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FRACTAL ANALYSIS OF LAND DEVELOPMENT ELEMENTS FOR SPATIAL PLANNING PURPOSES

Abstract: Spatial planning is a set of complex processes that aim to determine the correct location of objects in the established area subject to planning procedures. Planning documents, executed at all levels of detail, must consider the current state of land development, both in the area subject to land-use planning procedures and adjacent areas. Hence, the creation of such documents must be preceded by multiple analyses. These considerations should lead to the determination of conditions to be met by future land development elements in such a way as to take into account and use the existing ones. Because land use elements such as river network, road networks, buildings or forest areas are considered examples of random fractals, it was considered that the detection of fractal structures in area subject to land-use planning procedures could facilitate decision-making processes during creation of planning documents on a regional scale. This paper checks if it is possible to mathematically describe the clear chaos that prevails in the existing area subject to land-use planning procedures and if the use of fractal analysis in spatial planning is possible. The research was based on data from the Database of General Geographical Objects and the Bank of Local Data. Analyses were conducted within the borders of provinces.

Keywords: box-counting dimension, fractal structures, land-use, planning procedures, regional planning

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Introduction

Spatial planning is one of the issues, the correct development of which has a fundamental impact on the intensification of economic development (Bieda et al., 2012). It is a set of complex creative and decision-making processes, whose aim is to determine the correct distribution of objects in the established area subject to planning procedures. The optimal organisation of space management created on the basis of relevant planning documents, significantly facilitates its management that in turn leads to the spatial order and sustainable development in the adopted planning space (Popović et al., 2021).

In Poland, the legal basis for all planning activities is the Act on Spatial Planning and Development (Act, 2003). It expresses the principles for developing spatial policy and the scope and methods of proceeding during the determination of the designation of land for specific purposes, as well as during the determination of the principles for developing such land.

It (Act, 2003) takes spatial order and sustainable development as the basis for planning activities. According to its provisions, spatial planning should be conducted in such a way that the planning space (Adamczyk et al., 2014):

1. Creates a harmonious whole and consider, in orderly relations, all functional, socio-economic, environmental, cultural and compositional and aesthetic conditions and requirements (spatial order).
2. Ensure socio-economic development, which is a process of integrating political, economic and social activities, maintaining natural balance and sustainability of basic natural processes to ensure the possibility of satisfying basic needs of particular communities or citizens of both the present and future generations (sustainable development).

Despite such simple formulations, shaping the space we live in is an extremely difficult task (Adamczyk et al., 2014). This is because no planning work is suspended in a vacuum. Emerging planning documents must consider existing spatial developments. New planning documents created at all levels of detail (national, regional and local) must consider the current state of land use, both in the area covered by spatial planning and in neighbouring areas. Hence, the creation of such documents must be preceded by numerous analyses. These considerations should lead to the definition of conditions to be met by future land development elements in such a way as to consider and use those already existing.

In the last few decades, in research on the structure of planning space, as in other fields, fractal geometry (Frankhauser, 2018) has been increasingly used to analyse the distribution of the components of this structure at different scales (which Euclidean geometry does not allow). It seems that the use of fractal geometry can allow the ordering of chaos and the representation of the complex nature of the planning space and its elements in terms of simple numerical measures (Patuano & Tara, 2020). Because land use elements such as river, road or settlement networks (Bieda, 2016; Pászto, 2014; Shan, 2000), are considered examples of fractals, it was considered that the detection of fractal structures in the planning space could facilitate decision-making processes when creating planning documents at central and regional levels.

Research background

The history of fractal geometry is not long and dates back only to the late 1970s. It was initiated by a Warsaw-born French mathematician, Benoit Mandelbrot (Furmanek, 2002). In one of his works, he wrote "Clouds are not spheres, mountains are not cones, coastlines are not circles, bark is not flat, nor does lightning move in a straight line" (Mandelbrot, 1982). To capture the irregularity of these real objects, Mendelbrot described new geometric forms, which he called fractals (Latin: fractus – broken).

Despite the great interest in fractals by researchers from many fields, there is still no strict mathematical definition of them. Currently, fractals are defined as sets for which the topological dimension is different (smaller) than the Hausdorff dimension and for which this dimension is not an integer (Kudrewicz, 2015). They are characterised by the following geometric and algebraic properties (Ratajczak, 1998):

1. The lack of a unique length scale. Magnified or reduced fractal objects do not change their shapes.
2. Self-similarity at any level of observation (measurement). If you cut out any small part of a fractal object and enlarge it, an object that mimics the whole (faithfully or approximately) will be created.
3. They are described only by recursive relations, not by mathematical formulas.

Fractal structures cannot always be studied by strict methods. However, there are simple empirical methods allow to identify them by determining the fractal dimension derived directly from Hausdorff's work.

The fractal dimension of the set of points A in the n-dimensional space Rn is called such a number D that satisfies the relation:

$$\lim_{s\to 0} N(s) \approx s^{-D} \tag{1}$$

where N(s) is the number of spheres of diameter s needed to cover the set A.

The fractal dimension can be defined in several ways. It depends on the type of object for which it is determined. The universal method for determining the dimension can be considered the one based on determining the box dimension. With its help, it is possible to determine the dimension of any type of object, including the most varied ones. Its determination consists in placing the examined structure on a regular grid of squares with the mesh size s, and then counting those "boxes" (meshes), which contain fragments of the examined structure N(s). The measurement is made several times, each time reducing the size of s. The resulting pairs of numbers: s and N(s) are placed on a logarithmic chart. The number, which is the directional coefficient of the line fitted into these points, is the box dimension of the analysed structure.

An example of determining the box dimension of a von Koch curve is given in Figure 1 (DS = 1.23).

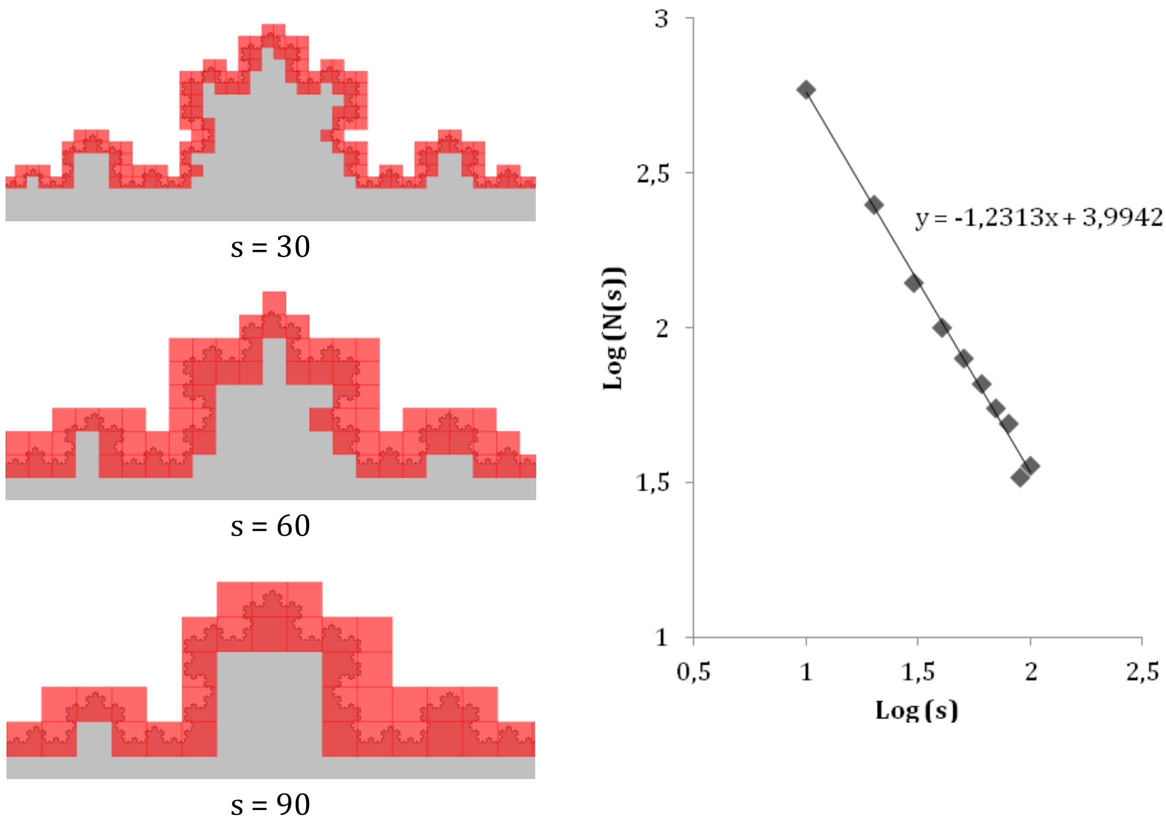

Fig. 1. Determination of the fractal dimension of the Koch curve
Source: own study

The fractal dimension may take fractional values. It means that there are fractal objects on the plane with dimension being any real number from 0 to 2. It also carries information about how much the fractal fills the space in which it is located (Gromada, 2003). Thus, the dimension of a Cantor set is 0.63, that of a Sierpiński triangle 1.58 and that of a Sierpiński carpet 1.89.

Fractal geometry and the fractal analysis developed on its basis are now powerful tools of science (Ratajczak, 2013). Many disciplines, including those related to geography and spatial management in the broadest sense, draw on its achievements. One well-known example is the issue of measuring the length of coastlines, first mentioned by Mandelbrot (Husain et al., 2021). However, most applications refer to the analysis of spatial distributions that are generated by asymmetric processes of interaction between the centre and its periphery and reproduce the same pattern of alternation between vacant and occupied places at different geographical scales (Tannier & Pumain, 2005).

Previous research indicates that concepts from fractal geometry can be used to optimise the spatial structure in future planning. In particular, fractal dimensions of land use forms are important for spatial planning, as they provide key clues to the direction of future development (Jevrić et al., 2014; Purevtseren et al., 2018). However, the fractal dimension of road networks is related to the characteristics of the land through which they pass (population, area of built-up areas, number of buildings and their type)

(Abid et al., 2021) and is a measure of the complexity of road transport infrastructure (Lu et al., 2016).

Materials and methods

To confirm fractal features for such elements of planning space as river, transport and settlement networks, their arrangement at the regional scale was analysed. The studies were based on data from the Database of General Geographic Objects (BDOO) functioning in Poland. Therefore, this will also be a test of the usefulness of this data source and in particular of the impact of generalisation of the shape of objects. The downloaded data were current as of 28 September 2015.

Additionally, it was assumed that if elements such as roads, rivers or buildings are arranged fractally then areas characterised by a higher possibility of change, such as agricultural land or forests, should also show the self-similarity they acquire by adapting to these more fixed objects.

The BDOO content includes 9 thematic areas (Regulation, 2011):

- water network,
- transportation network,
- utility network,
- land cover,
- buildings, structures and facilities,
- land use complexes,
- protected areas,
- territorial subdivisions,
- other objects.

Each listed theme is recorded on several layers. Out of many layers, 5 have been selected (Table 1).

Table 1. BDOO layers selected for fractal analysis

Level 1		Level 2	
Code	Name of object class category	Code	Name of object class
SW	Water network	SWRS	River and stream
SK	Transportation network	SKDR	Road
PT	Land cover	PTZB	Buildings
		PTLZ	Wooded and forested area
		PTTR	Grassland and arable farming

Source: (Regulation, 2011)

Since the basic unit for which a single BDOO data set is maintained in the voivodship, it was for this area that the analyses were conducted. Data from the layer were saved in raster form. The printout to the graphic file occurred with the resolution of 300 dpi. The scale of the printout was 1:250 000. The actual size of one pixel was about 20 m. A total of 80 images (5 layers for 16 voivodeships) were analysed. Data for the Mazowieckie Voivodship are shown in Figure 2.

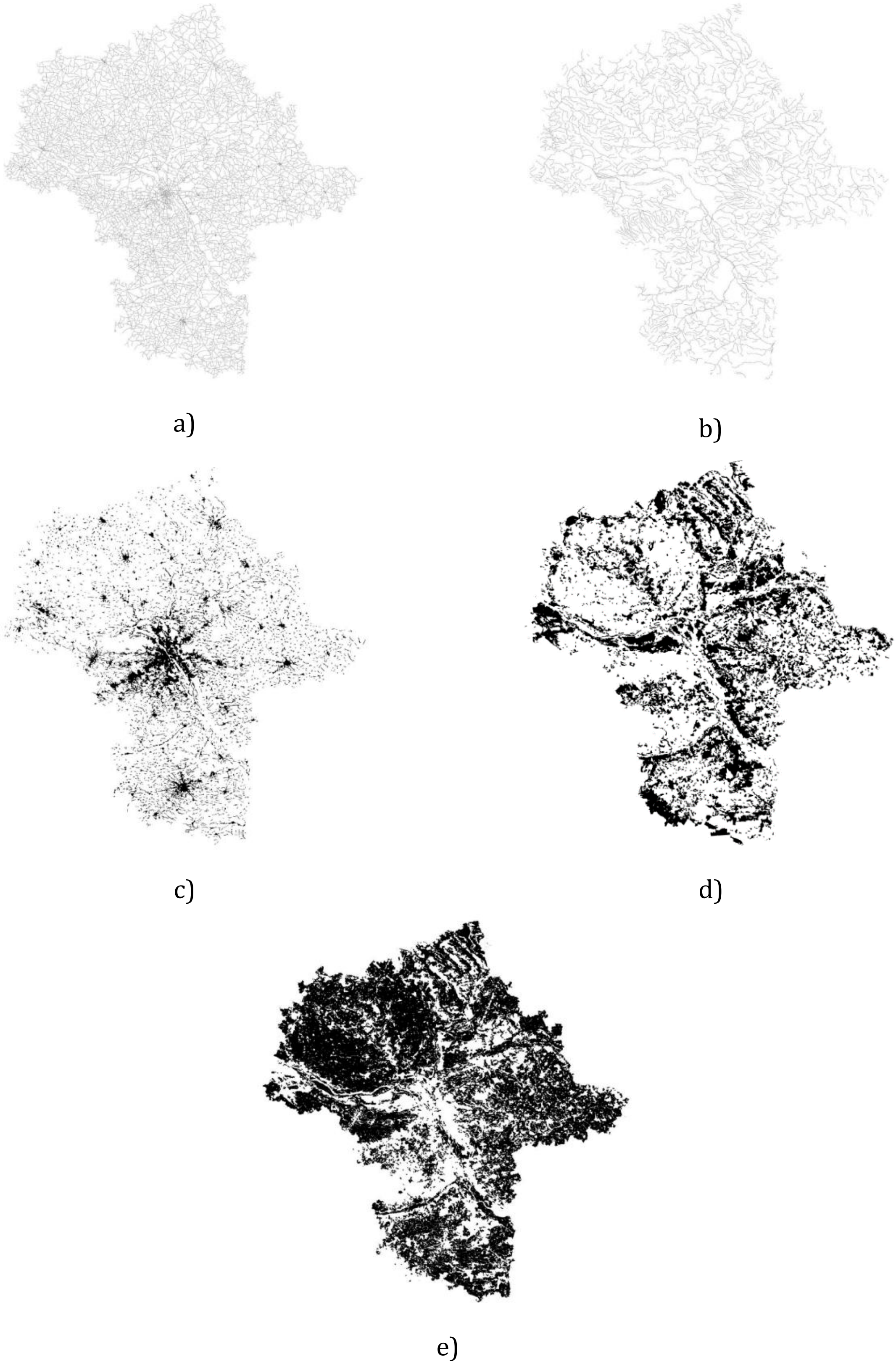

Fig. 2. BDOO data for the Mazowieckie voivodeship: a) SKDR layer – roads, b) SWRS layer – rivers, c) PTZB layer – buildings, d) PTLZ layer – forests, e) PTTR layer – farming
Source: BDOO

For all images, their box size was determined. To determine it, the images were covered with regular meshes of 2, 5, 10, 20, 35, 50, 75 and 100 pixels. After counting the number of boxes in which each analysed set is located, graphs were plotted, from which the values of their box dimension were read

Results and discussion

Obtained results are summarised in Table 2. These data are also illustrated on the graph in Figure 3. Additionally, for a better interpretation of the results obtained, cartograms were made, which present the size of the dimension obtained, for each analysed element of the planning space in each voivodeship (Figure 4).

Table 2. Box dimensions of the analysed elements of the planning space within individual voivodeships

Voivodship	Rivers	Forest	Roads	Buildings	Farming
Dolnośląskie	1.24	1.77	1.28	1.42	1.88
Kujawsko-pomorskie	1.18	1.77	1.28	1.40	1.90
Lubelskie	1.19	1.73	1.26	1.42	1.89
Lubuskie	1.20	1.86	1.24	1.38	1.79
Łódzkie	1.18	1.73	1.30	1.44	1.88
Małopolskie	1.24	1.78	1.29	1.61	1.80
Mazowieckie	1.20	1.74	1.30	1.46	1.87
Opolskie	1.26	1.76	1.26	1.40	1.89
Podkarpackie	1.23	1.85	1.25	1.53	1.80
Podlaskie	1.18	1.77	1.27	1.33	1.89
Pomorskie	1.19	1.82	1.24	1.43	1.84
Śląskie	1.21	1.79	1.31	1.64	1.79
Świętokrzyskie	1.19	1.79	1.30	1.46	1.82
Warmińsko-mazurskie	1.20	1.77	1.23	1.32	1.87
Wielkopolskie	1.21	1.75	1.33	1.41	1.89
Zachodniopomorskie	1.21	1.81	1.23	1.35	1.85
Mean	1.21	1.78	1.27	1.44	1.85
Standard deviation	0.02	0.04	0.03	0.09	0.04

Source: own study

It is clearly visible that, according to the definition, the analysed objects can be classified as fractals. What is important, the box dimension of the same layers in individual voivodeships is similar. The highest value of standard deviation for box dimension in voivodeships is 6%. It was obtained for the PTZB layer (buildings). Such a high value, compared with others, is caused by the dependence of the density of buildings on the density of population in a given province. The Pearson's complete correlation coefficient for these two values is 0.88 (determined on the basis of data from the Local Data Bank for 2015 – see Table 3). Thus, for the two most densely populated voivodeships (Małopolskie and Śląskie) the box dimension is the largest and amounts to over 1.60, while in the two voivodeships with the lowest population density (Podlaskie and Warmińsko-Mazurskie) the box dimension is the smallest (just over 1.30).

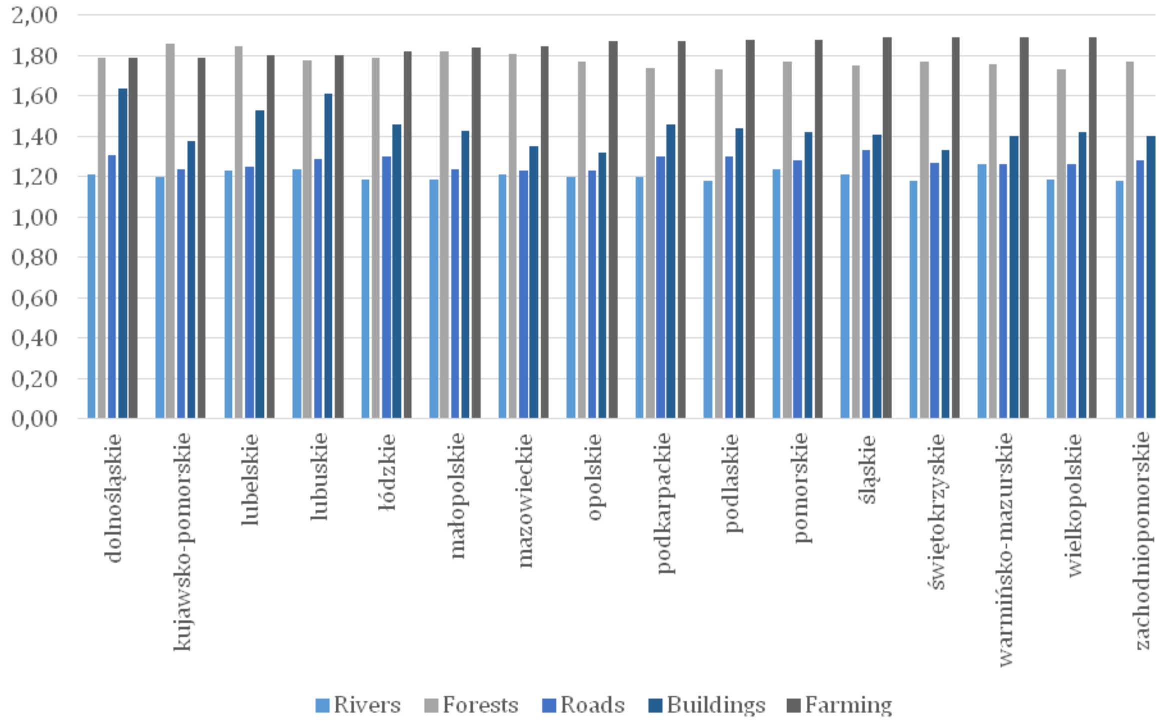

Fig. 3. The box dimension of the analysed elements of the planning space within the borders of individual voivodeships
Source: own study

From the spatial distribution of the fractal dimension (Fig. 4), it can be concluded that where certain elements fill the planning space more, others occupy less of it. In the case of roads (Fig. 4a), the boxed dimension is larger in the centre of the country. This may be related to the need to connect the centre of Poland with the rest of the country and thus a better developed road network in this part of the country. For rivers (Fig. 4b), the lowest values of the box dimension were obtained in voivodeships crossed by watersheds. However, the highest values for rivers were obtained in mountainous areas where the river network is much more developed.

The remaining elements of the planning space, i.e. buildings, forests and crops, in a way complement each other. Looking at the greyscale system in Figures 4c, 4d and 4e, one cannot also fail to notice the relationship of the spatial distribution of the values of the box dimension with the lie of the land. The highest values for forests and crops are in the north of the country, while the highest values for buildings are in the south.

Additionally, the analyses presented here confirm that the box dimension is a measure of the degree to which space is filled by the fractal object it contains – box dimensions are larger for layers that occupy a larger area.

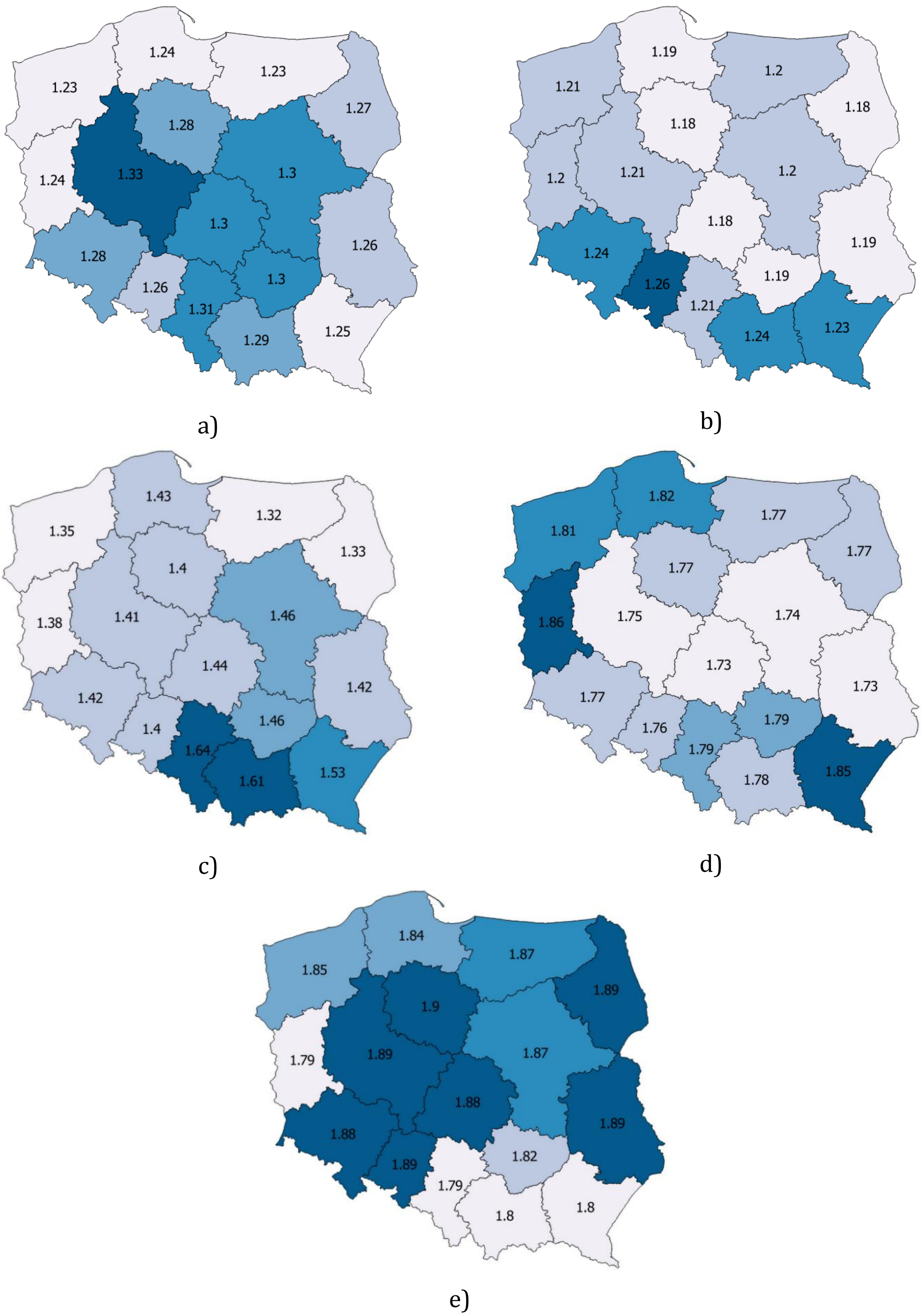

Fig. 4. The box dimension of the analysed elements of planning space within the borders of individual voivodeships: a) roads, b) rivers, c) buildings, d) forests, e) farming
Source: own study

To confirm the latter thesis, the area recorded in the real estate cadastre was determined using data from the Local Data Bank:
- land under flowing surface waters – "Wp",
- forests – "Ls",
- roads – "dr",
- buildings, i.e., the total of developed agricultural land – "Br", residential areas – "B", industrial areas – "Ba" and other developed areas – "Bi",
- crops, i.e. the total of arable land – 'R', permanent meadows – 'Ł' and permanent pastures – 'Ps'.

Individual values are presented in Table 3.

Table 3. Voivodeships' characteristics based on the Local Data Bank

Voivodship	Area [m^2]	Population density [per./km^2]	Waters [m^2]	Forests [m^2]	Roads [m^2]	Buildings [m^2]	Farming [m^2]
Dolnośląskie	1994674	146	15633	610968	62505	75883	1129463
Kujawsko-pomorskie	1797134	116	43162	428491	42612	54237	1125099
Lubelskie	2512246	85	11988	582405	60670	83428	1644570
Lubuskie	1398789	73	22089	710350	33924	30756	538959
Łódzkie	1821895	137	8967	390950	47880	77474	1201786
Małopolskie	1518279	222	18462	440664	41353	78971	849452
Mazowieckie	3555847	150	39105	824660	91936	163117	2215565
Opolskie	941187	106	11820	258846	26799	31372	576732
Podkarpackie	1784576	119	19373	683462	45624	65687	877665
Podlaskie	2018702	59	24640	629184	50557	45628	1171611
Pomorskie	1831034	126	51494	681537	44854	53945	884635
Śląskie	1233309	371	12960	402607	41987	103291	592958
Świętokrzyskie	1171050	107	7807	334796	27161	46070	680364
Warmińsko-mazurskie	2417347	60	118285	769824	53864	49086	1268498
Wielkopolskie	2982650	117	36751	785998	77102	95404	1848971
Zachodniopomorskie	2289248	75	68027	834760	50198	49165	1085677

Source: Local Data Bank

Having determined the percentage share of land area used specifically in the total area of voivodeships (Table 4), Pearson correlation coefficients of these values and corresponding box dimensions can be determined (Table 2). The obtained correlation coefficients are presented in Table 5.

As can be easily observed, the correlations for forests, buildings and crops are quantities that are considered strong in the literature (Czaja, 1997). The correlation for roads is neither strong nor weak. Its average value may be the effect of presenting roads in BDOO only as their axis. The situation is similar for rivers. The weak correlation of the surface area of flowing waters with the fractal dimension of the river network is most probably a consequence of even greater generalisation of these objects than in the case of roads.

Table 4. Share of land area in total area of voivodships

Voivodeship	Rivers [%]	Forests [%]	Roads [%]	Buildings [%]	Farming [%]
Dolnośląskie	0.8	30.6	3.1	3.8	56.6
Kujawsko-pomorskie	2.4	23.8	2.4	3.0	62.6
Lubelskie	0.5	23.2	2.4	3.3	65.5
Lubuskie	1.6	50.8	2.4	2.2	38.5
Łódzkie	0.5	21.5	2.6	4.3	66.0
Małopolskie	1.2	29.0	2.7	5.2	55.9
Mazowieckie	1.1	23.2	2.6	4.6	62.3
Opolskie	1.3	27.5	2.8	3.3	61.3
Podkarpackie	1.1	38.3	2.6	3.7	49.2
Podlaskie	1.2	31.2	2.5	2.3	58.0
Pomorskie	2.8	37.2	2.4	2.9	48.3
Śląskie	1.1	32.6	3.4	8.4	48.1
Świętokrzyskie	0.7	28.6	2.3	3.9	58.1
Warmińsko-mazurskie	4.9	31.8	2.2	2.0	52.5
Wielkopolskie	1.2	26.4	2.6	3.2	62.0
Zachodniopomorskie	3.0	36.5	2.2	2.1	47.4

Source: own study based on the data from Local Data Bank

Table 5. Dependence of the box dimension on the area of the planning space element

Elements of planning space	Rivers	Forests	Roads	Buildings	Farming
Correlation coefficient	-0.13	0.92	0.48	0.89	0.74

Source: own study

Given that the fractal dimension of rivers and roads is not as strongly correlated with the area these elements occupy in the planning space as the dimension of forests, buildings and crops, they were omitted from further considerations.

Subsequent analyses concerned comparison of the size of the box-size dimension of buildings with the dimension of forests and crops for individual voivodeships. The relationship between the two is shown in the graphs in Figures 5 and 6.

The voivodeships on the diagrams were arranged in order, depending on the box dimension of buildings that characterised them (from the largest to the smallest). For each voivodeship, apart from the size of the dimension for buildings, the dimension for crops (Fig. 5) and forests (Fig. 6) is also presented. As can be easily noticed, with decreasing value of the box-size dimension for buildings, the value of the dimension for crops increases slightly. Only three voivodeships: Pomorskie, Lubuskie and Zachodniopomorskie diverge from the general tendency.

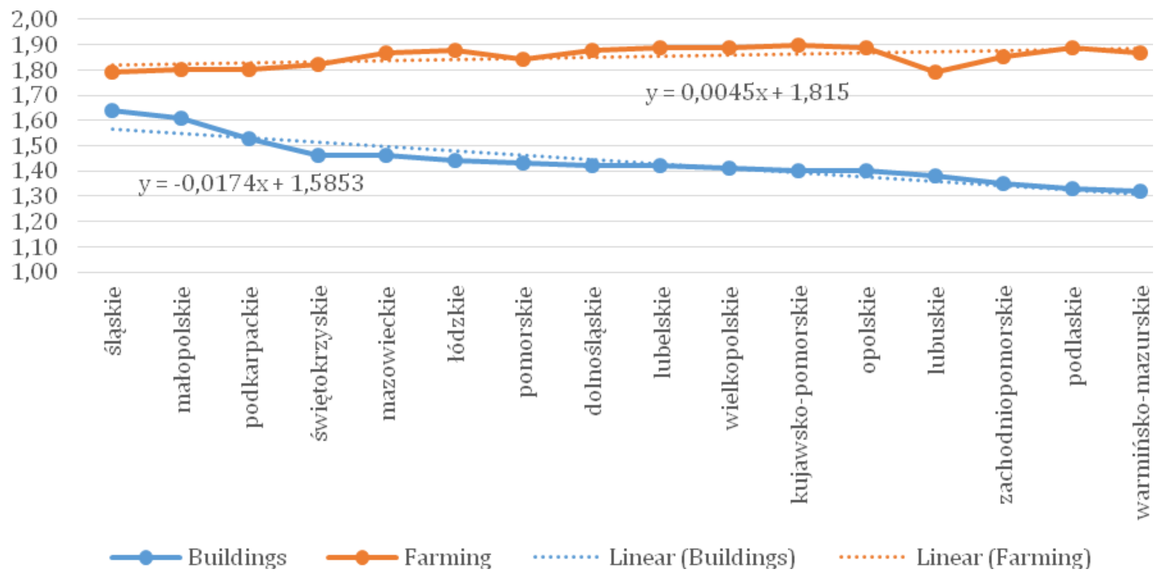

Fig. 5. Comparison of fractal dimensions of building and farming areas
in various voivodeships
Source: own study

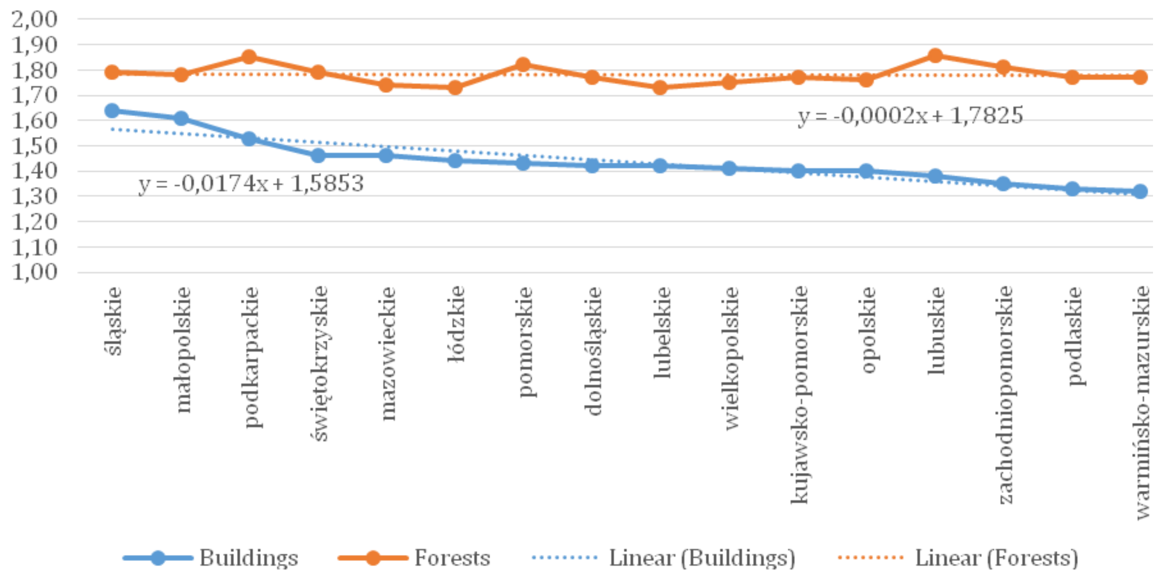

Fig. 6. Comparison of fractal dimensions of building and forest areas
in various voivodeships
Source: own study

In the case of forests, such no relationship is noticeable, so it was decided to check how the values of fractal dimensions of forests and crops relate to each other (Fig. 7).

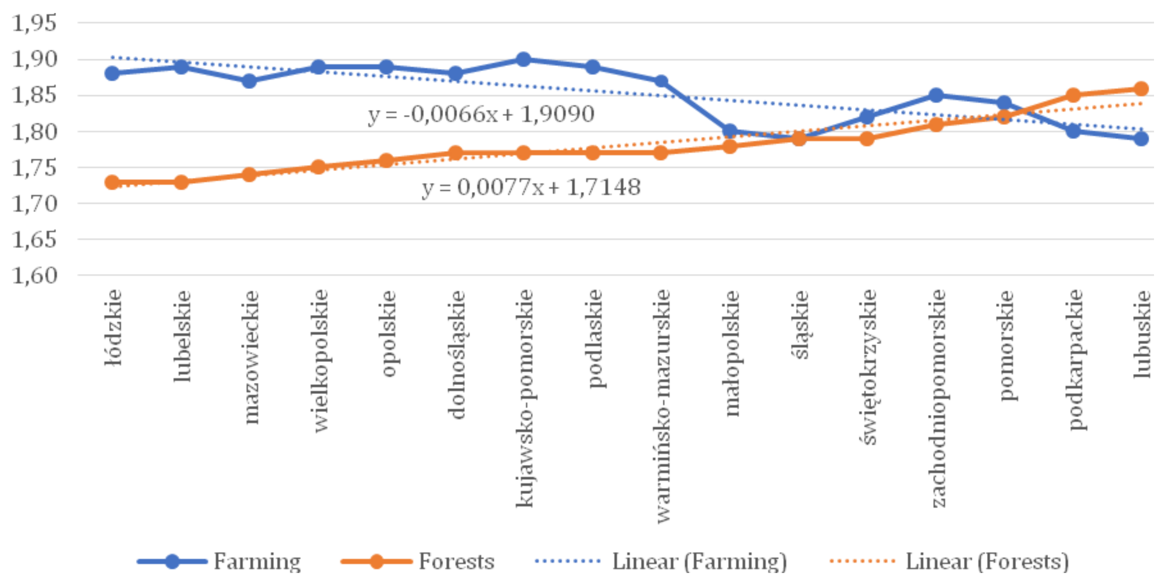

Fig. 7. Comparison of fractal dimensions of farming and forest areas in various voivodeships
Source: own study

In the diagram of fractal dimensions of crops and forests in individual voivodeships, the order of voivodeships depends on the value of the box dimension for forests. The voivodeships have been arranged in ascending order of the value of this dimension. The general tendency of the corresponding values of the box dimension for crops is decreasing. In most voivodships the box dimension of crops is larger than that of forests. The exceptions are Podkarpackie and Lubuskie voivodships.

Summary

The study of spatial development elements for the presence of regularities in the planning space should provide a foundation for the creation or modification of spatial plans. Spatial planning should be based on the reduction of uncertainty in dynamic spatial systems as objects of spatial planning interest (Bajerowski, 2003). In this context, it is necessary to know the rules and regularities occurring in the past, and such information can be provided by, among others, the analysis of fractal features. The ordering and optimisation of the planning space is connected with its rational management, which ultimately is to lead to the desired states of spatial development. The analysis of land use is therefore an indispensable element in the aspect of the directions of further development of areas included in development plans relating to the future.

In conclusion it may be noted that:

1. The analyses presented in the paper can be used in the area subject to land-use planning procedures at the national and regional level. However, the use of fractal analysis in spatial planning is only possible to determine the general directions of development of a specific planning space.

2. Confirmation of fractal features for selected elements in the area subject to planning procedures was possible by determining their box dimensions. All the analysed factors shaping the planning space (road network, river network, buildings, forests and crops) fulfil the basic criterion from the definition of a fractal object.
3. The strong correlations of box dimensions for BDOO layers containing buildings, forests and crops with the corresponding geodetic area from the Local Data Bank confirm the fact that the fractal dimension indicates to what extent the figure fills the space in which it is located. The lack of strong relationships for the river network and the road network is a result of the generalisation of the data in the BDOO database.

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