VERSITA

Arch. Min. Sci., Vol. 57 (2012), No 3, p. 741–768

Electronic version (in color) of this paper is available: http://mining.archives.pl

DOI 10.2478/v10267-012-0048-8

YOUSUE AZIMI*, MONTZA OSANLOO*, AKBAR ESFAHANIPOUR**

SELECTION OF THE OPEN PIT MINING CUT-OFF GRADE STRATEGY UNDER PRICE UNCERTAINTY USING A RISK BASED MULTI-CRITERIA RANKING SYSTEM

WYBÓR STRATEGII OKREŚLANIA WARUNKU OPŁACALNOŚCI WYDOBYCIA W KOPALNIACH ODKRYWKOWYCH W WARUNKACH NIEPEWNOŚCI CEN W OPARCIU O WIELOKRYTERIALNY SYSTEM RANKINGOWY Z UWZGLĘDNIENIEM CZYNNIKÓW RYZYKA

Cut-off Grade Strategy (COGS) is a concept that directly influences the financial, technical, economic, environmental, and legal issues in relation to exploitation of a mineral resource. A decision making system is proposed to select the best technically feasible COGS under price uncertainty. In the proposed system both the conventional discounted cash flow and modern simulation based real option valuations are used to evaluate the alternative strategies. Then the conventional expected value criterion and a multiple criteria ranking system were used to rank the strategies based on the two valuation methods. In the multiple criteria ranking system besides the expected value other stochastic orders expressing abilities of strategies in producing extra profits, minimizing losses and achieving the predefined goals of the exploitation strategy are considered. Finally, the best strategy is selected based on the overall average rank of strategies through all ranking systems. The proposed system was examined using the data of Sungun Copper Mine. To assess the merits of the alternatives better, ranking process was done at both high (prevailing economic condition) and low price conditions. Ranking results revealed that at different price conditions and valuation methods, different results would be obtained. It is concluded that these differences are due to the different behavior of the embedded option to close the mine early, which is more likely to be exercised under low price condition rather than high price condition. The proposed system would enhance the quality of decision making process by providing a more informative and certain platform for project evaluation.

Keywords: Open pit mining, Cut-off grade strategy, Least-squares Monte Carlo Real options valuation, Multiple criteria ranking system, Metal price uncertainty

Strategia doboru granicy opłacalności (COGS) jest koncepcją mająca bezpośredni wpływ na kwestie finansowe, techniczne, ekonomiczne, środowiskowe oraz prawne związane z eksploatacją surowców naturalnych. Zaproponowano system decyzyjny umożliwiający wybór najkorzystniejszej fizycznie wykonalnej strategii doboru opłacalności wydobycia w warunkach niepewności cen. W proponowanym systemie do

* AMIRKABIR UNIVERSITY OF TECHNOLOGY, DEPRATMENT OF MINING AND METALLURGY, HAFEZ AVE., TEHRAN, IRAN

analizy alternatywnych strategii wykorzystuje się konwencjonalne metody oparte o analizy przepływu strumienia gotówki oraz nowoczesne techniki symulacji rzeczywistych opcji. Następnie zastosowano tradycyjny system oparty o kryterium wartości oczekiwanej oraz system rankingu wielokryterialnego do określenia rankingu strategii, w oparciu o dwie metody oceny. W systemie wielokryterialnym obok wartości oczekiwanej uwzględnia się inne dane stochastyczne określające zdolność poszczególnych strategii do generowania dodatkowych zysków, do ograniczania strat i osiągania wcześniej zdefiniowanych celów. W etapie końcowym dokonuje się wyboru optymalnej strategii w oparciu o całkowity ranking strategii uwzględnionych w systemie. Proponowane podejście testowano w oparciu o dane uzyskane z kopalni miedzi Sungun. Aby ocenić zalety najlepszej alternatywy, ranking przeprowadzono przyjmując warunki wysokich i niskich cen. Wyniki rankingu wykazały, że w warunkach różnych cen i przy zastosowaniu różnych metod oceny, uzyskane rezultaty będą się różnić. Należy wnioskować, że różnice te spowodowane są różnicami w podejściu do wbudowanej opcji wczesnego zamknięcia kopalni, która ma większą szansę na realizację w warunkach niskich cen, a nie wysokich. Proponowany system podniesie jakość procesu decyzyjnego poprzez dostarczenie platformy dodatkowych informacji dla oceny przedsięwzięcia.

Słowa kluczowe: kopalnia odkrywkowa, strategia wyboru granicy opłacalności wydobycia, metoda najmniejszych kwadratów, metoda Monte Carlo, opcje rzeczywiste, ocena, ranking wielokryterialny, niepewność cen metali

1. Introduction

Taylor (1972), defines the cut-off grade as an operating control, which could be used to choose one of the either two choices of actions that are going to be taken place on a specified amount of rock. In other words, it determines the routing of the specific materials inside a reserve, for example to mine or leave, to dump as a waste or stockpile for future processing or immediate processing, send it to heap leaching or milling and so on. Conventionally, two types of cut-off grades are defined in the open pit mine planning and design. The first one that is known as break-even mine cut-off grade, is used to determine if a block of material deserves to be mined or not while trying to determine the ultimate pit limits. The second one that is called the milling cut-off grade (the interest of this study) is defined to determine how the mined block should be treated, (e.g. send for further processing or taken to the waste dump), (Dagdelen, 1992). This cut-off grade, in its general form, is also known as the optimum Cut-Off Grade Strategy (COGS) or exploitation strategy. COGS has a determinant role in optimality of the mining operation process. In this study it is tried to incorporate uncertainty aspects in determination of COGS.

To indicate the significance of this study, it is necessary to declare the role of COGS in the mine planning process. The goal of an open pit mine planning is to find an optimum production plan resulting in the highest Net Present Value (NPV), while meeting several technical and operational constraints. Hence, once the geologic block model of an orebody is built, a mine planner tries to perform the mine planning by answering the three following questions (Dagdelen, 2007). (1) whether a given block in the model should be mined or not? (2) if it is to be mined, when it should be done? (3) once it is mined, how should it be processed (OR: where should it be sent?)?

Conventionally, these questions are solved by dividing the whole mine scheduling problem into sub-problems similar to the one shown in Figure 1. Explanation of the above-mentioned questions is out of the scopes of this paper since discussed in the literature thoroughly (Lane, 1988, Camus and Jarpa, 1996, Dagdelen, 2007). However, answering to the first two questions solving the third question is equivalent to solving the optimum COGS problem. The other important point is the existing relationship between optimization of the extraction scheduling sub-problem (second question) and the COGS (third question), as shown in Figure 1.

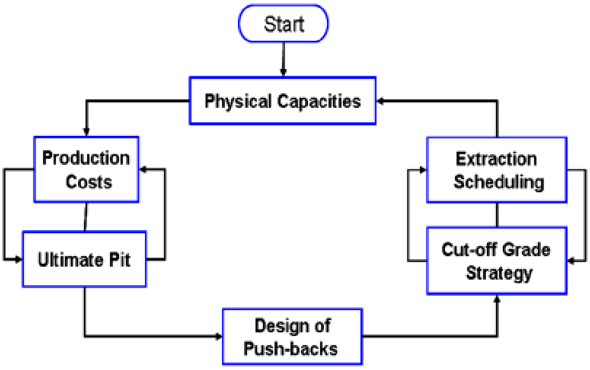


Fig. 1. Steps of traditional planning by circular analysis (Dagdelen, 2007)

Hence, any wrong decision in determining COGS will create substantial mistakes in mine planning process and subsequently causes the inefficient performance of the different stages of the mining operation especially the concentrating plant. Among the lots of the research works have been done during almost one century, the most comprehensive model of COGS was developed by K.F. Lane (1988). His model reveals that optimum cut-off grade depends not only on the economic parameters, but also on all the salient technological features of mining, such as the capacity of extraction (mine) and of milling and refining/marketing, the geometry and geology of the orebody and the optimum grade of the concentrate to send to the smelter (Cairns & Shinkuma, 2003). Traditionally, a constant price trend and a grade tonnage curve estimated from the kriged grades obtained, are employed in calculation of the optimum COGS. But, the real situation in mining industry is much more complex than this simple assumption.

In practice, as mine planners do not know the values of the aforementioned variables (with certainty), they usually estimate them from the available data, ignoring the extreme uncertainties inherent in the estimated values (Dowd, 1997). That is why the mining investments are known as a high risk business among practitioners, where lots of fortune could be made or lost. The most important uncertainties in minerals industry are the geological and market related uncertainties. As stated by Vallee (2000), the adverse effects of geological and market uncertainty on the real mine operations in monetary terms have caused substantial losses of multimillion dollars.

Meanwhile the losses due to market uncertainty could be either due to missing extra profits that could have been gained during the favorable market condition or because of capital losses as a result of unexpected weak market condition. In this respect, mine planners try to take advantage of the uncertainty evaluation methods such as scenario and sensitivity analysis and Monte Carlo simulation to explore and assess the corresponding risks. But, one of the main problems with the conventional value and risk analysis is that it is not always possible to added up their results to form, an objective decision making tool. Therefore, in such cases, these results are subjectively interpreted that may result in different decisions when handled by different decision makers.

Recently, to resolve the market uncertainty, the modern stochastic valuation method known as Real Option (RO) valuation has been found increasing use. It is a specific tool used to value the strategic and managerial flexibilities in response to the uncertain market conditions. Recently, RO valuation is considered as an alternative valuation approach to the Discounted Cash Flow

(DCF) valuation, that provides a basis to developing a flexible mine plan in interaction with the uncertain market conditions. Many researchers have been confirmed that under conditions of uncertainty, RO valuation performs better than DCF methods such as NPV and the internal rate of return (IRR) (Mardones, 1993; Trigeorgis, 1996; Samis & Poulin, 1998).

Generally, the value of an option can be estimated using either analytical or numerical methods. Analytical methods are only suitable for valuing simple options as the one developed by Black and Scholes (1973). But, numerical methods started to generate greater interest in assessing complex options such as the American type options with multiple uncertain state variables. The most widely known numerical techniques are finite difference (Schwartz, 1977), binomial lattice (Cox et al., 1979), and Monte Carlo simulation (Boyle, 1977). Amongst, the Monte Carlo technique has superior performance over the two other techniques, in dealing with dimensionality difficulties encountered within the problems with multiple and complex uncertain state variables.

Brennan and Schwartz (1985), were the first to apply the RO analysis in the mineral industry. Since then, RO analysis has been extended by valuing various types of managerial flexibilities inherent in the mineral industry using different valuation methods, (Mardones, 1993, Abdel Sabour & Poulin, 2006, Samis et al., 2006, Dimitrakopoulos & Abdel Sabour, 2007; Topal, 2008, Cortazar et al., 2008; Akbari et al., 2009, Gligoric et al., 2011).

However, even though the RO analysis provides important information about the response of mining investment to market uncertainty by valuing operating flexibilities, but just same as the conventional methods it does not prescribe a certain design as the best from the available alternative designs. Usually, the expected value generated by each alternative is used to select an alternative. Therefore, such as the conventional methods, results needed to be subjectively interpreted in decision making process. One way to minimize the subjective judgments in decision making process is to employ multiple criteria ranking system. In a multiple criteria ranking system, to identify the most economical alternative, other aspects of the economic abilities of the alternatives rather than the expected value criterion are considered in decision making process. These aspects may include the abilities of the alternatives in producing extra profits, minimizing losses at unfavorable economic conditions and meeting the predefined goals of the mining operation, (Abdel sabour et al., 2008).

Although, it is about two decays that uncertainty based mine planning has been found increasing use and developments in minerals industry, but unfortunately there are very little works on uncertainty based determination of COGS. One of the early works on incorporating metal price uncertainty in calculation of COGS is the work done by Dowd (1976). He developed a model based on stochastic dynamic programming in which price uncertainty was modeled as a stochastic Markov process. Despite the some operational simplifications considered in regard with concentrating and refining plant capacities it seems that Dowd's model is capable to be extended to a more general model. Krautkraemer (1988), studied the optimal response of COGS based on anticipated and unanticipated price changes using optimal control theory. He provided some useful theoretical results and discussions but his model suffers from either lack of generality and non applicability in real life mining conditions. Recently, McIsaac (2008) and Thompson (2010), employed golden search optimization inside a simulation based models considering metal price uncertainty to find a robust fixed cut-off grade and a fixed mining rate for whole life of the mining operation for metalliferous underground and open pit mining operation respectively. However, employing a fixed cut-off grade throughout the whole mining life time, will lead to non-optimal operations and mis-use of mining, milling and refining capacities.

Therefore, according to the substantial effects of COGS as well as metal price uncertainty in profitability of mining operation, in this paper a risk based ranking system is proposed to select an optimum COGS under metal price uncertainty. The decision making process in the proposed ranking system is based on the four ranking systems namely; RO valuation based multiple criteria ranking system, DCF valuation based multiple criteria ranking system, ranking according to the expected value of strategies using RO valuation and ranking according to the expected value of strategies using DCF valuation. It is worth to mention that the value of operating flexibility to close the mine early is also considered by RO valuation. To generate different technically feasible alternative COGS, the Lane (1988), comprehensive algorithm, in which adaptation to the economic variables variations is respected in calculation of COGS, is used. In this respect, the Lane (1988), comprehensive algorithm, in which the variations of economic parameters effect is respected in calculation of optimum COGS, is used to generate sufficient number of technically feasible alternative COGSs. While the metal price uncertainty is quantified using stochastic process. To assess the applicability of the proposed methodology it is applied for Sungun Copper Mine (SCM) data, to determine the best optimum exploitation strategy under copper price uncertainty and the results are discussed. At the two following sections the mechanism of the proposed ranking system is outlined.

2. The proposed COGS selection method

As mentioned the proposed ranking system is based on the integrate performance of four different ranking systems that are composed of two multiple criteria ranking systems and two ranking systems based on expected value of alternatives. In the proposed system the multiple value statistics and cash flows characteristics incorporating the value of management flexibility in reaction to the new information are considered in defining multiple criteria ranking system while the well known expected value, is employed by single criterion ranking system. The geological uncertainty is not considered in this study. Therefore, throughout this study both the ore grade and the ore reserves are assumed to be known with certainty. As outlined in Figure 2, the employed methodology in this study contains four main steps. The first step of the analysis is related to the modeling of metal price evolution. In this respect, the stochastic processes are usually used to quantify the metal price uncertainty. Then the Lane optimization algorithm is used in conjunction with the simulated equally probable realizations of metal prices to generate the technically feasible COGS. At the next step the newly developed simulation based RO valuation known as least squares Monte Carlo is used along with conventional DCF valuation model to valuate the alternatives, while the operating flexibility to close the mine early is also considered by RO valuation. The fourth step of the proposed methodology is to employ the statistics of cash flows obtained in the previous step separately for both valuation methods in the format of the multiple and single criteria ranking systems to select the best strategy. The following sections provide the theoretical basis of the mentioned features.

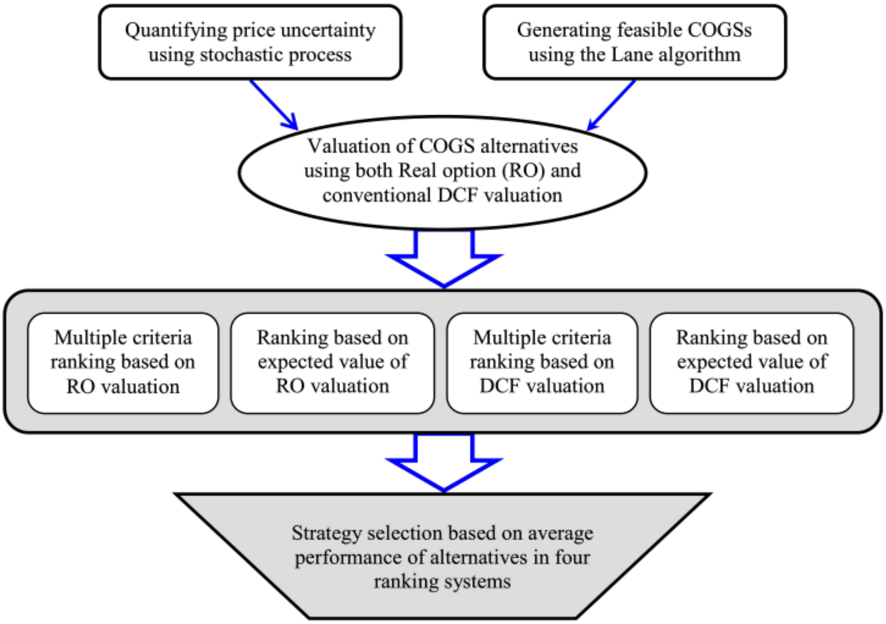


Fig. 2. The proposed multiple criteria ranking system for selection of the best COGS under price uncertainty

2.1. Price uncertainty quantification

There are many stochastic processes that can be used in quantifying market uncertainty of commodity prices. The most commonly used processes are the Geometric Brownian Motion (GBM), equation (1), and Mean Reverting Process (MRP), equation (2), (Dixit & Pindyck, 1994);

$$dP = \alpha P dt + \sigma P dz \tag{1}$$

$$dP = \eta(\ln\overline{P} - \ln P)Pdt + \sigma P dz \tag{2}$$

where P is the price of a metal, α is the expected trend, σ is the standard deviation, dz is an increment in a standard wiener process, dt is an increment of time and η is the speed at which the price reverts to its long term equilibrium level $\overline{P}$. Generally, GBM is suitable for variables that exhibiting a constant trend, such as precious metal prices and security prices while MRP is appropriate for modeling variables that has a long term equilibrium level, such as base metal prices (Schwartz, 1997). The advantage of stochastic process modeling is that the selection of appropriate process and calculating corresponding parameters are based on the analyzing the historical market prices of a metal. After selecting the appropriate model and its parameters, it is important in the simulation process to generate sample paths according to the Markov property. According to the Markov property only the current information is sufficient to forecast the future value. Simulating price paths based on this property enhances time connectivity, reduces the unrealistic shifts throughout the single Markov and simultaneously guarantees the naturally respected confidence level according to the probability function at each time interval, (Lemelin et al., 2007).

As assumed by Schwartz (1997), the copper price P evolves according to the MRP process. Assuming that the spot price of the copper (P), at each specified time is log-normally distributed, thus it can be written, $Y = \ln P$. Applying Ito's lemma to the equation (2) allows characterization of the log price by a simpler mean reverting process as equation (3),

$$dY = \eta(\bar{Y} - Y)dt + \sigma dz \tag{3}$$

$$\bar{Y} = \ln(\bar{P}) - \sigma^2/2\eta \tag{4}$$

Giving that the correct discrete-time format for a continuous-time process of MRP is the stationary first-order autoregressive process (Dixit and Pindyck, 1994) and using $P = e^Y$, samples of future prices for P_t can be generated as follows:

$$P_t = \exp\left\{ ln(P_{t-1})e^{-\eta t} + \left(ln(\bar{P}) - \frac{\sigma^2}{2\eta} \right)(1 - e^{-\eta t}) + N(0,1)\sigma\sqrt{\frac{1 - e^{-2\eta t}}{2\eta}} \right\} \tag{5}$$

where $N(0, 1)$ is the standard normal distribution random variable.

2.2 Generating alternative feasible COGS

Berry (1922) was the first to apply present value theory to cut-offs but missed the relationship between declining cut-offs and PV maximization. Fixed cut-off over the whole life time of the project was the standard assumption until Henning (1963) considered time interest to obtain a declining cut-off over time. Lane (1996, 1988) has developed the most comprehensive theory of cut-off grade optimization, based on Bellman (1957), foundational Dynamic Programming (DP) methodology. Lane's model has been considered as the basis for future works by many researchers (Whittle & Wharton, 1995; King, 2001; Bascetin, 2007; Osanloo et al. 2008). Furthermore, some outstanding works done by extending the Lane model for determination of COGS in multi mineral deposits by Ataei and Osanloo (2003a, 2003b), Osanloo and Ataei (2003), Ataei and Osanloo (2004) and Asad (2005). Some other methods have also been used to model the cut-off grade problem such as optimal control theory, stochastic DP, linear and non-linear programming and genetic algorithm, (Dowd, 1976; Akaike et al. 1999, Cetin & Dowd, 2002; Cairns & Shinkuma, 2003; Azimi & Osanloo, 2011). In this regard, Azimi and Osanloo (2011) using mathematical programming showed that the COGS problem is a nonlinear and a non-convex optimization problem and solve it by using augmented Lagrangian genetic algorithm.

The Lane's work that is regarded as the landmark in cut-off grade optimization, considers that the mining operation consists of three main stages of mining, milling/treatment and refining/marketing along with their distinctive costs and capacities. The Lane's model consists in making the best use of capacities of mining operation to maximize the PV and confirms the need for declining COGS under the fixed economic condition. Figure 3 illustrates the interactions between COGS and other parts of the mine planning and design.

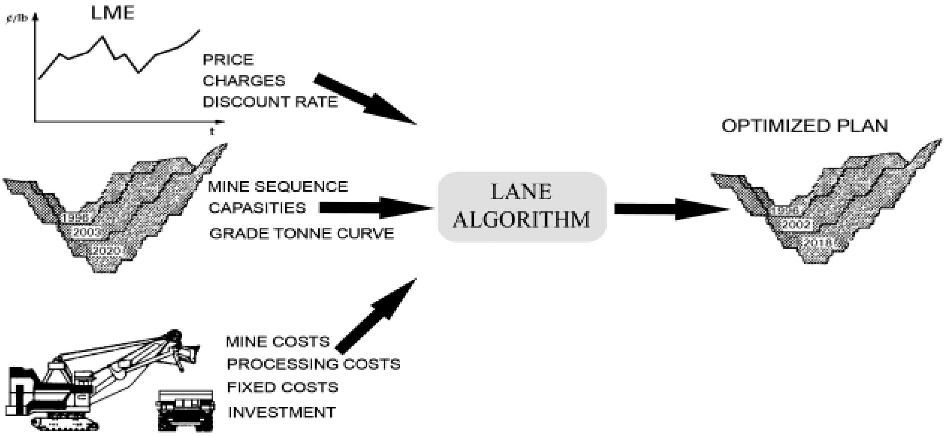


Fig. 3. Fundamentals of Lane algorithm (Azimi & Osanloo, 2011)

The present value (V) of the mining operation at each particular time (T) can be expressed as a function of available amount of resource (R) at time T and whole operating strategy (Ω), which defines the setting of variables affecting V, (equation (6)). Meanwhile, there is only one operating strategy (Ω^*) for which V is maximized. Despite the V, the optimum present value (V^*) is no longer a function of the strategy Ω, Lane (1988).

$$V = V(T, R, \Omega) \tag{6}$$

$$Max_{\Omega} V(T, R, \Omega) = V^*(T, R) \tag{7}$$

Consider for a small decrement (r) (indicating the extraction) in resource R, ω is the strategy to exploit the decrement r during Time (t), which will provide the (c) amount of cash flow per unit of extracted resource. After extracting fraction r the remaining resource will be $R - r$ and the new time will be $T + t$ where the strategy to exploit the remaining resource from this time onward will be (Ω). Thus the present value at the beginning of time T can be expressed as follows:

$$V(T, R, \omega + \Omega') = rc(\omega, t) + V(T, R - r, \Omega')/(1 + \delta)t \tag{8}$$

Taking advantage of the sequential nature of the mining operation and maximizing present value in equation (8) with respect to ω and using equation (7), the maximum present value (V*) of the mining operation can be written as;

$$V^*(R,T) = \underset{all\ \omega}{0 \le \overset{Max}{r} \le R} \left\{ c(t, r, \omega) + \frac{V^*(R - r, T + t)}{(1+\delta)^t} \right\} \tag{9}$$

where the δ is the discount rate. Assuming r and t are small and replacing binomial expansion of discounting term $(1/(1 + \delta)^t)$ and first order approximation of Taylor expansion for the term $V^*(R - r, T + t)$, the fundamental formula for optimum COGS determination can be expressed as:

$$\frac{dV}{dR} = Max_{\omega}\{c - \tau F\} \tag{10}$$

$$F = \left(\delta V + \frac{dV}{dT}\right) \tag{11}$$

where, $\tau = t/r$, is the time taken to work through one unit of resource, dV/dR is the economic value added when mining and processing a unit of resource and F is the opportunity cost.

Equation (10) shows that the optimum exploitation strategy should provide the greatest possible change in present value of mining operation due to the decrease in the resource amount.

In conclusion, the optimum exploitation strategy to maximize the present value of mining operation could be determined, at any stage, by maximizing the equation (10) with respect to ω, and considering the capacity limiting constraints of three main stages namely; mining, refining and marketing capacities. It is worth to note that the only variables affecting the strategy are c and τ, while F is independent of the ω because V is already at its optimum condition.

Hence, the optimization problem in equation (10) is divided into six sub-problems considering six different possible combinations of capacity limiting constraints of mining, refining and marketing. Therefore, the final optimum cut-off grade in each mining period is one of the six grades, which provides the greatest change in the present value of mining operation. The set of selected cut-off grades for the whole mining lifespan is known as the optimum COGS.

The flowchart of the Lane algorithm used by the authors to develop a Matlab based code for determination of optimum COGS is shown in Figure 4.

Where V_i is the present value of the resource remaining at the beginning of year i, W_i, is the present value of the same amount of resource at the end of year i. F_i and c_i, are the opportunity cost and cash flow in year i respectively for stream V. All the variables assigned by a prime refer to the stream W except the g'_i that is the average grade of the ore in both streams. It is also worth noting that for stream W the economic parameters of the following year must be used instead of the current year. Finally, the term μ is the correlation for excess use of reserve in initial W sequence that calculated as;

$$\mu = (r_i - r_i') \cdot \frac{V_{i+1} - W_i}{r_i} \tag{12}$$

where r is the reserve consumed during the period i, in the stream V and r', is the reserve consumed in the period i in the stream W. More details on how to calculate the optimum cut-off grades are presented in Appendix (A).

2.3. Valuation methods

As the conventional DCF valuation method is widely available in the literature, in this study only the employed RO valuation method is briefly described.

2.3.1. Stochastic RO valuation

Monte Carlo simulation was originally proposed by Boyle (1977) for pricing European options as a forward intuition technique and extended for pricing American options by combin-

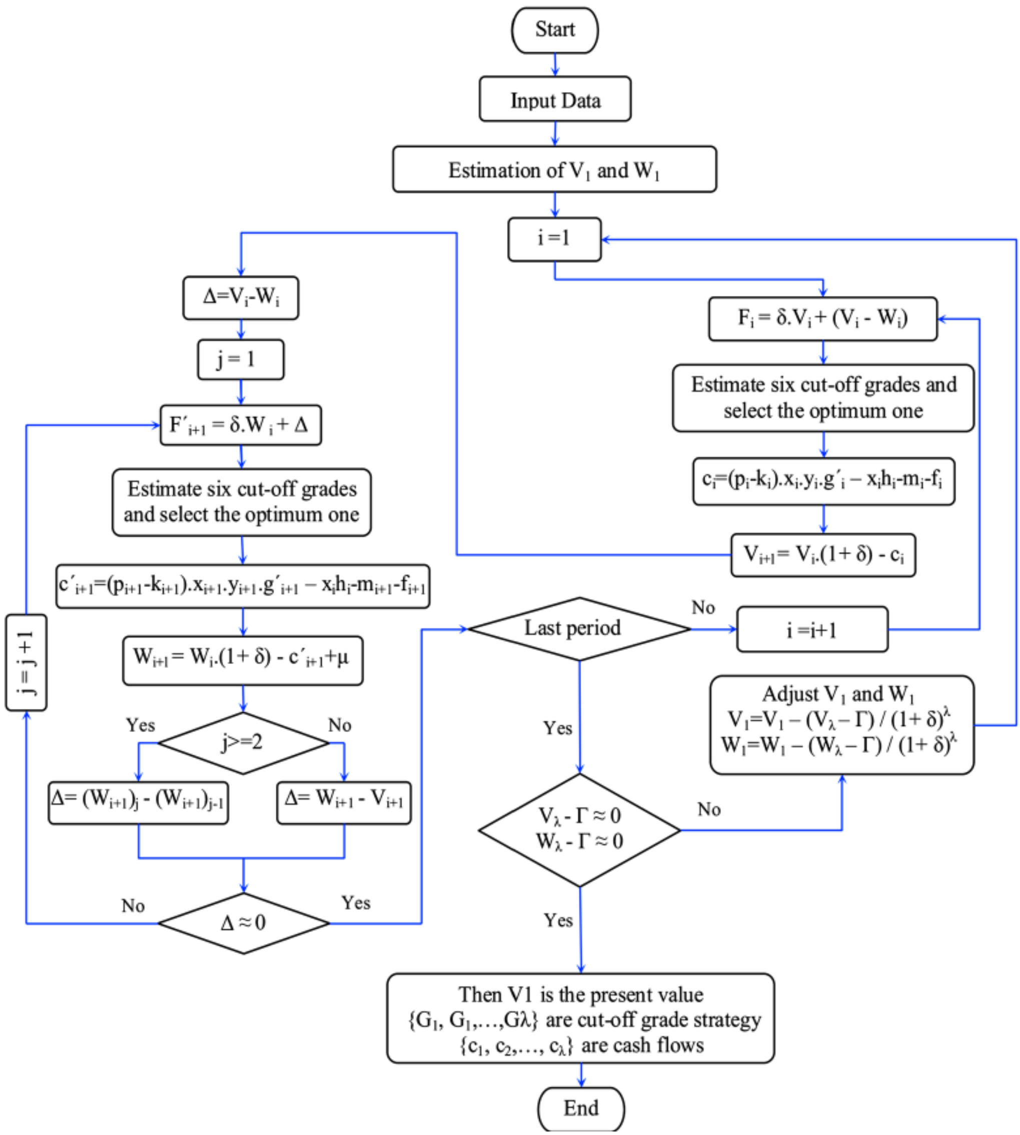


Fig. 4. Flowchart of determination of optimum cut-off grade strategy

ing the simplicity of forward induction with the ability of backward induction in determining the optimal option exercise policy. In this regard, Longstaff and Schwartz (2001), extended a promising method of Least Squares Monte Carlo (LSM) simulation for valuing path dependant American options.

Recently, the Monte Carlo RO valuation method is introduced as an alternative to the DCF valuation technique in real capital investment. One of the most promising new approaches for valuing real capital projects is the least squares Monte Carlo RO that is an extension to the method proposed by Longstaff and Schwartz (2001), for valuing financial options. Longstaff and Schwartz (2001), assumed that American options can be exercised at discrete early exercise dates. The time to maturity T is divided into K discrete times ($0 < t_1 \leq t_2 \leq ... \leq t_K = T$) and N simulated paths are generated under the risk-neutral measure. Like in any American option valuation procedure, the optimal exercise policy decision at any point in time t_k, is obtained as the maximum between the immediate exercise value $I(\omega \,;\, t_k)$ and the expected continuation value $F(\omega \,;\, t_k)$. Given that the expected continuation value depends on future outcomes, the procedure must work its way backwards, starting from the end of the time horizon, T. The put option payoff $C\,(\omega \,;\, t_k)$ can be interpreted as the Bellman equation of dynamic programming as follow;

$$C(\omega \,;\, t_k) = max[I(\omega, t_k), F(\omega, t_k)] \tag{13}$$

The immediate exercise value $I(\omega;\, t_k)$ along the path ω is simply the difference between the exercise price H and the stock price $S(\omega, t_k)$. At the expiration date along the sample path ω the expected continuation value is zero so that,

$$C(\omega \,;\, t_k = T) = max[(H - S(\omega, T)), 0] \tag{14}$$

But at any early exercise time $t_k < T$, under the risk-neutral pricing measure Q, the expected continuation value $F(\omega \,;\, t_k)$ based on the information β at time t_k assuming that the option is not exercised until after t_k and the option holder following the optimal stopping strategy for all x, $t_k < x \leq T$ is

$$F(\omega;t_k) = E_Q\left[\sum_{j=k+1}^{k} \exp\left[-\int_{t_k}^{t_j} r(\omega,x)\,dx\right] C(\omega, t_j\,; t_k, T \,|\, \beta_{tk}\right] \tag{15}$$

where $C(w, t_j\,;\, t_k, T)$ is the remaining cash flows and $r(\omega,x)$ is the risk-free discount rate. The main contribution of the LSM method is to estimate the expected conditional continuation value at $t_K - 1$, $t_K - 2$, ..., t_1 by regressing the discounted future option values on a linear combination of M basis functions $L_j(S)$ such as Laguerre polynomials, trigonometric series, or simple powers of the stock price S such as

$$F(\omega;t_k) = \sum_{j=0}^{M} a_j L_j(S) \tag{16}$$

After the regression, the optimal obtained coefficients a_j are used to estimate the expected continuation value $\hat{F}(\omega \,;\, t_k)$. For the all in-the-money paths at time t_k the optimal exercise policy is obtained by choosing the maximum between the immediate exercise value and the estimated continuation value at time t_k and consequently the cash flows along the path ω from time t_k to time T are revised. For the out-of-the-money paths the option payoff is zero. The recursion proceeds backward until time t_1. After the exercise decisions at each exercise date along all the paths are determined, the value of the option is estimated by discounting the cash flows to time 0 at the risk-free rate and averaging over the number of paths N.

Abdel Sabour and Poulin (2006) extended the LSM to value mining investments under multiple market uncertainties and further developed by Abdel Sabour et al., (2008) to evaluate mining investments under both multiple market and geological uncertainties. Cortazaret et al. (2008), have also extended the approach to cover the three-factor risk models more suitable for long-term commodity real options. Gligoric et al. (2011), proposed a hybrid model of the evaluation based on Monte Carlo simulation and fuzzy numbers. Totally, the extended LSM method is mainly based on simulating multiple realizations of the uncertain variables and making the optimal decision at each period using value expectations.

In this work, the operating flexibility to close the mine early is integrated into the valuation of open pit mining COGS and considering the metal prices uncertainty. In order to evaluate mining operations under the flexible operating model, it is assumed that the mining operation status can be changed at the beginning of each yea. After generating N simulated metal price paths according to the Markov property described above, the cash flows at each discrete date along the simulated metal price paths are defined based on the amount of production, the metal price and the unit production cost.

Then recursively starting from the last time step towards the first period at any period t (between 0 and T) a status that maximizes the mining expected value is chosen as the optimum operation policy conditional to a sample price path $n \in N$. Let CV_O denotes the continuation value of "open mining" status. As described in Longstaff and Schwartz (2011), CV_O is a function of the uncertain state variable and can be estimated as the linear combination of basis functions of uncertain variables. Given that the P denotes the price of the metal. Then the expected continuation value of mining is approximated as

$$E(CV_O \mid n,t) = \sum_{i=0}^{N} a_i L_i (P_{n,t}) \qquad (17)$$

The parameters of these functions are estimated at each time period by regressing the sum of the discounted values of cash flow beyond time t onto their basis functions. After estimating the basis functions parameters, the expected mine value at time t for each simulated sample path n is approximated as the sum the expected continuation value with the cash flow of mining operation $CF(n,t)$, during the time period t.

$$E(V|n,t) = E(CV_O|n,t) + CF(n,t) \qquad (18)$$

At any time t based on the expected mine value $E(V)$ the decision whether to keep the mine open or close it early is as follows.

Keep the mine open if

$$E(V|n,t) > K_a \qquad (19)$$

Abandon the mine if

$$E(V|n,t) \leq K_a \qquad (20)$$

where K_a is the abandonment cost. This optimization process is carried out for all simulated realizations of metal price throughout all time periods. After revising the cash flows according to the determined optimum operating policy at the discrete exercise points throughout all the simulated metal price paths, the mining value at time 0 is determined by discounting the cash flows at the risk-free rate and averaging over the number of simulated metal price paths. Besides the expected value various statistics for the overall value as well as annual cash flows of mine value can be generated throughout this analysis. These statistics will be employed in a multiple criteria ranking system in order to rank the different exploitation strategies. The flowchart of the LSM algorithm used in this paper is shown in Figure 5.

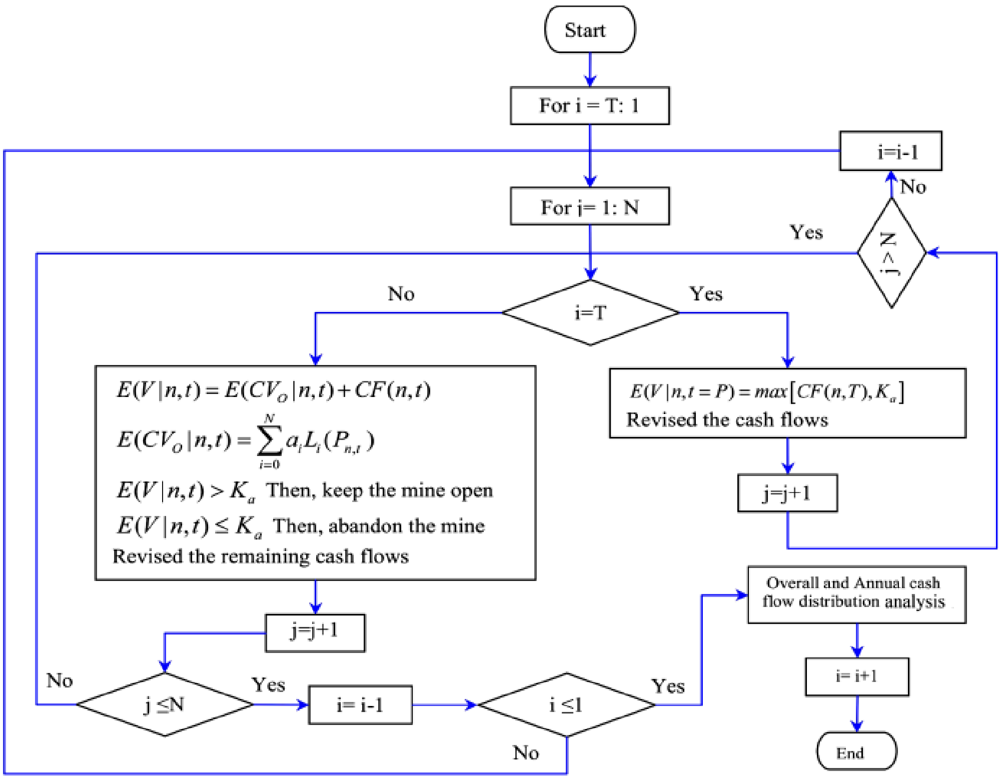


Fig. 5. Flowchart of the LSM algorithm considering managerial flexibility to close the mine early under metal price uncertainty

2.4. Ranking systems

In this study two ranking methods namely; single and multiple criteria are used to select the best alternative. The single criteria ranking system is based on the expected value of the each alternative, while the multiple criteria ranking system gathers multiple value and risk analysis indicators into one quantitative measure integrating real industry complexities such as uncertainty and operating flexibilities. The multi criteria ranking system used is as follow.

2.4.1. Multi-criteria ranking system

Multi-criteria ranking/decision system is an effective tool enhancing the role of objective interpretations against the subjective interpretations when a decision making problem exists. In this paper, based on the available information at the initial planning time the following aspects are considered to select the best exploitation strategy.

(1) Upside potential that indicates the ability of alternative strategies to capture possibly more profits than those expected if outcomes were favorable.
(2) Downside risk that reflects the difference between strategies in minimizing negative cash flows risk throughout mine life.
(3) Statistics of the estimated operation values which includes the average, lower and upper limit at a certain confidence level.
(4) Probability of completion, which is the probability that the mine will be operate as planned throughout its planned life.

For simplicity, these criteria are considered to have equal weights in this paper while according to policy of a mining company different weights can be adopted. At the following, the way of the calculation of the corresponding indicators are explained.

2.4.1.1. Upside potential indicator

The upside potential measures the ability of designs to obtain possibly more profits than those expected if outcomes were favorable. Therefore, the upside potential indicator (*UPI*) for a mine design *m,* during a production year *t* can be computed as follows;

$$UPI_{m,t} = 100\left(DCF^{+}_{m,t}\,p^{+}_{m,t} - \left(\sum_{m=1}^{M} DCF^{+}_{m,t}\,p^{+}_{m,t}\,/M\right)\right) / \left(\sum_{m=1}^{M} EV_m\,/M\right) \tag{21}$$

where $DCF+$ is the expected annual discounted positive cash flow, $p+$ is the probability that those cash flows will be positive and EV_m is the overall expected value of design m calculated by the simulation based RO model outlined above. A positive indicator indicates that a design has the potential to generate positive net cash flows during year *t* higher than the average of all designs and vice versa. This process is repeated for all production years and the total *UPI* for design m is the summation of the annual indicators, as follows (Abdel Sabour et al., 2008);

$$UPI\ m = \sum_{t=1}^{T} UPI_{m,t} \tag{22}$$

2.4.1.2. Downside risk indicator

Considering the uncertainties inherent in the minerals industry, there is a probability of producing negative cash flows during some periods of mine life time, although expecting positive value at the mine design time. The downside risk indicator (*DRI*) accounts for such probabilities and reflects the difference between designs in minimizing risk of negative cash flows throughout mine life. The expected negative cash flows DCF^{-} along with their corresponding probabilities p^{-} that are resulted from the economic valuation model, are used to calculate the *DRI* for the designs. During a production year *t*, the *DRI* of a design *m* is calculated as;

$$DRI\ m,t = 100\left(DCF^{-}_{m,t}\,p^{-}_{m,t} - \left(\sum_{m=1}^{M} DCF^{-}_{m,t}\,p^{-}_{m,t}\,/M\right)\right) / \left(\sum_{m=1}^{M} EV_m\,/M\right) \tag{23}$$

The DRI value for a design could be negative or positive. A greater DRI for design indicates higher capability of minimizing negative cash flow risks. The overall *DRI* for design *m* is (Abdel Sabour et al., 2008);

$$DRI\ m = \sum_{t=1}^{T} DRI_{m,t} \tag{24}$$

2.4.1.3. Value statistics indicator

The value statistics indicator (*VSI*) accounts for the overall expected present value of future cash flows as well as the confidence limits estimated at a specified confidence level. As with the above described indicators, the performance of each mine design is compared to the average of all designs and an indicator is assigned reflecting whether the performance of each design is above or below the average. The indicator related to the expected value for a design *m*, EVI_m is calculated such as;

$$EVI\ m = 100\left(EV_m - \sum_{m=1}^{M} EV_m / M\right) / \left(\sum_{m=1}^{M} EV_m / M\right) \tag{25}$$

The upper limit indicator (ULI_m) of the expected value at a predefined confidence level is,

$$ULI_m = 100\left(UL_m - \sum_{m=1}^{M} UL_m / M\right) / \left(\sum_{m=1}^{M} EV_m / M\right) \tag{26}$$

ULI_m indicates to what extent the difference in performance between design *m* and the average of all designs is significant compared to the overall expected mine value. The lower limit indicator (*LLI*) of the expected value at a predefined confidence level is,

$$LLI_m = 100\left(LL_m - \sum_{m=1}^{M} LL_m / M\right) / \left(\sum_{m=1}^{M} EV_m / M\right) \tag{27}$$

The value statistics indicator *VSI* for each design is then estimated by summing up the indicators corresponding to the expected value, the upper limit and the lower limit as following, (Abdel Sabour et al., 2008);

$$VSI_m = EVI_m + ULI_m + LLI_m \tag{28}$$

2.4.1.4. Probability of completion indicator

The insurance option such as early abandonment of mine that limits further losses, may lead to uncompleted production plans can result in economic and social inconveniences. Therefore, the relative abilities of various mine designs to sustain unfavorable economic conditions and perform as planned throughout the anticipated project life, is the criteria that is expressed by Probability of completion indicator (*PCI*). The *PCI* for a mine design *m* in a production year *t* is calculated as follows (Abdel Sabour et al., 2008);

$$PCI_{m,t} = 100\left(\left(PO_m - \sum_{m=1}^{M} PO_{m,t} / M\right) / \left|\sum_{m=1}^{M} PO_{m,t} / M\right|\right) \times \left(\left|\sum_{m=1}^{M} DCF_{m,t} / M\right| / \sum_{m=1}^{M} EV_m / M\right) \qquad (29)$$

where $PO_{m,t}$ is the probability that the mine will not produce the scheduled amount of metal in year *t* if the design *m* is selected, $DCF_{m,t}$ is the expected discounted cash flow in year *t*. The overall PCI indicator of design m throughout the mine life is

$$PCI_m = \sum_{t=1}^{T} PCI_{m,t} \qquad (30)$$

A positive *PCI* indicator means that design *m* will perform better than the average of all designs in year *t* and vice versa. It is important to note that since at the planning time in the conventional DCF valuation method operating flexibilities to revised the predefined strategies are not considered, this indicator is ignored in the calculation of DCF based multiple ranking system.

After estimating the above described four indicators for each mine design, the total ranking indicator *TRI* is simply the summation of these four indicators. The designs are then ranked according to the *TRI* and the design with the highest indicator should be selected.

3. Implementation of the proposed selection system at Sungun Copper Mine

To demonstrate the application of the proposed methodology for selection of an exploitation strategy under price uncertainty, the described ranking system was applied using the data form Sungun Copper Mine (SCM). The SCM is one of the Iranians largest porphyry copper mines that is located in East Azerbaijan, Iran. The mineralization in it occurs in the Cenozoic Sahand-Bazman orogenic belt. Open pit mining method is used to extract the material from the mine and the froth flotation method is used in beneficiation plant to obtain the copper concentrate. Then the produced copper concentrate meets the electorwinning refinery plant to result in 99.99% pure copper products. The production in the mine has been commissioned in the mid of the year 2006. Resource estimation and optimization of mine planning and design was done using Datamine® and NPV Scheduler®. The blocks dimensions in the model were selected as 25 m × 25 m × 12.5 m, and ordinary kriging was used to estimate the copper grade in the ore blocks. Performing production sequencing using NPV Scheduler® four practical increments were developed inside the optimum ultimate pit (Rashidinejad et al., 2008). The other necessary economic and technical parameters are listed in Table 1. For simplicity throughout this analysis, free market conditions considered and taxation, inflation, salvage value and capital expenditure are ignored. Furthermore, no stockpile option is considered to store intermediate grade material for later treatment.

3.1. Generation of technically possible COGS alternatives

Different strategies could be generated by a number of ways according to the objectives of the mining operation, economic and social conditions. One way, which was applied in this study, is to use the Lane algorithm considering different realizations of metal prices. At the section 2.2 it was shown that the optimum COGS is completely dependent to the prevailing economic parameters. Therefore 200 different technically feasible COGSs were generated for SCM by applying the Lane algorithm for 200 different independent realizations of copper price paths. These paths were generated through Monte Carlo simulation using mean reversion stochastic process, presented in equation (2). Using the Lane algorithm each alternative exploitation strategy adapted a life of mine (LOM) and a COGS corresponding to its economic condition. The ranges of LOM and PV for generated alternatives were between 22 to 36 years and $US 3460 million to $US 8450 million respectively. To have a comparison with the common practice in the industry, another alternative COGS is calculated based on constant expected price value of 4500 ($/t), that is equal to the expected long term equilibrium price level of copper price according to price behavior at year 2006. This COGS was named as strategy 201.

TABLE 1

Economical and technical information of Sungun Copper Mine

Costs		Mining cost ($/ton)	1.60
		Milling cost ($/ton)	2.50
		Refinery ($/Kg)	315.00
		Fixed cost (M$/year)	10.20
Capacities	Phase 1	Mining throughput (Mt/year)	25
		Milling throughput (Mt/year)	7
		Marketing throughput (Kt/year)	150
	Phase 2	Mining throughput (Mt/year)	40
		Milling throughput (Mt/year)	14
		Marketing throughput (Kt/year)	300

In order to select the best strategy under metal price uncertainty using the information available at the planning time, these strategies were ranked according to the following four ranking measures.

- the expected value estimated by the conventional DCF valuation method
- the expected value estimated by the RO valuation
- the DCF based indicator explained above, summation of the defined indicators except the PCI since the conventional DCF cannot consider the managerial flexibilities.
- the RO valuation based indicator.

Furthermore, in order to investigate the effect of price level, two high and low price conditions were considered to model the price uncertainty.

3.2. COGS ranking at SCM

As mentioned two economic scenarios were considered. A high price scenario, which belongs to the planning time year 2006 and the low price scenario in which the initial and long term equilibrium prices were considered nearly a half of the high price scenarios. Both of the high and low price scenarios were modeled with the mean reverting process in equation (2). The relevant economic data are listed in Table 2.

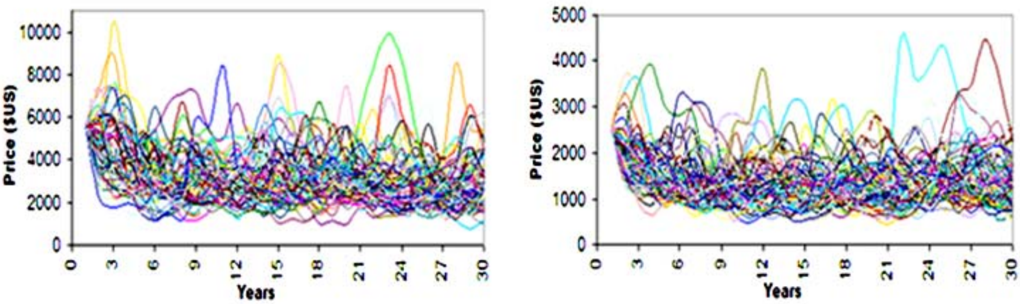
TABLE 2

Economic and stochastic process parameters for copper mine valuation

Stochastic process parameters	High price scenario	Low price scenario
P (Initial price)	5600	2500*
$\bar{P}$ (long term price level)	4500	2000*
σ (Volatility)	0.30	0.30
η (Reversion speed)	0.369	0.369
Risk free interest rate	10 %	10 %

* These are not based on the statistical analysis of copper price behavior

Except the initial copper price, these parameters have been retrieved from statistical analysis of metal price movements based on historical data from 1980 to 2006. An illustrative example of generated copper price paths for both high and low prices is shown in Figure 6. Using Monte Carlo method, each of the 201 strategies has been evaluated considering 20 000 copper price realizations for the low and high price scenarios.



a) b)

Fig. 6. copper price uncertainty. (a) high price, (b) low price

Two valuation techniques of conventional DCF and RO valuation were applied to each of the alternatives. For this purpose both valuation methods were coded in Matlab environment. In the RO valuation the operating flexibility to abandon the mine early was considered and the optimum decision at each period was taken according to the expected mining value conditional on the simulated price paths.

In addition to the expected value, the Monte Carlo simulation provides the planner with the multiple statistics for the overall value as well as annual cash flows to carry out more insight risk analysis such as the multiple criteria ranking system explained in the previous section. Using the empirical distribution of annual cash flows as well as the PV of alternative strategies produced by Monte Carlo simulation a DCF based indicator and a RO based indicator have been calculated for each strategy. The only difference between the DCF and the RO valuation indicators is that the *DCF* consists of three sub-indicators since the probability of completion indicator is the same for all designs based on DCF valuation.

Table 3 lists the 10 best and 10 worst strategies based on their expected values estimated by the *DCF* and the real options valuation for both high and low price scenarios. Table 4 is also shows the 10 best and the 10 worst strategies ranked according to the calculated TRI based on the valuation results of the DCF and the RO valuation for both high and low price scenarios. As shown in Table 3 and Table 4 under high price scenario strategies 39, 121, 189 and 189 are the best strategies based on RO based TRI, DCF based TRI, RO expected value and DCF expected value respectively. While under low price scenario strategies 153, 199 are the best strategies based on RO and DCF based TRI and RO and DCF expected value respectively. The word 'best' throughout this work refers to the strategy that has the maximum discounted value or the highest

TABLE 3

Ten best and worst strategies under high and low price scenarios based on expected value

Rank	High price condition				Low price condition			
	RO*		DCF		RO		DCF	
	strategy	E(PV)**	strategy	E(PV)	strategy	E(PV)	strategy	E(PV)
1	189	1587.44	189	1587.39	153	235.69	199	177.66
2	121	1587.42	121	1587.38	199	235.26	112	173.71
3	58	1585.86	58	1585.82	121	235.03	153	172.75
4	39	1585.14	39	1585.10	98	234.91	124	172.52
5	149	1584.78	149	1584.74	59	234.48	121	171.90
6	30	1584.78	30	1584.73	168	234.43	158	171.80
7	82	1584.63	82	1584.58	112	234.28	59	171.77
8	198	1584.24	198	1584.19	130	234.08	168	171.73
9	85	1584.04	85	1583.98	124	233.86	130	171.67
10	47	1584.01	47	1583.96	28	233.72	66	170.89
192	55	1552.75	55	1552.73	122	223.75	126	146.55
193	5	1551.29	5	1551.17	32	223.74	99	146.54
194	199	1545.90	199	1545.88	43	223.74	128	146.54
195	168	1544.23	168	1544.21	71	223.74	154	146.49
196	112	1542.22	112	1542.20	90	223.74	105	146.47
197	136	1542.15	136	1542.14	94	223.74	122	146.37
198	63	1541.16	63	1541.13	163	223.74	183	146.32
199	67	1530.69	67	1530.67	5	223.56	143	145.86
200	59	1523.71	59	1523.69	180	223.36	5	145.80
201	130	1515.99	130	1515.99	201	219.92	201	127.43

* Real options valuation.

** The expected values are in million $US.

rank. It is worth noting that strategies 153 and 199, the best strategies under low price scenario, are of the worst strategies under high price scenario. This is due to the effect of an insurance option such as the early abandonment of mining operation, which is expected to be exercised more under unfavorable economic condition to limits further losses. But the strategies 39, 121, and 189 that are the best strategies under high price scenario, also have good performance, (especially strategy 121) under low price scenario, (they are between the 20 best strategies).

TABLE 4

Ten best and worst strategies under high and low price scenarios based on TRI

Rank	**High price condition**				**Low price condition**			
	RO*		**DCF**		**RO**		**DCF**	
	strategy	**TRI****	**strategy**	**TRI**	**strategy**	**TRI**	**strategy**	**TRI**
1	39	3.6618	121	3.7086	153	21.6800	199	61.78
2	189	3.6614	39	3.6370	121	20.0031	112	50.04
3	121	3.6512	189	3.5943	98	19.6499	121	47.49
4	82	3.5311	38	3.5192	199	18.1542	153	47.24
5	38	3.5157	82	3.4786	124	17.5397	124	47.24
6	142	3.4127	142	3.3872	28	17.4600	158	46.34
7	74	3.3876	74	3.3820	112	16.4744	59	45.68
8	58	3.3848	47	3.3434	158	15.3621	168	45.48
9	47	3.3771	58	3.3319	66	14.3544	130	44.88
10	179	3.3162	179	3.3111	168	14.0866	66	43.64
192	44	-5.1559	44	-5.1153	110	-3.7732	99	-18.12
193	55	-5.5247	55	-5.5360	40	-3.9306	32	-18.34
194	199	-7.2921	199	-7.0252	26	-4.0169	43	-18.36
195	168	-7.8299	168	-7.7040	25	-4.0213	71	-18.36
196	136	-8.2843	136	-8.2538	135	-4.0361	90	-18.36
197	112	-8.3863	112	-8.3138	9	-4.1161	94	-18.36
198	63	-8.6587	63	-8.6668	196	-4.2294	122	-18.55
199	67	-11.2386	67	-11.1746	180	-4.3280	143	-18.71
200	59	-13.0715	59	-12.8679	67	-4.8086	5	-21.53
201	130	-15.2100	130	-14.9450	201	-11.0804	201	-66.82

* Real options valuation.
** Total Ranking Indicator.

The ranks of strategy number 201 based on different ranking methods for both high and low price scenarios are shown in the Table 5. It is seen that at low price scenario the strategy number 201 is the worst strategy incorporating price uncertainty. Given that the strategy number 201 was obtained based on the constant price of US $ 4500, such a low performance seems logical under low price condition. However this means that if the actual behavior of copper price became worse than assumed constant price (the usual practice in the real mining operation), the worst strategy is to continue with the strategy such as strategy number 201. In addition, the strategy number 201 has low performance at the high price scenario. According to the Table 5, the strategy number 201 is the 17^{th} and 11^{th} worst strategy based on expected value and TRI methods respectively. Although, Lane (1988), proved that employing declining COGS in exploitation of a reserve instead of a constant break-even cut-off grade would maximize the profitability of the mining operation more. But the results of the analysis in this study show that under such the uncertain market environment, the declining strategy calculated based on a fix price is highly probable to be failed or lead to a low profitable operation.

TABLE 5

Ranks of the strategy number 201 under both high and low price condition

	High price condition	Low price condition
E(DCF)	184	201
TRI (DCF)	190	201
E(RO)	184	201
TRI(RO)	190	201

To assess the ranking methods a term "rank-difference" was defined as the absolute difference between ranks of a strategy in two the alternative ranking methods. Then the average rank-difference calculated as an indicator to compare the ranking systems with each others. Table 6 shows the average rank-difference between the four ranking methods for both high and low price scenarios. A low rank-difference indicates that the two ranking methods perform similarly while a high one indicates that the two ranking methods act differently to each other.

As can be seen from Table 6, the level of the differences between the pairs of the ranking methods at low price scenario is higher than the high price scenario.

TABLE 6

Ranking difference between the ranking methods for both high and low price condition

Price level	Methods	E(DCF)		TRI (DCF)		E(RO)		TRI(RO)	
		RD*	SR**	RD	SR	RD	SR	RD	SR
High price	E(DCF)	0	201	9.30	11	0.13	177	–	–
	TRI(DCF)	9.30	11	0	201	–	–	1.15	78
	E(RO)	0.13	177	–	–	0	201	8.88	12
	TRI(RO)	–	–	1.15	78	8.88	12	0	201
Low price	E(DCF)	0	201	3.55	38	22.80	5	–	–
	TRI(DCF)	3.55	38	0	201	–	–	37.23	4
	E(RO)	24.62	5	–	–	0	201	22.80	13
	TRI(RO)	–	–	37.23	4	24.62	13	0	201

* Ranking Difference.
** Number of alternatives with zero ranking deference.

At high price scenario, the similarity between the pairs of the identical the ranking methods based on valuation method is higher than the similarity between the pairs of different ranking methods based on identical valuation method. But, at the low price scenario, the case is the reverse of the high price scenario. Thus, the similarity between the pairs of the identical raking methods in both valuation methods is lower than the similarity between the two ranking methods using the same valuation methods. Amongst, the difference between the RO based TRI and DCF based TRI is the highest one.

These ranking differences at low and high price scenario can be due to the existence of the embedded insurance option to abandon the mine early in the RO valuation, which is expected to be exercised more at the low price scenario. In such the situation using the RO valuation, exploitation strategies tend to close the operation earlier than the planned time to avoid extra losses at the unfavorable economic condition. While the conventional DCF method does not consider, such a preventive action in the valuation of the alternatives. But at high price scenario, insurance option such as the early abandonment of the mine is not so exercised due to the favorable market condition. Therefore, RO and conventional DCF valuations perform almost identically and consequently similar ranking results based on both valuation methods seem straightforward. Furthermore, as expected the value of mining operation estimated based on the RO valuation was higher than the conventional DCF valuation especially at low price.

3.3. Ranking based on average performance of the four rankings methods

As shown in Tables 3 and 4 using different ranking systems and valuation methods under different economic conditions different rates for strategies were obtained. Hence in order to make the best decision according to the above shown conditions the Average Rank Performance (ARP) was calculated as the mean rank of the each strategy over the four raking systems under low and high price scenarios. According to the ARP values the strategy with the highest ARP was selected as the best strategy that contains the more stable and compatible performance under different economic conditions. Table 7 lists the 10 top strategies with highest ARP for low and high price scenarios as well as total ARP as the average of the alternatives ARP at low and high price scenarios.

TABLE 7

Ten strategies with highest ARP

Low price		High price		Low and high price	
Strategy No.	**ARP**	**Strategy No.**	**ARP**	**Strategy No.**	**ARP**
199	2	189	1.75	121	2.62
153	2.25	121	2	189	7.25
121	3.25	39	2.75	149	12.87
112	4.5	58	5.75	58	13.75
124	5.75	82	5.75	30	15.5
158	7.75	142	8.5	85	17.87
168	8	38	8.75	39	18.87
59	8.75	47	9.25	38	21.5
28	9.5	198	9.75	15	26
66	10.25	179	11	182	28.75

In conclusion according to the overall performance of alternative strategies shown at the Tables 3, 4 and 7, under price uncertainty, different ranking methods and market conditions, strategy number 121 has the most stable and compatible performance. Therefore strategy number 121 is selected as the best COGS based on the data available at the planning time. The details of the strategy 121 are presented in Figure 7. At last, it is experiences in this study that using multiple criteria ranking systems will provide for the mine planner a more informative and certain environment in mine planning and designing procedure.

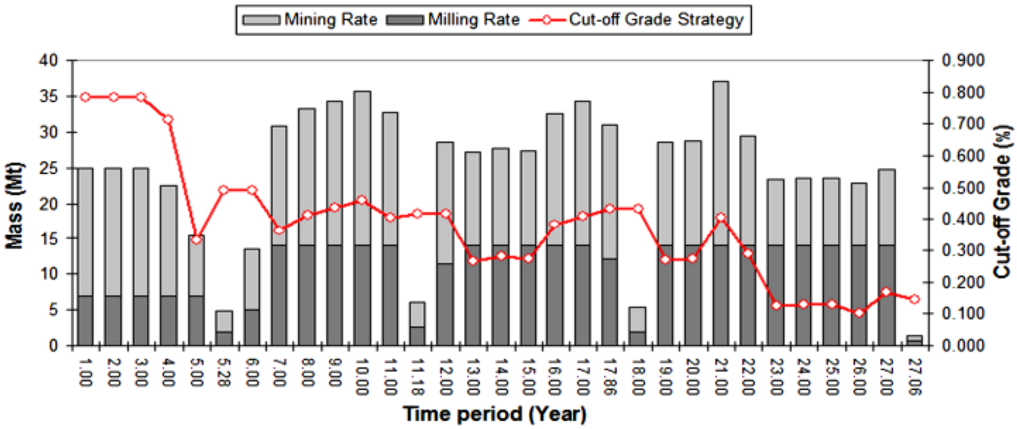


Fig. 7. Cut-off grade strategy and mine and concentrator throughput of the strategy 121, which is selected as the best strategy at SCM

4. Conclusion

Cut-off grade is a geological/technical concept that embodies important economic aspects of mining venture, and plays an influential role in optimality of mine planning and design. Hence, in this article, with regard to importance of cut-off grade and uncertain nature of the design parameters especially metal price, it is tried to select a mining exploitation strategy under price uncertainty. Hence a combination of single and multiple criteria ranking systems based on both Real Option (RO) and conventional Discounted Cash Flow (DCF) valuation were used simultaneously to select a COGS integrating market uncertainty. In this regard, some technically feasible Cut-Off Grade Strategies (COGS) were generated by Lane comprehensive algorithm (1988) using an open pit mining data and stochastically simulated equally probable realization of price paths as the possible future prices. Furthermore, to evaluate the alternative strategies better two high and low price scenarios were considered for DCF and RO valuations. In other to avoid unwanted losses at unfavorable market condition an insurance option to close the mine early was also considered in RO valuation.

The proposed methodology was applied using Sungun Copper Mine data to select an exploitation strategy under copper price uncertainty. In order to investigate the merits of each alternative better the analysis was done under two economic conditions, once for low price trend and once for high price trend. Comparing the ranking systems results revealed that their priorities at high and low price scenarios are different. It is seen form the results that at high price scenario, the two valuation methods act similar to each other and so do the corresponding ranking systems. Unlike, at low price scenario, the two valuation methods act differently. In this case the similarity between the ratings of alternatives based on one valuation is higher than the similarity of ratings based on same ranking method employing different valuation methods. These different performances should be due to this fact that insurance options such as to close the mine early are more expected to be exercised under low price economic condition rather than high price economic condition. At last according to the average performance of alternatives in the four ranking systems the Average Rank Performance (ARP) was defined. According to the ARP values the strategy number 121 was selected as the best strategy with the most stable and compatible performance under low and high prices conditions. It worth noting that the results of this study are specific to the presented case study, different results might be obtained in other circumstances. Although the results of this study were not validated according to the actual market condition due to the lack of sufficient data, but it is clear that the results are completely dependent to the selected stochastic model of the metal price and its parameters. Hence, it is recommended that a professional econometrician be retained to parameterize metal price processes for any valuation exercise. Furthermore, it is proposed to investigate the response of COGS to other operating flexibilities such as the expansion of mine and concentration plan capacities that will provide extra opportunities for mining operation profitability.

References

Abdel Sabour S.A., Dimitrakopoulos R., Kumral M., 2008. *Mine plan selection under uncertainty*. Mining Technology: IMM Transactions Section A 117, 2, 53-64.

Abdel Sabour S.A., Poulin R., 2006. *Valuing real capital investments using the least-squares Monte Carlo method*. The Engineering Economist 51, 2, 141-160.

Akaike A., Dagdelen K., 1999. *A strategic production scheduling method for an open pit mine*. In: Proceedings of the 28th APCOM, Golden Colorado, USA, 729-738.

Akbari A.D., Osanloo M., Shirazi M.A., 2009. *Minable reserve estimation while determining ultimate pit limits, UPL) under price uncertainty by real option approach, ROA)*. Arch. Min. Sci., Vol. 54, No 2, p. 321-339.

Asad M.W.A., 2005. *Cutoff grade optimization algorithm with stockpiling option for open pit mining operations of two economic minerals*. Int. J. Surf. Min. Reclamation Environ., 19, 3, 176-187.

Ataei M., Osanloo M., 2004. *Using a Combination of Genetic Algorithm and the Grid Search Method to Determine Optimum Cutoff Grades of Multiple Metal Deposits*. International Journal of Surface Mining Reclamation and Environment (IJSM) 18, 1, 60-78.

Ataei M., Osanloo M., 2003a. *Determination of optimum cutoff grades of multiple metal deposits by using golden section search*. Journal of the South African institute of mining and metallurgy (SAIMM) 103, 8, 493-500.

Ataei M., Osanloo M., 2003b. *Methods for Calculation of Optimal Cutoff Grades in Complex Ore Deposits*. Journal of Mining Science 39, 5, 499-507.

Azimi Y., Osanloo M., 2011. *Determination of open pit mining cut-off grade strategy using combination of nonlinear programming and genetic algorithm*. Arch. Min. Sci., Vol. 56, No 2, p. 189-212.

Bascetin A., 2007. *Determination of optimal cut-off grade policy to optimize NPV using a new approach with optimization factor*. Journal of the South African Institute of Mining and Metallurgy 107, 2, 87-94.

Bellman R., 1957. *Dynamic Programming*. Princeton University Press New Jersey, USA.

Berry E.S., 1922. *Present value in its relation to ore reserves, plant capacity and grade of ore*. The American Institute of Mining and Metallurgical Engineers 187, 11-16.

Black F., Scholes M., 1973. *The pricing of options and corporate liabilities*. J. Political Econom., 81, 3, 637-654.

Boyle P., 1997. *Options: a Monte Carlo approach*. J. Financial Economics 4, 3, 323-338.

Brennan M.J., Schwartz E.S., 1985. *Evaluating natural resource investments*. Jbus 58, 2, 135-157.

Cairns R.D., Shinkuma T., 2003. *The choice of the cutoff grade in mining*. Resour. Policy 29, 3-4, 75-81.

Camus J.P., Jarpa S.G., 1996. *Long range planning at Chuqicamata mine*. In: Proceedings of the 26th APCOM, Penn State University, USA, 237-241.

Cetin E., Dowd P.A., 2002. *The use of genetic algorithms for multiple cutoff grade optimization*. In: Proceedings of the 30th APCOM , Littleton, Colorado, USA, 769-780.

Cortazar G., Gravet M., Urzua J., 2008. *The valuation of multidimensional American real options using the LSM simulation method*. Computers & Operations Research 35, 1, 113-129.

Cox JC., Ross SA., Rubinstein M., 1979. Option pricing: a simplified approach. J. Financ. Econ., 7, 3, 229-263.

Dagdelen K., 2007. *Open Pit Optimization — Strategies for Improving Economics of Mining Projects through Mine Planning ore body*. In: Orebody Modelling and Strategic Mine Planning (ed: R Dimitrakopoulos), Perth, Western Australia, 145-148.

Dagdelen K., 1992. *Cut-off grade optimization*. In: Proceedings of the 23th APCOM, Littleton, Colorado, USA, 157-165.

Dimitrakopoulos R., Abdel Sabour S.A., 2007. *Evaluating mine plans under uncertainty: can the real options make a difference?* Resour. Policy 32, 3, 116-125.

Dixit A.K., Pindyck R.S., 1994. *Investment under uncertainty*. Princeton University Press Princeton.

Dowd P.A., 1997. *Risk in minerals projects analysis, perception and management*. Mining Technology: IMM Transactions Section A 106, 9-18.

Dowd P.A., 1976. *Application of dynamic and stochastic programming to optimize cutoff grades and production rates*. Mining Technology: IMM Transactions Section A 85, 22-31.

Gligoric Z., et al., 2011. *Hybrid model of evaluation of underground lead – zinc mine capacity expansion project using Monte Carlo simulation and fuzzy numbers*. Simulation: Transactions of the Society for Modeling and Simulation International 87, 8, 726-742.

Henning U., 1963. *Calculation of cut-off grade*. Can. Min. J., 84, 3, 54-57.

King B.M., 2001. *Optimal mine scheduling policies*. Dissertation, Royal School of Mines, London University.

Krautkraemer J.A., 1988. *The cut-off grade and the theory of extraction*. Canadian Journal of Economics 21, 1, 146-160.

Lane K.F., 1988. *The Economic Definition of Ore: Cutoff Grades in Theory and Practice*. Mining Journal Books Ltd, London, UK .

Lane K.F., 1964. *Choosing the optimum cut-off grade*. Quarterly of the Colorado School of Mines 59, 811-829.

Lemelin B.M., Abdel Sabour S.A., Poulin R., 2007. *A flexible mine production model based on stochastic price simulations: application at Raglan mine, Canada*. Mining Technology: IMM Transactions Section A 116, 4, 158-166.

Longstaff F.A., Schwartz E.S., 2001. *Valuing American options by simulation: a simple least-squares approach*. RevFinancial Stud 14, 1, 113-147.

Mardones J., 1993. *Option valuation of real assets: Application to a copper mine with operating flexibility*. Resour. Policy 19, 1, 51-65.

McIsaac G., 2008. *Strategic design of an underground mine under conditions of metal price uncertainty*. Dissertation, Queen's University, Kingston, Ontario, Canada.

Osanloo M., Rashidinejad F., Rezai B., 2008. *Incorporating environmental issues into optimum cut-off grades modeling at porphyry copper deposits*. Resour. Policy 33, 4, 222-229.

Osanloo M., Ataei M., 2003. *Using equivalent grade factors to find the optimum cutoff grades of multiple metal deposits*. Minerals Engineering 16, 8, 771-776.

Rashidinejad F., Osanloo M., Rezai B., 2008. *Cutoff grade optimization with environmental management; a case study: Sungun copper mine*. IUST International Journal of Engineering Science 19, 5-1, 1-13.

Samis M., Davis, G.A., Laughton D., Poulin R., 2006. *Valuing uncertain asset cash flows when there are no options: a real options approach*. Resour. Policy 30, 4, 285-298.

Schwartz E.S., 1997. *The stochastic behaviour of commodity prices: implications for valuation and hedging*. J. Finance 52, 3, 923-973.

Schwartz E.S., 1977. *The valuation of warrants: Implementing a new approach*. Journal of Financial Economics 4, 1-2, 79-93.

Taylor H.K., 1972. *General background theory of cutoff grades*. Mining Technology: IMM Transactions Section A 81, 160-179.

Thompson J., 2010. *Test of an innovative stochastic design system on an open pit*. Dissertation, Queen's University, Kingston, Ontario, Canada.

Topal E., 2008. *Evaluation of a mining project using Discounted Cash Flow analysis, Decision Tree analysis, Monte Carlo Simulation and Real Options using an example*. International Journal of Mining and Mineral Engineering 1, 1, 62-76.

Trigeorgis L., 1996. *Real options: managerial flexibility and strategy in resource allocation*. MIT Press: Cambridge 28, 1, 1-20.

Vallee M., 2000. *Mineral resource + engineering, economic and legal feasibility = ore reserve*. CIM Bull 93, 1038, 53-61.

Whittle J., Wharton C., 1995. *Optimizing cut-offs over time*. In: Proceedings of the 25th APCOM, Brisbane, Australia, 261-265.

Received: 28 November 2011

Appendix

A.1. Cut-off grade formula

Avoiding mathematical calculations and complexities, only the necessary formula and explanations are presented here. Considering that, there is one unit of mineralized material then the technical coefficients to convert mineralized material into ore and ore into metal as well as the three component stages of the mining process are as shown in Table A.1.

TABLE A.1

Three component stages of the mining process

Stage	Basic throughput	Quantity	Variable cost per unit of throughput	Capacity through-put per year
Mining	Mineral material	1	m	M
Treatment	Ore	x	h	H
Refining	Metal	$x.g'.y$	k	K

As mentioned based on the each of the three stages and combination of them, three economical and three balancing cut-off grades are calculated at each period of mining operation. For economic cut-off grades, only one of the stages limits the operation and works with full capacity and they denote the material that at the margin just pays for downstream processing. The corresponding formula for limiting economic cut-off grades are as following:

$$g_m = \frac{h}{(p-k)\cdot y}\ ;\ g_h = \frac{h+(f+F)/H}{(p-k)\cdot y}\ ;\ g_r = \frac{h}{p-k-(f+F)/K} \tag{A.1}$$

where g_m, g_h and g_r are the mine, milling and refining limiting economic cut-off grades respectively. Balancing cut-off grades are values at which capacities are fully utilized in pairs. There is no exact mathematical formula to calculate balancing cut-off grades. These cut-off grades are determined according to the grade-tonnage distribution characteristics of the reserve and the ratio between capacities of stages. In order to calculate these balancing cut-off grades the following criteria must be satisfied by balancing cut-off grades.

- mine/milling balancing cut-off grade, g_{mh}, should satisfy; $x = H/M$.
- mine/refining balancing cut-off grade, g_{mr}, should satisfy; $x.g'.y = K/M$.
- milling/refining balancing cut-off grade, g_{hr}, should satisfy; $g'.y = K/H$.

A schematic determination of balancing cut-off grades are shown at the Figure A.1.

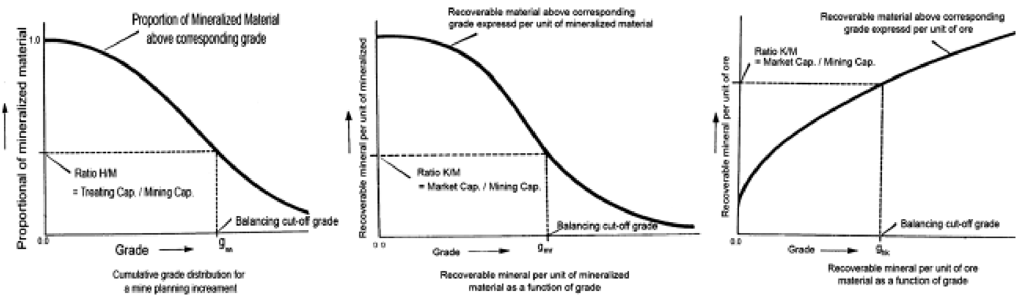


Fig. A1. Schematic representation of determination of balancing cut-off grades

A.1.2. Optimal cut-off grade

At last the optimum cut-off grade is determined according to the equations (A.2) and (A.3).

$$G_{mh} = \begin{cases} g_m & \text{if } g_{mh} < g_m \\ g_h & \text{if } g_{mh} > g_h \\ g_{mh} & otherwise \end{cases} \text{ and } G_{mr} = \begin{cases} g_m & \text{if } g_{mr} < g_m \\ g_r & \text{if } g_{mr} > g_h \\ g_{mr} & otherwise \end{cases} \text{ and } G_{hr} = \begin{cases} g_r & \text{if } g_{hr} < g_r \\ g_h & \text{if } g_{hr} > g_h \\ g_{hr} & otherwise \end{cases} \tag{A.2}$$

$$G = \text{middle of } (G_{mh}, G_{mr}, G_{hr}) \tag{A.3}$$

where G is the overall optimum cut-off grade at the period.

A.2. Cut-off grade strategy

The set of calculated optimum cut-off grades for the whole mining lifespan is known as the optimum cut-off grade strategy. To calculate the optimum COGS (especially g_h and g_k), it is necessary to first estimate the opportunity cost (F), at every period of mining lifespan. The opportunity cost F, is a fixed cost that incurred by the investor because of being tied up in the mining operation. The presence of the term F in the optimization formula ensures that the object of the optimization is to maximize the present value of the future profits and not the profit per period.

The opportunity cost is composed of two pseudo costs namely, the cost of capital being tied up in the operation (δV^*), and the capital gain ($-dV^*/dT$) that expresses the rate of variations in the present value assuming the same quantity of resource during the period. The second pseudo cost can be a decline (penalty) in value, because of deteriorating economic condition or a bonus if the economic condition improves. It can be estimated as the difference between the present value one period forward, and the present value now for the same quantity of resource remaining. However, both terms of the opportunity cost depend on the present value of operation and present value of operation is calculated after determining the optimum COGS. To overcome this circular dependency, the opportunity cost is estimated as a function of the difference between the present value one period forward, and the present value now for the same quantity of resource remaining. Therefore, two streams of exploitation policies (V and W), are calculated simultaneously, in which it is supposed that the second stream (W) will be commenced at the end of the first year of the stream V. Assuming an initial present value for each of the two streams, opportunity cost as well as the optimum cut-off grade are calculated for every period of mine's life in two streams Simultaneously. Then, at the end of the reserve life, the present value of the reserve must be equal to zero or any specified terminal value in both of the streams. If it is not, then the assumed initial present values will be adjusted and the calculations will be repeated until the mentioned condition is met. The set of the cut-off grades for the last iteration in the stream V is the optimum COGS of mining operation. The flowchart of the Lane algorithm for determination of optimum COGS is shown in Fig. 4.