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OPTIMAL DOCKING PROBLEM OF UAV AT DETECTED MOVING OBJECT

Summary. In the article, the problem of detecting a suspicious object in the control by unmanned air vehicle (UAV) and tracking it by reaching and changing its direction in the shortest period of time is explored. To solve this optimal control problem, it is considered that the flight of UAV is described with simple motion equations. In the beginning, known quantities are current coordinates and speed of UAV, equation of motion of detected suspicious object.
Keywords: drone, algorithm, optimal control, uav, docking

1. INTRODUCTION

Due to their sufficient processing and low cost, unmanned aerial vehicles are currently used to solve various problems. One of such issues is related to the protection of territories and borders. An unmanned aerial vehicle (UAV) equipped with a video camera flies along a certain route in the area where it is deployed and monitors the area. As a rule, the analysis of video images is performed in automatic mode. When the UAV detects a suspicious object moving in the area, it first reports it to the control center. Such UAVs with electric motors fly "quietly" from a sufficient height, therefore, without attracting attention, they change their trajectory and approach a suspicious object and fly over it. The main requirement put forward during this maneuver is the minimum docking time.

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In the scientific and technical literature, these types of issues are often related to the control of the process of docking ships to bridges (for example, [1, 2, 3]), connecting space vehicles to each other (for example, [4, 5, 6]) or reaching an asteroid [7]. in connection with and in other cases. Depending on the requirements, the formalization of the relevant issues and the methods of solving them are different.

In the article, the issue of optimal control of the UAV, which realizes the process of docking to the detected moving object in the minimum time without changing the flight height, is studied. It is believed that the aircraft can move in different directions while maintaining the same height while performing the video-monitoring process. The area control process is carried out by means of a video camera attached to the aircraft. If a suspicious object is detected in the video image, the movement trajectory of the detected object is determined based on those images. The essence of the docking issue is that, regardless of the direction in which the UAV is moving, if it detects a suspicious object moving on the territory at any time, it should change its direction and reach the suspicious object in the shortest possible time and start tracking it.

From a mathematical point of view, this problem is an optimal control problem with phase constraints or on time-optimal control (for example, [8, 9, 10]). Mathematical formalization of this type of problems differs from each other depending on the characteristics of the problem and the proposed conditions. Mathematical formalization of this type of problems differs from each other depending on the characteristics of the problem and the proposed conditions.

Below is given a mathematical formalization of the problem of docking an object which motion is described by simple equations on a plane to another object moving in a straight line along a known trajectory, a stable control function and a suitable optimal solution are established.

2. STATEMENT OF THE ISSUE

The investigated issue is related to the tracking of the suspicious object detected in the video camera image. From this point of view, the geographical scale of the issue is such that the area can be considered a flat part. Since the flight altitude of the UAV does not change, it flies parallel to the ground, so we will make the flight plane the same as the Earth's plane.

In order to describe the mutual position of the UAV and the suspect object, let us introduce a rectangular OXY coordinate system with respect to the ground. Let's mark the time of the docking process with $t \geq 0$ and the coordinates of the UAV as a function of time with $\mathbf{x}(t) = (x_1(t), x_2(t))$. In a simple case, the governing equations of the UAV can be written as follows:

$$\begin{cases} x_1''(t) = u_1(t), \\ x_2''(t) = u_2(t). \end{cases} \tag{1}$$

Here is the control function, which physically represents the ratio (momentum) of the propulsion force generated by the UAV's engines to its mass. As a rule, the controllability of the aircraft is limited, which means that there is a known $u_0 > 0$ quantity determined by the power of the UAV engine that,

$$u_1^2(t) + u_2^2(t) \leq u_0. \tag{2}$$

For the sake of simplicity, we can assume that the UAV is located at the origin of coordinates at the moment $t=0$, and let us denote its speed $\mathbf{V}(0)=(v_{x,1}, v_{x,2})$ at that moment. These conditions can be written as follows:

$$\begin{cases} x_1(0)=0, \\ x_2(0)=0, \end{cases} \tag{3}$$

$$\begin{cases} x_1'(0)=v_{x,1}, \\ x_2'(0)=v_{x,2}. \end{cases} \tag{4}$$

As mentioned above, the movement trajectory of the suspicious object is determined based on the camera images. We will consider that the movement of this object has the character of straight-line uniform speed movement. If we mark the coordinates of the suspicious object as a function of time, its trajectory can be expressed as follows:

$$\begin{cases} x_{1,n}(t)=a_{x,1}+b_{x,1}t, \\ x_{2,n}(t)=a_{x,2}+b_{x,2}t. \end{cases} \tag{5}$$

Here $a_{x,1}, a_{x,2}, b_{x,1}, b_{x,2}$ coefficients are known quantities. In order to track the detected object, it is necessary to choose control $\mathbf{u}(t)\equiv(u_1(t), u_2(t))$ in such a way that regardless of where the aircraft is at the moment of detection, it will change its trajectory and reach the suspicious object and begin to move along with it (parallel) and follow it. This can happen when the speed of the aircraft is greater than the speed of the detected object, i.e.

$$\sqrt{b_{x,1}^2+b_{x,2}^2}<\sqrt{2}|\mathbf{u}(t)|\leq\sqrt{2}u_0.$$

Let us mark the moment when the UAV will be controlled and reach the suspicious object with T. As the case may be, the UAV should be managed in such a way that, at the moment T its coordinates and velocity should coincide with the current coordinates and velocity of the suspect object, in other words, the following equations should be satisfied.

$$\begin{cases} x_1(T)=a_{x,1}+b_{x,1}T, \\ x_2(T)=a_{x,2}+b_{x,2}T, \end{cases} \tag{6}$$

$$\begin{cases} x_1'(T)=b_{x,1}, \\ x_2'(T)=b_{x,2}. \end{cases} \tag{7}$$

The optimal docking problem can be formulated as follows:

− It is necessary to find a control $\mathbf{u}(t)$ function that satisfies the inequality (2) so that the solution of the system of equations (1) satisfies the conditions (3)-(4), (6)-(7) and T is minimal. In other words, the UAV whose movement is given by equations (1) should be controlled in such a way that it reaches the suspect object in the shortest possible time, so $T\to\min$.

3. PROBLEM SOLUTION

In order to apply the mathematical apparatus of optimal control theory with phase constraints, let us write the system (1) as a system of first order ordinary differential equations. If we substitute $x_1'(t) = x_3(t),\ x_2'(T) = x_4(t)$, problem (1)-(7) will be as follows:

$$\begin{cases} x_1'(t) = x_3(t), \\ x_2'(t) = x_4(t), \\ x_3'(t) = u_1(t), \\ x_4'(t) = u_2(t), \end{cases} \tag{8}$$

$$\begin{cases} x_1(0) = 0, \\ x_2(0) = 0, \\ x_3(0) = v_{x,1}, \\ x_4(0) = v_{x,2}, \end{cases} \tag{9}$$

$$\begin{cases} x_1(T) = a_{x,1} + b_{x,1}T, \\ x_2(T) = a_{x,2} + b_{x,2}T, \\ x_3(T) = b_{x,1}, \\ x_4(T) = b_{x,2}. \end{cases} \tag{10}$$

Note that,

$$J \equiv \int_0^T 1 \cdot dt \tag{11}$$

$T \to \min$ requirement can be written as $J \to \min$ by including (11) functional. Then, the problem of optimal docking considering its functions is formulated as follows with respect to $x_1(t),\ x_2(t),\ x_3(t),\ x_4(t)$ functions:

– It is necessary to find such a $\mathbf{u}(t)$ control function that ensures the solution of the problem (8)-(10) so that the functional J takes a minimal value.

In order to check whether the control function is optimal, it is necessary to study the necessary conditions for optimality. For this purpose, let's construct the Lagrange function [11, p.125]:

$$\begin{gathered} L(x_1,..,x_4,x_1',..,x_4',u_1,u_2) \equiv \\ \equiv \int_0^T [\lambda_0 + y_1(t)(x_1' - x_3) + y_2(t)(x_2' - x_4) + y_3(t)(x_3' - u_1) + y_4(t)(x_4' - u_2)]dt + \\ + \lambda_1 x_1(0) + \lambda_2 x_2(0) + \lambda_3 (x_3(0) - v_{x_1,0}) + \lambda_4 (x_4(0) - v_{x_2,0}) + \\ + \lambda_5 [x_1(T) - (a_{x_1} + b_{x_1}T)] + \lambda_6 [x_2(T) - (a_{x_2} + b_{x_2}T)] + \lambda_7 x_3(T) + \lambda_8 x_4(T). \end{gathered} \tag{12}$$

Here $\lambda_0, \lambda_1, \ldots, \lambda_8$ are positive Lagrange multiples, $y_1(t), y_2(t), \ldots, y_4(t)$ are is the solution of the conjugate system of equations (13).

$$\begin{cases} -y_1'(t) = 0, \\ -y_2'(t) = 0, \\ y_3'(t) = -y_1(t), \\ y_4'(t) = -y_2(t). \end{cases} \tag{13}$$

From the transversality guarantees on (x_1, x_2, x_3, x_4), we can write values of $y_1(t), y_2(t), \ldots, y_4(t)$ functions at the moments $t = 0$ and $T = 0$.

$$\begin{cases} y_1(T) = -\lambda_5, \\ y_2(T) = -\lambda_6, \\ y_3(T) = -\lambda_7, \\ y_4(T) = -\lambda_8, \end{cases} \qquad \begin{cases} y_1(0) = \lambda_1, \\ y_2(0) = \lambda_2, \\ y_3(0) = \lambda_3, \\ y_4(0) = \lambda_4. \end{cases} \tag{14}$$

The optimality condition on *u* becomes the extremum condition written as follows for the appropriate Hamiltonian function:

$$\begin{gathered} H(x_1, x_2, x_3, x_4, y_1, y_2, y_3, y_4, u_1, u_2) \equiv . \\ \equiv y_1(t)x_1(t) + y_2(t)x_2(t) + y_3(t)u_1(t) + y_4(t)u_2(t) - \lambda_0 \to \max . \end{gathered} \tag{15}$$

If we solve (13)-(14) problem, we find $\lambda_5 = -\lambda_1, \lambda_6 = -\lambda_2, \lambda_7 = \lambda_1 T + \lambda_3, \lambda_8 = \lambda_2 T + \lambda_4$, and get:

$$\begin{cases} y_1(t) = \lambda_1, \\ y_2(t) = \lambda_2, \\ y_3(t) = -\lambda_1 t + \lambda_3, \\ y_4(t) = -\lambda_2 t + \lambda_4, \end{cases} \tag{16}$$

From (16) equalities it seems that, there are $t = \tau_1 \equiv \dfrac{\lambda_3}{\lambda_1}$ və $t = \tau_2 \equiv \dfrac{\lambda_4}{\lambda_2}$ moments, correspondingly $y_3(t)$ and $y_4(t)$ functions change their signs. This means that for the function (15) to have its maximum value, the *u* vector function must also change at those moments of time. According to the nature of the matter, $\tau_1, \tau_2 \in (0, T)$. In other words, $u_1(t), u_2(t)$ control functions can not be equal to the same constant in whole $(0, T)$ interval, so they must change their values at least one time. Let us assume that both functions change their value at the same moment $\tau \equiv \tau_1 = \tau_2$, knowing that it does not lead to a contradiction later. In other words, there is a $\tau \in (0, T)$ point that flying control $\mathbf{u}$ changes at the $t = \tau$ moment when bringing a flying object from state (8) to state (9).

3. CONSTRUCTION OF THE OPTIMAL SOLUTION

In $(0,\tau)$ and (τ,T) intervals, let us denote the values of control vectors $(u_{x_1,0}, u_{x_2,0})$ and $(u_{x_1,T}, u_{x_2,T})$ accordingly. From the management theory of extremal problems, it is known that the solution of the optimization problem within the constraint (2) is realized in the case of equality, in other words, for $u_{x_1,0}, u_{x_2,0}, u_{x_1,T}, u_{x_2,T}$ quantities:

$$\begin{cases} u_{x_1,0}^2 + u_{x_2,0}^2 = u_0^2, \\ u_{x_1,T}^2 + u_{x_2,T}^2 = u_0^2. \end{cases} \tag{18}$$

We can write the general solution of system (8) as follows:

$$\left.\begin{array}{l} x_1(t) = \begin{cases} \frac{1}{2}u_{x_1,0}t^2 + v_{x,1}t, & 0 < t < \tau, \\ \frac{1}{2}u_{x_1,T}t^2 + c_{x,1}t + e_{x,1}, & \tau < t < T, \end{cases} \\ x_2(t) = \begin{cases} \frac{1}{2}u_{x_2,0}t^2 + v_{x,2}t, & 0 < t < \tau, \\ \frac{1}{2}u_{x_2,T}t^2 + c_{x,2}t + e_{x,2}, & \tau < t < T, \end{cases} \\ x_3(t) = \begin{cases} u_{x_1,0}t + v_{x,1}, & 0 < t < \tau, \\ u_{x_1,T}t + c_{x,1}, & \tau < t < T, \end{cases} \\ x_4(t) = \begin{cases} u_{x_2,0}t + v_{x,2}, & 0 < t < \tau, \\ u_{x_2,T}t + c_{x,2}, & \tau < t < T. \end{cases} \end{array}\right\} \tag{19}$$

Here $c_{x,1}$, $c_{x,1}$, $e_{x,1}$, $e_{x,1}$ are constant quantities. Taking account conditions (10):

$$\begin{cases} \frac{1}{2}u_{x_1,T}T^2 + c_{x,1}T + e_{x,1} = a_{x,1} + b_{x,1}T, \\ \frac{1}{2}u_{x_2,T}T^2 + c_{x,2}T + e_{x,2} = a_{x,2} + b_{x,2}T, \\ u_{x_1,T}T + c_{x,1} = b_{x,1}, \\ u_{x_2,T}T + c_{x,2} = b_{x,2}. \end{cases} \tag{20}$$

On the other hand, from continuity condition of (x_1, x_2, x_3, x_4) functions at $t = \tau$ moment:

$$\begin{cases} \frac{1}{2}u_{x_1,0}\tau^2 + v_{x,1}\tau = \frac{1}{2}u_{x_1,T}\tau^2 + c_{x,1}\tau + e_{x,1}, \\ \frac{1}{2}u_{x_2,0}\tau^2 + v_{x,2}\tau = \frac{1}{2}u_{x_2,T}\tau^2 + c_{x,2}\tau + e_{x,2}, \\ u_{x_1,0}\tau + v_{x,1} = u_{x_1,T}\tau + c_{x,1}, \\ u_{x_2,0}\tau + v_{x,2} = u_{x_2,T}\tau + c_{x,2}. \end{cases} \tag{21}$$

If we eliminate variables $c_{x,1}$, $c_{x,1}$, $e_{x,1}$, $e_{x,1}$ from (20), (21) systems, by taking linear combinations of different rows, we get:

$$\begin{cases} u_{x_1,T}\left(\tau^2-T^2\right)-u_{x_1,0}\tau^2=2a_{x,1}, \\ u_{x_2,T}\left(\tau^2-T^2\right)-u_{x_2,0}\tau^2=2a_{x,2}, \\ u_{x_1,T}(T-\tau)+u_{x_1,0}\tau=b_{x,1}-v_{x,1}, \\ u_{x_2,T}(T-\tau)+u_{x_2,0}\tau=b_{x,2}-v_{x,2}. \end{cases} \tag{22}$$

Based on (22) system, $u_{x_1,0}, u_{x_1,T}, u_{x_2,0}, u_{x_2,T}$ quantities can be expressed depending on τ and T:

$$\begin{cases} u_{x_1,0}=\tau^{-1}T^{-1}\left[2a_{x,1}+(T+\tau)\left(b_{x,1}-v_{x,1}\right)\right], \\ u_{x_2,0}=\tau^{-1}T^{-1}\left[2a_{x,2}+(T+\tau)\left(b_{x,2}-v_{x,2}\right)\right], \\ u_{x_1,T}=-(T-\tau)^{-1}T^{-1}\left[2a_{x,1}+\tau\left(b_{x,1}-v_{x,1}\right)\right], \\ u_{x_2,T}=-(T-\tau)^{-1}T^{-1}\left[2a_{x,2}+\tau\left(b_{x,2}-v_{x,2}\right)\right]. \end{cases} \tag{23}$$

If we consider the expressions (23) in (18), considering the variables, the following system of nonlinear algebraic equations of the 4th order is obtained depending on τ and T:

$$\begin{cases} \left[2a_{x,1}+(T+\tau)\left(b_{x,1}-v_{x,1}\right)\right]^2+\left[2a_{x,2}+(T+\tau)\left(b_{x,2}-v_{x,2}\right)\right]^2=\tau^2T^2u_0^2, \\ \left[2a_{x,1}+\tau\left(b_{x,1}-v_{x,1}\right)\right]^2+\left[2a_{x,2}+\tau\left(b_{x,2}-v_{x,2}\right)\right]^2=(T-\tau)^2T^2u_0^2. \end{cases} \tag{24}$$

As can be seen from the system (24), the value of τ and T variables depends only on the initial data of the problem - known $v_{x,1}, v_{x,2}, a_{x,1}, b_{x,1}, a_{x,2}, b_{x,2}$ quantities and u_0. Given these quantities, the system of equations (24) can be solved by approximate calculation methods, for example, simple iterations or Newton's method. ([12, 13]).

Thus, in order to solve the problem of optimal control of the UAV, which realizes the process of landing on the detected moving object in a minimum period of time without changing the flight height, first, according to the data of the problem, from the solution of the system (24), the minimum landing time and the control change moment are calculated. Then the values of the optimal control are calculated from isa (23) system. Then the values of the optimal control are calculated from the system (23).

When it is required to construct the flight trajectory of the UAV, the values of the variables $c_{x,1}$, $c_{x,1}$, $e_{x,1}$, $e_{x,1}$ are first found from the system of linear algebraic equations (20). Then the optimal landing trajectory of the UAV is given by the system (19).

4. AN EXAMPLE OF THE DOCKING PROBLEM

Suppose that the equations of motion of the detected object are given by the following system of equations:

$$\begin{cases} x_{1,n} = 600 + 4.2 \cdot t, \\ x_{2,n} = 100 + 5.6 \cdot t. \end{cases}$$

The speed of the UAV performing the control process $v_{x,1} = -11\,[m/\sec]$, $v_{x,2} = 0$, maximum value of control $u_0 = 10.4\,[m/\sec^2]$. Solving the docking problem means that, first of all, the moment of change of the control mode according to the data τ and the moment process ends T -must be determined from (24) systems, then the values of control $(u_{x_1,0}, u_{x_2,0})$ and $(u_{x_1,T}, u_{x_2,T})$ in intervals $[0,\tau]$ and $[\tau, T]$ accordingly must be calculated with (23) formulas.

If we apply Newton iterations method to solve (24) equations systems, then $\tau = 9.4\,[\sec]$, $T = 15.6\,[\sec]$ will be found. If these values are taken into account in (23):

$$\begin{cases} u_{x_1,0} = 10.77\,[m/\sec^2], \\ u_{x_2,0} = 2.31\,[m/\sec^2], \\ u_{x_1,T} = -13.88\,[m/\sec^2], \\ u_{x_2,T} = -2.61\,[m/\sec^2]. \end{cases}$$

Thus, the optimal value of control for UAV to approach suspicious object determined in monitoring zone is calculated based on initial data of problem.

7. CONCLUSION

Thus, based on the detection of UAV with simple equations of motion, the problem of optimal approach to the object is formulated in the order of problem (1)-(5) with phase controls. The solution of the problem in the class of piecewise stable control functions is given by formulas (6). The minimum arrival time is determined from the system of equations (7) using numerical calculation methods.

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