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Investigation of temperature field in a three-phase high voltage cable system

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A modified "Pareto principle" has been applied to the analysis of the temperature field in three-phase high-voltage cables. In the paper the temperature field is described by thirteen distinct parameters including the geometrical dimensions of the system. By solving the thermal conductivity equation 2p times, it is possible to determine the parameters belonging to set A, which exert a decisive influence on the temperature of the cable core. To describe the *ABC* sets elements of linear algebra are applied. The paper's concluding notes present a preliminary optimization procedure of the system.

KEYWORDS: Pareto principle, high voltage cable, 3-phase system, core temperature

1. Introduction

The need to distribute electrical energy in urban areas is the main reason for placing power cables below the ground's surface. The maximum allowable temperature of the inner core of the power cable determines the amount of energy that can be supplied by three-phase cable lines.

Papers [1, 2, 3] present analysis of the impact of the parameters of the threephase high-voltage cable system located in the ground on the temperature of the system. Paper [1] discusses temperature distribution in the cable core as well as in the screen and on its outer sheath at different distances from the ground's surface h and in dependence on the thermal conductivity of the ground. In turn, paper [2] presents the application of both the *Pareto Principle* [8, 9] and the resultant *ABC* charts to the analysis of the temperature field in a single HV cable core located at different depths in the ground. The paper also presents the influence of a number of parameters such as air temperature above the ground $T_{\rm p}$, thermal conductivity of the ground $\lambda_{\rm z}$ as well as current load *I* on the temperature of the cable core. Paper [3] shows the distribution of the temperature field in the three-phase system by additionally analysing the impact of thermal conductivity $\lambda_{\rm b}$ of the concrete block and its air duct in which the cable is laid.

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In this paper the scope of the research relating to the three-phase high-voltage cable system has been extended to include the influence of the geometrical dimensions of the system on the temperature field.



Fig. 1. a) Three-phase high-voltage cable system (110 kV) laid in a concrete block, b) Structure of 64/110 kV copper cable [11]

Figure 1a shows the design parameters of the tested system where u_1 , u_2 , u_3 stand for the dimensions of the concrete block, R is the radius of the air duct in the concrete block, (x_1, y_1) , (x_2, y_2) and (x_3, y_3) represent the coordinates of the centres of the air ducts in which the cables are laid. The following assumption $x_3 = x_2/2$ and $\Delta w = x_2 = y_3$, $x_1 = y_1 = y_2 = 0$ has been made for the system.

2. Temperature field analysis of the system with regard to its geometric dimensions

Laying an electric cable in the ground requires a number of protective measures against such hazards as mechanical damage, rodents and unpredictable interference resulting from earthwork carried out by humans. However, placing HV cables in concrete embedded ducts additionally increases to thirteen the number of factors affecting the distribution of temperature field in the tested system.

In the cases analysed so far, the basic parameters that have exerted a decisive impact on the temperature of the cable core included the following: air temperature over the ground's surface T_p , thermal conductivity of the ground λ_z and current carrying capacity of the cables.

Additionally, this paper analyses the effect of cable placement geometry in a concrete embedded duct (concrete blocks) filled with air or partially with water. It is interesting to examine mutual impact of the distance between the individual cores on their temperature. The testing involves the use of a modified method of multi-parameter system analysis resulting from the Pareto principle which uses ABC charts to define A, B, and C sets with a decisive, medium, and minimum impact on the temperature of the core respectively [2, 7, 8, 9]. Based on the above principle, the paper investigates the impact of the thirteen parameters on the system's temperature. It follows from the previous research that the basic parameters affecting the temperature of the cable core comprise the following: $T_{\rm p}$, $\lambda_{\rm z}$ and I, thermal conductivity of dielectric $\lambda_{\rm d}$ and concrete block λ_b , thermal conductivity of air duct λ_p , thermal conductivity of cable core λ_{Cu} , convective heat transfer coefficient over the ground's surface ε . In the present research these parameters are supplemented by five other parameters that define the geometrical dimensions of the system, namely the parameters describing the distances of the air ducts from the side surfaces of the concrete block, i.e. u_1, u_2, u_3 , the radius of the air ducts in the concrete block as well as the ducts' configuration Δw (Fig. 1a). Hence, the temperature of the three-phase cable system T_r will ultimately depend on the following factors:

$$T_r = f(\lambda_z, T_p, I, \varepsilon, \lambda_d, \lambda_{Cu}, \lambda_p, \lambda_b, u_1, u_2, u_3, R, \Delta w)$$
(1)

To carry out the analysis of the three-phase system we used a 64/110 kV (IEC 60840) power cable (made by *Tele-Fonika Kable*, LTD) with the following basic specifications: current and long-term rating I = 1140 A, maximum allowable core temperature 90^oC.

Figure 1a shows the structure of the cable [11]. The steady state temperature field of the system shown in Fig. 1a is described by the equation for thermal conductivity [4]:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = -\frac{g}{\lambda}$$
(2)

where: $g = j^2 \cdot \rho \, [W/m^3]$ is the efficiency of spatial heat sources in the cable cores, $\lambda \, [W/(m \cdot K)]$ represents the thermal conductivity of individual components and T = T(x, y) denotes the temperature distribution in the system.

To analyse the system, equation (2) has been resolved 2p times using a finite element method, where p stands for the number of the analysed parameters [5]. In the analysis of equation (2) we used a numerical model presented in Fig. 2 which also shows the boundary conditions discussed in papers [1, 2, 3]. The computational model assumed $T_0 = 8^{0}$ C at the bottom edge, whereas T_{p} and T_{s} at the top edge represent air temperature and the temperature of the ground's surface respectively, ε [W/(m²·K)] is the convective heat transfer coefficient (describing wind speed above the ground's surface [9]), Figure 1a.

An exemplary distribution of the system's temperature field with the parameters presented in Table 1 located at the depth of 1 m is shown in Fig. 3.



Fig. 2. Three-phase HV cable laid underground in a concrete block. Diagram of the numerical model with marked boundary conditions



Fig. 3. Temperature field distribution of the three-phase system (h = 1 m)

3. The Pareto principle and ABC charts of the system

The analysis of the temperature distribution in the system uses a modified method based on the *Pareto Rule* [2, 7, 8]. The assumed ranges of parameter changes affecting the temperature of the cable core have been called *base change ranges* of the analysed parameters.

Def. 1 Changes of basic parameters affecting the temperature distribution in the cable core resulting from varying physical conditions occurring in Polish conditions or assumed percentage changes resulting from the production technology are is called "base change ranges".

Temperature distribution occurring in the HV cable core due to a very small gradient is characterized by maximum temperature and is called cable core temperature.

Making use of the *Pareto Rule* and *ABC* charts, we performed a *Pareto-ABC* analysis of the power system using the following dependences [2]:

$$T_{k,w} = \frac{T_{k,max} - T_{k,min}}{T_{k,max}}, \quad (k = 1,...,p)$$
(3)

where $T_{k,w}$ is the relative temperature of the top core of the k^{th} parameter, $T_{k,max}$, $T_{k,min}$ are maximum and minimum temperature values of the core resulting from the base change ranges of the parameters.

$$T_{s} = \sum_{k} T_{k,w} = \sum_{k} \frac{T_{k,\max} - T_{k,\min}}{T_{k,\max}}$$
(4)

where T_s is the sum of relative core temperature changes

$$S_k = \frac{T_{k,w}}{\sum_k T_{k,w}}$$
(5)

 S_k is the element of the cumulative temperature value being, at the same time, the weight coefficient b_k .

$$S = \sum_{k} S_{k} = \sum_{k} b_{k} = 1, \qquad (6)$$

where S is the relative cumulative temperature value in the cable core.

The values of the parameters used for the analysis of the temperature field shown in Fig. 1a are presented in Table 1. They represent the system's rated values.

The placement depths of the three-phase system varied from 1 m to 20 m [2]. The ranges of base parameter changes assumed for the computations as well as the denotations of particular parameters (k = 1, ..., 13) is shown in Table 2.

Symbol	Value	Unit	Parameter		
T_p	+30	⁰ C	Air temperature		
λ_z	1	W/(m·K)	Thermal conductivity of the ground		
З	16.6	$W/(m^2 \cdot K)$	Convective heat transfer coefficient above		
			the ground		
Ι	1140	А	Long-term cable load		
λ_d	3	W/(m·K)	Thermal conductivity of the dielectric		
λ_{cu}	360	W/(m·K)	Thermal conductivity of the cable core		
λ_p	0.6	W/(m·K)	Thermal conductivity of the air duct [10]		
λ_b	0.7	W/(m·K)	Thermal conductivity of concrete [10]		
P_{Cu}	1.75E-8	Ω·m	Copper resistivity		
R	0.16	m	Radius of air ducts		
x_2	0.43	m	Geometric distance x from the duct centre (axis)		
<i>Y</i> 3	0.43	m	Geometric distance y from the duct centre (axis)		
u_{I}	0.2	m	Distance of air ducts from the side surface		
			of the concrete block - bottom		
u_2	0.2	m	Distance of air ducts from the side surface		
			of the concrete block – lateral		
u_3	0.2	m	Distance of air ducts from the side surface		
			of the concrete block - top		

Table 1. Parameters used for the for the analysis of the temperature field of the three-phase system

By changing the placement of the cable system under the surface of the ground in the range from 1 m to 20 m, it was possible to calculate cumulative temperature values *S* and relative temperatures T_w by using both the procedure developed for a single cable laid underground [2] and equations (3) to (6). On this basis we developed matrix $\boldsymbol{B} = [b_{ij}]$, where i = 1, ..., 7 is, at the same time, distance *h* of the system from the ground's surface, whereas j = 1, ..., 13 stands for the number of the considered parameters.

*a*1 a2а3 a4а5 a6 а7 *a*8 a9 a10 *a*11 *a*12 a13 0.4138 0.0008 0.1179 0.2547 0.0079 0.0000 0.0669 0.1118 0.0044 0.0054 0.0065 0.0011 0.0087 $h_1 = 1 \,\mathrm{m}$ $h_2 = 2m$ $0.3262 \quad 0.0009 \quad 0.1406 \quad 0.3018 \quad 0.0087 \quad 0.0000 \quad 0.0747 \quad 0.1203 \quad 0.0047 \quad 0.0059 \quad 0.0070 \quad 0.0012 \quad 0.0082 \quad 0.0082 \quad 0.0012 \quad 0.0012 \quad 0.0082 \quad 0.0012 \quad 0$ $h_3 = 3m$ $0.2730 \quad 0.0006 \quad 0.1548 \quad 0.3275 \quad 0.0091 \quad 0.0000 \quad 0.0796 \quad 0.1268 \quad 0.0050 \quad 0.0062 \quad 0.0087 \quad 0.0012$ 0.0074 $h_4 = 4m$ 0.0078 $0.0093 \mid h_5 = 5m$ $0.2066 \quad 0.0005 \quad 0.1712 \quad 0.3549 \quad 0.0116 \quad 0.0000 \quad 0.0854 \quad 0.1352 \quad 0.0053 \quad 0.0066$ 0.0106 0.0027 $0.1292 \quad 0.0002 \quad 0.1890 \quad 0.3826 \quad 0.0107 \quad 0.0000 \quad 0.0917 \quad 0.1494 \quad 0.0157 \quad 0.0072 \quad 0.0100 \quad 0.0014 \quad 0.0129 \quad \left| \begin{array}{c} h_6 = 10m_{10} + 10m_{10} +$ $0.0139 h_7 = 20m$ $0.0762 \quad 0.0000 \quad 0.2024 \quad 0.4031 \quad 0.0116 \quad 0.0002 \quad 0.0972 \quad 0.1553 \quad 0.0169 \quad 0.0078 \quad 0.0108 \quad 0.0047$

(7)

Coefficients b_k are equal to the elements of the cumulated temperature values $b_k = S_k$ and $\sum_k b_k = S = I$. It should be noted that, for instance, for $h_1 = 1$ m,

 b_1 is equal to element b_{11} of matrix **B**, $(b_1 = b_{11})$.

Para- meter	Name	Symbol	Unit	Minimum value	Maximum value
al	Air Temperature	T_n	⁰ C	-30	+30
a2	Convective heat transfer coefficient [10]	е З	$W/(m^2 \cdot K)$	16.6	150
a3	Long-term cable load	Ι	А	1026	1254
a4	Thermal conductivity of the ground	λ_z	W/(m·K)	0.2	1.4
a5	Thermal conductivity of the dielectric	λ_d	W/(m·K)	2	5
a6	Thermal conductivity of the cable core	λ_{Cu}	W/(m·K)	360	400
a7	Thermal conductivity of concrete [6]	λ_b	W/(m·K)	0.7	1.4
a8	Thermal conductivity of the air duct [6]	λ_p	W/(m·K)	0.2	1.4
a9	Geometric distances of air duct centres $\Delta x[m]$ (20% change)	$\triangle w$	m	0.36	0.5
a10	Distances of air duct centres from side surfaces of concrete block - bottom	<i>u</i> ₁	m	0.16	0.24
a11	Distances of air duct centres from side surface of concrete block - lateral	u_2	m	0.16	0.24
a12	Distances of air duct centres from side surface of concrete block - top	U_3	m	0.16	0.24
a13	Radius of air ducts	R	m	0.12	0.19

Table 2. The range of base parameter changes for the three-phase system laid at the depths from 1 m to 20 m

Exemplary weight coefficients b_k and their share of the cumulative value *S* is shown below for $h_2 = 2$ m. In brackets are the numbers of the parameters (Table 2):

$$S = \sum_{k} b_{k} = 0.3262(a_{1}) + 0.0009(a_{2}) + 0.1406(a_{3}) + 0.3018(a_{4}) + 0.0087(a_{5}) + 0.0087(a_{$$

$$0,0000(a_6) + 0,0747(a_7) + 0,1203(a_8) + 0,0047(a_9) + 0,0059(a_{10}) + 0,0070(a_{11}) + 0,0012(a_{12}) + 0,0082(a_{13}) = 1.$$

Assuming values $b_k \leq 0.05$ and denoting them by *R*, we obtain matrix **B** containing the values of the elements of sets *A* and *B*.

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$$\boldsymbol{B} = \begin{bmatrix} 0,4138 & 0 & 0,1179 & 0,2547 & 0 & 0 & 0,0669 & 0,1118 & 0 & 0 & 0 & 0 \\ 0,3262 & 0 & 0,1406 & 0,3018 & 0 & 0 & 0,0747 & 0,1203 & 0 & 0 & 0 & 0 \\ 0,2730 & 0 & 0,1548 & 0,3275 & 0 & 0 & 0,0796 & 0,1268 & 0 & 0 & 0 & 0 \\ 0,2361 & 0 & 0,1651 & 0,3450 & 0 & 0 & 0,0833 & 0,1321 & 0 & 0 & 0 & 0 \\ 0,2066 & 0 & 0,1712 & 0,3549 & 0 & 0 & 0,0854 & 0,1352 & 0 & 0 & 0 & 0 \\ 0,1292 & 0 & 0,1890 & 0,3826 & 0 & 0 & 0,0917 & 0,1494 & 0 & 0 & 0 & 0 \\ 0,0762 & 0 & 0,2024 & 0,4031 & 0 & 0 & 0,0972 & 0,1553 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} k_1 = 1m \\ k_2 = 2m \\ k_3 = 3m \\ k_4 = 4m \\ k_5 = 5m \\ k_6 = 10m \\ k_7 = 20m \end{bmatrix}$$

(9)

Figures 4 and 5 show examples of ABC charts and Lorenz curve developed for the analysed system using matrix **B** at distances h equal to 2 m and 10 m from the ground's surface.



Fig. 4. *ABC* charts of a three-phase cable system laid at the depth h = 2 m

It should be noted that the scale of drawings on the left relates to the bar charts of relative temperature Tw, whereas the scale on the right refers to the increases of cumulative values *S* expressed by $S_k = b_k$ (increments on the *Lorenz curve*). The charts of relative changes of temperature T_w in the cable core for the assumed range of base parameter changes listed in Table 2 and different values

of h are shown in Fig. 6 For clarity's sake, the charts have been made only for some selected parameters of the system, i.e. *a*1, *a*4, *a*3, *a*7 and *a*8



Fig. 5. ABC charts of a three-phase cable system laid at the depth h = 10 m



Fig. 6. Dependencies of relative temperature changes of cable core T_w in the function of the distance of the cable system from the ground's surface for the parameters in A and B sets

Analysing Fig. 7 we can define sets A, B and C depending on the placement depths of the cable system. Up to the depth of 5 m set A is composed of a1 i.e. the temperature over the ground's surface and a4 i.e. thermal conductivity of the ground. Set B includes a3, i.e. long-term cable load, a7, i.e. concrete's thermal conductivity, a8, i.e. thermal conductivity of air duct, $B = \{a3, a7, a8\}$. In

contrast, a minimum impact on the core temperature of the cable is exerted by the elements of set $C = \{a2, a5, a6, a9, a10, a11, a12, a13\}$.

With the depth *h* exceeding 10 m the situation changes. Set *A* will contain thermal conductivity of the ground, long-term cable load and thermal conductivity of the air duct, $A = \{a4, a3, a8\}$, set *B* will include air temperature, and thermal conductivity of concrete block, $B = \{a1, a7\}$, whereas set *C* remains unchanged, $C = \{a2, a5, a6, a9, a10, a11, a12, a13\}$.

In examining the impact of parameter a1 (temperature above the ground), we can observe its decisive role in the *A* and *B* sets of the system. It has the greatest impact on the temperature of the cable wires up to the depth of h = 2.2 m. Over this depth we observe a change where thermal conductivity of the ground (i.e. parameter a4) becomes dominant.

The impact of some selected parameters such as a9 - geometrical distances of air duct centres Δw , a12 - concrete conduit width (top) and a13 - radius of air ducts connected with the geometry and dimensions of concrete conduit can be seen in Fig. 7. All of them belong to set C parameters of the analysed system and have a negligible effect on the temperature of cable cores throughout the whole depth range of cable location in the ground.



Fig. 7. Dependencies of relative temperature changes of cable core T_w in the function of the distance of the cable system from the ground's surface for parameters $\Delta w (a_{9y}u_3(a12)) R(a13)$

By analysing matrix (8), we can notice that at specified depths *h* the following five parameters exert major impact on T_r i.e. the temperature of the cable: T_{p_r} -temperature above the ground's surface, λ_z - thermal conductivity of the ground, I - cable load λ_p , - thermal conductivity of air duct, λ_{b_r} - thermal conductivity of concrete, namely

$$T_r = f(T_p, \lambda_z, I, \lambda_p, \lambda_b) \tag{10}$$

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4. Comparison of the temperature of three-phase cable core system laid directly in the ground with the temperature of the core placed in a concrete conduit

Figure 8 shows the influence of thermal conductivity λ_z varied from 0.8 to 1.4 [W/(m·K)] on the temperature of the cable core system placed in a concrete block as well as the one placed directly in the ground.



Fig. 8. Core temperature changes T_r with the assumed ground's thermal conductivity changes of the cable system placed in a concrete block and directly in the ground. Other parameters as in Table 1, h = 1 m

Figure 9. shows the temperature changes of the cable core system placed in a concrete block and the other one placed directly in the ground both with long-term cable load I from 1026 A to 1254 A.



Fig. 9. Temperature changes of the core T_r with long-term cable load changes of the analysed system laid in a concrete block and the other one placed directly in the ground. Other parameters as in Table 1, h = 1 m

Effect of air temperature T_p on the of the temperature of the cable core system laid in a concrete block and the other one placed directly in the ground is presented in Fig. 10.



Fig. 10. Temperature changes of the core T_r with the assumed air temperature changes of the analysed system laid in a concrete block and the other one placed directly in the ground. Other parameters as in Table 1, h = 1 m

5. Comments and conclusions

The results of the investigations allowed us to formulate the following conclusions:

A major impact on the temperature of a three-phase cable core laid underground in a concrete block is exerted by the following: temperature of the air above the ground, thermal conductivity of the ground, current-carrying capacity of the system *I*, thermal conductivity of the air duct, thermal conductivity of concrete

- in the analysed system at depths up to 2.2 m a significant impact on the core temperature is exerted primarily by air temperature and thermal conductivity of the ground. However, below a depth of 2.2 m we can observe a dominant influence of the thermal conductivity of the ground. At a depth of h = 10 m there is a change in the elements of sets A and B in relation to smaller depths, namely, $A = \{a4, a3, a8\}$, and $B = \{a1, a7\}$,
- a minimal effect on the temperature of the core is exerted by the parameters of the physical properties of the materials used for cable construction and also the rate of convective heat transfer characterizing, by an approximation, the speed of wind over the ground,

- all the constructional parameters: u1, u2, u3, R, △w belong to set C and have a minimal effect on the temperature of the cable core at all depths of its location underground,
- the temperature difference of cable cores in the concrete block system and also the cables placed directly in the ground, depending on the basic parameters (λ_z , I, T_p) varies within the range of a few degrees for the assumed base changes of the analysed parameters.

The elaborated method of temperature analysis in a three-phase HV cable system can be applied to carry out a preliminary optimization of the system. The optimization model takes the following form:

using the genetic algorithm we have

$$\min\frac{1}{3}\sum_{k}T_{r,l} \quad l=1,..,3$$
(11)

with respect to

 $\nabla^2 T = -\frac{g}{\lambda}$ with the assumed boundary conditions

$$m_k \le p_k \le n_k, \quad k = 1, \dots, s, \quad p_k \in A$$

$$T_r \le 90^{\circ} C$$
(12)

where: $T_{r,l}$ is the temperature of the three-phase core system, m_k and n_k determine the basic change ranges of individual parameters, *s* is the number of elements in set *A* including additionally adopted distance *h* from the ground's surface. In the consecutive steps of the algorithm analysing the temperature field we made use of a professional software *NISA/Heat Transfer* employing the finite element method (*FEM*) supplemented with *macro-definitions*.

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