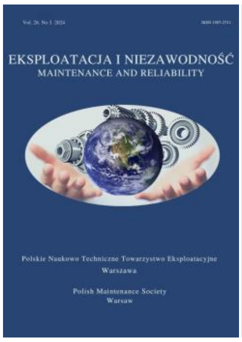


Eksploatacja i Niezawodnosc – Maintenance and Reliability

Volume 26 (2024), Issue 3

journal homepage: http://www.ein.org.pl

Article citation info:
Ziółkowski J, Oszczypała M, Lęgas A, Konwerski J, Małachowski J, A method for calculating the technical readiness of aviation refuelling vehicles, Eksploatacja i Niezawodnosc – Maintenance and Reliability 2024: 26(3) http://doi.org/10.17531/ein/187888

A method for calculating the technical readiness of aviation refuelling vehicles

Indexed by: Web of Science Group

Jarosław Ziółkowski[a,*], Mateusz Oszczypała[b], Aleksandra Lęgas[b], Jakub Konwerski[b], Jerzy Małachowski[b]

[a] Military Academy, Warsaw, Poland
[b] Faculty of Mechanical Engineering, Military University of Technology, Poland

Highlights

- The operation process of an aviation refuelling vehicle was developed.
- Calculations have been made for the several (five) scenarios according to the planned flight table.
- The results for Markov chain and Markov process were discussed.
- The technical readiness of aviation refuelling vehicles supplying fuel to aircraft was investigated.
- The limit probabilities for the Markov process were compare with the total presence times of the tested sample.

Abstract

In the paper a mathematical model of the process of operating aviation refuelling vehicles supplying fuel to aircraft before flight was developed. The present work is a continuation and supplement to the model contained in [52]. The phase space of the process under study was mapped by a 7-state directed graph of the operation process. To calculate the technical readiness index (K_{gt}) Markov chains and processes were used. Also, in Section 3, *Results and discussions*, optional methods for determining the technical readiness coefficient of a vehicle were provided (k_{gt}) , based on the total time of the object in individual operating states. This is an alternative in a situation where the analysed process cannot reach a stable average state indefinitely. Two types of measures were used to determine the readiness, i.e. border probabilities and average times of the object in individual states. In both cases, the basis was statistical databases with operational vehicle data, which enabled the calculation of the readiness index and coefficient.

Keywords
Markov chains and processes, technical readiness, mathematical model, operation, vehicles

1. Introduction

The subject of modelling methods for the operation processes of objects is often discussed in the source literature. Stochastic methods, supplemented with predictive methods [2, 3], and performance evaluation [29, 44] are widely used. These methods are interpreted ambiguously in the source literature [19, 21, 28]. The authors of this work presuppose that the stochastic methods are currently defined as classical probabilistic methods, extended and supplemented with modern methods of analysing stochastic processes and time series, often having deterministic components. Stochastic methods are constantly being developed, improved and standardised to a small degree, while many of them are still the subject of scientific debate. They can be applied to the theory of reliability, operations research, renewal theory and statistical analysis methods. Referring them to technical objects, it should be stated that the actual operation of a vehicle is usually described in the source literature using stochastic processes, and such a process is often a combination of deterministic and random components. In the reliability theory, deterministic components are reflected with the presupposed and orderly arranged organisation of the process, which is reflected in operational plans. However, random components must be introduced due to the unpredictability of phenomena such as damage, failures, delivery delays, lack of spare parts or free

(*) Corresponding author.
E-mail addresses: J. Ziółkowski (ORCID: 0000-0001-8880-5142) jaroslaw.ziolkowski@wat.edu.pl, M. Oszczypała (ORCID: 0000-0002-1194-6913) mateusz.oszczypala@wat.edu.pl, A. Lęgas (ORCID: 0000-0002-4835-1163) aleksandra.legas@wat.edu.pl, J. Konwerski (ORCID: 0000-0002-7527-1177) jakub.konwerski@wat.edu.pl, J. Małachowski (ORCID: 0000-0003-1300-8020) jerzy.malachowski@wat.edu.pl

capacity, and they usually cause disruptions in the proper execution of the assigned tasks.

Readiness [26] and system reliability tests [22, 34, 49], as well as technical objects [9, 16, 25] or processes tests [5] are the subject of a number of publications and may cover multiple areas. For example, Wawrzyński et al. in [43] presented a reliability assessment of aircraft commutators, while the authors of the work [13] created an application of a selected pseudorandom number generator for the reliability of farm tractors. As far as logistics is concerned, the authors of the work [7] applied it to the modelling of delivery sequences, Żurek et al. [53] applied it to ensure the required amount of supplies within a specific period in an enterprise, while Aulin et al. [4] proposed to implement the logistic approach in the international cargo delivery system. In paper [33], the authors applied reliability analysis of a small solar system installed in a house. There is also a body of work on reliability analysis applied to transport systems [1, 12, 27, 42] or networks [24, 50]. For example, Gołda et al. in work [15] described the evaluation of efficiency and reliability of airport processes using simulation tools, while Yeh et al. proposed [46] the reliability evaluation of a multistate railway transportation network from the perspective of a travel agent. The Markov reliability and safety model of the railway transportation system was presented in [35].

Markov and semi-Markov theory is the subject of a number of publications on methods of analysing and assessing the reliability of technical objects. Balanced reliability systems under the Markov process have been described in [11]. In [48] research on the model of a multistate railway transportation network, an aggregated Markov repairable system has been studied. Xiong et al. in [45] presented an analytical approach based on stochastic dynamic programming to optimise coordinated vehicle platooning. The aim of the Markov decision making process was to minimise the discounted cumulative travel costs over an infinite time horizon. An analysis of the refuelling behaviour of hydrogen fuel vehicles through a stochastic model using the Markov Chain Process was used in [20]. A unified algorithm framework for mean-variance optimisation in discounted Markov decision processes was set in paper [30]. In the case of semi-Markov processes, the requirements for exponential durations of individual states are rejected [41]. Implementation mean time to failure index in the control of the logistical support of the operation process has been presented in paper [54], while Çekyay et al. in paper [8] determined MTTF and availability of semi-Markov missions with non-identical generally distributed component lifetimes. The subject of research by some authors is hidden [32, 36, 40] or muddy [31] semi-Markov processes [6, 38].

The literature related to technical objects with their own specific functioning used in military systems constitutes a slightly smaller set of publications. Their characteristic feature is high functionality and reliability, which translates into the effective execution of combat missions. The basic measure for analysing and assessing the reliability of the system is the availability of human resources, e.g. students [10, 37] or military surgeons [47] or maintaining performance during military training [23]. The technical and equipment readiness is assessed slightly differently [14] - the readiness can be referred to both a single object [18, 51] or the entire system [17, 39].

Based on the above-mentioned works, it can be concluded that there are a number of available studies regarding modelling of reliability, including the readiness of technical objects. The source literature review confirmed the thesis that studies on military facilities still constitute a limited subset of the entire source literature. It should be noted that the models developed in the reviewed papers are, on the one hand, so universal that they can be extrapolated to the so-called to all models of Su military aircraft understood as a group with the same or similar features. On the other hand, reflecting the specificity of a given operating environment, they are so characteristic that they cannot be applied to objects with different operating characteristics. The readiness of the objects used in military systems is carefully examined and monitored. Its optimisation criterion is the maximisation of the object's ability to perform tasks, which is equivalent to minimising time losses for renovation, maintenance and downtime. Therefore, the necessary conditions for testing and optimising the readiness of aircraft refuelling vehicles are the proper specification of operational states and reliable records of their duration. Therefore, the authors of this paper, based on the flight table (Fig. 1), proposed a realistic and composed of 7 states model of the refuelling vehicle operation model (Section 2) supplying aircraft in combat conditions with 5 different scenarios. The calculations were performed individually for each scenario and then for the tested sample. In both cases, the models enabled to perform the readiness analysis and assessment. The model created, the obtained results and the conclusions constitute the authors' original achievement, because, as shown in the source literature review, none of the previous publications concerned such a specific area of research.

This paper is a supplement and continuation of the research conducted in work [52]. The layout of the paper is presented below. Section 1 includes the latest source literature review on the use of mathematical methods used to analyse and assess the reliability parameters, including readiness. It was emphasised that a significant part of the publications contains broadly understood stochastic processes that utilise the Markov theory. Moreover, the purpose of developing this publication was justified. Section 2 Includes assumptions on the model of the process of refuelling combat aircraft before flight (Fig. 1). Further in this section, a Markov theory-based model was developed for discrete time (Point 2.2) and continuous time (Point 2.3). The model reflects a directed graph of the operation process (Fig. 2), where the object moves in the phase space between individual operational states. Section 3 The obtained solutions were summarised and discussed, noting the factors affecting the obtained results.

2. Material and methods

2.1. Assumptions for building the mathematical model

According to the methodology of constructing and analysing event models for operational processes, the description of the examined operation process of the vehicles refuelling aircraft before flight is based on the assumptions discussed below:

Assumption 1. At any time, the vehicle may be in only one of the possible operating states, the set of which (the so-called phase space) is determined by the tasks performed and the object's operating instructions.

Assumption 2. The vehicle's phase space is a finite set of disjoint operating states. The number of states is determined by successive approximations in accordance with the assumed modelling goal.

Assumption 3. Interstate crossings are time-sensitive and their moments are recorded. In exploitation theory, direct returns to the state ($S_i \rightarrow S_i$ transitions) are prohibited.

Assumption 4. The times of changing object states are measured with arbitrary accuracy. This is equivalent to assuming that the process time is a continuous random variable. The phase trajectory of the object is a step function, right-continuous at times of interstate transitions.

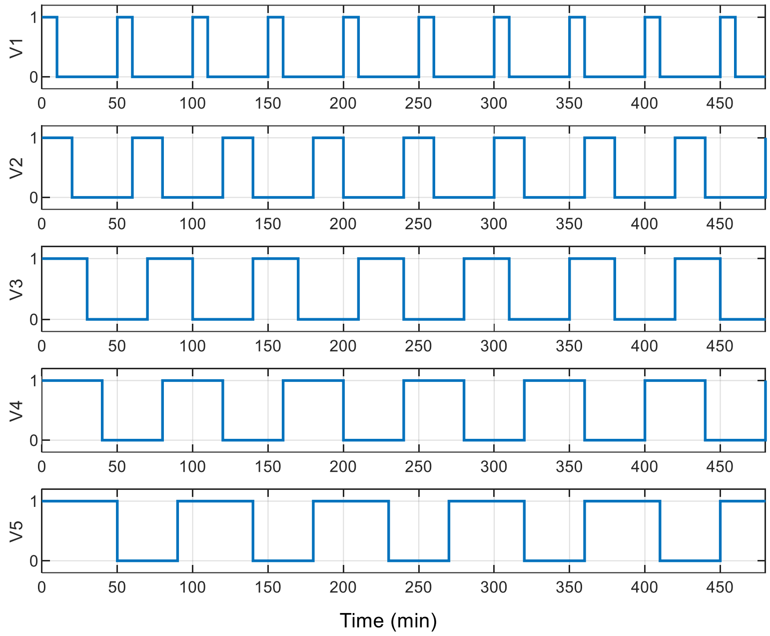
Assumption 5. The environmental and external process conditions, as well as the deterministic factors affecting the operation process are known.

In practice, combat flights of Su-22 aircraft are determined by the nature of the missions to be performed, which determines their duration. The scheduled flight table reflects all of the above. The time intervals between individual operations are the intervals necessary to restore readiness, during which the supplies are replenished, and maintenance/diagnostic operations are carried out.

The authors of this paper assumed that the flight combat missions are carried out with the maximum frequency of operations. For a single aircraft, it means the minimum time to restore readiness for the next mission, which for Su-22 is technically 40 minutes. Five scenarios were considered, in which for each of them the length of a single flight was a random step variable. Scenarios V_1–V_5 presented in Fig. 1 represent the lengths of a single operation and the corresponding fuel consumption factor K_{zu}, i.e.

a) V_1 - 10 minutes flight ($K_{zu} = 0.165$);
b) V_2 - 20 minutes flight ($K_{zu} = 0.33$);
c) V_3 - 30 minutes flight ($K_{zu} = 0.5$);
d) V_4 - 40 minutes flight ($K_{zu} = 0.66$);
e) V_5 - 50 minutes flight ($K_{zu} = 0.83$).

The presented scenarios are implemented within eight working hours ($T_0 = 480$ minutes). During this time, depending on the length of a single operation (Fig. 1) the aircraft can perform six (V_5) to ten (V_1) combat flights.



where:
1 – Execution of the flight mission
0 – Aircraft maintenance

Fig. 1. Planned flight table taking into account scenarios V_1–V_5.

2.2. Markov model for discrete time

To develop a Markov model reflecting the researched operation process, its phase space must be determined in the first place. The space is created by a set of operational states that are important in terms of the adopted modelling objective. Isolated operating states of aviation refuelling vehicles were systematically functionally aggregated for the purpose of calculating readiness indexes. The seven-state phase space satisfies requirements 1-5 described in subsection 2.1:

- S_1 – vehicle access to the airport apron;
- S_2 – fuel settling;
- S_3 – checking the purity of fuel in the vehicle;
- S_4 – aircraft refuelling procedure (including the total time components of the arrival of the vehicle to the aircraft, the process of refuelling the aircraft and the return to the airport apron);
- S_5 – vehicle tank refuelling cycle (including total times of travel to the pump, vehicle refuelling and return to the airport apron);
- S_6 – non-operability of the vehicle (replacement with a working one);
- S_7 – vehicle waiting for refuelling of the aircraft (depending on the flight table).

The model of the above-described process is a directed graph presented in Fig. 2, for which the operation is understood as a transfer of an object through individual states.

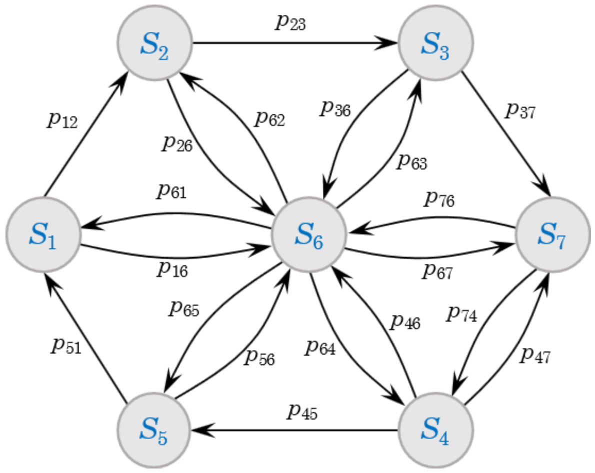


Fig. 2. Directed graph of the operation process of an aviation refuelling vehicle.

As presented in Fig. 2, the vehicle non-operability S_6 is the most closely related to the other states, which symbolises the randomness of the process described. In practice, the square matrix of the allowed transfers represents $P = [p_{ij}]$ a more clear form of the model. It is described by the dependence (1):

$$P = [p_{ij}]_{7\times 7} = \begin{bmatrix} 0 & p_{12} & 0 & 0 & 0 & p_{16} & 0 \\ 0 & 0 & p_{23} & 0 & 0 & p_{26} & 0 \\ 0 & 0 & 0 & 0 & 0 & p_{36} & p_{37} \\ 0 & 0 & 0 & 0 & p_{45} & p_{46} & p_{47} \\ p_{51} & 0 & 0 & 0 & 0 & p_{56} & 0 \\ p_{61} & p_{62} & p_{63} & p_{64} & p_{65} & 0 & p_{67} \\ 0 & 0 & 0 & p_{74} & 0 & p_{76} & 0 \end{bmatrix} \quad (1)$$

The matrix $P = [p_{ij}]$ reflects the process of supplying fuel to aircraft performing aviation missions. For the analysed process, the matrix represents a set of 19 allowed transitions, according to the process arrangement. In Table 1, theoretically possible (permitted) interstate transitions (symbol 1) and prohibited transitions (symbol 0) are marked in green.

The tested sample (Fig. 1), taking into account scenarios V_1-V_5 in practice narrowed the set of allowed transitions to those executed in the analysed process. The latter (marked in red) are listed in Table 2.

Table 1. A matrix of theoretically allowed transitions.

ω_{ij}	S_1	S_2	S_3	S_4	S_5	S_6	S_7
S_1	0	1	0	0	0	1	0
S_2	0	0	1	0	0	1	0
S_3	0	0	0	1	0	1	1
S_4	0	0	0	0	1	1	0
S_5	1	0	0	0	0	1	0
S_6	1	1	1	1	1	0	1
S_7	0	0	0	1	0	1	0

Table 2. Matrix of transitions completed by a sample of objects performing tasks in accordance with the scenarios V_1-V_5.

ω_{ij}	S_1	S_2	S_3	S_4	S_5	S_6	S_7
S_1	0	1	0	0	0	1	0
S_2	0	0	1	0	0	0	0
S_3	0	0	0	1	0	0	1
S_4	0	0	0	0	1	0	0
S_5	1	0	0	0	0	1	0
S_6	1	1	1	0	0	0	1
S_7	0	0	0	1	0	0	0

As can be seen from the comparison of the data presented in Table 1 and Table 2, the majority of the transitions were completed in the tested sample, i.e. 13 out of 19 permitted transitions, which is 68.4%. Not completed transitions only concern the state (S_6) of the vehicle's non-operability, for which the cases of failure are usually random and in fact vary in terms of the labour demand. For this reason, aviation refuelling vehicles are not repaired during the performance of tasks, but redundant equipment is kept on the airport apron. When a vehicle brakes, it is replaced with a fully operational one that is on duty at the airport as redundant equipment. The process of replacing the vehicle is therefore quick and does not pose a threat to the mission performed by the aircraft. In practice, the failure of the vehicle determines the results obtained for discrete time (Markov chain), but does not significantly affect the readiness of the vehicle (Markov process).

Vehicle readiness testing was performed separately for each scenario (Fig. 1) Depending on the length of the single flight of the aircraft in the range $V_1 - V_5$. Its first stage of the testing was preparing individual interstate transition matrices, on the basis of which systems of linear equations were written along with the normalisation condition. Then, using the Mathematica software, limiting probabilities were calculated. The obtained results in discrete time are presented in Fig. 3.

The results obtained individually for each scenario (Fig. 3) reflect the planned and orderly arranged organisation of the process under study. The refuelling vehicle that provides the fuel to the aircraft before flights performs a strictly defined sequence of tasks. They are represented by the probabilities of entering particular states that for scenarios V_4 and V_5 are equal in value and almost equal, as is the case with scenarios V_1–V_3. Regardless of the value, they represent the planned organisation of the process, for which the differences are determined by the planned flight table (Fig. 1), as well as the randomness of the state S_6 indicating the failure of the vehicle.

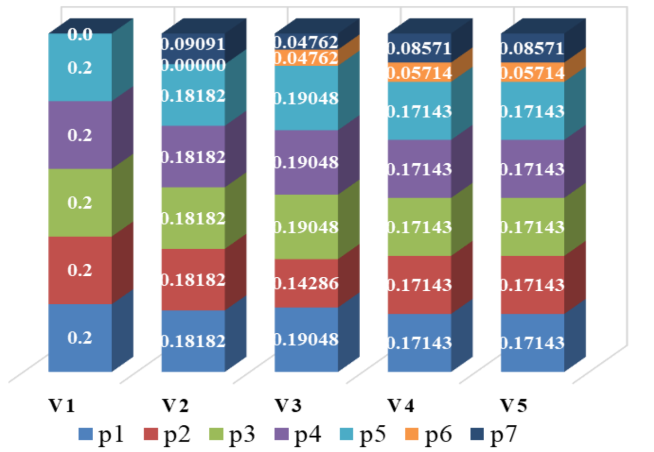


Fig. 3. Limiting probabilities for discrete time $p_j(n)$ obtained individually for the considered scenarios V_1–V_5.

Discrete-time Markov models provide two basic types of information necessary for detailed analyses and optimisation of the operation process. The first type of information is the numerical data on the number of objects in each operational state, the corresponding percentages and the number of interstate transitions during the period under study. The second type of information is the forecasts for the limiting probabilities of the object states that allow one to assess the degree of process deviation from equilibrium in a defined phase space (set of states). Deviations from the balance also have a double meaning. Firstly, they represent the defects in the design of operational processes, secondly, they constitute optimisation tips. A preliminary examination of the Markov chain in terms of the operationally useful interpretation has the number of states ($S_1 - S_7$), the number $[N_i]$ of entries into the state S_i and the number $[N_j]$ of exits from the state S_j and the corresponding percentages $[\%N_i]$, $[\%N_j]$. The summary of the above data set (Table 3) allows for a preliminary assessment of the properties of the process under study.

Table 3. The number of the interstate transitions between the N_{ij}, initial and final states N_i N_j and their corresponding percentages $[\%N_i]$ i $[\%N_j]$ for the sample taking into account scenarios $V_1 - V_5$.

$[N_{ij}]$	S_1	S_2	S_3	S_4	S_5	S_6	S_7	$[N_j]$	$[\%N_j]$
S_1	0	272	0	0	0	16	0	288	18.2
S_2	0	0	288	0	0	0	0	288	18.2
S_3	0	0	0	192	0	0	112	304	19.2
S_4	0	0	0	0	272	0	0	272	17.2
S_5	208	0	0	0	0	32	0	240	15.2
S_6	32	16	16	0	0	0	16	80	5.1
S_7	0	0	0	112	0	0	0	112	7.1
$[N_i]$	240	288	304	304	272	48	128	1584	Nd
$[\%N_i]$	15.2	18.2	19.2	19.2	17.2	3.1	8.1	Nd	Nd

In Table 3, the most numerous transitions are highlighted in green, and the least numerous in red. The total number of interstate transitions for all scenarios was satisfactory and amounted to N = 1.584. No single or small-number transitions (less than 10) were recorded. Among the 13 transitions, small-number transitions (N = 16) and two multiple transitions (N = 32) were recorded. Moreover, there was quite a large dynamic in the number of transitions, which was 1:18 (min 16: max 288). The largest number ($n_{23} = 288$) had a transition from the fuel settling state S_2 to control the purity of the fuel S_3. The coupling of both states is not accidental and results from the applicable procedures, according to which the quality of aviation fuel is always checked before the aircraft is refuelled. The second largest number, in both cases amounting to, were the transitions ($S_1 \rightarrow S_2$) from the status of arrival to the apron and the status of fuel settling and, respectively, the transitions ($S_4 \rightarrow S_5$) from the status of the aircraft aviation fuel supply and the vehicle tank refuelling cycle. In both cases, they constitute obligatory activities in the organisation of the examined process, and therefore the above-mentioned numerical values prove that the organisation of the process was properly planned and executed. The lowest frequency of observations amounting only to $N_6 = 3.1\%$ had the state S_6 of short-term non-operability of the vehicle. This can be considered a manifestation of the correct organisation of the process, for which damage to the vehicle supplying aviation fuel to the aircraft before flights occurred randomly and sporadically.

The next stage of examining the process in discrete time was the calculation of empirical frequencies of interstate transitions for the sample under study. They are summarised in Table 4.

Table 4. Estimators of transitions probabilities ω_{ij} for the sample including the scenarios V_1–V_5.

ω_{ij}	S_1	S_2	S_3	S_4	S_5	S_6	S_7
S_1	0	0.944	0	0	0	0.056	0
S_2	0	0	1	0	0	0	0
S_3	0	0	0	0.632	0	0	0.368
S_4	0	0	0	0	1	0	0
S_5	0.867	0	0	0	0	0.133	0
S_6	0.4	0.2	0.2	0	0	0	0.2
S_7	0	0	0	1	0	0	0

Empirical interstate transition rates are used to prepare systems of equations enabling the calculation of cut-off probabilities for discrete time as:

$$\begin{cases} 0.867p_5 + 0.4p_6 - p_1 = 0 \\ 0.944p_1 + 0.2p_6 - p_2 = 0 \\ p_2 + 0.2p_6 - p_3 = 0 \\ 0.632p_3 + p_7 - p_4 = 0 \\ p_4 - p_5 = 0 \\ 0.056p_1 + 0.133p_5 - p_6 = 0 \\ 0.368p_3 + 0.2p_6 - p_7 = 0 \end{cases} \tag{2}$$

along with the condition of the system normalisation:

$$\sum_{j=1}^{7} p_j = 1 \,. \tag{3}$$

Condition (3) of the system normalisation is an additional but necessary condition because it excludes the case of obtaining zero values of the limit probabilities. After solving the above systems of equations for the sample, the results obtained are presented in Fig. 4.

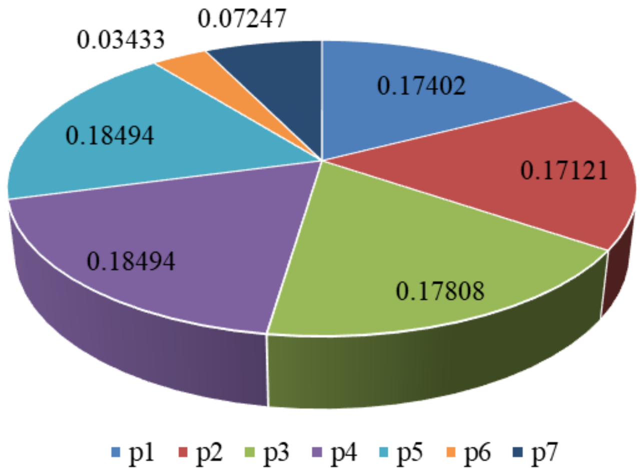


Fig. 4. Values of limiting probabilities $p_j(n)$ of the Markov chain for the tested sample including the scenarios V_1–V_5.

The highest and most balanced values of the entry probabilities were observed for five of the seven individual operational states (Fig. 4). They reflect the sequence of activities performed sequentially, i.e. arrival at the airport (S_1), fuel settling (S_2), control of the fuel purity in the vehicle (S_3), supplying aircraft with aviation fuel (S_4) and the vehicle tank refuelling cycle (S_5). The probabilities of entering the mentioned states are equal (for p_4 and p_5) or almost equal (as in the case of p_1, p_2 and p_3). In practice, it is the above states that directly affect the continuity and systematic implementation of the planned aviation missions. The smallest value of $p_6 = 0.03433$ had the probability of entering a state of non-operability and the vehicle waiting for refuelling $p_7 = 0.07247$. The low probability of entering the state of non-operability S_6 is a desirable phenomenon and indicates a relatively low failure rate of the vehicle refuelling the aircraft before flight. As is the probability of entering the state S_7, waiting of the vehicle for refuelling means translates into the vehicle availability but does not increase the efficiency of its use.

2.3. Markov model for continuous time

The evolution of a homogeneous, state-discrete Markov process in continuous time t is described by the systems of Chapman-Kolmogorov equations, assuming the existence of parameters called intensities in the process transition matrix. Therefore, when the Markov model is studied in real-time, it is important to determine the empirical quadratic matrix $\Lambda = [\lambda_{ij}]$ of the intensity of the process transition with the elements λ_{ij} and m degree. This matrix is always singular ($\det[\Lambda] = 0$) due to the way of calculating the off-diagonal intensities of transitions from the state i to the state j in the time t. The transition intensities λ_{ij} are defined as the right-hand derivatives of the transition probabilities $\dot{p}_{ij}(t)$ with respect to time as:

$$\lambda_{ij} = \frac{d}{dt} p_{ij} = \dot{p}_{ij}(t) = \lim_{\Delta t \to 0} \frac{p_{ij}(t+\Delta t) - p_{ij}(t)}{\Delta t}, \tag{4}$$

while exiting the state i is the opposite event to entering the state j, hence:

$$\lambda_{ii} = -\frac{d}{dt} p_{ii} = -\dot{p}_{ii}(t) = \lim_{\Delta t \to 0} \frac{1-(p_{ii}(t+\Delta t) - p_{ii}(t))}{\Delta t}, \tag{5}$$

the balance of transitions to all states $S_{j \neq i}$ and process exits S_i is zero, due to the way diagonal values are calculated:

$$\lambda_{ii} = -\sum_{j \neq i} \lambda_{ij}. \tag{6}$$

The name of the transition intensity matrix $\Lambda = [\lambda_{ij}]$ is misleading for a number of novice scientists and usually causes difficulties in interpreting individual components because its elements λ_{ij} have been untowardly called transition intensities by theorists, suggesting that they reflect the power or strength of the phenomenon. In practice, the mentioned intensities do not have a substantive interpretation and do not represent the intensity of the examined process. However, their inverses have such an interpretation, which, according to the definition, means the average duration of a given state before transition to the next state, in accordance with the following dependence (8).

The theoretical transition intensity matrix of the studied process was created as follows (7):

$$\Lambda_{ij} = \begin{bmatrix} -\lambda_{11} & \lambda_{12} & 0 & 0 & 0 & \lambda_{16} & 0 \\ 0 & -\lambda_{22} & \lambda_{23} & 0 & 0 & \lambda_{26} & 0 \\ 0 & 0 & -\lambda_{33} & \lambda_{34} & 0 & \lambda_{36} & \lambda_{37} \\ 0 & 0 & 0 & -\lambda_{44} & \lambda_{45} & \lambda_{46} & 0 \\ \lambda_{51} & 0 & 0 & 0 & -\lambda_{55} & \lambda_{56} & 0 \\ \lambda_{61} & \lambda_{62} & \lambda_{63} & \lambda_{64} & \lambda_{65} & -\lambda_{66} & \lambda_{67} \\ 0 & 0 & 0 & \lambda_{74} & 0 & 0 & -\lambda_{77} \end{bmatrix}. \tag{7}$$

For a stochastic process that is a Markov process $X(\mathrm{t})$, the off-diagonal intensities are calculated according to the formula:

$$\lambda_{ij} = \frac{1}{\bar{t}_{ij}}, \tag{8}$$

where: $i, j \in \{1, \ldots 7\}$, whereas $\bar{t}_{ij}$ is the average residence time of the process $X(t)$ in the state i before the transition to state j, calculated according to the dependence (9):

$$\bar{t}_{ij} = \frac{\sum_{n=1}^{N} {}_{n}\bar{t}_{ij}}{N}, \tag{9}$$

where: ${}_{n}\bar{t}_{ij}$ means average time spent in state i before transition to state j for the vehicle number n; N – number of vehicles in the tested sample $N \in \{1, \ldots, 16\}$.

As already mentioned above, the discrete-in-state and continuous-in-time Markov model is represented by the systems of Chapman-Kolmogorov equations, the normalisation condition and the initial distribution vector. In its canonical form, the Chapman-Kolmogorov system of equations for the analysed 7-state process is a homogeneous system of seven first-order ordinary linear differential equations with constant coefficients of the following form:

$$\begin{cases} \frac{d}{dt} p_1 = \lambda_{11} p_1 + \lambda_{21} p_2 + \lambda_{31} p_3 + \lambda_{41} p_4 + \lambda_{51} p_5 + \lambda_{61} p_6 + \lambda_{71} p_7 \\ \frac{d}{dt} p_2 = \lambda_{12} p_1 + \lambda_{22} p_2 + \lambda_{33} p_3 + \lambda_{42} p_4 + \lambda_{52} p_5 + \lambda_{62} p_6 + \lambda_{72} p_7 \\ \frac{d}{dt} p_3 = \lambda_{13} p_1 + \lambda_{23} p_2 + \lambda_{33} p_3 + \lambda_{43} p_4 + \lambda_{53} p_5 + \lambda_{63} p_6 + \lambda_{73} p_7 \\ \frac{d}{dt} p_4 = \lambda_{14} p_1 + \lambda_{24} p_2 + \lambda_{34} p_3 + \lambda_{44} p_4 + \lambda_{54} p_5 + \lambda_{64} p_6 + \lambda_{74} p_7 \\ \frac{d}{dt} p_5 = \lambda_{15} p_1 + \lambda_{25} p_2 + \lambda_{35} p_3 + \lambda_{45} p_4 + \lambda_{55} p_5 + \lambda_{65} p_6 + \lambda_{75} p_7 \\ \frac{d}{dt} p_6 = \lambda_{16} p_1 + \lambda_{26} p_2 + \lambda_{36} p_3 + \lambda_{46} p_4 + \lambda_{56} p_5 + \lambda_{66} p_6 + \lambda_{76} p_7 \\ \frac{d}{dt} p_7 = \lambda_{17} p_1 + \lambda_{27} p_2 + \lambda_{37} p_3 + \lambda_{47} p_4 + \lambda_{57} p_5 + \lambda_{67} p_6 + \lambda_{77} p_7 \end{cases} \tag{10}$$

The above equations are complemented by: the normalisation condition of the system (11) and the initial distribution vector (12):

$$\sum_{j=1}^{7} p_j = 1, \tag{11}$$

$$p_j = [1,0,0,0,0,0,0]. \tag{12}$$

Analytical solutions for the numerical values of the elements of the intensity matrix can be found using mathematical programs such as Mathematica, Matlab, Maple and others In practice, programs do not generate analytical solutions for symbolic matrix elements of degree greater than 2. The analytical solutions formulas of Mathematica program for the seven-state model developed in this publication are so complex that their testing is only possible numerically. Real-time normalised probabilities $p_j(t)$ of the presence of the vehicle in particular states for the adopted scenarios V_1-V_5 are presented in Fig. 5.

When analysing the results presented in Fig. 5, it should be noted that the values of the limiting probabilities in continuous time are significantly different from the results obtained for discrete time (see Fig. 3). The obtained data presented in Fig. 3 show the probabilities of entering individual states, while the results shown in Fig. 5 (solutions to the systems of Chapman-Kolmogorov equations) represent the probability of the vehicle being in a given state at an instant of continuous time in infinity. The values $\mathrm{p_j(t)}$ for the Markov process, differ for each state within a given scenario, but only slightly. This means that the analysed process is homogeneous and ergodic and can be considered stable. The differences in numerical values are mainly determined by the randomness of the occurrence of two states, i.e. S_6 (non-operability of the vehicle) and S_7 (vehicle waiting for refuelling *sp*). Moreover, the average presence times in the mentioned states have variable characteristics, which causes a disproportion in the intensity of transitions in the matrices $[\Lambda_{ij}]$ individually for each scenario. Similarly to the case of discrete time, a collective model for continuous time (Markov process) was developed, taking into account all considered scenarios. Its first element was the calculation of the components of the collective matrix of interstate transition intensity $\Lambda = [\lambda_{ij}]$ for the tested sample:

$$\Lambda = \begin{bmatrix} -0.140 & 0.073 & 0 & 0 & 0 & 0.067 & 0 \\ 0 & -0.018 & 0.018 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.314 & 0.114 & 0 & 0 & 0.2 \\ 0 & 0 & 0 & -0.055 & 0.055 & 0 & 0 \\ 0.010 & 0 & 0 & 0 & -0.028 & 0.018 & 0 \\ 0.125 & 0.125 & 0.125 & 0 & 0 & -0.5 & 0.125 \\ 0 & 0 & 0 & 0.026 & 0 & 0 & -0.026 \end{bmatrix} \tag{13}$$

In order to determine the limiting probabilities, the transformed transition intensity matrix must be multiplied with the probability vector Λ to the equation $\Lambda^T \cdot [p_j] = 0$, the following equation in matrix form was obtained for the examined operation process:

$$\begin{bmatrix} -0.140 & 0 & 0 & 0 & 0.010 & 0.125 & 0 \\ 0.073 & -0.018 & 0 & 0 & 0 & 0.125 & 0 \\ 0 & 0.018 & -0.314 & 0 & 0 & 0.125 & 0 \\ 0 & 0 & 0.114 & -0.055 & 0 & 0 & 0.026 \\ 0 & 0 & 0 & 0.055 & -0.028 & 0 & 0 \\ 0.067 & 0 & 0 & 0 & 0.018 & -0.5 & 0 \\ 0 & 0 & 0.2 & 0 & 0 & 0.125 & -0.026 \end{bmatrix} \cdot \begin{bmatrix} p'_1 \\ p'_2 \\ p'_3 \\ p'_4 \\ p'_5 \\ p'_6 \\ p'_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{14}$$

Values of limiting probabilities $p_j(t)$ for the Markov process were calculated using the Mathematica program, the results are presented in Fig. 6.

The results presented in Fig. 6 for the tested sample are significantly different from the results obtained for discrete time (see Fig. 3). In continuous time, the dominant role of four states became visible, i.e. S_2, S_4, S_5 oraz S_7, the states in which the vehicle, on average in the sample, spent the most time (over 92%) related to the research period understood as the time of the flights. The remaining three states are short-lived and will not have a decisive impact on the calculated object readiness index.

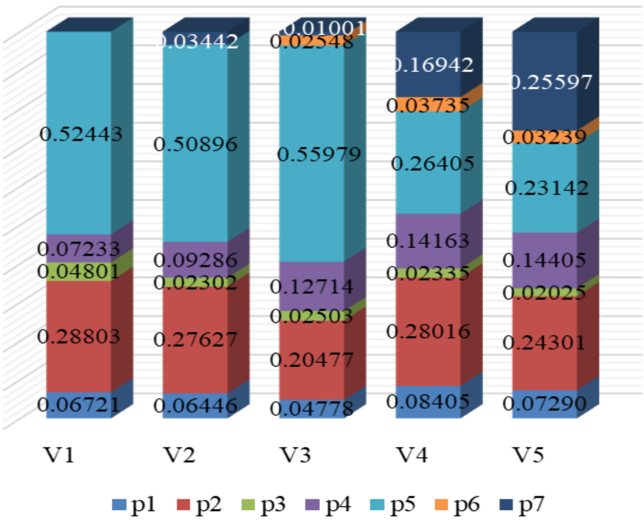


Fig. 5. Values of limiting probabilities $p_j(t)$ of the Markov process for the adopted scenarios V_1-V_5.

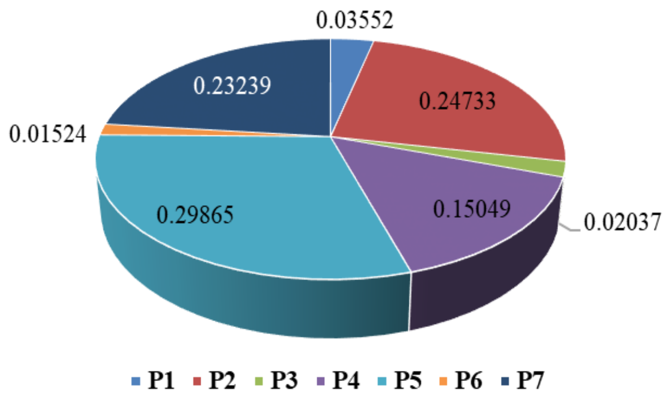


Fig. 6. Values of limiting probabilities $p_j(t)$ of the Markov process for the tested sample including the scenarios V_1-V_5.

3. Results, discussion and conclusions

The paper presents a method for calculating the technical readiness of aviation refuelling vehicles supplying fuel to aircraft before combat flights. Five scenarios (Fig. 1) were investigated, reflecting the variable length of a single flight [min] in the range of V_1 - $V_5 \in \{10,20,30,40,50\}$. For this purpose, a 7-state model of the process of operation was developed, for which, under each scenario, the limit probabilities were calculated for discrete time (Markov chain - Fig. 3) and continuous time (Markov process - Fig. 6).

Analysing the values of marginal probabilities $p_j(n)$ shown in Fig. 3 regarding discrete time, the following conclusions can be drawn:

1. The analysed stochastic process is a composition of two sub-processes, i.e. deterministic and random. The deterministic part results from the applicable procedures reflected by the planned flight table (Fig. 1). The organisation of vehicle operation involves performing a sequence of activities carried out in a specific order, i.e. after each aircraft supply process (S_4) the vehicle always (regardless of the amount of fuel left in its tank) automatically completes its tank refuelling cycle (S_5) and then, in accordance with the regulations, the fuel must then undergo a settling cycle (S_2) and cleanliness control (S_3) before the next aircraft refuelling process (S_4). The phenomenon described above is reflected in a sequence of states $S_4 \rightarrow S_5 \rightarrow S_2 \rightarrow S_3$ for which the values of the limiting probabilities are almost the same for a given scenario, although they differ in value between individual scenarios (V_1-V_5). The mentioned sequence reflects the deterministic part of the process under study, while the random components represent two states, i.e. vehicle unsuitability S_6 and waiting for refuelling S_7.
2. The Markov chain boundary probabilities presented in Fig. obtained for the scenarios under study, from the standpoint of technical readiness indicators should be interpreted quantitatively, not qualitatively. They indicate the limits of the number of entries to a given state against the background of transitions to all states constituting the phase space of the analysed process. Their constant values indicate balance and organisational order related to the implementation of aviation operations.
3. In the case of script V_1, the vehicle was in 5 out of 7 operating states (Fig. 3), for which the calculated entry probabilities were the same and amounted to 0.2. No damage to the vehicle was recorded (state S_6), as well as waiting of the aircraft for refuelling (state S_7). It can therefore be concluded that this scenario has purely deterministic components. The probability values for scenario V_2 should be interpreted similarly, with the difference that in this case the vehicle waiting for refuelling its tank was recorded(S_7), which took place at the expense of a proportional reduction in the share of entries to the remaining states, i.e. $S_1 - S_5$.
4. The elements of randomness reflect the other three scenarios (V_3-V_5) of the process under study, for which observations were recorded in all states in the examined sample, but in the case of V_3 the damage occurred after the vehicle reached the airport apron (transition $S_1 - S_6$). This event had an impact on reducing the probability of entering the fuel settling state S_2 (see Fig. 3).
5. For scenarios V_4 and V_5, the calculated boundary probabilities have exactly the same values of entering individual states in discrete time. This is related to the adopted length of a single exit for the mentioned scenarios, which is: V_4=40 and V_5= 50 minutes and in both cases significant degrees of emptying the tank of the Su-22 aircraft, respectively $K_{zu} = 0.66$ and $K_{zu} = 0.83$, for which the vehicle performed exactly the same number of refuelling cycles.

The values of probabilities in continuous time have a different interpretation (Fig. 5), which should be interpreted qualitatively in terms of the required readiness. They represent the duration of individual operational states. They are summarised in Table 5.

Table 5. Values of limiting probabilities $p_j(t)$ for scenarios V_1-V_5.

$p_j(t)$	V_1	V_2	V_3	V_4	V_5
p_1	0.06721	0.06446	0.04778	0.08405	0.07290
p_2	0.28803	0.27627	0.20477	0.28016	0.24301
p_3	0.04801	0.02302	0.02503	0.02335	0.02025
p_4	0.07233	0.09286	0.12714	0.14163	0.14405
p_5	0.52443	0.50896	0.55979	0.26405	0.23142
p_6	0.00000	0.00000	0.02548	0.03735	0.03239
p_7	0.00000	0.03442	0.01001	0.16942	0.25597

The real-time normalised probabilities for continuous time allow us to draw the following conclusions:

1. The states with the lowest variability in the calculated values of marginal probabilities compared under individual scenarios were:
   a) travel to the airport apron p_1;
   b) fuel settling p_2;
   c) checking the purity of fuel in the vehicle p_3;

d) vehicle non-operability p_6.

This is because the above states have relatively constant (but different) average execution times. Moreover, the states S_2 and S_3 are organisationally interconnected, which means that after the fuel has settled, its purity (quality) is always checked, symbolised by the transitions $S_2 \rightarrow S_3$. The above phenomenon is presented by exactly the same values of the limiting probabilities for discrete time (see Fig. 3), regardless of the adopted scenario. The exception is the scenario V_3, in which the vehicle was randomly damaged upon arrival at the airport. The state of non-operability of the vehicle S_6 is short-lived (replacement of the vehicle with a fully operational one) and generally does not determine the readiness index.

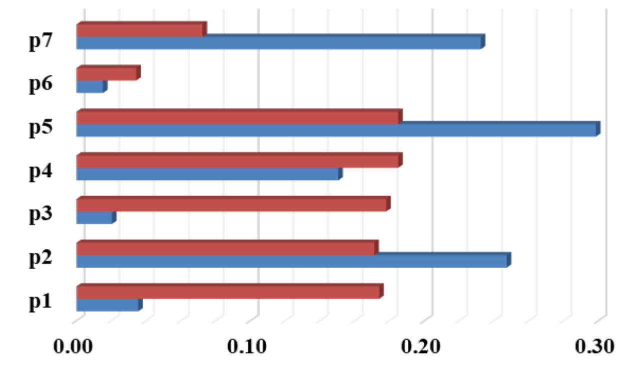
2. Much greater disproportions between individual scenarios were shown in the probability values for the remaining states, i.e.:
   a) refuelling the aircraft p_4;
   b) refuelling the vehicle's tank;
   c) waiting for refuelling the vehicle's tank p_7.

Probability values of the state of aircraft fuel supply p_4 changed abruptly for individual scenarios V_1-V_5, as appropriate within a range of $p_4 \epsilon \{0.07233;\ 0.09286;\ 0.12714;\ 0.14163;\ 0.14405\}$ - Table 5. Systematically increasing values of probabilities are dictated by the coefficient of emptying the aircraft fuel tanks, which varies stepwise in the range of $K_{zu} = \{0.165;\ 0.33;\ 0.5;\ 0.66;\ 0.83\}$. The above probabilities depend on the length of a single departure, and thus on the amount of aviation fuel used, which must be replenished after each flight.

Probabilities p_5 of the vehicle refuelling cycles depend on the number of completed cycles and the average duration of each of them. Their values decrease in steps within a range of $p_5 \epsilon \{0.52443;\ 0.50896;\ 0.55979;\ 0.26405;\ 0.23142\}$-Table 5, except scenario V_3, where the vehicle has completed one more cycle compared to the scenarios V_1 and V_2 in time T_0 of the flights execution.

However, the state of waiting for the vehicle to be refuelled has variable probability values, depending on a given scenario, which is equal to zero (as for V_1), close to zero (as for V_2 and V_3) and slightly higher respectively for scenarios V_4 and V_5. This is because for higher values of a single flight, the aircraft complete fewer of them in the assumed time T_0. This results in a reduction in the refuelling frequency for a single aircraft (Fig. 1), while the vehicle waits statistically longer for refuelling.

Then within individual scenarios V_1-V_5 probabilities were determined for the tested sample. The obtained results are shown in Fig. 7.



	p1	p2	p3	p4	p5	p6	p7
pj(n)	0.17402	0.17121	0.17808	0.18494	0.18494	0.03433	0.07247
pj(t)	0.03552	0.24733	0.02037	0.15049	0.29865	0.01524	0.23239

Fig. 7. Values of the limiting probabilities of the Markov chain $p_j(n)$ and process $p_j(t)$ for the tested sample including the scenarios V_1-V_5.

Analysing the results regarding the limiting probabilities p_j in discrete time (Markov chain) and continuous time (Markov process), the following conclusions can be formulated:

1) for discrete time:
   - the highest and, at the same time, the same in terms of entry probability value $p_4, p_5 = 0.18494$ observed for states S_4 (supply aircraft in aviation fuel) and S_5 (vehicle refuelling cycle), which is a normal phenomenon from the standpoint of the main purpose of the tested process; these are the two most important states that determine the role of the vehicle during flights;
   - slightly lower and equal to two decimal places entry probabilities of $p_1, p_2, p_3 = 0.17$ obtained for the states: travelling to the airport apron (S_1), fuel settling (S_2) and checking the fuel purity in the vehicle (S_3). The above states are positively correlated and if one of them occurs, the others must be executed;
   - the lowest entry probability was observed for two states, i.e. vehicle unsuitability (S_6), for which damage occurred sporadically in the tested sample and waiting for the vehicle to be refuelled

(S_7), which proves the good operational efficiency of the object;

2) for continuous time:
   - the calculated technical readiness index of the vehicle delivering aviation fuel is $K_{gt} = 0.382$ and was calculated as the sum of the probability values p_4 and p_7. One might get the impression that it is too low for combat missions. It should be borne in mind however that this index is understood as the vehicle's ability to perform tasks at a randomly selected moment. Taking into account the fact that the adopted flight structure is a process fully covered by the plan, which is reflected in the planned flight table, it should be stated that the calculated value of the index fully secures fuel supplies for aircraft;
   - the calculated value of the readiness index can be justified by the necessary organisational activities that reflect the following states in the model: the arrival of the vehicle to the airport apron S_1, fuel settling, S_2, checking fuel purity in the vehicle S_3 and the vehicle's tank refuelling cycle S_5. The above states are organisationally interconnected in the model and none of them can be omitted. Damage, which statistically occurs very rarely, is a short-term condition and does not have a significant impact on the calculated vehicle readiness index.

In practice, the studied stochastic process may turn out to be non-homogeneous and non-ergodic. In the latter case, this means that the process will never (indefinitely) reach an average stable state. As a result, it will be impossible to calculate the limiting (ergodic) probabilities. In such a case, an alternative qualitative measure of the assessment, which is also a method of verifying the technical readiness index, is the calculation of the total presence times of the vehicle in individual operational states. Technical readiness coefficient (k_{gt}) is in such a case defined as the quotient of the total arrival time in states of readiness and the total time of stay in states of readiness and states of non-readiness to complete the task, according to the dependence:

$$k_{gt} = \frac{\sum_{i=1}^{n} T_G(t)}{\sum_{i=1}^{n} T_G(t) + \sum_{j=1}^{m} T_N(t)} \tag{15}$$

where:

i – number of the operational state determining readiness;

j – number of the operational state determining readiness;

$\sum_{i=1}^{n} T_G(t)$ – sum of times spent in readiness states;

$\sum_{i=1}^{m} T_N(t)$ – sum of times spent in non-readiness states.

The time of effective flight execution, in accordance with Fig. 1 was T_0 = 480 minutes. The total duration of individual states S_i were different for each scenario. For this reason, it was necessary to calculate the total presence times of a sample of 16 vehicles in the states $S_1 - S_7$ - they are summarised in Table 6.

Table 6. Total presence times of the sample of 16 vehicles in the states S_1-S_7 (in minutes).

T_i	T_1	T_2	T_3	T_4	T_5	T_6	T_7	Sum
[min.]	150	540	95	227.65	1004.267	40	343.083	2400
[%]	6.25	22.5	3.958	9.485	41.84	1.667	14.295	100

The value of the technical readiness factor for a sample of 16 vehicles, calculated on the basis of the total presence times in individual operating states, was $k_{gt} = \frac{T_4+T_7}{\sum_{i=1}^{7} T_i} = 0.2378$ and it was also not very high. Moreover, it was lower (by approx. 14.42%) compared to the value of the readiness index determined on the basis of the value of the limiting probabilities $K_{gt} = 0.382$. The differences in interpretation are the readiness factor k_{gt} reflects the results for the tested sample, its value may understandably vary between samples. The value of the readiness index K_{gt} is theoretical and represents the probability at infinity to which the readiness factor is predicted to tend. A positive premise is the forecast of an increase in the value of the index (k_{gt}) to the index value (K_{gt}) calculated for infinity).

The total presence times of the sample of vehicles in individual states, summarised in Table 6, also present optimisation tips for the readiness index value. According to them, the total times of the vehicle's refuelling cycle have the greatest optimisation potential T_5 and fuel separation T_2.

To sum up, the results obtained in this study confirm the validity of the use and effectiveness of Markov processes for the analysis and assessment of basic reliability indexes (including readiness). The obtained results show that in military systems, generally high values of the object readiness are not required or achievable in all processes. The above proposition is documented by the relatively low values of the coefficient and readiness index of vehicles supplying aviation fuel to aircraft during combat flights obtained in this study. Their values result from the planned organisation of the process under study, containing necessary states that are at the same time undesirable because they ultimately reduce the readiness of the object. Despite the relatively low readiness, the vehicles effectively ensure the continuity of aviation fuel supplies to aircraft. This is due to maintaining equipment redundancy (in practice, a surplus number of such vehicles is kept on duty during combat flights), as described in more detail in [52]

Acknowledgements

This research was funded by the Military University of Technology, grant number UGB 22-835.

References

1. Adjetey-Bahun K, Birregah B, Châtelet E, Planchet J-L. A model to quantify the resilience of mass railway transportation systems. Reliability Engineering & System Safety 2016; 153: 1–14, https://doi.org/10.1016/j.ress.2016.03.015.
2. Ahmad F, Tang X-W, Qiu J-N et al. Prediction of slope stability using Tree Augmented Naive-Bayes classifier: Modeling and performance evaluation. Mathematical Biosciences and Engineering 2022; 19(5): 4526–4546, https://doi.org/10.3934/mbe.2022209.
3. Ahmad M, Ahmad F, Wróblewski P et al. Prediction of ultimate bearing capacity of shallow foundations on cohesionless soils: A gaussian process regression approach. Applied Sciences (Switzerland) 2021. doi:10.3390/app112110317, https://doi.org/10.3390/app112110317.
4. Aulin V, Lyashuk O, Pavlenko O et al. Realization of the logistic approach in the international cargo delivery system. Communications - Scientific Letters of the University of Žilina 2019; 21(2): 3–12. https://doi.org/10.26552/com.C.2019.2.3-12
5. Blank C, Park D K, Petruccione F. Quantum-enhanced analysis of discrete stochastic processes. npj Quantum Information 2021. doi:10.1038/s41534-021-00459-2, https://doi.org/10.1038/s41534-021-00459-2.
6. Borucka A. Method of testing the readiness of means of transport with the use of semi-markov processes. Transport 2021; 36(1): 75–83, https://doi.org/10.3846/transport.2021.14370.
7. Borucka A, Kozłowski E, Parczewski R et al. Supply Sequence Modelling Using Hidden Markov Models. Applied Sciences 2022; 13(1): 231, https://doi.org/10.3390/app13010231.
8. Çekyay B, Özekici S. MTTF and availability of semi-Markov missions with non-identical generally distributed component lifetimes. Stochastic Models 2023; 39(2): 414–447, https://doi.org/10.1080/15326349.2022.2112225.
9. Cevasco D, Koukoura S, Kolios A J. Reliability, availability, maintainability data review for the identification of trends in offshore wind energy applications. Renewable and Sustainable Energy Reviews 2021. doi:10.1016/j.rser.2020.110414, https://doi.org/10.1016/j.rser.2020.110414.
10. Cole R, Egan S J, Schwartz J, Rudinsky S L. The Impact of High-fidelity Simulations on Medical Student Readiness. Military Medicine 2023; 188: 7–14, https://doi.org/10.1093/milmed/usac382.
11. Cui L, Chen J, Li X. Balanced reliability systems under Markov processes. IISE Transactions 2019; 51(9): 1025–1035, https://doi.org/10.1080/24725854.2018.1536304.
12. Durán O, Aguilar J, Capaldo A, Arata A. Fleet resilience: evaluating maintenance strategies in critical equipment. Applied Sciences (Switzerland) 2021; 11(1): 1–17, https://doi.org/10.3390/app11010038.
13. Durczak K, Rybacki P, Sujak A. Application of a Selected Pseudorandom Number Generator for the Reliability of Farm Tractors. Applied Sciences (Switzerland) 2022. doi:10.3390/app122312452, https://doi.org/10.3390/app122312452.
14. Englezos P. Technology readiness level of gas hydrate technologies. Canadian Journal of Chemical Engineering 2023; 101(6): 3034–3043, https://doi.org/10.1002/cjce.24673.
15. Gołda P, Zawisza T, Izdebski M. Evaluation of efficiency and reliability of airport processes using simulation tools. Eksploatacja i Niezawodnosc 2021; 23(4): 659–669, https://doi.org/10.17531/ein.2021.4.8.
16. Gourieroux C, Jasiak J. Time varying Markov process with partially observed aggregate data: An application to coronavirus. Journal of Econometrics 2020. doi:10.1016/j.jeconom.2020.09.007, https://doi.org/10.1016/j.jeconom.2020.09.007.
17. Hoang T N, Nguyen T P, Pham M P et al. Assessment of availability, readiness, and challenges for scaling-up hypertension management

services at primary healthcare facilities, Central Highland region, Vietnam, 2020. BMC Primary Care 2023. doi:10.1186/s12875-023-02092-8, https://doi.org/10.1186/s12875-023-02092-8.

18. Horning S, Leung P, Fitzgerald A, Mrad N. Operational readiness simulator: Optimizing operational availability using a virtual environment. International Journal of Aerospace Engineering 2012. doi:10.1155/2012/425075, https://doi.org/10.1155/2012/425075.
19. Huang F, Chen Y, Chen Y, Sun H. Stochastic collocation for optimal control problems with stochastic PDE constraints by meshless techniques. Journal of Mathematical Analysis and Applications 2024. doi:10.1016/j.jmaa.2023.127634, https://doi.org/10.1016/j.jmaa.2023.127634.
20. Isaac N, Saha A K. Analysis of refueling behavior of hydrogen fuel vehicles through a stochastic model using Markov Chain Process. Renewable and Sustainable Energy Reviews 2021. doi:10.1016/j.rser.2021.110761, https://doi.org/10.1016/j.rser.2021.110761.
21. Jiang Y, Lan G. Convergence and exponential stability of modified truncated Milstein method for stochastic differential equations. Journal of Computational and Applied Mathematics 2024. doi:10.1016/j.cam.2023.115288, https://doi.org/10.1016/j.cam.2023.115288.
22. Klimczak T, Paś J, Duer S et al. Selected Issues Associated with the Operational and Power Supply Reliability of Fire Alarm Systems. Energies 2022. doi:10.3390/en15228409, https://doi.org/10.3390/en15228409.
23. Koltun K J, Bird M B, Forse J N, Nindl B C. Physiological biomarker monitoring during arduous military training: Maintaining readiness and performance. Journal of Science and Medicine in Sport 2023; 26: S64–S70, https://doi.org/10.1016/j.jsams.2022.12.005.
24. Kou L, Qin Y, Jia L, Fu Y. Multistate Reliability Evaluation of Bogie on High Speed Railway Vehicle Based on the Network Flow Theory. International Journal of Software Engineering and Knowledge Engineering 2018; 28(04): 431–451, https://doi.org/10.1142/S0218194018400053.
25. Kováč J, Gregor I, Melicherčík J, Kuvik T. Analysis of the Operating Parameters of Wood Transport Vehicles from the Point of View of Operational Reliability. Forests 2023. doi:10.3390/f14071511, https://doi.org/10.3390/f14071511.
26. Kozłowski E, Borucka A, Świderski A. Application of the logistic regression for determining transition probability matrix of operating states in the transport systems. Eksploatacja i Niezawodnosc 2020; 22(2): 192–200, https://doi.org/10.17531/ein.2020.2.2.
27. Lech B, Paweł S. Resilience Assessment Of Heterogeneous Complex Transport Networks – A General Framework And A Case Study. Proceedings of the 29th European Safety and Reliability Conference (ESREL), Research Publishing Services: 2019: 1381–1388, https://doi.org/10.3850/978-981-11-2724-3_0336-cd.
28. Li Y, Vo L, Wang G. Higher order time discretization method for a class of semilinear stochastic partial differential equations with multiplicative noise. Journal of Computational and Applied Mathematics 2024. doi:10.1016/j.cam.2023.115442, https://doi.org/10.1016/j.cam.2023.115442.
29. Lopes D, Ferreira J, Rafael S et al. High-resolution multi-scale air pollution system: Evaluation of modelling performance and emission control strategies. Journal of Environmental Sciences (China) 2024; 137: 65–81, https://doi.org/10.1016/j.jes.2023.02.046.
30. Ma S, Ma X, Xia L. A unified algorithm framework for mean-variance optimization in discounted Markov decision processes. European Journal of Operational Research 2023; 311(3): 1057–1067, https://doi.org/10.1016/j.ejor.2023.06.022.
31. Maan V S, Saini M, Kumar A. Investigation of fuzzy semi-Markovian model for single unit systems with partial failure and Weibull distributed random laws. International Journal of Information Technology (Singapore) 2022; 14(6): 2971–2980, https://doi.org/10.1007/s41870-022-01070-0.
32. Maruotti A, Punzo A. Initialization of Hidden Markov and Semi-Markov Models: A Critical Evaluation of Several Strategies. International Statistical Review 2021; 89(3): 447–480, https://doi.org/10.1111/insr.12436.
33. Musa I M, Yusuf I. Reliability analysis of a small solar system for a home. International Journal of Quality and Reliability Management 2023; 40(1): 267–279, https://doi.org/10.1108/IJQRM-10-2020-0336.
34. Paś J. Issues Related to Power Supply Reliability in Integrated Electronic Security Systems Operated in Buildings and Vast Areas. Energies 2023. doi:10.3390/en16083351, https://doi.org/10.3390/en16083351.
35. Restel F J. The Markov reliability and safety model of the railway transportation system. 2015: 303–311, https://doi.org/10.1201/b17399-46.
36. Shen H, Zhang Y, Wang J et al. Observer-Based Control for Discrete-Time Hidden Semi-Markov Jump Systems. IEEE Transactions on Automatic Control 2022: 1–7, https://doi.org/10.1109/TAC.2022.3229959.

37. Song Y-K, Chung E K, Lee Y S et al. Objective structured clinical examination as a competency assessment tool of students' readiness for advanced pharmacy practice experiences in South Korea: a pilot study. BMC Medical Education 2023. doi:10.1186/s12909-023-04226-z, https://doi.org/10.1186/s12909-023-04226-z.
38. Świderski A, Borucka A, Grzelak M, Gil L. Evaluation of machinery readiness using semi-Markov processes. Applied Sciences (Switzerland) 2020. doi:10.3390/app10041541, https://doi.org/10.3390/app10041541.
39. Vindel E, Akinci B, Bergés M. A critical investigation of the readiness of VAV systems to adopt grid-interactive capabilities. Energy and Buildings 2023. doi:10.1016/j.enbuild.2023.112974, https://doi.org/10.1016/j.enbuild.2023.112974.
40. Vorapojpisut S, Agrawal K. Modeling of Manufacturing Processes using Hidden Semi-Markov Model and RSSI data. 2022. doi:10.1109/iSAI-NLP56921.2022.9960270, https://doi.org/10.1109/iSAI-NLP56921.2022.9960270.
41. Wang J, Miao Y. Optimal preventive maintenance policy of the balanced system under the semi-Markov model. Reliability Engineering and System Safety 2021. doi:10.1016/j.ress.2021.107690, https://doi.org/10.1016/j.ress.2021.107690.
42. Wang N, Wu M, Yuen K F. A novel method to assess urban multimodal transportation system resilience considering passenger demand and infrastructure supply. Reliability Engineering & System Safety 2023; 238: 109478, https://doi.org/10.1016/j.ress.2023.109478.
43. Wawrzyński W, Zieja M, Tomaszewska J, Michalski M. Reliability assessment of aircraft commutators. Energies 2021. doi:10.3390/en14217404, https://doi.org/10.3390/en14217404.
44. Wu L, Shen Y, Che F et al. Evaluating the performance and influencing factors of three portable black carbon monitors for field measurement. Journal of Environmental Sciences (China) 2024; 139: 320–333, https://doi.org/10.1016/j.jes.2023.05.044.
45. Xiong X, Sha J, Jin L. Optimizing coordinated vehicle platooning: An analytical approach based on stochastic dynamic programming. Transportation Research Part B: Methodological 2021; 150: 482–502, https://doi.org/10.1016/j.trb.2021.06.009.
46. Yeh C-T, Lin Y-K, Yeng L C-L, Huang P-T. Reliability evaluation of a multistate railway transportation network from the perspective of a travel agent. Reliability Engineering & System Safety 2021; 214: 107757, https://doi.org/10.1016/j.ress.2021.107757.
47. Yonge J, Schaetzel S, Paull J et al. Optimizing combat readiness for military surgeons without trauma fellowship training: Engaging the "voluntary faculty" model. The journal of trauma and acute care surgery 2023; 95(2S Suppl 1): S31–S35, https://doi.org/10.1097/TA.0000000000004040.
48. Zhang Q, Yu S, Han Y, Li Y. Research on the model of a multistate aggregated Markov repairable system. Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability 2022; 236(2): 266–276, https://doi.org/10.1177/1748006X19887651.
49. Zieja M, Ważny M, Jasztal M, Stępień S. Estimation of reliability for aircraft systems as regards the impact of destructive ageing processes. 2020: 2372–2377, https://doi.org/10.3850/978-981-11-2724-3_0630-cd.
50. Ziółkowski J, Lęgas A, Szymczyk E et al. Optimization of the Delivery Time within the Distribution Network, Taking into Account Fuel Consumption and the Level of Carbon Dioxide Emissions into the Atmosphere. Energies 2022. doi:10.3390/en15145198, https://doi.org/10.3390/en15145198.
51. Ziółkowski J, Małachowski J, Oszczypała M et al. Simulation model for analysis and evaluation of selected measures of the helicopter's readiness. Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering 2022. doi:10.1177/09544100211069180, https://doi.org/10.1177/09544100211069180.
52. Ziółkowski J, Żurek J, Małachowski J et al. Method for Calculating the Required Number of Transport Vehicles Supplying Aviation Fuel to Aircraft during Combat Tasks. Sustainability (Switzerland) 2022. doi:10.3390/su14031619, https://doi.org/10.3390/su14031619.
53. Żurek J, Zieja M, Ziółkowski J. Reliability of supplies in a manufacturing enterprise. 2018: 3143–3148. https://doi.org/10.1201/9781351174664-393
54. Żyluk A, Zieja M, Grzesik N et al. Implementation of the Mean Time to Failure Indicator in the Control of the Logistical Support of the Operation Process. Applied Sciences (Switzerland) 2023. doi:10.3390/app13074608, https://doi.org/10.3390/app13074608.