Arch. Min. Sci., Vol. 58 (2013), No 2, p. 591–600

Electronic version (in color) of this paper is available: http://mining.archives.pl

DOI 10.2478/amsc-2013-0041

JERZY MICHALCZYK*

MATHEMATICAL MODEL OF A GENERAL MOTION OF VIBRATORY MACHINES DRIVEN BY MOTORS OF A LIMITED POWER

MODEL MATEMATYCZNY RUCHU OGÓLNEGO MASZYN WIBRACYJNYCH NAPĘDZANYCH SILNIKAMI OGRANICZONEJ MOCY

The method of creating the mathematical model of over-resonant vibratory machines driven in a general motion by the inertial vibrator of a limited power is presented in the paper. This model can constitute the basis for digital simulation of single- or multi-drive vibratory machines used in industry. It allows to investigate the steady and transient states of these machines as well as to investigate the advanced problems of the free synchronisation of drives.

Keywords: vibratory machines of a limited power, vibrations in general motion, self-synchronisation, transient states

W pracy przedstawiono sposób budowy modelu matematycznego nadrezonansowej maszyny wibracyjnej w ruchu ogólnym, napędzanej wibratorem inercyjnym ograniczonej mocy. Model ten może stanowić podstawę dla symulacji cyfrowej jedno-, lub wielonapędowych maszyn wibracyjnych używanych w przemyśle. Pozwala on na badanie zarówno pracy ustalonej tych maszyn jak i ich stanów przejściowych oraz na badanie zaawansowanych problemów synchronizacji swobodnej napędów.

Słowa kluczowe: maszyny wibracyjne ograniczonej mocy, drgania w ruchu ogólnym, samosynchronizacja, stany przejściowe

1. Introduction

Vibratory machines, such as vibrating screens (Banaszewski, 1990) and vibratory conveyors (Czubak, 1964) used in mining industry, shake out grids used for casting knocking out and vibrating tables used in building industry for compacting and forming concrete mixtures (Michalczyk,

* AGH UNIVERSITY OF SCIENCE AND TECHNOLOGY, FACULTY OF MECHANICAL ENGINEERING AND ROBOTICS, A. MICKIEWICZA 30 AVE., 30-059 KRAKOW, POLAND

1995), constitute usually solid bodies placed on systems of elastic elements, excited for vibrations by means of unbalanced masses driven be electric motors.

Despite the simplicity of construction these systems are very complicated objects of analysis, among others, in consideration of a strongly non-linear character of effects occurring there.

From among the non-linear effects the basic ones are:

- strong influence of forced vibrations of the machine frame on running of inertial vibrators, deciding on the time-history of transient processes and on obtaining the self-synchronisation of vibrators,
- occurrence of gyroscopic effects and Euler's couplings in between individual components of the machine angular motion,
- strong dependence of the horizontal stiffness of the elastic support system on the vertical load of elastic elements.

The last of the mentioned factors is not usually a problem, since vibrations occur in the vicinity of the static balance and the elastic characteristics can be linearised near this position (Michalczyk, 1995; Rivin, 2003).

Making allowances for gyroscopic effects also does not cause difficulties in practice, e.g. (Sokołowska, 1999).

The fact of dependence of the unbalance rotor running on vibrations of the oscillator driven by this rotor, noticed by Sommerfeld, was several times analysed in references. The basic and survey works dealing with this problem are e.g. (Kononienko, 1964; Blechman ,1994; Ławendel, 1981). Newer works, such as e.g. (Fidlin, 2006) are either based on classic works or do not undertake such problems (Brown, 2002). A common feature of the majority of these works is limiting the analysis to various systems of only one degree of freedom. In relation to systems of several degrees of freedom the description is limited to the formulation of the general form of equations of motion and does not provide the method of the determination of vibratory moments in case of the unbalanced rotor mounted to the machine frame performing the general motion e.g. (Blechman, 1994). This does not create the possibility of building effective simulation models for investigating e.g. the transient resonance effect or the advanced problems of self-synchronisation of drives.

As it is known from the transient resonance investigations (Michalczyk, 2012a) or the advanced self-synchronisation problems (Michalczyk, 2012b), basing on models, which are not taking into account this effect, leads to the results completely useless in practice.

The aim of this study is preparing the grounds for creation the mathematical models of vibratory machines in general motion, with taking into account the limited power of the driving motor, allowing for effective investigations of the advanced self-synchronisation problems and transient resonance by means of the digital simulation.

2. Equations of natural vibrations of the machine frame

Let us consider the machine frame as a solid body of 6 degrees of freedom, described in the absolute co-ordinate system *Oxyz*, which centre *O* overlaps in the static equilibrium state with the machine mass centre *S* and axis *z* is vertical. Let the co-ordinates of the points of fastening elastic elements to the frame in the static equilibrium state are x_j, y_j, z_j, and their coefficients of elasticity k_{xj}, k_{yj}, k_{zj} and viscous damping ratio b_{xj}, b_{yj}, b_{zj}, generally different, were determined by the linearisation around the static equilibrium position, $j = 1...\kappa$.

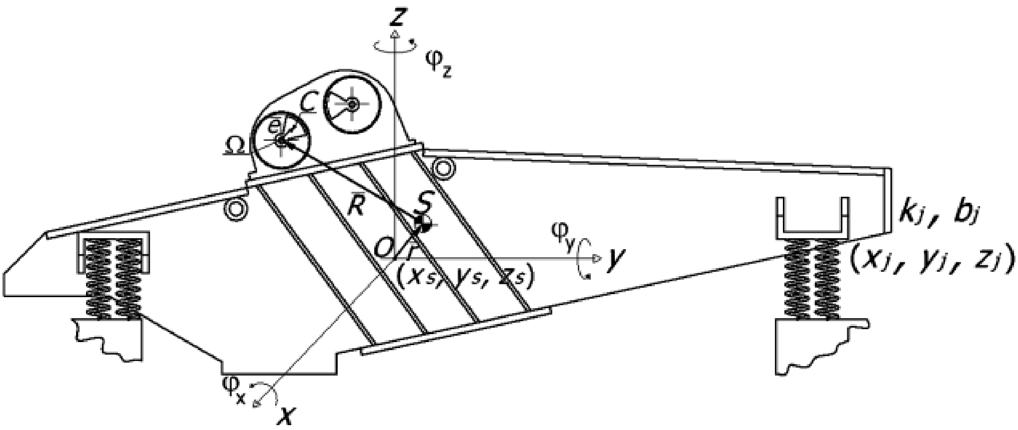

Fig. 1. Calculation model of vibratory machine
where: k_{xj}, k_{yj}, k_{zj} – coefficients of elasticity, b_{xj}, b_{yj}, b_{zj} – viscous damping ratios,
S – machine body mass centre, C – mass centre of unbalanced mass of vibrator, Ω – pivoting point of vibrator,
x_s, y_s, z_s, φ_x, φ_y, φ_z – generalized coordinates of machine body with respect to the absolute coordinate system

As can be easily proved, for small displacements of the frame the approximate relations occur in between co-ordinates of the reaction elasticity forces P_j of the j^{th} element and the machine frame co-ordinates:

$$\begin{aligned} P_{xj} &= -k_{xj}(x_s + \varphi_y z_j - \varphi_z y_j) \\ P_{yj} &= -k_{yj}(y_s + \varphi_z x_j - \varphi_x z_j) \\ P_{zj} &= -k_{zj}(z_s + \varphi_x y_j - \varphi_y x_j) \end{aligned} \tag{1}$$

where: x_s, y_s, z_s and φ_x, φ_y, φ_z denote small displacements of the mass centre S in the absolute system xyz, and small angles of rotation of the solid body with respect of the axes of this system, respectively.

Moments of these reactions with respect to the frame mass centre S is determined by the dependence:

$$\overline{M} = \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ x_j & y_j & z_j \\ P_{xj} & P_{yj} & P_{zj} \end{vmatrix} \tag{2}$$

where: $\overline{i}$, $\overline{j}$, $\overline{k}$ – versors of the axes: x, y, z.

From there:

$$\begin{aligned} M_x &= y_j P_{zj} - z_j P_{yj} \\ M_y &= z_j P_{xj} - x_j P_{zj} \\ M_z &= x_j P_{yj} - y_j P_{xj} \end{aligned} \tag{3}$$

Dependences (1) and (3) after summing with respect of $j = 1...\kappa$, are describing the elasticity matrix **K** in the general case.

If for each elastic element: $k_{xj} = k_{yj} = k_{xyj}$, the matrix **K** is of a form:

$$\mathbf{K} = \begin{bmatrix} \Sigma k_{xyj} & 0 & 0 & 0 & \Sigma k_{xyj} z_j & -\Sigma k_{xyj} y_j \\ & \Sigma k_{xyj} & 0 & -\Sigma k_{xyj} z_j & 0 & \Sigma k_{xyj} x_j \\ & & \Sigma k_{zj} & \Sigma k_{zj} y_j & -\Sigma k_{zj} x_j & 0 \\ & & & \Sigma k_{zj} y_j^2 + \Sigma k_{xyj} z_j^2 & -\Sigma k_{zj} x_j y_j & -\Sigma k_{xyj} x_j z_j \\ & sym. & & & \Sigma k_{zj} x_j^2 + \Sigma k_{xyj} z_j^2 & -\Sigma k_{xyj} y_j z_j \\ & & & & & \Sigma k_{xyj} x_j^2 + \Sigma k_{xyj} y_j^2 \end{bmatrix} \tag{4}$$

The viscous damping matrix **B** can be determined, in the simplest way, when assuming the model of the material damping for elastic elements (Michalczyk & Cieplok, 1999).

In this model the ratio of the energy loss for the period of steady state vibrations ΔL, to the potential energy maximum value U in a deformation cycle – is assumed as a constant:

$$\frac{\Delta L}{U} = \Psi = \text{const} \tag{5}$$

Equating the energy loss for the vibration period with amplitude $q_{\max}$ and frequency ω of the linear system with damping coefficient b – with the loss in a system with material damping determined in (5) – we obtain:

$$\pi q_{\max}^2 b\omega = \Psi \cdot \frac{1}{2} k q_{\max}^2 \tag{6}$$

Thus, the equivalent coefficient (in a sense of energy losses for a period) of viscous damping equals:

$$b = \frac{\Psi k}{2\pi\omega} \tag{7}$$

Generalising the above for the system of several degrees of freedom, in which the individual elastic elements are made from the same material of the material damping coefficient Ψ, vibrating with frequency ω, it is possible to present the damping matrix as the matrix of the proportional damping:

$$\mathbf{B} = \frac{\Psi}{2\pi\omega} \mathbf{K} \tag{8}$$

In order to take into account a structural damping, dominating in the case of e.g. coil springs, coefficient Ψ determined for the sum of material and structural damping, should be introduced into (8) (Michalczyk, 1995).

<u>The inertia matrix</u> in a general case, when axes of the system xyz – in the static equilibrium state – do not coincide with the main central solid body inertia axes, is of the form:

$$\mathbf{M} = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ & m & 0 & 0 & 0 & 0 \\ & & m & 0 & 0 & 0 \\ & & & J_{xx} & -J_{xy} & -J_{xz} \\ & sym. & & & J_{yy} & -J_{yz} \\ & & & & & J_{zz} \end{bmatrix} \tag{9}$$

Thus, the equation of natural vibrations can be written in the following form:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{B}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = 0 \tag{10}$$

where

$$\mathbf{q} = \mathrm{col}\{x_s, y_s, z_s, \varphi_x, \varphi_y, \varphi_z\} \tag{11}$$

3. Equations of forced vibrations of the machine frame, at forces *a priori* given

Let us presently consider the problem of a vibratory machine motion with an inertial drive, which means taking into account the vector of forces exciting the machine frame by means of the inertial vibrators – unbalanced statically or dynamically – attached to the frame.

We will assume for the moment, that the vibrators running is not disturbed by frame vibrations, which can be accepted e.g. in the analysis of a steady motion outside the resonance zone.

In such case the linearised equation of motion will take the form:

$$\mathbf{M}\ddot{\mathbf{q}} + (\mathbf{B} + \mathbf{G})\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{Q} \tag{12}$$

where vector $\mathbf{Q}$ describes components of inertial forces (centrifugal and tangent) and moments from forces originated from unbalanced vibrators, exciting system vibrations. Matrix $\mathbf{G}$ takes into account the gyroscopic influences. The way of constructing this matrix will be explained on the example of the typical inertial vibrator.

As usual , the inertial vibrator component elements can be divided into two kinds (Banaszewski, 1990; Michalczyk, 1995):

- balanced, axially-symmetrical with respect of the axis of rotation,
- unbalanced, of a plane of symmetry perpendicular to the axis of rotation.

Let us name the point, being the intersection of the plane – perpendicular to the axis of rotation and passing via the mass centre of the given i^{th} element – with the axis of rotation, as the 'rotation point' Ω_i.

Calculating the angular momentum of individual i^{th} component originated from rotations of a 'fast' angular velocity $\overline{\omega}_o$, we will assume for each of them the absolute reference system: $\xi_i \eta_i \zeta_i$ of the beginning in its rotation point Ω_i and axis ζ_i directed along the axis of rotation, compatibly with the direction of angular velocity of all elements $\overline{\omega}_o$. Since each element is of a symmetry allowing the zeroing of deviation moments $J_{i\zeta\xi}$ and $J_{i\zeta\eta}$, and the angular velocity of a fast rotation $\overline{\omega}_o$ has a component only along axis ζ_i, the angular momentum of the i^{th} element with respect of its centre of rotation lies on axis ζ_i and is equal to: $\overline{K}_i = J_{i\zeta\zeta} \cdot \overline{\omega}_o$.

If the angular velocity of the 'slow' rotation resulting from the machine frame motion will be marked $\overline{\omega}(\omega_x, \omega_y, \omega_z)$, the gyroscopic moment, with which the analysed rotor influences the machine frame, can be written as:

$$\overline{M} = -\Sigma \frac{d}{dt} \overline{K}_i = -\Sigma \overline{\omega} \times \overline{\omega}_o J_{i\zeta\zeta} = \overline{\omega}_o \times \overline{\omega} \cdot \Sigma J_{i\zeta\zeta} = \overline{\omega}_o \times \overline{\omega} \cdot J_{\zeta\zeta} \tag{13}$$

where $J_{\zeta\zeta}$ denotes the moment of inertia of the whole vibrator with respect of its axis of rotation, while the rotational speed $\overline{\omega}_o$ is treated as a constant value.

Dependence (13) can be for analysed vibrator presented in the form:

$$\overline{M} = J_{\zeta\zeta} \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ \omega_{ox} & \omega_{oy} & \omega_{oz} \\ \omega_x & \omega_y & \omega_z \end{vmatrix} \tag{14}$$

or, after the determinant expansion, in the form:

$$\begin{aligned} M_x &= J_{\zeta\zeta}(\omega_{oy}\omega_z - \omega_{oz}\omega_y) \\ M_y &= J_{\zeta\zeta}(\omega_{oz}\omega_x - \omega_{ox}\omega_z) \\ M_z &= J_{\zeta\zeta}(\omega_{ox}\omega_y - \omega_{oy}\omega_x) \end{aligned} \tag{15}$$

On this basis the matrix **G** in (12) can be written for single vibrator as:

$$\mathrm{G} = -J_{\zeta\zeta} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\omega_{oz} & \omega_{oy} \\ 0 & 0 & 0 & \omega_{oz} & 0 & -\omega_{ox} \\ 0 & 0 & 0 & -\omega_{oy} & \omega_{ox} & 0 \end{bmatrix} \tag{16}$$

where $J_{\zeta\zeta}$ – moment of inertia of the whole vibrator with respect of its axis of rotation.

Vector **Q**, should be determined only for unbalanced elements of a vibrator, on the basis of forces of inertia of these elements resulting from their angular motion (steady or accelerated). Rotating vectors of these normal and tangential forces of inertia should be applied in their points of rotation.

Components of vector **Q** constitute projections of these forces on the axis of the system xyz, and their moments with respect of the axis, when the influence of the body displacing is omitted for the simplification. The determination of these components by classic methods is usually easy.

It should be mentioned that matrix of the system masses **M** should be calculated with taking into account vibrators, while masses of unbalanced elements must be concentrated in their rotation points.

In case when the vibrators driving motors are attached to the machine frame, not to a separate foundation, vector **Q** should be supplemented – by being on the axis ζ – moments $\overline{M}_\zeta$ resulting from the change of the vibrator rotational speed:

$$\overline{M}_\zeta = -\frac{d}{dt}(J_{\zeta\zeta}\,\overline{\omega}_o) = -J_{\zeta\zeta}\,\frac{d}{dt}\,\overline{\omega}_o = -J_{\zeta\zeta}\,\overline{\varepsilon}_o \tag{17}$$

Analysis of dependencies (13) and (17) leads to conclusion that in two-drive systems, of identical counter rotating vibrators, gyroscopic moments as well as moments resulting from the change of the rotors rotational speed, are mutually zeroing, which allows not taking into account these effects in equations (12).

Thus, equations (12) do not take into account the influence of vibrations of vibrators axes, (originated due to the machine frame motion) on vibrators running.

4. Dynamic equations of motion taking into account the influence of frame vibrations on running of unbalanced rotors

Equations of motion (12) should be presently interpreted differently, assuming that the time-history of angular velocity $\overline{\omega}_o$ and angular acceleration $\overline{\varepsilon}_o$ of a 'fast' motion of individual rotors is not given a priori in vector **Q**, but results from solving the additional dynamic equations of angular motion.

Additional equations of angular motion for each unbalanced rotor can be written in the form of Euler's equation, assuming the life-axes system $C\xi\eta\zeta$, related to the main central inertial axes of the investigated rotor. Instantaneously we assume, that one of these axes, e.g. ζ is parallel to the rotor axis of rotation, (which occurs for rotors of static unbalancing e.g. inertial vibrators of a 'force' type). Then:

$$J_\varsigma \dot{\omega}_\varsigma - \omega_\eta \omega_\xi (J_\eta - J_\xi) = M_\varsigma \tag{18}$$

where:

J_ξ, J_η, J_ζ — main, central moments of inertia of the rotor,
$\omega_\varsigma = \omega_o$ — 'fast' rotational speed of the rotor, while
ω_ξ, ω_η — projections of the machine frame angular velocity on the life-axis system,
M_ς — sum of driving, anti-torque moment and a moment with respect of axis ζ, originated from a reaction in rotor bearings.

Reaction forces in individual bearings, as concurrent force system, can be presented in a form of a sum of the axial components and a component in the plane perpendicular to the axis of rotation, while the axial components do not give any moment with respect of axis ζ. The remaining components can be reduced to the 'rotation point' Ω (intersection point of the rotor axis of rotation with the rotation plane of the rotor mass centre C), obtaining a resultant force perpendicular to the rotor axis and a couple of forces of a moment vector being in the plane perpendicular to the rotor axis. Thus, these couple does not give the moment with respect of axis ζ and the moment with respect of this axis is originated solely from the resultant force. This resultant force of the bearings reaction (sum of axial and radial components) originates from the rotor inertial forces system. These forces are generated by components of acceleration of the rotor mass centre C: Coriolis and relative acceleration.

The Coriolis inertial force, as perpendicular to the relative velocity vector, lies in the plane passing via the axis of rotation and axis ζ, thus the bearing reaction originated from it also lies in this plane not giving any moment with respect of axis ζ.

The rotor dynamic influence being the result of its relative motion consists of a radial force, which does not give a moment – with respect of axis ζ passing via the rotor mass centre C – and of a tangent force, from which the bearing reaction gives – with respect of axis ζ – a moment of a value:

$$M_{\varsigma(\tau)} = -m_n e^2 \varepsilon_o \tag{19}$$

where:

m_n — rotor mass,

e — rotor unbalanced radius,

$\varepsilon_o = \dfrac{d\omega_o}{dt}$.

Taking into account the components – described above – equation (18) can be presented in the form:

$$J_\varsigma \dot{\omega}_o - \omega_\eta \omega_\xi (J_\eta - J_\xi) = -m_n e^2 \dot{\omega}_o + M_{\varsigma(u)} + M_n - M_o \tag{20}$$

where: ω_η, ω_ξ – originate from the machine frame angular motion and constitute angular velocity functions $\dot{\varphi}_x$, $\dot{\varphi}_y$, $\dot{\varphi}_z$, while $M_{\varsigma(u)}$ denotes a moment with respect of axis ζ from bearing reaction generated by the force of transportation inertia, M_n and M_o – driving and anti-torque moments – respectively, applied to the rotor, usually dependent on $\omega_o(t)$.

Determination of $M_{\varsigma(u)}$ will be carried out in the following way:

a) Vectors will be marked: $\overline{OS} = \overline{r}$, $\overline{S\Omega} = \overline{R}$, where vector $\overline{R}$ determines the approximate (averaged for the rotor motion period) location – with respect of the machine mass centre S- of the point performing the transportation motion. $\overline{R}$ is a function of φ_x, φ_y, φ_z, while $\overline{r}$ is defined by x_s, y_s, z_s.

b) Then, the approximate absolute position of the point performing the transportation motion of the rotor mass centre C is determined by a sum:

$$\overline{OC} \cong \overline{r} + \overline{R} \tag{21}$$

c) Differentiating two times sum (21) versus time the transportation acceleration of point C can be obtained. The second derivative of the first component determines acceleration of the machine mass centre in system *Oxyz*, while the acceleration resulting from the change of $\overline{R}$ constitutes the sum of the rotational and centrifugal accelerations in rotation about $\overline{S}$.

Low values of the machine frame angular velocity (resulting from very small angles of rotation of the frame, of the order 10^{-3}) usually allow to omit the centrifugal acceleration. In such case the dependence, determining transportation acceleration $\overline{a}_u$, obtains the approximate form:

$$\overline{a}_u \cong \overline{i}\ddot{x}_s + \overline{j}\ddot{y}_s + \overline{k}\ddot{z}_s + \overline{\varepsilon} \times \overline{R} \cong \overline{i}\ddot{x}_s + \overline{j}\ddot{y}_s + \overline{k}\ddot{z}_s + \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ \ddot{\varphi}_x & \ddot{\varphi}_y & \ddot{\varphi}_z \\ x_\Omega & y_\Omega & z_\Omega \end{vmatrix} \tag{22}$$

where $\overline{i}, \overline{j}, \overline{k}$ mark system versors: *Oxyz*, $\overline{\varepsilon}$ is the frame angular acceleration in its rotation about mass centre *S* while x_Ω, y_Ω, z_Ω are point Ω coordinates in the system *Oxyz* determined in the static balance point of the machine frame.

d) Bearing reaction to the force of transportation inertia equals:

$$\overline{P}_{(u)} = m_n \cdot \overline{a}_u \tag{23}$$

e) In order to calculate the moment of this reaction with respect of axis ζ, let us determine versor $\overline{n}$ being simultaneously perpendicular to axis ζ and rotating vector $\overline{\Omega C} = \overline{e}(\varphi_o)$:

$$\overline{n}(\varphi_o) = \frac{\overline{\omega}_o}{\omega_o} \times \frac{\overline{e}}{e} \tag{24}$$

where the vector $\overline{\omega}_o$ direction, approximately constant, corresponding to the position of the frame static balance, was assumed while by $\varphi_o(t)$ the angle – determining the rotor rotation with respect of the selected initial position – was marked.

That time the moment from reaction $\overline{P}_{(u)}$ with respect of axis $\varsigma \| \overline{\omega}_o$ equals:

$$M_{\varsigma(u)} = -e \cdot \overline{P}_{(u)} \cdot \overline{n} \tag{25}$$

Equation (20) can be written in a simpler form, using the dependence:

$$J_\varsigma + m_n e^2 = J_o \tag{26}$$

where J_o constitutes the rotor moment of inertia with respect of its axis of rotation (see comment below), and omitting the angular velocities product $\omega_\eta \omega_\xi$ as being very small:

$$J_o \dot{\omega}_o = M_{\varsigma(u)} + M_n - M_o \tag{27}$$

As it can be seen from this form, the rotor is moving under an influence of the driving and anti-torque moments as well as the moment being the bearing reaction to the force of transportation inertia.

This last component, often of a value dominating at circum-resonant states, occurs due to the rotor axes vibrations related to the frame motion and can be determined on the bases of equations (22) to (25).

Comment

In the case when the motor rotor does not constitute the integral whole with the unbalanced rotor and was not taken into account at the determination of J_o, the reduced moment of the motor rotor and other rotating power transmission elements should be counted in to the value of J_o.

It should be also remembered that driving and anti-torque moments exerted to the rotor are related to applying the opposite moments to carrying bodies and if this is the machine frame they should be applied to it.

In the case of rotors balanced statically but not dynamically or in a general unbalancing case equation (27) is valid, however, the rotor should be divided into two elements (or in case of need into more) for which the main central axis parallelism to the axis of rotation can be assumed. The moment of inertia J_o and the moment from forces of transportation inertia $M_{\varsigma(u)}$ constitute then sums of the respective expressions for each of this mass.

5. Conclusions

The mathematical model of the over-resonant vibratory machine driven in a general motion by the inertial vibrator of a limited power, was formulated in the study. This model can constitute the basis for the digital simulation of single- or multi-drive vibratory machines used in industry. It allows to investigate their steady and transient states as well as to investigate the advanced problems of the free synchronisation of drives.

References

Banaszewski T., 1990. *Przesiewacze*. Wyd. Śląsk.

Blechman I.I., 1994. *Vibracjonnaja Mechanika*. Nauka, Moskwa.

Czubak A., 1964. *Przenośniki wibracyjne.*Wyd. Śląsk.

Brown S. (editor), 2002. *Encyclopedia of Vibration*. Academic Presss, N. York.

Fidlin A., 2006. *Nonlinear Oscillations in Mechanical Engineering*. Springer-Verlag, Berlin.

Kononienko V.O., 1964. *Kolebatielnyje systemy s ograniczonnym vozbuzdienijem*. Nauka, Moskva.

Ławendel E.E. (editor), 1981. *Vibracji w Technikie* t.2. Maszynostrojenije, Moskwa.

Michalczyk J., 1995. *Maszyny wibracyjne. Obliczenia dynamiczne, drgania, hałas*. WNT, Warszawa.

Michalczyk J., 2012a. *Transient Resonance of Machines and Devices in General Motion,* Journal of Theoretical and Applied Mechanics, No 2/50.

Michalczyk J., 2012b. *Angular oscillations of vibratory machines of independent driving systems caused by a non-central direction of the exciting force operations*. Arch. Min. Sci., Vol. 57, No 1, p. 169-177.

Michalczyk J., Cieplok G., 1999. *Wysokoefektywne układy wibroizolacji i redukcji drgań*. Collegium Columbinum, Kraków.

Rivin E., 2003. *Passive Vibration Isolation*, ASME PRESS, New York.

Sokołowska R., 1999. *Modelowanie i analiza dynamiczna pojemnikowych maszyn wibracyjnych do obróbki powierzchniowej*. X Jubileuszowe Sympozjum Dynamiki Konstrukcji, Rzeszów.

Received: 24 September 2012