

# IMPROVEMENT IN ACCURACY OF DETERMINING A VESSEL'S POSITION WITH THE USE OF NEURAL NETWORKS AND ROBUST M-ESTIMATION

Krzysztof Czaplewski

Department of Geodesy and Oceanography, Gdynia Maritime University, Poland

Mariusz Wąż

Institute of Navigation and Hydrography, Polish Naval Academy

## ABSTRACT

*In the 21st century marine navigation has become dominated by satellite positioning systems and automated navigational processes. Today, global navigation satellite systems (GNSS) play a central role in the process of carrying out basic navigational tasks, e.g. determining the coordinates of a vessel's position at sea. Since satellite systems are being used increasingly more often in everyday life, the signals they send are becoming more and more prone to jamming. Therefore there is a need to search for other positioning systems and methods that would be as accurate and fast as the existing satellite systems. On the other hand, the automation process makes it possible to conduct navigational tasks more quickly. Due to the development of this technology, all kinds of navigation equipment can be used in the process of automating navigation. This also applies to marine radars, which are characterised by a relatively high accuracy that allows them to replace satellite systems in performing classic navigational tasks. By employing M-estimation methods that are used in geodesy as well as simple neural networks, a software package can be created that will aid in automating navigation and will provide highly accurate information about a given object's position at sea by making use of radar in comparative navigation.*

*This paper presents proposals for automating the process of determining a vessel's position at sea by using comparative navigation methods that are based on simple neural networks and geodetic M-estimation methods.*

**Keywords:** navigation, neural network, radar, M-estimation

## INTRODUCTION

For over 20 years, satellite positioning systems have been an indispensable part in the lives of humans, not only with regard to determining a given object's position on the Earth's surface but also as regards other spheres of human life. The rapid development of satellite positioning systems has been accompanied by the process of automating and robotising a majority of tasks that have, until recently, been carried out by human beings. Human life that has been changing this way is always fraught with many new dangers. On the one hand, the process of automating everyday life increasingly leads humans to lose their influence over the functioning of the devices around them. On the other hand, the temptation to exert control over various systems and devices in the world has been growing ever stronger as part of the struggle between different groups of influence which set various objectives for themselves. Increasingly more often it can be observed that

the changing political situation in the world and the rapid development of technology, which is becoming more and more widely available to all social groups, are both creating conditions that make it easier to interfere with and jam, for example, satellite positioning systems. Over the last several years, certain initiatives have been developed to counteract such practices. To this end, one should also search for and compare new, automated and accurate positioning methods which would be based on existing or new positioning systems.

A GNSS receiver is the basic tool for determining a vessel's position at sea. This receiver must work properly and the system it utilises must be reliable. Additionally, radars are also commonly used on vessels. Navigation that employs radar-based positioning methods is full of errors that affect the quality of the obtained position fix. These errors are indirectly related to the resolving power of a given navigation radar system. Moreover, many distortions in radar images, which are characteristic of radar observations, result from radar

radiation, the duration of the probing pulse and beam width, i.e. a directional characteristic. There are also distortions that are caused by the non-linearity of time-base pulses as well as distortions resulting from variability in wave propagation conditions and hydrometeorological conditions. As a result of the development of techniques and technologies for building radars, the above-mentioned gross errors now affect radar observations to a much lesser extent than they did 10–15 years ago. The resolutions that have been adopted by the IMO require that more than one available positioning system be used on vessels, which is why it seems very natural to also employ radar.

Comparative navigation, which is currently developing very rapidly, is one of the new branches of navigation. This discipline deals with searching for non-satellite positioning methods. According to the authors of this paper, the comparative methods that are used in navigation can form the basis for alternative systems for determining vessel positions. As for radar navigation, comparative methods are used to precisely match radar images with the nautical chart. It is important to demonstrate the mutual relations between radar and nautical chart images when adopting these methods. Thus it is necessary to convert radar images into a form that is as close as possible to that of nautical chart images because this will facilitate and speed up the process of matching these images. Another solution is to create a radar map based on archival radar images showing a selected sea area, which would make it possible to search for the best possible match between radar images and the given radar map by omitting the corresponding nautical chart with its different cartometric properties. In the authors' opinion, a perspective azimuthal projection with a positive projection point is "the best" cartographic representation of a nautical chart that makes it similar to a radar image (Wąż, 2009).

Comparative navigation is mainly based on the so-called minimum-distance methods which implement a range of similarity functions, i.e. distance, closeness and correlation functions, in order to calculate the coefficient of the best match between radar and nautical chart images (Czaplewski & Wąż, 2009). In other words, this coefficient should point to the radar image that is the "closest" to a nautical chart image in terms of a given similarity function. In comparative navigation, it is also possible to use neural networks to determine the position of the radar images in relation to the nautical chart images.

The comparative navigation methods that the authors of the present paper are familiar with are mainly based on adapted neural networks which are not able to meet the high accuracy requirements that are set for marine navigation without additional support. Modern M-estimation methods allow to increase the accuracy of the determinations. Those that are used in geodesy have proved to be interdisciplinary as well as to have a wide range of applications in navigation, which is closely related to geodesy. Moreover, robust adjustment methods allow to make the results of observations that are generally fraught with gross errors robust. Previous studies, i.e. (Czaplewski, 2014), have shown that these methods can

easily be adapted so that they can suit the purpose of different positioning techniques in marine navigation. In the studies presented herein, there are used the Artificial Neural Networks of the following types: multilayered perceptron and GRNN networks (General Regression Neural Networks). The authors indicate a possibility of combining the neural networks with robust adjustment methods into the applications, which can be successfully used in the automated ECDIS systems. Independence from the external devices of the suggested interdisciplinary method for comparative navigation results in its high resistance to jamming and suppression of the navigation systems' functioning.

## NEURAL NETWORK METHODS FOR APPROXIMATING THE POSITIONS OF RADAR IMAGES

The idea of applying artificial neural networks to determine a vessel's position by using comparative navigation methods involves using nautical chart images at the neural network training stage (Praczyk, 2006A; 2006B). A purpose-built training set (training images) can be expanded by including several radar images that cover a given sea area so as to improve the representativeness of that training set. The preliminary study that was conducted by the authors of this paper showed that it is necessary to first compress the images to a smaller size (Praczyk, 2006A; 2006B). When images were used in their original sizes, the structure of the neural network was much more complex, whereas the network training process was very long and did not produce the desirable results. Figure 1 shows this concept:

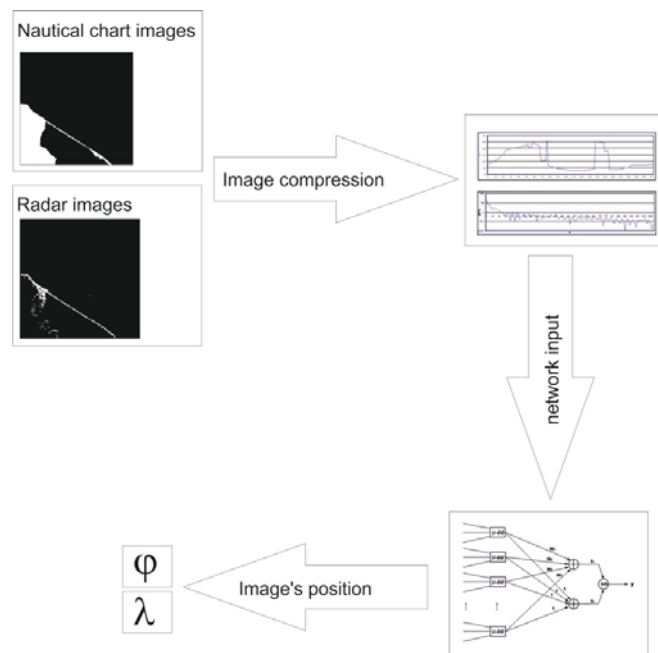


Figure 1. Using artificial neural networks to determine a vessel's position in radar navigation

## IMAGE COMPRESSION

Neural networks must be fed with images that consist of an equal number of elements at each input. Therefore, the compression of images is not only aimed at minimising the number of input data items but also at establishing a fixed value of this number which is responsible for the structure of an ANN's input layer (Praczyk, 2006B).

Projection is a simple form of image compression. When processing an original image the values of the image points along the directions of projection have to be added up. It is recommended that an image be projected in two directions for the sake of accuracy of that image's later reconstruction, e.g. recorded images can be 1000×1000 square matrices (1000000 pixels) in their original form, with each element representing a pixel's gain (level of brightness). For binary radar images these values would indicate that an elementary echo was observed (a pixel's gain = 1) or that there was no such echo (a pixel's gain = 0). Compression is aimed to reduce the number of elementary data items that are inputted into an ANN. A projection is made along two straight lines, i.e. the x- and y-axes, so that the compressed images are not similar to one another, then an image is obtained that consists of 2000 elements. The degree of radar image compression which is achieved by means of a projection along these straight lines is 1:500 (for 1000x1000 pixels).

A compressed image is inputted into a neural network. First, the values that are projected onto the x-axis are fed into the network, and then those that are projected onto the y-axis. Thus the network should have 2000, and not 1 million, input neurons for uncompressed images. An example compression process is shown in Figure 2.

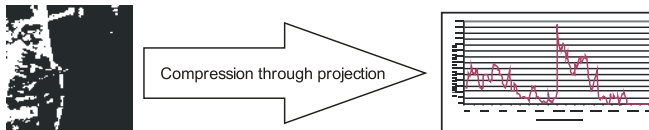


Figure 2. Compression of radar images

## TRAINING SET

A training set is the collection of images that will represent the entire range of variation among radar images for a selected sea area at the system's training stage.

Therefore, it is very important to properly select the elements of this set so as to minimise the generalisation error. A training set should be composed of radar images showing the entire radar-observed area as well as of representations of nautical chart images whose structure, scale, sizes and resolution should correspond to those of the radar images. This approach to creating a training set increases its representativeness.

## THE USE OF NEURAL NETWORKS TO DETERMINE A POSITION

In the studies presented further in the article, multilayered perceptron (MLP – multilayered perceptron) and a GRNN network were used. An MLP network has a layered architecture consisting of the input layer, output layer and one or more hidden layers. A task of the element in the input layer is the initial processing of the input signal, which may include image normalisation or scaling. The essential neural processing of the input signal takes place in the hidden layers as well as the output layers. (Stateczny, 2001; Masters, 1996).

During studies, the input MLP layer contained as many neurons, as the number of elements of which the compressed radar image consists. The output layer consisted of two neurons, the output signal of which corresponded to a value of the normalised shift vector ( $x, y$ ) of a position in relation to a position of the left lower vertex of the analyzed sea area (nautical chart), for which the positioning system was elaborated (Fig. 4). Perceptron with one and two hidden layers was analyzed. A number of neurons in the hidden layers was assorted stepwise.

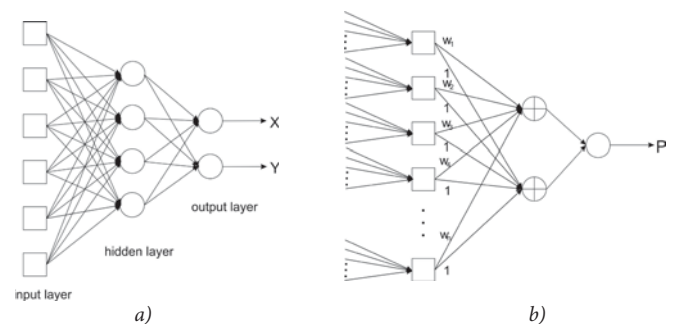


Figure 3. Exemplary diagrams of the neural networks:  
a) the MLP network b) the GRNN network

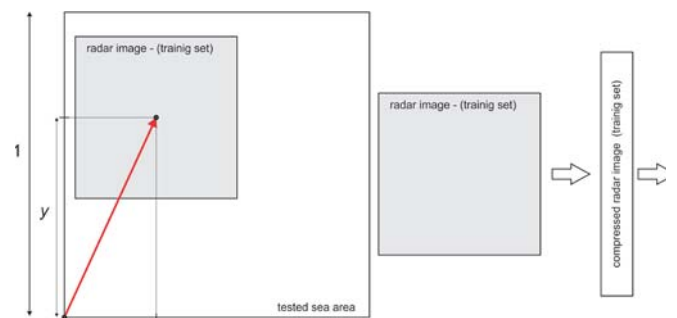


Figure 4. The use of the MLP network to determine a position of the radar image

Further studies were carried out for the GRNN network. The GRNN network is a typical approximation network, usually with one output. It is a memory network and learning is based on loading of the training images to the "network's memory".

In the studies, there was used a network with one output indicating a number of pixel P of an image of the tested sea area corresponding to a position of the radar image (Fig. 5):

$$P = (y - 1) \cdot n + x \quad (1)$$

where:

$x, y$  – coordinates of the pixels of an image of the tested sea area in the levorotatory Cartesian coordinate system,  
 $n$  – a number of pixels in the columns and rows in the image of the tested sea area.

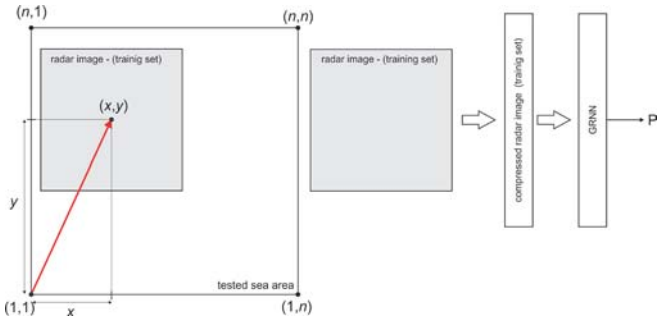


Figure 5. The use of the GRNN network to determine a position of the radar image

## ESTABLISHING THE ADJUSTMENT PROBLEM

The M-estimation methods that are employed in geodesy are also used in a similar manner to carry out navigation tasks. The bearing or distance measurements perform the function of observations in radar navigation. As for the present research problem, radar bearings were used to illustrate the process of adapting and using the neural network method and the M-estimation method together as well as to determine a vessel's position ( $X_p, Y_p$ ) at sea. Bearings  $NR_i$  were taken to distinct radar echoes. In this way,  $n$  observations were obtained (where  $n > 2$ ). The coordinates, of objects that produce distinct echoes were known. This geometrical arrangement makes it possible to first create linear matrix equations and then matrix adjustment equations, as in the papers written for example by (Czaplewski, 2004) and by (Wiśniewski, 2003):

$$NR_i + v_i = F_i(\hat{X}_p, \hat{Y}_p) = \arctg \frac{X_i - \hat{X}_p}{Y_i - \hat{Y}_p} \Bigg|_{i=1, \dots, n} \Leftrightarrow \mathbf{D} + \mathbf{V}_p = \mathbf{F}(\hat{\mathbf{X}}_p) \quad (1)$$

where:

$\mathbf{V}_p = [v_1, v_2, \dots, v_n]^T$  – vector of adjustments to the measured bearings,

$\hat{\mathbf{X}}_p = [\hat{X}_p, \hat{Y}_p]^T$  – adjusted coordinates of a vessel at sea.

It is assumed that the vector of approximate coordinates of the given vessel,  $\mathbf{X}_p^o = [X_p^o, Y_p^o]^T$ , is known. Coordinates that were determined by using the neural network method were adopted as the vessel's approximate coordinates. Then, after writing the function  $\mathbf{F}(\hat{\mathbf{X}}_p)$  in linear form by expanding it

into the Taylor series and only using the first terms of this series (Wiśniewski, 2002, 2003, 2014), we obtain

$$\mathbf{F}(\hat{\mathbf{X}}_p) = \mathbf{F}(\mathbf{X}_p^o) + \mathbf{A}_p \mathbf{d}\hat{\mathbf{X}}_p \quad (2)$$

where:

$$\mathbf{A}_p = \partial_x \mathbf{F}(\mathbf{X}_p^o), \text{ whereas } \hat{\mathbf{X}}_p = \mathbf{X}_p^o + \mathbf{d}\hat{\mathbf{X}}_p.$$

When taking into account the above assumption, the system of adjustment equations can be written as:

$$\mathbf{D} + \mathbf{V}_p = \mathbf{F}(\hat{\mathbf{X}}_p) \Leftrightarrow \mathbf{D} + \mathbf{V}_p = \mathbf{F}(\mathbf{X}_p^o) + \mathbf{A}_p \mathbf{d}\hat{\mathbf{X}}_p \Leftrightarrow \mathbf{V}_p = \mathbf{A} \mathbf{d}\hat{\mathbf{X}}_p + \mathbf{L}_p \quad (3)$$

where:

$$\mathbf{L}_p = \mathbf{F}(\mathbf{X}_p^o) - \mathbf{D}.$$

Let us assume that the mean errors  $m_1, m_2, \dots, m_i$  of the mutually independent measurements of the bearings  $NR_1, NR_2, \dots, NR_n$  are known. A diagonal cofactor matrix of the results of the measurement  $\mathbf{Q}_D = \text{Diag}(m_1^2, m_2^2, \dots, m_n^2)$  will be such an approximation of the covariance matrix  $\mathbf{C}_D = \text{Diag}(\sigma_1^2, \sigma_2^2, \dots, \sigma_n^2)$  that (Wiśniewski, 2003):

$$\mathbf{C}_D = m_0^2 \mathbf{Q}_D = m_0^2 \mathbf{P}^{-1} \quad (4)$$

where:

$m_0^2$  – is an unknown coefficient of variance,

$$\mathbf{P} = \mathbf{Q}_D^{-1} = \begin{bmatrix} p_1 & & & \\ & p_2 & & \\ & & \ddots & \\ & & & p_n \end{bmatrix} \quad (5)$$

– is a known weight matrix ( $p_n$  – weight of the  $n$ -th observation)

Let us also assume that  $\Phi(\mathbf{d}\hat{\mathbf{X}}_p) = \mathbf{V}_p^T \mathbf{C}_D^{-1} \mathbf{V}_p = \min$  is the least squares estimation criterion. Then the process of determining an unknown vector of a vessel's own position can be identified by solving the classical optimisation problem by using the method of least squares (Wiśniewski, 2003, 2014):

$$\begin{cases} \mathbf{V}_p = \mathbf{A}_p \mathbf{d}\hat{\mathbf{X}}_p + \mathbf{L}_p \\ \mathbf{C}_D = m_0^2 \mathbf{Q}_D = m_0^2 \mathbf{P}^{-1} \\ \Phi(\mathbf{d}\hat{\mathbf{X}}_p) = \mathbf{V}_p^T \mathbf{P} \mathbf{V}_p = \min \end{cases} \quad (6)$$

This is the solution to the problem (provided that  $|\mathbf{A}_p^T \mathbf{P} \mathbf{A}_p| \neq 0$ ):

$$\mathbf{d}\hat{\mathbf{X}}_p = -(\mathbf{A}_p^T \mathbf{P} \mathbf{A}_p)^{-1} \mathbf{A}_p^T \mathbf{P} \mathbf{L}_p \quad (7)$$

Moreover, since

$$\mathbf{V}_p = -\mathbf{A}_p (\mathbf{A}_p^T \mathbf{P} \mathbf{A}_p)^{-1} \mathbf{A}_p^T \mathbf{P} \mathbf{L}_p + \mathbf{L}_p = \mathbf{Q}_v \mathbf{P} \mathbf{L}_p,$$

where:  $\mathbf{Q}_v = \mathbf{P}^{-1} - \mathbf{A}_p (\mathbf{A}_p^T \mathbf{P} \mathbf{A}_p)^{-1} \mathbf{A}_p^T$  is an adjustment cofactor matrix,  $\mathbf{V}_p$

Cofactor matrix  $\mathbf{Q}_V$  is understood here as such an approximation of the covariance matrix  $\mathbf{C}_V$  of vector  $\mathbf{V}_p$  that

$$\mathbf{C}_V = m_0^2 \mathbf{Q}_V \quad (8)$$

The coefficient of variance is given by:

$$m_0^2 = \frac{\mathbf{V}_p^T \mathbf{P} \mathbf{V}_p}{n-2} \quad (9)$$

However, for the purpose of the present discussion it is assumed that  $m_0^2 = 1$ ; then  $\mathbf{C}_V = \mathbf{Q}_V$ .

Let us now assume that one of the bearings is fraught with gross error, e.g. because a radar echo was identified incorrectly. The so-called equivalent weight  $\hat{p}_i$  will be assigned to this observation; this weight will result from attenuating the original weight  $p$  (resulting from the assumed mean measurement error). The attenuation process will occur in accordance with the following formula:

$$\hat{p}_i = t(\bar{v}_i) p_i \quad (10)$$

where  $t(\bar{v}_i), i=1, \dots, n$  is an attenuation function having the following basic properties:

$$\begin{cases} t(\bar{v}_i) = 1 & \text{for } \bar{v}_i \in \Delta \bar{v}_i = \langle -k, k \rangle \\ 0 < t(\bar{v}_i) < 1 & \text{for } \bar{v}_i \notin \Delta \bar{v}_i = \langle -k, k \rangle \end{cases}$$

Ranges  $\Delta v_i = \langle -k; k \rangle$  (e.g.  $k = 2, 5$  see Wiśniewski 2014) are acceptable ranges for standardized variables  $\bar{v}_i = \frac{v_i}{\sigma_{v_i}}$  where  $\sigma_{v_i} = \sqrt{[\mathbf{C}_V]_{ii}}$  standard deviation for  $i$ -th adjustment.

For the purpose of this study, the Danish attenuation function in the following form was used in the analysis (Jianjun 1996, Hampel at all, 1986):

$$t(\bar{v}) = \begin{cases} 1 & \text{for } \bar{v} \in \langle -k, k \rangle \\ \exp\{-\lambda(|\bar{v}| - k)^g\} & \text{for } |\bar{v}| > k \end{cases} \quad (11)$$

Therefore, equivalent weight values will be determined according to the formula:

$$\hat{p}_i = t(\bar{v}_i) p_i = \begin{cases} p_i & \text{for } \bar{v}_i \in \langle -k, k \rangle \\ \exp\{-\lambda(|\bar{v}_i| - k)^g\} p_i & \text{for } |\bar{v}_i| > k \end{cases} \quad (12)$$

It is usually assumed that  $l = 0.01 \div 0.1, g = 2$ . However, the values of parameters  $l$  and  $g$  should be selected experimentally. If the values of these parameters are chosen incorrectly, this unnecessarily increases the number of steps in the iterative process which is aimed to solve the robust adjustment problem.

By using an attenuation function it is possible to formulate the following equivalent weight matrix:

$$\hat{\mathbf{P}} = \mathbf{T}(\bar{\mathbf{V}}_p) \mathbf{P} = \begin{bmatrix} t(\bar{v}_1) p_1 & & & \\ & t(\bar{v}_2) p_2 & & \\ & & \ddots & \\ & & & t(\bar{v}_n) p_n \end{bmatrix} \quad (13)$$

where:

$\mathbf{T}(\bar{\mathbf{V}}_p)$  is a diagonal attenuation matrix with elements  $[\mathbf{T}(\bar{\mathbf{V}}_p)]_i = t(\bar{v}_i)$ .

Optimisation problem Eq.(6) could then be replaced with an equivalent problem, as proposed in this paper, which would be expressed in the following form (Yang at all 2002; Zhong, 1997):

$$\begin{cases} \mathbf{V}_p = \mathbf{A}_p \mathbf{d}\hat{\mathbf{X}}_p + \mathbf{L}_p \\ \mathbf{C}_D = m_0^2 \mathbf{Q}_D = m_0^2 \mathbf{P}^{-1} \\ \hat{\mathbf{P}} = \mathbf{T}(\bar{\mathbf{V}}_p) \mathbf{P} \\ \Phi(\mathbf{d}\hat{\mathbf{X}}_p) = \mathbf{V}_p^T \hat{\mathbf{P}} \mathbf{V}_p = \mathbf{V}_p^T \mathbf{T}(\bar{\mathbf{V}}_p) \mathbf{P} \mathbf{V}_p = \min \end{cases} \quad (14)$$

and which would have an iterative solution:

$$\begin{cases} \mathbf{d}\hat{\mathbf{X}}_p^z = -(\mathbf{A}_p^T \mathbf{T}(\bar{\mathbf{V}}_p^{z-1}) \mathbf{P}^{z-1} \mathbf{A}_p)^{-1} \mathbf{A}_p^T \mathbf{T}(\bar{\mathbf{V}}_p^{z-1}) \mathbf{P}^{z-1} \mathbf{L}_p \\ \mathbf{V}_p^z = \mathbf{Q}_V^{z-1} \mathbf{T}(\bar{\mathbf{V}}_p^{z-1}) \mathbf{P}^{z-1} \mathbf{L}_p \end{cases} \quad (15)$$

It should be assumed that the iterative process consists of no more than 5 steps  $z = 1, 2, \dots, 5$ .

## SIMULATION STUDY

In order to verify the correctness of the conducted theoretical analyses, it was decided to reproduce the following navigational situation: a vessel manoeuvred on the sea area of the Bay of Gdansk, the positions of the observed vessel were determined on the basis of a sea radar. From the analysis of the radar image it was concluded that a visible image of the seashore and marking displayed in the swimming region is suitable for use in the process of determining the own position. It has adequately characteristic echoes. Therefore, the radar image was transferred to a computer in order to conduct analyses on the basis of the neural network.

The first step of the studies was a process of training the neural network and selection of its topology. In case of the MLP, there was used an algorithm of the backpropagation (backpropagation algorithm) while the GRNN network was prepared by entering the compressed radar images into its "memory". Apart from the training process, in case of the MLP it was also necessary to determine the optimal network topology. On the one hand, it was necessary to indicate the size of an input layer (different size of the compressed radar images), while on the other hand – to determine a number of hidden layers and of neurons in these layers.

During the initial studies, the aim of which was to determine a final optimal MLP topology, the following networks were checked: 2000-U1-2, 2000-U1-U2-2, 200-U1-2, where  $U_i$  is an  $i$ - hidden layer with the following number of

neurons: 20, 40, 60, 80, 100. The studies were conducted in the following conditions:

- the tested sea area, a nautical chart with a scope of 12 to 12 nautical miles, a region of the Bay of Gdansk.
- images of a training set (140 of the real radar images with a scope of observation of 6 nautical miles and 10000 simulated radar images evenly distributed on the whole tested sea area.; The simulated radar images, generated for the needs of the training set, were created from the nautical chart with a resolution of 1000×1000 pixels and covered an area of 6 to 6 nautical miles. The images were distributed evenly on the whole sea area with an interval of 10 pixels. With the assumed research sea area, it gave 100 rows of images with 100 columns in each row (10000 images).
- GSD = 158 m; GSD (Ground Sampling Distance) means the distance between two successive pixel centers measured on the ground. The bigger the value of GSD, the lower the spatial resolution of an image and less visible details. GSD will be related to the scope of observation: the bigger the scope, the bigger the value of GSD. A position error is stated in pixels and depends mainly on the resolution of the original radar images (radar images before compression) and on the scope of radar observation. GSD=158 m corresponds to the radar images of a resolution of 100×100 pixels.

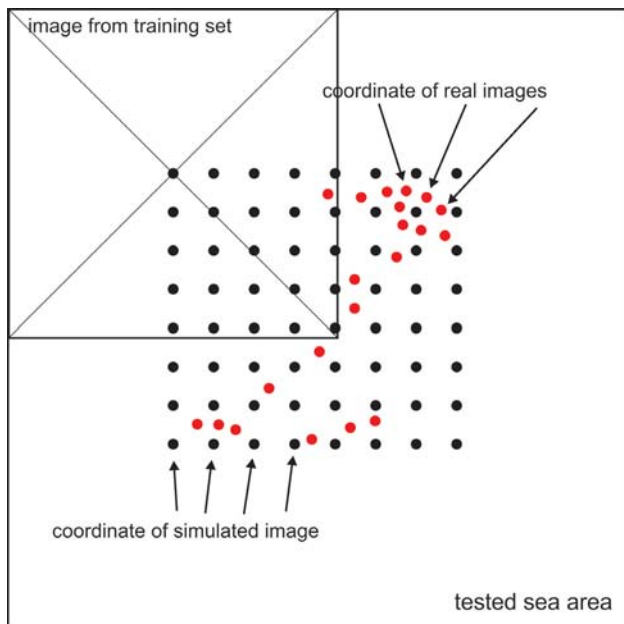


Figure 6. Structure of a training set of neural network

For further test, there was chosen one MLP network, with the lowest value of the learning error. In this case, no generalization tests were conducted on the basis of the testing set. The chosen network had the following topology: 200-20-2. The learning process of the MLP network with 2000 inputs failed.

The aim of the next stage of the studies was to determine one neural network, which will be connected with M-estimation. For this purpose, both the GRNN network of 2000 and 200

inputs, as well as the above mentioned MLP network were checked. At this stage of the studies, a criterion of a choice of the specific network was a generalization error and the same learning data were used for research as on the previous stage. Moreover, in order to verify the generalization capacities of each network, there was used a testing set consisting of 1000 randomly generated radar images and 100 real radar images not included in the training set. The obtained results are presented on the Fig. 7 and they include mean errors and maximum errors of a position, obtained for a testing set. The test results led to a situation that the only neural network which was tested with M-estimation was the GRNN network of 2000 inputs (Fig. 7b).

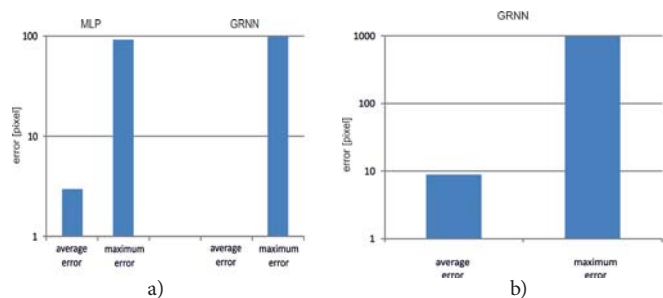


Figure 7. Obtained mean errors and maximum errors of positioning the radar images:

- a) for the input images with 200 elements;
- b) for the input images with 2000 elements

In the essential verification part of the studies, a position mean error for the GRNN network was determined at the very beginning. This error was determined on the basis of the images from the testing sequence and it amounted to 8 pixels for GSD= 15.8 m. Then, a final verification of a connection of the neural network with M-estimation was conducted. It consisted in determining the vessel's position by means of the suggested method for one real radar image, which did not occur either in the training set or in the testing sequence. This process proceeded in three stages. The first stage was a determination of an initial position observed by means of the GRNN network. Then, knowing the position's mean error for the GRNN network, there were determined the bearings and distances to the characteristic points described in the Table 1. The next stage was an application of the M-estimation for equalization of the determined bearings and distances and the finally estimated, equalized position of a vessel.

The network's response, during operation, is the observed position of a vessel at the time of recording the radar image. The entirety is transferred to a center position of the recorded radar image. Thus, it indirectly influences the position value of all the pixels of the recorded image, as well as those which are responsible for characteristic points (echoes) (e.g. the pier ends, breakwaters, buildings etc.).

On the images used during the studies, there were identified the characteristic points, the data of which are given in the Table 1 and showed on the Figure 8

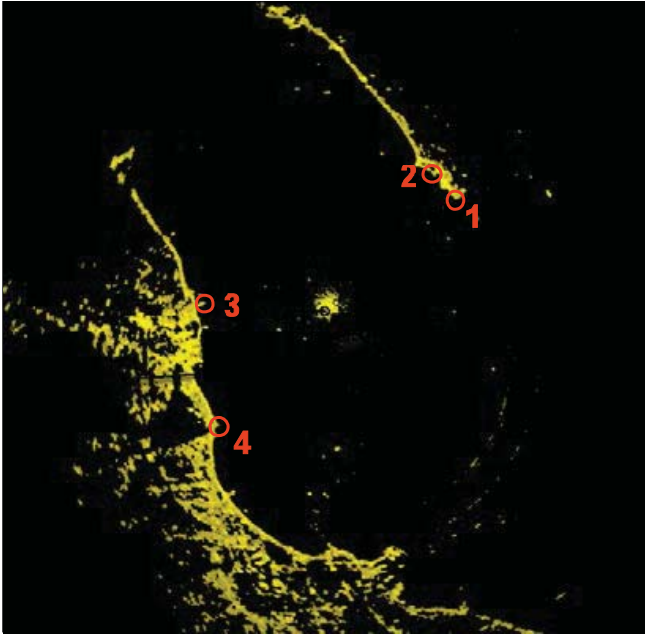


Figure 8. Radar image of the Bay of Gdansk; the echoes that were used when solving the problem are marked

Table 1. Results of the identification of echoes in the radar image

Item no.	Name of echo	Geographic coordinates		Radar bearing [°]
		$\varphi$ [°]	$\lambda$ [°]	
1	Cape Hel	54°35.6'N	018°48.6'E	059.5
2	Head of the eastern breakwater in the port of Hel	54°36.9'N	018°47.0'E	041.0
3	Degaussing station building	54°33.0'N	018°34.2'E	272.5
4	Cape Redlowo	54°29.2'N	018°34.3'E	230.0

The geographic coordinates of these echoes were read from a nautical chart. Moreover, the bearings and distances to the distinct points were determined by using artificial neural networks. Also, the coordinates of the vessel at sea were determined and were later treated as approximate coordinates of that vessel, i.e.  $\mathbf{X}_p^o = [X_p^o \ Y_p^o]^T$ . These coordinates are presented in Table 2.

Table 2. Vessel's coordinates determined at the neural network stage

Geographic coordinates		Gauss-Krüger coordinates	
$\varphi$ [°]	$\lambda$ [°]	X [m]	Y [m]
54.54918167	18.69014333	6049604.54	350542.21

The error of determination of the distance between two elements of an image is constant for a given observational range and independent of the distance between these points, while the bearing error depends on the distance between two points, which influences its value. The bearing error decreases with the increasing distance. Therefore, the navigator should make measurements relative to objects that are located at the edge of the radar range.

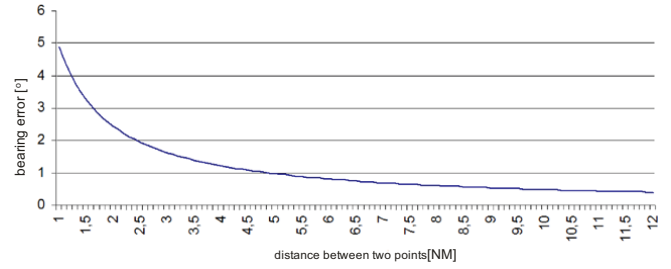


Figure 9. Error of a radar's bearing measurement made by using a GRNN neural network for a radar range of 6 nmile

While analysing the obtained data, it was decided to correct the own position using resistant bearings. For the purposes of further determinations, the adopted value of the mean error of the bearing that was determined at a distance of 6 nautical miles was  $m_{NR} = 0,8^\circ$ . In order to comprehensively illustrate the navigation problem that is presented in this paper, the solution was achieved in three ways, i.e. by:

- classically adjusting those measurements that were not fraught with gross errors;
- using observations that had been made by employing a neural network;
- using M-estimation when adjusting the observations that had been obtained by employing a neural network.

We assumed that there were bearings that were not fraught with gross error. Then the classical adjustment that was made by using the method of least squares Eq.(6) yielded the following results:

$$d\hat{\mathbf{X}}_p = \begin{bmatrix} d\hat{X}_p \\ d\hat{Y}_p \end{bmatrix} = \begin{bmatrix} -0.504 \\ -0.992 \end{bmatrix},$$

which gave the vessel's adjusted position with the coordinates:

$$\hat{\mathbf{X}} = \begin{bmatrix} \hat{X}_p \\ \hat{Y}_p \end{bmatrix} = \mathbf{X}_p^o + d\hat{\mathbf{X}}_p = \begin{bmatrix} 6049604.54 \\ 350542.21 \end{bmatrix} + \begin{bmatrix} -0.504 \\ -0.992 \end{bmatrix} = \begin{bmatrix} 6049604.36 \\ 350541.215 \end{bmatrix}.$$

However, this is a hypothetical situation since actual observations are obviously always fraught with gross errors. Therefore, when actual determinations which were obtained by employing the neural network method were used, the following results were obtained:

$$d\hat{\mathbf{X}}_p^n = \begin{bmatrix} d\hat{X}_p^n \\ d\hat{Y}_p^n \end{bmatrix} = \begin{bmatrix} 57.14 \\ 451.93 \end{bmatrix},$$

which gives the following position of the vessel at sea with these Gauss-Krüger coordinates:

$$\hat{\mathbf{X}}_p^n = \begin{bmatrix} \hat{X}_p^n \\ \hat{Y}_p^n \end{bmatrix} = \mathbf{X}_p^o + d\hat{\mathbf{X}}_p^n = \begin{bmatrix} 6049604.54 \\ 350542.21 \end{bmatrix} + \begin{bmatrix} 57.14 \\ 451.93 \end{bmatrix} = \begin{bmatrix} 6049661.68 \\ 350994.14 \end{bmatrix}.$$

A position that is obtained in this way is obviously fraught with gross error, and if it does not meet the accuracy standards that are defined by the IMO then the navigator must repeat the measurements. However, by using formulas Eq. (10)–Eq. (15) one can use observations that are fraught with gross errors

and can lower their impact on the final determinations. When setting out to make a robust adjustment in the navigational situation that is discussed here, one must determine:

a) vector of adjustments of observations:

$$\mathbf{V} = \begin{bmatrix} 1.13085 \\ -1.91204 \\ 0.57677 \\ -1.37003 \end{bmatrix}$$

b) adjustment covariance matrix (it is assumed that  $m_0^2 = 1$ ):

$$\mathbf{C}_V^{m_0^2=1} = \mathbf{Q}_V = \begin{bmatrix} 0.4563 & -0.1928 & 0.1300 & 0.1723 \\ -0.1928 & 0.3275 & -0.0986 & 0.2353 \\ 0.1300 & -0.0986 & 0.0448 & -0.0055 \\ 0.1723 & 0.2353 & -0.0055 & 0.4513 \end{bmatrix}$$

c) accepted range for standardised adjustments:

According to the IMO's recommendations, a vessel's position fix should be determined at a confidence level of at least 95%. For this confidence level  $k = 20$  Therefore:  $\Delta v = \langle -2.0; 2.0 \rangle$ ;

d) values of standardised adjustments:

$$\bar{v}_1 = \frac{v_1}{\sigma_{v_1}} = 1.6740 \in \Delta v, \quad \bar{v}_2 = \frac{v_2}{\sigma_{v_2}} = -3.3411 \notin \Delta v, \\ \bar{v}_3 = \frac{v_3}{\sigma_{v_3}} = 2.7245 \notin \Delta v, \quad \bar{v}_4 = \frac{v_4}{\sigma_{v_4}} = 0.000 \in \Delta v$$

where:  $\sigma_{v_i} = \sqrt{\left[ \mathbf{C}_V^{m_0^2=1} \right]_{ii}}$

e) values of the Danish attenuation function:

As can be observed, the standardised adjustments  $\bar{v}_2, \bar{v}_3$  fall outside the assumed acceptable range. Therefore the weights for the first three observations must be attenuated. For the successive steps in the iterative process, the Danish attenuation function will assume the following values for particular measurements:

Table 3. Values of the standardised adjustments and the attenuation function for particular observations

Iteration step	Parameters of the attenuation function		Values of the attenuation function for particular observations				Values of the standardised adjustments for particular observations			
	1	g	$t(\bar{v}_1)$	$t(\bar{v}_2)$	$t(\bar{v}_3)$	$t(\bar{v}_4)$	$\bar{v}_1$	$\bar{v}_2$	$\bar{v}_3$	$\bar{v}_4$
1	0.009	2	0.9999	0.9839	0.9952	0.9883	1.6740	-3.3411	2.7245	0.8584
2	0.09	2	1	0.8534	0.9557	1	1.6814	-3.3268	2.7090	0.8564
3	0.09	2	1	0.8784	0.9711	1	1.5499	-3.2001	2.5708	0.8360
4	1	2	1	0.0493	0.4870	1	1.5734	-3.2266	2.5997	0.8344
5	1	2	1	1	1	1	0.1734	-1.0114	0.3982	0.1991

The following vector of the vessel's coordinates is the result of a robust adjustment:

$$\hat{\mathbf{X}}_P^r = \begin{bmatrix} \hat{X}_P^r \\ \hat{Y}_P^r \end{bmatrix} = \mathbf{X}_P^o + \mathbf{d}\hat{\mathbf{X}}_P^r = \begin{bmatrix} 6049604.54 \\ 350542.21 \end{bmatrix} + \begin{bmatrix} -5.890 \\ 230.230 \end{bmatrix} = \begin{bmatrix} 6049598.65 \\ 350772.44 \end{bmatrix}$$

It should be noted that the obtained result is largely free from gross error.

## ANALYSIS OF THE TASK'S ACCURACY. TESTING THE THEORETICAL ASSUMPTIONS

In order to assess the suitability of the described method, we calculated the average errors of the fixed position. The test presented here indicates the possibility of improving the accuracy of fixing the observed position with the use of bearings to 4 navigational aids. First, the coordinates of the given vessel's position fix were determined:

a) for bearings that were not fraught with gross error:

$$\hat{\mathbf{X}}_P = \begin{bmatrix} \hat{X}_P^o \\ \hat{Y}_P^o \end{bmatrix} = \mathbf{X}_P^o + \mathbf{d}\hat{\mathbf{X}}_P^o = \begin{bmatrix} 6049604.036 \\ 350541.215 \end{bmatrix}$$

b) for observations that had been made by employing a neural network:

$$\hat{\mathbf{X}}_P^n = \begin{bmatrix} \hat{X}_P^n \\ \hat{Y}_P^n \end{bmatrix} = \mathbf{X}_P^o + \mathbf{d}\hat{\mathbf{X}}_P^n = \begin{bmatrix} 6049661.68 \\ 350994.14 \end{bmatrix}$$

c) for a robust adjustment of observations that had been obtained by employing a neural network:

$$\hat{\mathbf{X}}_P^r = \begin{bmatrix} \hat{X}_P^r \\ \hat{Y}_P^r \end{bmatrix} = \mathbf{X}_P^o + \mathbf{d}\hat{\mathbf{X}}_P^r = \begin{bmatrix} 6049598.65 \\ 350772.44 \end{bmatrix}$$

Next, the values of adjustments to the bearings for each of the three cases were determined:

$$\mathbf{V}_P = \begin{bmatrix} 0.0036 \\ 0.0018 \\ 0.0004 \end{bmatrix}, \quad \mathbf{V}_P^n = \begin{bmatrix} 1.1308 \\ -1.9120 \\ 0.5768 \end{bmatrix}, \quad \mathbf{V}_P^r = \begin{bmatrix} 0.1016 \\ -3.5526 \\ 0.1076 \end{bmatrix}$$

as well as the covariance matrices of the measurements:

$$\mathbf{C}_X = \begin{bmatrix} 0.977 & 0.399 \\ 0.399 & 0.365 \end{bmatrix}, \quad \mathbf{C}_X^n = \begin{bmatrix} 148682.322 & 60824.196 \\ 60824.196 & 55496.253 \end{bmatrix}, \quad \mathbf{C}_X^r = \begin{bmatrix} 32098.783 & 13832.646 \\ 13832.646 & 10394.835 \end{bmatrix}$$

Given the previous assumption, i.e.  $m_0^2 = 1$ , the covariance matrix takes the following form:

$$\mathbf{C}_{\hat{X}} = \begin{bmatrix} m_x^2 & \text{cov}(x, y) \\ \text{cov}(y, x) & m_y^2 \end{bmatrix}$$

Therefore, the average error of the position fix can be determined based on the formula:

$$M_X = \sqrt{\text{Tr}[(\mathbf{C}_X) (Tr - \text{matrix trace})]}$$

Thus it can be stated that the mean error of particular positions was:

a) for bearings that were not fraught with gross error:

$$M_X = [0.9771 + 0.3647]^{\frac{1}{2}} = 1,158m$$

b) for observations that were made by employing a neural network:

$$M_X^n = [148682.322 + 55496.253]^{\frac{1}{2}} = 451.861m$$

c) for a robust adjustment of observations that were obtained by employing a neural network:

$$M_X^r = [32098.783 + 10394.835]^{\frac{1}{2}} = 206.139m$$



## CONCLUSIONS

The accuracy of a position that is determined by using an ANN depends on, e.g. the resolution of the recorded images, on their quality as well as on the radar observation range.

Neural network methods are fast due to parallel information processing. A neural network with optimised parameters must be trained sufficiently in advance. Proper selection of the training set ensures that the positions of radar images will be determined with an accuracy of up to one pixel. A training set should be representative of all the radar images that are available for a given sea area.

It can be concluded from the analysis of the above application of a selected neural network and the robust adjustment methods that although it is not possible to determine a vessel's position as precisely as for error-free measurements, the quality of the determinations can still be very significantly improved without the need to repeat observations.

The results presented in this paper show that the use of artificial neural networks allows to automate the process of fixing the ship's position and that the application of the M-estimation method further enhances the quality of the observations obtained. It is obvious that the number of observed navigational aids affects the accuracy of the position. The authors conducted a study using a many system configuration of navigational aids which differed in their number of leads and geometrical arrangement. All studies point to an increase in the accuracy of the final determinations in a range comparable to the test described in section 5 of this article.

## REFERENCES

1. Alippi C. 1995. Real-time analysis of ships in radar images with neural networks, *Pattern Recognition*, Volume 28 (12), pp. 1899-1913. New York, NY, USA.
2. Caspary W., Haen W. 1990. Simultaneous Estimation of Location and Scale Parameters in the Context of Robust M-estimation. *Manuscripta Geodaetica* no. 15, pp 273–282, Italy.
3. Czaplewski, K., 2004. Positioning with interactive navigational structures implementation. *Annual of Navigation*, no. 7/2004. Gdynia, Poland.
4. Czaplewski, K., 2014. Development of the IANS chain using a satellite system. *TransNav*, pp. 485–493, Gdynia, Poland.
5. Czaplewski, K., Wąż, M., 2009. Automation of radar navigation. *European Journal of Navigation*, no 7 (2), pp. 33–39. the Netherland.
6. Granger E., Rubin M., Grossberg S., Lavoie P. 2001. A What-and-Where fusion neural network for recognition and tracking of multiple radar emitters', *Neural Networks* 14/2001, pp. 325-344, Bandera, TX, USA.
7. Hampel F.R., Ronchetti E.M., Rousseeuw P.J, Stahel W.A. 1986. *Robust Statistics. The Approach Based on Influence Functions*. John Wiley & Sons, New York, USA.
8. Jianjun Z. 1996. Robustness and the Robust Estimate. *Journal of Geodesy* no. 70, pp 586-590, Heidelberg, Germany.
9. Masters T., 1996. *Sieci neuronowe w praktyce. Programowanie w języku C++*. Wydawnictwo Naukowo Techniczne WNT, Warszawa, Poland.
10. Osowski S., 1996. *Sieci neuronowe w ujęciu algorytmicznym*. Wydawnictwo Naukowo Techniczne WNT, pp. 161–186, Warszawa, Poland.
11. Praczyk T., 2006A. Application of Neural Networks and Radar Navigational Aids of Shore Area to Positioning. *Computational Methods in Science and Technology* 12(2), DOI: 10.12921/cmst.2006.12.02. pp.149-155. Poznań, Poland.
12. Praczyk T., 2006B. Better Kohonen Neural Network in Radar Images Compression, *Computational Methods In Science And Technology* 12(2), DOI:10.12921/cmst.2006.12.02, pp.157-164, Poznań, Poland.
13. Tadeusiewicz R., 1993. *Sieci neuronowe*. Akademicka Oficyna Wydawnicza RM, pp.121–125, Warszawa, Poland.
14. Vicen-Bueno R., Carrasco-Álvarez R., Rosa-Zurera M, Nieto-Borge J. 2009. Sea Clutter Reduction and Target Enhancement by Neural Networks in a Marine Radar System, *Sensors* 9(3), pp. 1913-1936; DOI:10.3390/s90301913. Basel, Switzerland.
15. Wąż M., 2009. Precise matching of radar display with the nautical chart. *International Conference ENC-GNSS 2009*, pp. 329–334. Naples, Italy.
16. Wąż M., Meller R., 2003. Determination of ship's position by way of matching a radar image with a nautical chart using a multilayer perceptron. *International Radar Symposium IRS*, pp. 701–706, Dresden, Germany.
17. Wiśniewski Z., 2002. *Koncepcje opracowania wyników pomiarów nawigacyjnych*. Naval University of Gdynia, pp. 47–50. Gdynia, Poland.
18. Wiśniewski Z., 2003. *Metody opracowania wyników pomiarów w nawigacji i hydrografii*. Naval University of Gdynia, pp. 224 –235, 249–263. Gdynia, Poland.

19. Wiśniewski Z., 2014. Zaawansowane metody opracowania obserwacji geodezyjnych z przykładami. Wydawnictwo UWM, Olsztyn, Poland.
20. Yang Y., Song L., Xu T. 2002. Robust Estimator for Correlated Observations Based on Bifactor Equivalent Weights. Journal of Geodesy nr 76, pp 353 – 358, Heidelberg, Germany.
21. Zhong D. 1997. Robust Estimation and Optimal Selection of Polynomial Parameters for the Interpolation of GPS Geoid Heights. Journal of Geodesy no. 71, pp 552 – 561, Heidelberg, Germany.
22. Żurada J., Barski M., Jędruch W., 1996. Sztuczne sieci neuronowe. Wydawnictwo Naukowe PWN. Warszawa, Poland

#### CONTACT WITH THE AUTHOR

**Krzysztof Czaplewski**

*e-mail: krzysztof@czaplewski.pl*

Akademia Morska w Gdyni, Morska 81-87,  
81-225 Gdynia

**POLSKA**

**Mariusz Wąż**

*e-mail: m.waz@amw.gdynia.pl*

Akademia Marynarki Wojennej,  
Śmidowicza 69,  
81-103 Gdynia

**POLSKA**