

AN ALGORITHM FOR IMPROVING THE ACCURACY OF SYSTEMS MEASURING PARAMETERS OF MOVING OBJECTS

Dimitar Dichev¹⁾, Hristofor Koev¹⁾, Totka Bakalova²⁾, Petr Louda³⁾

1) *Technical University of Gabrovo, Faculty of Machine and Precision Engineering, Hadji Dimitar 4, 5300 Gabrovo, Bulgaria*
(✉ dichevd@abv.bg, +359 06 682 7273, koevh@abv.bg)

2) *Technical University of Liberec, Institute for Nanomaterials, Advanced Technologies and Innovation, Studentska 2, 46117 Liberec, Czech Republic* (tbakalova@seznam.cz)

3) *Technical University of Liberec, Department of Material Science, Studentska 2, 46117 Liberec, Czech Republic* (petr.louda@tul.cz)

Abstract

The paper considers an algorithm for increasing the accuracy of measuring systems operating on moving objects. The algorithm is based on the Kalman filter. It aims to provide a high measurement accuracy for the whole range of change of the measured quantity and the interference effects, as well as to eliminate the influence of a number of interference sources, each of which is of secondary importance but their total impact can cause a considerable distortion of the measuring signal. The algorithm is intended for gyro-free measuring systems. It is based on a model of the moving object dynamics. The mathematical model is developed in such a way that it enables to automatically adjust the algorithm parameters depending on the current state of measurement conditions. This makes possible to develop low-cost measuring systems with a high dynamic accuracy. The presented experimental results prove effectiveness of the proposed algorithm in terms of the dynamic accuracy of measuring systems of that type.

Keywords: Kalman filter, adaptive algorithm, dynamic measurements, dynamic error, adaptive measuring systems.

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1. Introduction

A characteristic feature of instruments measuring parameters of moving objects is their operation in dynamic conditions. They are caused by motion of moving objects, shaking of ships, fluctuations of aircraft and vehicles, as well as by vibrations taking place in the location of the measuring instruments. Those motions generate inertial forces and moments which act upon the measuring instruments and systems causing a dynamic error in the measurement result [1]. If there are no appropriate solutions in the metrological chains and procedures of the measuring instruments to deal with the problem, the dynamic error could be considerable, which leads to a high inaccuracy of the measurement result [2]. Another important feature of the metrological problems to be solved in these cases is that the measured quantities and the interference effects are characterized with parameters (intensity and frequency of the maximum in the spectrum) that are changing within a wide range.

It is a serious obstacle in the synthesis of measuring instruments that provide an acceptable measurement accuracy for the wide range of changing the parameters of the inertial effects.

This problem could be solved by using adaptive algorithms integrated in the metrological chain of measuring systems [3, 4]. The mathematical models of those algorithms can be developed so as to make possible the automatic adjustment of their parameters depending on the current state of measurement conditions. In addition, the structure, parameters and measurement procedures of those algorithms can be developed in such a way that any parameter accepted as a measure of divergence between the measurement result and the measured quantity can have a function minimum for given measurement conditions. One of such parameters can

be the root-mean-square deviation of the error [5]. It enables to solve the problem related to the synthesis of similar measuring systems operating in optimal conditions of the output solutions on the basis of the minimum error criterion.

Development of such algorithms is possible if only there is an accurate and comprehensive mathematical model which is logically consistent in relation to the physical processes. The latter have their own specific features depending on the type and characteristics of the moving objects, measured quantities and interference effects, as well as on the dynamic properties of measuring instruments. With regard to the above mentioned there are various types of adaptive algorithms which differ in their computational complexity, behavior patterns, used output data, structure of the adapting systems themselves [6, 7]. Integration of such algorithms in the metrological chain of systems measuring parameters of moving objects is a good perspective for the enhancement of measuring equipment in this area since it considerably increases the measurement accuracy in the dynamic mode without using expensive elements and units in the measuring system structure [8]. Therefore, the paper presents an adaptive algorithm based on the Kalman filter, whose aim is to increase the accuracy of a system measuring the parameters that define the space-temporal position of a ship.

2. A block diagram of the measuring system

The proposed algorithm is part of the metrological chain of a system measuring the roll, pitch, heel and trim of a ship [9]. However, it is universal enough because it is based on the real model of the moving object dynamics so that it can be used in other similar measuring systems.

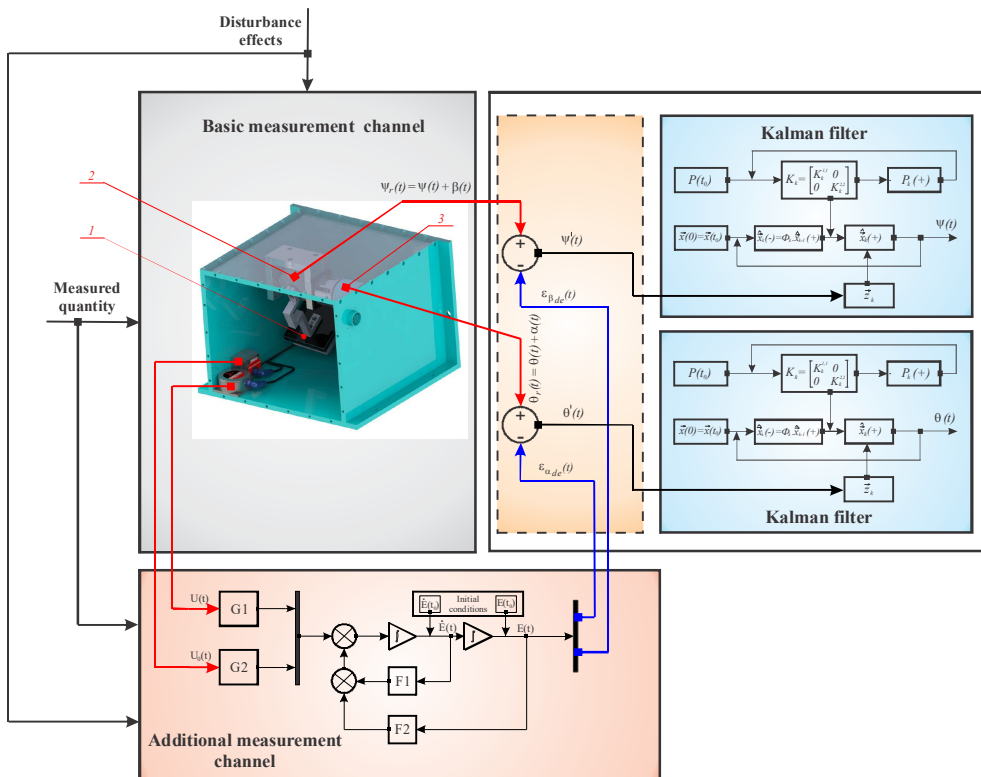


Fig. 1. A block diagram of the measuring system.

The block diagram of the system is shown in Fig. 1. The measuring system consists of two measurement channels operating in parallel. The physical pendulum 1 in the main measurement channel is used for modeling the local vertical. It enables to obtain information on the current values of the ship pitch and roll by means of two absolute encoders 2 and 3. Due to instability of the physical pendulum in the inertial field of force the information obtained from the main measurement channel contains unacceptably high dynamic errors. The latter are caused by deviations of the pendulum 1 from the actual position of the local vertical.

Accuracy of a measuring system in the dynamic mode is increased by operation of an additional measurement channel whose main purpose is to eliminate the above mentioned dynamic errors. The additional measurement channel is based on a measuring calculation-based method which assumes that information on the system state can be obtained by means of a theoretical model of its dynamics, whose input vector is formed as a result of the current measurement values. The vector of the system state is described by the following system of differential equations [9]:

$$\frac{d^2 \mathbf{E}(t)}{dt^2} = F1(t) \cdot \frac{d\mathbf{E}(t)}{dt} + F2(t) \cdot \mathbf{E}(t) + G1(t) \cdot \mathbf{U}_\theta(t) + G2(t) \cdot \mathbf{U}(t), \quad (1)$$

where: $\mathbf{U}_\theta(t)$ and $\mathbf{U}(t)$ – the r -dimensional vectors defining the linear and angular inertial effects, respectively; $F1(t)$ and $F2(t)$ – matrices of $n \times n$ type; $G1(t)$ and $G2(t)$ – matrices of $n \times r$ type; $\mathbf{E}(t)$ – the n -dimensional vector of the dynamic system state, whose mathematical model is reduced in [9] to the dynamic error defined by the functions of the deviations of the physical pendulum when measuring roll and pitch, respectively, *i.e.*:

$$\mathbf{E}(t) = \begin{pmatrix} \alpha(t) \\ \beta(t) \end{pmatrix} = \begin{pmatrix} \varepsilon_{\alpha_{de}}(t) \\ \varepsilon_{\beta_{de}}(t) \end{pmatrix}, \quad (2)$$

where the deviations of the physical pendulum $\alpha(t)$ and $\beta(t)$ are equal to the dynamic errors $\varepsilon_{\alpha_{de}}(t)$ and $\varepsilon_{\beta_{de}}(t)$ when measuring roll and pitch, respectively.

In a linearized form of the dynamic model of the main measurement channel all elements of matrices $F1$, $F2$, $G1$ and $G2$ are constant. Information on the current values of elements of the $\mathbf{U}_\theta(t)$ and $\mathbf{U}(t)$ vectors is provided by measurement results from two 3-axis *Micro-Electromechanical Systems* (MEMS). A block diagram of the additional measurement channel presented in Fig. 1 reduces the mathematical model (1) to the consistent logic of a metrological procedure that gives the output information on the current values of dynamic errors $\varepsilon_{\alpha_{de}}(t)$ and $\varepsilon_{\beta_{de}}(t)$. Since $\varepsilon_{\alpha_{de}}(t)$ and $\varepsilon_{\beta_{de}}(t)$ functions are obtained as a result of operation of an algorithm being a measuring calculation-based analog of the current physical pendulum motion, they can be used as correction signals of the information obtained from the main measurement channel (Fig. 1).

3. A structural model of the algorithm

Accuracy of a measuring system can be considerably increased in the dynamic operating mode if a module with an adaptive algorithm based on the Kalman method is placed at the end of the metrological chain (Fig. 1). In this case the Kalman filter is based on the dynamic models and measurement of a moving object (the ship). The symbols presented in Table 1 are used to describe the nature of the developed algorithm.

A block diagram of the algorithm is shown in Fig. 2. In the paper the structure of the algorithm for the channel measuring the ship's roll $\theta(t)$ is considered. However, an analogous filter of identical structure and mathematical model is connected to the metrological chain of the

second measurement channel (to measure the pitch). To obtain the forecast estimate $\hat{\bar{x}}_k(-)$, it is necessary to integrate the differential equation [10, 11]:

$$\frac{d\bar{x}(t)}{dt} = F(t) \cdot \bar{x}(t), \tag{3}$$

under the initial condition $\bar{x}(0) = \bar{x}(t_0)$.

Table 1. The symbols used in the algorithm design.

Symbols	Description
$\bar{x}(t)$	the vector of the measured quantity
Φ_k	the transition function of the $[n \times n]$ -dimensional state defining the dynamics of the ship at the roll
$\hat{\bar{x}}_{k-1}(+)$	the estimate of the vector of the state at the moment t_{k-1}
$\bar{w}(t)$	the r -dimensional vector representing the signal at the system input
$\bar{y}(t)$	the m -dimensional vector characterizing the error-free signal at the output of the measurement channel before the Kalman filter
$\bar{z}(t)$	the m -dimensional vector defining the measurement result before the Kalman filter
$\bar{v}(t)$	the m -dimensional vector determining the measurement error
F, G, H	the $[n \times n]$, $[n \times r]$, $[m \times n]$ -dimensional matrices
$\bar{\varepsilon}(t)$	the filter error
$Q(t)$	the $[r \times r]$ -dimensional symmetric matrix defining the intensity of white noise
$\delta(t-\tau)$	the delta-function
$R(t)$	the $[m \times m]$ -dimensional symmetric matrix
$P(t)$	the correlation matrix defining the estimate error
K_k	the matrix amplification coefficient
$\theta(t)$	the function defining the roll
ν_θ	$\nu_\theta = \mu_1 / 2 \cdot (J_{yy'} + \lambda_1)$ – the relative damping factor of the roll
ω_θ	$\omega_\theta = \sqrt{G_g \cdot h_0 / (J_{yy'} + \lambda_1)}$ – the natural frequency of the roll
λ_1	the added moment of inertia
μ_1	the damping factor
$J_{yy'}$	the mass moment of inertia of the ship with regard to the central longitudinal axis
G_g	the ship weight
h_0	the transverse metacentric height
$\alpha(t)$	the angle of the wave slope

According to the block diagram presented in Fig.1 and the accepted processing procedure of the measuring system, $\bar{x}(t)$ is the n -dimensional vector determining shaking of the ship along the heel coordinate, *i.e.* the roll. Then, the forecast estimate transformed in discrete time will be:

$$\hat{\bar{x}}_k(-) = \Phi_k \cdot \hat{\bar{x}}_{k-1}(+). \tag{4}$$

The system model can be described by a vector-matrix differential equation determining the system dynamics and a second equation defining the measurement, *i.e.* [12]:

$$\frac{d\bar{x}}{dt} = F \cdot \bar{x}(t) + G \cdot \bar{w}(t), \tag{5}$$

$$\bar{z}(t) = \bar{y}(t) + \bar{v}(t) = H \cdot \bar{x}(t) + \bar{v}(t). \tag{6}$$

The components of vector $\vec{w}(t)$ are linearly linked to random functions of a *white noise* type. They have zero mathematical expectations and their correlation functions are determined by the equation [10]:

$$M[\vec{w}(t) \vec{w}^T(\tau)] = Q(t) \cdot \delta(t - \tau). \tag{7}$$

The vector defining the measurement errors $\vec{v}(t)$ has analogous properties, where:

$$M[\vec{v}(t) \vec{v}^T(\tau)] = R(t) \cdot \delta(t - \tau). \tag{8}$$

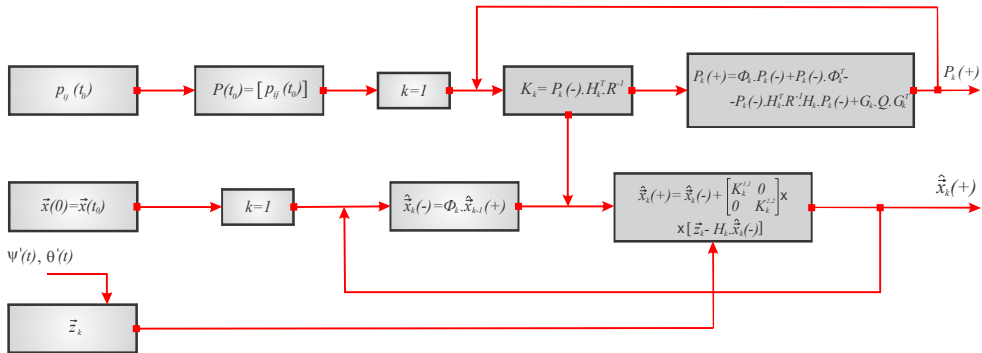


Fig. 2. A block diagram of the Kalman filter.

The problem being solved by the algorithm is to determine the optimal estimate $\hat{x}(t)$ of the state vector of the measured parameters. It is based on (5) and the signal $\vec{z}(t)$ obtained from measurement (6). The difference $\vec{e}(t)$ between vector $\vec{x}(t)$, defining the actual state of the measured parameter, and its estimate (the output signal from the module with the considered algorithm) $\hat{x}(t)$ determines the estimate error or the filter error, *i.e.*:

$$\vec{e}(t) = \vec{x}(t) - \hat{x}(t). \tag{9}$$

Within the Kalman algorithm the following condition is accepted as an estimate optimality criterion: the vector $\hat{x}(t)$ is a non-shifted estimate of $\vec{x}(t)$ that minimizes the variance of the error $\vec{e}(t)$. The correlation matrix $P(t)$ determining the estimate error is an $[n \times n]$ -dimensional symmetric matrix. It can be defined as a solution of the matrix differential equation [11]. The initial value $P(t_0)$ is a diagonal matrix whose elements are equal to the variances of the vector of the measured parameter, *i.e.* $P(t_0) = [p_{ij}(t_0)]$, $i, j = 1, 2, \dots, n$. Taking into account the stationary nature of the dynamic system and the discrete form of implementation of the algorithm, the solution of the differential equation can be written in the following form [13]:

$$P_k(+) = \Phi_k \cdot P_k(-) + P_k(-) \cdot \Phi_k^T - P_k(-) \cdot H_k^T \cdot R^{-1} \cdot H_k \cdot P_k(-) + G_k \cdot Q \cdot G_k^T. \tag{10}$$

In this way the error correlation matrix for each step is a function of the a priori value of that matrix. In the next step the matrix a posteriori value is used for correction purposes. The algorithmic sense of that operation is illustrated in Fig. 2. The error correlation matrix, whose value is updated in each time step, enables to correct the forecast estimate according to an optimality criterion regarding accuracy. Thus, the metrological criteria referring to the quality of the measuring process are satisfied. In the algorithm (Fig. 2) the estimate is corrected by the matrix amplification coefficient, whose discrete form is [13]:

$$K_k = P_k(-) \cdot H_k^T \cdot R^{-1}. \quad (11)$$

Then, the optimal estimate at the output of the algorithm will be [13]:

$$\hat{x}_k(+) = \hat{x}_k(-) + K_k \cdot [\bar{z}_k - H_k \cdot \hat{x}_k(-)]. \quad (12)$$

From (12) and Fig. 2 it follows that the algorithm is based on the model forming the actual measuring signal where, multiplied by the amplification coefficient K_k , the difference between the k -th value of the measured signal before filter \bar{z}_k and the respective estimate $\hat{y}_k = H_k \cdot \hat{x}_k(-)$ of its actual useful component is introduced.

4. Design of the algorithm elements

According to the block diagram in Fig. 1 the signals, obtained after the differential connection of outputs of the absolute encoders and the additional measurement channel, are transferred to the processing modules whose algorithms are based on the concept presented in Section 3. To obtain the forecast estimate $\hat{x}_k(-)$, it is necessary to integrate the following differential equation describing the ship roll [14]:

$$\ddot{\theta}(t) + 2 \cdot \nu_\theta \cdot \dot{\theta}(t) + \omega_\theta^2 \cdot \theta(t) = \omega_\theta^2 \cdot \alpha(t). \quad (13)$$

In order to maximally simplify the algorithm parameters, we reduce (13) to the following form:

$$\ddot{\theta}(t) + 2 \cdot \mu_1 \cdot \dot{\theta}(t) + b^2 \cdot \theta(t) = 2 \cdot b \cdot \sqrt{D(\theta)} \cdot \mu_1 \cdot \alpha(t), \quad (14)$$

where $D(\theta)$ – the variance of the roll; $b^2 = \mu_1^2 + \lambda_1^2$.

The correlation function of the random process defining the ship roll can be well approximated by the formula [15]:

$$K_\theta(\tau) = D(\theta) \cdot e^{-\mu_1 |\tau|} \cdot \left(\cos \lambda_1 \cdot \tau + \frac{\mu_1}{\lambda_1} \cdot \sin \lambda_1 \cdot |\tau| \right). \quad (15)$$

The correlation function (15) corresponds to the differential equation defining the relation between the random process $\theta(t)$ and the interference $w(t)$ introduced as white noise, *i.e.*:

$$\ddot{\theta}(t) + 2 \cdot \mu_1 \cdot \dot{\theta}(t) + b^2 \cdot \theta(t) = 2 \cdot b \cdot \sqrt{D(\theta)} \cdot \mu_1 \cdot w(t), \quad (16)$$

where $w(t)$ – the scalar white noise of intensity $Q = 1$.

The second order differential (16) can be written in the form of a system of two first order equations, which has the following form:

$$\begin{aligned} \dot{\theta}(t) &= \gamma(t), \\ \dot{\gamma}(t) &= -2 \cdot \mu_1 \cdot \gamma(t) - b^2 \cdot \theta(t) + 2 \cdot b \cdot \sqrt{D(\theta)} \cdot \mu_1 \cdot w(t). \end{aligned} \quad (17)$$

According to (5), the matrix form of vector $\bar{x}(t)$ will be:

$$\bar{x}^T(t) = [\theta \quad \gamma], \quad (18)$$

and matrices F and G will be in the form:

$$F = \Phi_k = \begin{bmatrix} 0 & 1 \\ -b^2 & -2 \cdot \mu_1 \end{bmatrix}, \quad G = G_k = \begin{bmatrix} 0 \\ 2 \cdot b \cdot \sqrt{D(\theta)} \cdot \mu_1 \end{bmatrix}. \quad (19)$$

In the solved problem the equation defining the measurement $z(t)$ can be written as follows:

$$z(t) = \theta(t) + v(t), \tag{20}$$

where $\theta(t)$ is the measurement result before the filter; $v(t)$ is the measurement error, which can be considered to be a white noise of intensity $R = 1$.

Then, taking into consideration the conditions of that problem, (6) can be written in the following form:

$$z(t) = H(t) \cdot \bar{x}(t) + v(t). \tag{21}$$

Therefore, the matrix of measurement H from (6), determined by the type of the input signal of the algorithm, will be:

$$H = H_k = [1 \ 0]. \tag{22}$$

The diagonal matrix $P(t_0)$, whose elements are equal to the variances $D(\theta)$ and $D(\dot{\gamma}) = D(\dot{\theta})$ of the components of vector $\bar{x}^T(t) = [\theta \ \gamma]$, is created on the basis of correlation functions of quantities $\theta(t)$ and $\dot{\theta}(t) = \dot{\gamma}(t)$. This is due to the equalities $D(\theta) = K_{\theta}(\tau)$ and $D(\dot{\theta}) = K_{\dot{\theta}}(\tau)$, where $K_{\theta}(\tau)$ and $K_{\dot{\theta}}(\tau)$ are the values of correlation functions $\theta(t)$ and $\dot{\theta}(t) = \dot{\gamma}(t)$, when $\tau = 0$.

The correlation function of the ship's roll $K_{\theta}(\tau)$ is determined by (15), resulting in the equality $K_{\theta}(\tau) = D(\theta)$, when $\tau = 0$. Since the function $\theta(t)$ is related to $\dot{\theta}(t)$ by a differentiation operator, its corresponding correlation functions will be as follows [16]:

$$K_{\dot{\theta}}(t_1, t_2) = \frac{\partial^2 K_{\theta}(t_1, t_2)}{\partial t_1 \cdot \partial t_2} = -\frac{d^2 K_{\theta}(\tau)}{d\tau^2} = K_{\dot{\theta}}(\tau), \tag{23}$$

where: $\tau = t_2 - t_1$.

From (23) and (15) it follows:

$$K_{\dot{\theta}}(\tau) = D_{\theta} \cdot (\mu_1^2 + \lambda_1^2) \cdot e^{-\mu_1 |\tau|} \cdot \left(\cos \lambda_1 \cdot \tau - \frac{\mu_1}{\lambda_1} \cdot \sin \lambda_1 \cdot |\tau| \right). \tag{24}$$

Then, on the basis of (15) and (24) we can write the initial state of the error correlation matrix $P(t_0)$, i.e.:

$$P(t_0) = \begin{bmatrix} D(\theta) & 0 \\ 0 & D(\theta) \cdot (\mu_1^2 + \lambda_1^2) \end{bmatrix}. \tag{25}$$

The equations obtained for matrices F , G , H and the established intensities of white noise $Q = 1$, $R = 1$ acting at the system input enable to solve the Kalman problem in the time domain by the algorithm presented in the block diagram in Fig. 2. This algorithm is based on a system of equations, formed by the dependences (10), (11) and (12) defining the Kalman algorithm. In this way, by using the Kalman algorithm, the optimal estimate $\hat{x}(t)$ of the measured quantity $\theta(t)$ is obtained at consecutive moments of system operation. The estimate optimality is determined by the minimum mean square error criterion defined by (10).

The above algorithm can considerably increase the measuring system accuracy because it is based on an actual model of the ship dynamics used for determining the estimate $\hat{x}(t)$, which can be significantly corrected as a result of analyzing each new measurement in the time sequence and the error estimate at the same moment.

5. Experiments

The experiments have been done using a specially developed dedicated stand. The stand is equipped with a six-degree-of-freedom hexapod, which enables to reproduce the ship fluctuations simulating real conditions [17–19]. To ensure the experiment accuracy, it is

calibrated and the metrological traceability of its unit to the length standard is ensured. A general view of the stand equipment is shown in Fig. 3. Its details can be found in [9].

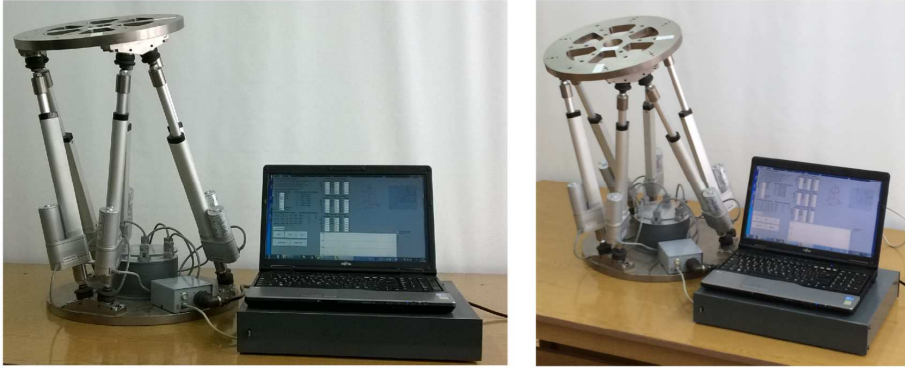


Fig. 3. Photos of the stand equipment.

The measurements have been performed in two operating modes of the measuring system – with and without the module containing the Kalman filter connected to the system processing procedure. Upon driving the stand operating platform by means of the first operating angular frequency ω_l and amplitude A_j , the absolute errors $\varepsilon_{l,i}^j$ at consecutive moments t_i are determined as:

$$\varepsilon_{l,i}^j = y_{l,i}^j - y_{0,i}^j, \quad (26)$$

where $y_{l,i}^j$ is the value, measured by the examined system, of the angular ordinate at the moment t_i when the operating platform produces a fluctuation of amplitude A_j and angular frequency ω_l ; $y_{0,i}^j$ – the value of the assignment controlling motion of the operating platform at the moment t_i with the input data: A_j and ω_l .

In this way the errors are determined when the Kalman filter is disconnected $\varepsilon_{de,i}^j$ and when it is connected $\varepsilon_{de,i}^{kf}$, respectively. The distributions of systematic error boundaries as functions of angular frequencies ω_l^θ (when reproducing the roll) and ω_l^ψ (when reproducing the pitch), with which the operating platform produces angular fluctuations when examining two measurement channels, are given in Fig. 4 and Fig. 5, respectively. Fig. 4 presents the testing results of the measurement channel accuracy for the roll and heel at the fluctuation amplitude $A^\theta=15^\circ$ of the operating platform, whereas Fig. 5 – those for the pitch and trim at the amplitude $A^\psi=6^\circ$. Both figures show the boundaries of changing the systematic error in two operating modes of the system – when the module processing the measuring signals in compliance with the Kalman algorithm is disconnected ($\tilde{\varepsilon}_{l,max}$ and $\tilde{\varepsilon}_{l,min}$) and when this module is connected ($\tilde{\varepsilon}_{l,max}^{kf}$ and $\tilde{\varepsilon}_{l,min}^{kf}$), respectively.

Figures 4 and 5 clearly show that the Kalman filter considerably increases the dynamic accuracy of the measuring system and the systematic error is decreased by at least 50%. When only the additional measurement channel without the Kalman filter is connected, an obvious trend towards increasing the systematic error value in the direction of increasing the fluctuation frequency of the operating platform can be observed [9]. Due to the non-linear characteristic of the Kalman filter a stable correlation between the systematic error and the frequency could not be obtained when this module is connected to the system metrological chain.

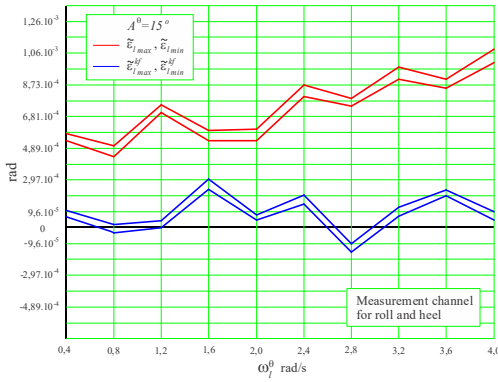


Fig. 4. The systematic error values, obtained upon measuring the roll and heel.

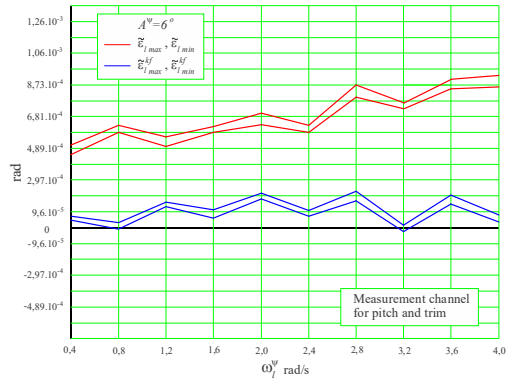


Fig. 5. The systematic error values, obtained upon measuring the pitch and trim.

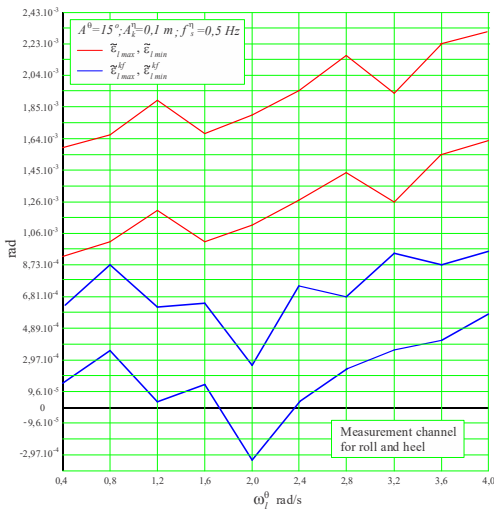


Fig. 6. Boundaries of the confident intervals of random component for the channel of the roll and heel.

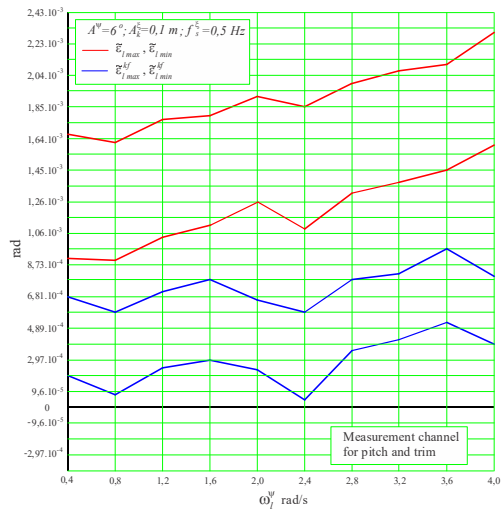


Fig. 7. Boundaries of the confident intervals of the random component for the channel of the pitch and trim.

The boundaries that can be reached by the error of the examined measuring system at the 95% confidence level are presented in Figs. 6 and 7. The experimental results show that the maximal values of the measurement error when the Kalman filter is disconnected do not exceed $0,16^\circ$ in the most unfavorable case of the measuring system operation. When the module is connected to the metrological chain, this error considerably decreases and in the experimental conditions it does not exceed $0,07^\circ$.

6. Conclusions

The proposed algorithm not only makes feasible the adaptability of the measuring system regarding a wide range of changing the parameters determining the measurement conditions but it also enables to eliminate the influence of a number of error sources. Those sources could

be internal interferences of random characteristics, additional external secondary processes of unpredictable behavior, as well as transformation processes in the system.

The experimental results confirm effectiveness of the proposed algorithm in relation to the dynamic accuracy of systems measuring parameters of moving objects. In addition, as a result of the algorithm operation, the accuracy characteristics of the measurement system in the conditions created by dynamic actions are considerably enhanced without using expensive stabilization elements and systems.

The algorithm can be successfully used for increasing the dynamic accuracy in gyro-free systems measuring parameters of moving objects. It can also be applied in a number of other measuring instruments and systems operating on ships, since the algorithm is based on a model of the moving object.

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