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Generalized multiresolution discrete orthogonal transforms

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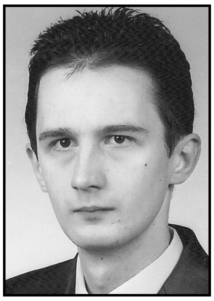

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Abstract

This paper presents an idea of the multiresolution discrete orthogonal transforms. One possible approach to realization of this multiresolution transform is implementation of the rationalized algorithm for computing the coefficients creating the consecutive resolution levels. The paper also presents an example of synthesis of the fast algorithm for computing the coefficients of the multiresolution discrete Hartley transform. For the description of the compuatational procedures we use a vector-matrix notation.

Keywords: multiresolution transform, fast transform, arithmetic complexity reduction, multiresolution analysis, vector-matrix notation.

Uogólnione wielorozdzielcze dyskretne transformacje ortogonalne

Streszczenie

W artykule przedstawiono uogólnioną wielorozdzielczą dyskretną transformację ortogonalną. Zdefiniowana w niniejszej pracy transformacja pozwala na analizę sygnału na wielu poziomach rozdzielczości. Poziomy te są stanowione poprzez współczynniki częstotliwościowe uzyskiwane w procesie realizacji szybkich dyskretnych transformat ortogonalnych np. dyskretnej transformaty Fouriera (DFT), dyskretnej transformaty kosinusowej (DCT), dyskretnej transformaty Hartley'a, czy też dyskretnej transformaty slant, w odniesieniu do kolejnych fragmentów badanego sygnału. Przedstawiony w niniejszym artykule schemat postępowania jest słuszny dla sygnałów o liczbie próbek będącej naturalną potęgą liczby dwa. Zastosowanie szybkich algorytmów realizacji poszczególnych przekształceń na kolejnych poziomach rozdzielczości, pozwala na uzyskanie znaczącej redukcji liczby wykonywanych działań arytmetycznych, w porównaniu do metody polegającej na bezpośrednim mnożeniu macierzy bazy i wektora kolumnowego danych wejściowych. W przedłożonej pracy, do opisu poszczególnych procedur obliczeniowych posłużono się rachunkiem wektorowo-macierzowym, który jest adekwatny do opisu przestrzenno-czasowych struktur procesów obliczeniowych, jak również umożliwia w sposób bezpośredni odwzorowanie tychże struktur w przestrzeni realizacji programowych i sprzętowych. W artykule zaprezentowano również przykład syntezy szybkiego algorytmu realizacji wielorozdzielczej dyskretnej transformaty Hartley'a dla sygnału jednowymiarowego o liczbie próbek wynoszącej osiem.

Słowa kluczowe: transformacja wielorozdzielcza, szybka transformacja, redukcja złożoności obliczeniowej, analiza wielorozdzielcza, notacja wektorowo-macierzowa.

1. Introduction

Orthogonal transforms play an important role in digital signal processing as they allow obtaining information which is not directly available in the original domain of the analyzed signal. Nevertheless, very often it is necessary to perform the multiresolution analysis of a signal, i.e. the analysis of frequency variations in time or space domain. Furthermore, in many cases there is a need to analyze the selected segments of the signal with different frequency resolution.

The traditional orthogonal transforms do not possess these properties. It should also be noted that these properties are particularly important in fields such as digital image processing and audio signal processing. In this regard, in recent years a lot of publications concerning the methods of the realization of the multiresolution Fourier transform [1, 2] and its possible applications [3, 4, 5, 6] have been presented. Nevertheless, in the literature there is still no generalized description of the realization of the multiresolution discrete orthogonal transforms. Therefore, the paper presents a generalized method for calculating coefficients of the multiresolution discrete orthogonal transform. The algorithm is suitable for the signals whose size is a natural power of two. It should also be emphasized that the presented generalized algorithm can be used with respect to the orthogonal transformations, creating consecutive resolution levels, for which fast algorithms for computing appropriate coefficients have been developed. These transformations include among other things: discrete Fourier transform, discrete cosine transform, discrete Hartley transform or discrete slant transform.

2. Definition of the vectorized form of the multiresolution discrete orthogonal transform

Let us consider a one-dimensional discrete signal, whose size is $N = 2^m$, for $m = 1,2,...,n$. This signal can be represented by a following column vector:

$$\mathbf{X}_{N\times 1} = [x_0, x_1, \ldots, x_{N-1}]^{\mathrm{T}}.$$

Let us define the multiresolution discrete orthogonal transform in a matrix form:

$$\mathbf{Y}_{N\times m} = \mathop{\blacksquare|\blacksquare}_{i=1}^{m} \left(\left(\mathbf{I}_{2^{i-1}} \otimes \mathbf{B}_{2^{m-i+1}} \right) \mathbf{X}_{N\times 1} \right),$$

where $\mathbf{Y}_{N\times m} = \left[\mathbf{Y}_{N\times 1}^{(m)}, \mathbf{Y}_{N\times 1}^{(m-1)}, \ldots, \mathbf{Y}_{N\times 1}^{(1)} \right]$ is the output vector, whose elements are vectors $\mathbf{Y}_{N\times 1}^{(i)} = \left[y_0^{(i)}, y_1^{(i)}, \ldots, y_{N-1}^{(i)} \right]^{\mathrm{T}}$ relating to all components of the signal at different resolution levels. The matrix $\mathbf{I}_N$ is an identity $N \times N$ matrix, in turn the index i indicates currently analyzed resolution level. The symbol $\blacksquare|\blacksquare$ denotes horizontal concatenation sign [7], and $\mathbf{B}_N$ is a basis matrix, characteristic for the considered transform, such as: DFT, DCT, DHT or DST. However, for simplicity of a description of the procedures, we use in the rest of this paper a vector form, in order to represent the coefficients of the multiresolution discrete orthogonal transform. For this representation the computational procedure takes the following form [1]:

$$\mathbf{Y}_{mN\times 1} = \bigoplus_{i=1}^{m}\left(\mathbf{I}_{2^{i-1}} \otimes \mathbf{B}_{2^{m-i+1}}\right)\mathbf{P}_{mN\times N}\mathbf{X}_{N\times 1}, \qquad (1)$$

where $\mathbf{P}_{mN\times N}$ is the data duplication matrix. This matrix is defined as follows:

$$\mathbf{P}_{mN\times N} = \mathbf{1}_{m\times 1} \otimes \mathbf{I}_N .$$

The symbols $\otimes$ and $\oplus$ denote a tensor product and a direct sum of two matrices, respectively [8], in turn $\mathbf{1}_{n\times k}$ is the matrix, whose elements are units. In order to better understand the described generalized transformation, Figure 1 shows the data-flow diagram of the process computing the coefficients of the multiresolution discrete orthogonal transform according to (1). In this paper the graph-structural models are oriented from left to right. Straight lines in the figure denote the operation of data transfer. The rectangles indicate the operation of multiplication by the matrix inscribed inside an element. It can be noted that the realization of that transformation according to formula (1) requires $2N(N-1)$ multiplications and $2N(N-1)-mN$ additions.

One way to realize the presented transform is implementation of a fast algorithm, characteristic for the used transformation, for the selected segments of a signal, e.g. the realization of the FFT algorithm for the multiresolution discrete Fourier transform. This approach allows for a significant reduction of the computational cost at each resolution level.

3. An example of the synthesis

Let us consider an example of the synthesis of the fast algorithm for computing the coefficients of the multiresolution discrete Hartley transform which is very often used in image recognition tasks. The Hartley orthogonal basis matrix is defined as:

$$\mathbf{B}_N^{DHT} = \left[b_{k,l}\right],\ b_{k,l} = \cos\left(\frac{2\pi kl}{N}\right) + \sin\left(\frac{2\pi kl}{N}\right).$$

Accordingly for $N = 8$ ($m = 3$) we can present the following matrices:

$$\mathbf{B}_8 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \sqrt{2} & 1 & 0 & -1 & -\sqrt{2} & -1 & 0 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & 0 & -1 & \sqrt{2} & -1 & 0 & 1 & -\sqrt{2} \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -\sqrt{2} & 1 & 0 & -1 & \sqrt{2} & -1 & 0 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 0 & -1 & -\sqrt{2} & -1 & 0 & 1 & \sqrt{2} \end{bmatrix},$$

$$\mathbf{B}_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix},\ \mathbf{B}_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}.$$

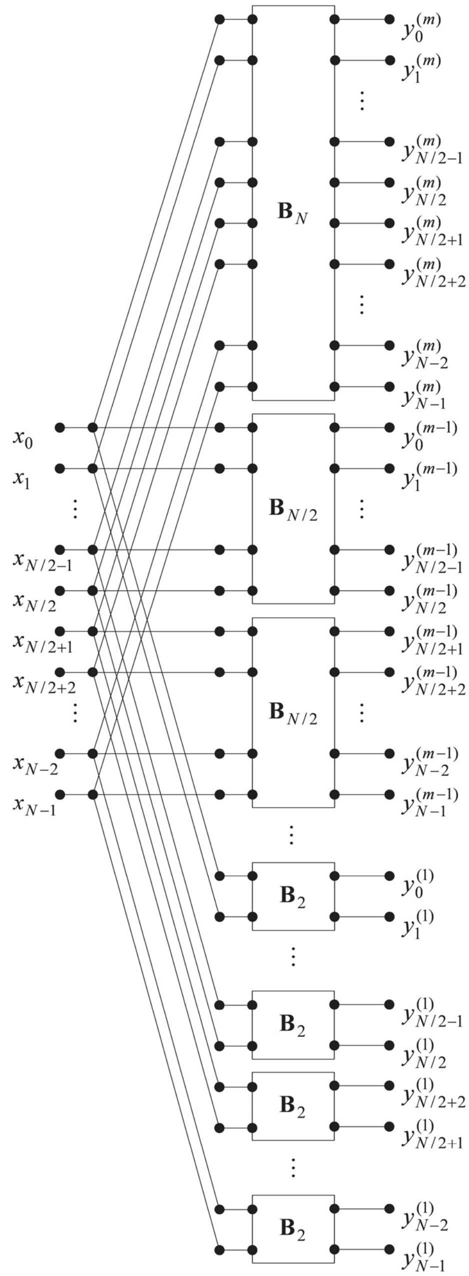

Fig. 1. The data-flow diagram of the process computing the coefficients of the multiresolution discrete orthogonal transform according to (1)

Rys. 1. Diagram przepływu danych procesu obliczającego współczynniki wielorozdzielczej dyskretnej transformaty ortogonalnej zgodnie z wyrażeniem (1)

Given the information above, we can write the naive computational procedure:

$$\mathbf{Y}_{24\times 1} = \mathbf{B}_8 \oplus \left(\mathbf{I}_2 \otimes \mathbf{B}_4\right) \oplus \left(\mathbf{I}_4 \otimes \mathbf{B}_2\right).$$

In turn, the procedure for fast computation of the multiresolution discrete Hartley transform takes the following form:

$$\mathbf{Y}_{24\times 1} = \mathbf{T}_{24}^{(2)}\mathbf{T}_{24}^{(1)}\mathbf{C}_{24}^{(3)}\mathbf{C}_{24}^{(2)}\mathbf{T}_{24}^{(1)}\mathbf{C}_{24}^{(1)}\mathbf{P}_{24\times 8}\mathbf{X}_{8\times 1}, \qquad (2)$$

where:

$$\mathbf{P}_{24\times 8} = 1_{3\times 1} \otimes \mathbf{I}_8,\ \mathbf{C}_{24}^{(1)} = \mathbf{U}_8^{(1)} \oplus \mathbf{V}_8^{(1)} \oplus \mathbf{W}_8^{(1)},$$

The graph-structural model for the realization of the presented algorithm for this example is illustrated in Figure 2. The circles in this figure show the operation of multiplication by a number inscribed inside the element. It can be noticed that the presented procedure given by (2) is characterized by a significant reduction in the computational cost compared to the naive method.

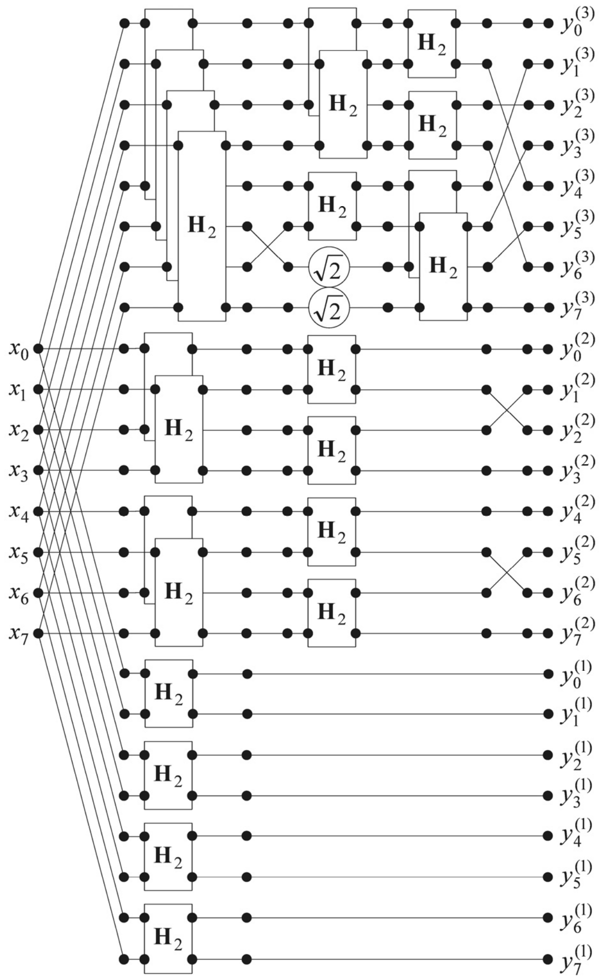

Fig. 2. The graph-structural model of computation process organization for vector $\mathbf{Y}_{24\times 1}$ elements calculation according to (2)

Rys. 2. Model grafostrukturalny organizacji procesu obliczeniowego elementów wektora $\mathbf{Y}_{24\times 1}$ zgodnie z wyrażeniem (2)

$$\mathbf{U}_8^{(1)} = \mathbf{H}_2 \otimes \mathbf{I}_4,\ \mathbf{V}_8^{(1)} = \mathbf{I}_2 \otimes \mathbf{H}_2 \otimes \mathbf{I}_2,\ \mathbf{W}_8^{(1)} = \mathbf{I}_4 \otimes \mathbf{H}_2,$$

$$\mathbf{H}_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix},$$

$$\mathbf{T}_{24}^{(1)} = \mathbf{I}_4 \oplus \widetilde{\mathbf{T}}_4^{(1)} \oplus \mathbf{I}_{16},\ \widetilde{\mathbf{T}}_4^{(1)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\mathbf{C}_{24}^{(2)} = \mathbf{U}_8^{(2)} \oplus \mathbf{V}_8^{(2)} \oplus \mathbf{I}_8,\ \mathbf{U}_8^{(2)} = (\mathbf{H}_2 \otimes \mathbf{I}_2) \oplus \mathbf{H}_2 \oplus \sqrt{2}\mathbf{I}_2,$$

$$\mathbf{V}_8^{(2)} = \mathbf{I}_4 \otimes \mathbf{H}_2,\ \mathbf{C}_{24}^{(3)} = \mathbf{U}_8^{(3)} \oplus \mathbf{I}_{16},$$

$$\mathbf{U}_8^{(3)} = (\mathbf{I}_2 \otimes \mathbf{H}_2) \oplus (\mathbf{H}_2 \otimes \mathbf{I}_2),$$

$$\mathbf{T}_{24}^{(2)} = \hat{\mathbf{T}}_8^{(1)} \oplus \left(\mathbf{I}_2 \otimes \widetilde{\mathbf{T}}_4^{(1)}\right) \oplus \mathbf{I}_8,$$

$$\hat{\mathbf{T}}_8^{(1)} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

4. Concluding remarks

This paper describes the generalized multiresolution discrete orthogonal transform. This transformation allows for a realization of the multiresolution signal analysis. The realization of the presented transform is based on the synthesis of the fast algorithms for computing the coefficients creating the consecutive resolution levels. This solution allows for a significant reduction in computational complexity compared to the method based on the multiplication of a basis square matrix by an input vector. Another advantage of the described transformation is the possibility of parallelization. Furthermore, the additional benefits such as targeting at the vectorization for calculating, that enable to reduce the delay of data processing, depend on the fast algorithms used to implement selected transforms. The main drawback of the presented approach is the requirement for the size of the signal, which should be 2^m, where m is a natural number. Due to the fact that a multiresolution analysis of signals plays an important role in many practical applications, we hope that the presented approach will be a useful and effective method for solving various kinds of digital signal processing tasks.

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otrzymano / received: 20.05.2013
przyjęto do druku / accepted: 03.07.2013 *artykuł recenzowany / revised paper*