\dot{a}^1 = F^+ w^1 = \begin{pmatrix} a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \end{pmatrix} = \begin{pmatrix} -2.08 \\ -4 \\ 3 \\ -1 \end{pmatrix}, \quad C_1 = e^{-2.08} = 0,125$$

and the polynomial u_1 is expressed as

$$u_1 = u_1(x) = 0,125 x_1^{-4} x_2^3 x_3^{-1}$$

Using the given calculate scheme for the risks u_2, u_3, u_4, we obtain

$$u_2 = u_2(x) = 0,8 x_1^2 x_2^{-2},$$
$$u_3 = u_3(x) = 6 x_1^{-2} x_2^3 x_3^2, u_4 = u_4(x) = 0,004 x_1^{-1} x_2^3 x_3^{-1}.$$

Thus, the common risk y at the interval [0,T] of time (T equals 1 day) is expressed as

$$y = f(x) = 0,125 x_1^{-4} x_2^3 x_3^{-1} + 0,8 x_1^2 x_2^{-2} + 6 x_1^{-2} x_2^3 x_3^2 + 0,004 x_1^{-1} x_2^3 x_3^{-1}$$

Coefficient of variation $\dot{V}$ is used for precision and sufficiency to empirical data:

$$\dot{V} = \frac{\dot{\sigma}}{\dot{y}} 100\%$$

Here

$$\dot{y} = \dot{u_1} + \dot{u_2} + \dot{u_3} + \dot{u_4}$$

Moreover $\dot{u_i}$, i=1,2,3,4, is a sample mean of observations u_{is}, s=1,2,...10; $\dot{\sigma}^2$ is the sum of the sample variance $\dot{\sigma_i}^2$:

$$\dot{\sigma_i}^2 = \| F\dot{a^i} - w^i \|^2 /(N - m - 1),$$

where N is the total number of observations, but m is the number of protection parameters (m=3); $\| \cdot \|$ means an euclidean norm of vector; $\dot{\sigma_i}^2$ is the sample variance according to the MLS-solutions

$$\dot{a^i} = F^+ w^i$$

for the algebraic system

$$Fa^i = w^i, \quad i = 1, 2, 3, 4.$$

In our case,

$$\dot{\sigma} = \sqrt{\dot{\sigma_1}^2 + \dot{\sigma_2}^2 + \dot{\sigma_3}^2 + \dot{\sigma_4}^2},$$

We obtained $\dot{V} = 9\%$ as a result of data processing for N=100. This result gives acceptable discrepancy between the experimental and calculated data [5].

It should be found the vector $x = x_* > 0$, with components x_{j^*} such that value $y_* = f(x_*)$ is minimal. Using exponents a_{ij}, we can write the exponent matrix A as

$$A = (a_{ij}) = \begin{pmatrix} B \\ H \end{pmatrix} = \begin{pmatrix} -4 & 3 & -1 \\ 2 & -2 & 0 \\ -2 & 3 & 2 \\ -1 & 3 & -1 \end{pmatrix},$$

where sub-matrices

$$B = \begin{pmatrix} -4 & 3 & -1 \\ 2 & -2 & 0 \\ -2 & 3 & 2 \end{pmatrix}, \quad H = (-1 \quad 3 \quad -1).$$

Note that $detB \neq 0$. It follows that exist an inverse matrix B^{-1} :

$$B^{-1} = \begin{pmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{pmatrix} = \begin{pmatrix} -2 & -\frac{9}{2} & -1 \\ -2 & -5 & -1 \\ 1 & 3 & 1 \end{pmatrix}.$$

In our case sub-matrix $H = (-1 \quad 3 \quad -1)$ contains one row of matrix A, which do not belong to the sub-matrix B. Hence the example has the first difficulty level (see [1]).

Using the formulas from [1], we get subsidiary variables δ_i. These ones are called dual variables and are found by the formula

$$\delta^T = (\delta_1 \quad \delta_2 \quad \delta_3 \quad \delta_4) = \frac{1}{\mu}\left(-H \cdot B^{-1}, 1\right) = \frac{1}{\mu}\begin{pmatrix} 5 & \frac{27}{2} & 3 & 1 \end{pmatrix},$$

where the number

$$\mu = 5 + \frac{27}{2} + 3 + 1 = \frac{45}{2}$$

Therefore

$$\delta^T = (\delta_1 \quad \delta_2 \quad \delta_3 \quad \delta_4) = \frac{1}{45}(10 \quad 27 \quad 6 \quad 2).$$

Thus,

$$\delta_1 = \frac{10}{45}, \quad \delta_2 = \frac{27}{45}, \quad \delta_3 = \frac{6}{45}, \quad \delta_4 = \frac{2}{45}.$$

Using the formulas of the paper [3], we can write the minimal value y_* multiplied by 10^3 of the common risk y due to

$$C_1 = 0.125,\ C_2 = 0.8,\ C_3 = 6.0,\ C_4 = 0.004.$$

In our case

$$(7) \quad y_* = \prod_{i=1}^{4} \left(\frac{C_i}{\delta_i}\right)^{\delta_i} = \left(\frac{0.125}{10}\right)^{\frac{10}{45}} \left(\frac{0.8}{27}\right)^{\frac{27}{45}} \left(\frac{6}{6}\right)^{\frac{6}{45}} \left(\frac{0.004}{2}\right)^{\frac{2}{45}} = 1.55,$$

i.e. minimal value of the common risk is 0,155%.

Then the protection parameters x_{j^*} , j=1,2,3, can be found from the relationships

$$x_{1*} = \prod_{i=1}^{3} \left(\frac{\delta_i \cdot y_*}{C_i}\right)^{k_{1i}} = \left(\frac{10 \cdot 1.55}{45 \cdot 0.125}\right)^{-2} \left(\frac{27 \cdot 1.55}{45 \cdot 0.8}\right)^{-\frac{9}{2}} \left(\frac{6 \cdot 1.55}{45 \cdot 6}\right)^{-1},$$

$$x_{2*} = \prod_{i=1}^{3} \left(\frac{\delta_i \cdot y_*}{C_i}\right)^{k_{2i}} = \left(\frac{10 \cdot 1.55}{45 \cdot 0.125}\right)^{-2} \left(\frac{27 \cdot 1.55}{45 \cdot 0.8}\right)^{-5} \left(\frac{6 \cdot 1.55}{45 \cdot 6}\right)^{-1},$$

$$x_{3*} = \prod_{i=1}^{3} \left(\frac{\delta_i \cdot y_*}{C_i}\right)^{k_{3i}} = \left(\frac{10 \cdot 1.55}{45 \cdot 0.125}\right) \left(\frac{27 \cdot 1.55}{45 \cdot 0.8}\right)^{3} \left(\frac{6 \cdot 1.55}{45 \cdot 6}\right),$$

Thus, we get optimal protection parameters (in hours)

$$x_{1^*} = 1.98, \quad x_{2^*} = 1.84, \quad x_{3^*} = 0.15$$

3. Final remarks

Thus, if the proposed model is acceptable with respect to the coefficient of variation, it allows to solve the problem of minimizing common risk of the object safety violation in a simple analytic form due to the choice of object protection parameter set.

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