The influence of concrete shrinkage on durability of reinforced structural members

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Abstract. The aim of the paper was to present the author's novel approach to the problem of the influence of concrete shrinkage on the static-strength performance of reinforced structures. The problem of concrete shrinkage has been known in concrete technology for years, mainly in the theoretical and experimental aspects. However, there are few works in which the effect of concrete shrinkage in real reinforced structural members and structures is shown. In the present article the author performs an analysis of these effects on a macro-scale, useful in the assessment of the influence of concrete shrinkage on limit states of bearing capacity and serviceability of reinforced concrete structures. An important distinction is made between shrinkage stresses imposed in RC members by external and internal (reinforcement induced) constraints and residual shrinkage stresses inside members (massive especially) resulting from non-stationary and non-linear moisture fields. The article concludes with a way of calculating the necessary, near-surface anti-shrinkage reinforcement in such members.

Key words: unrestrained shrinkage of concrete, calculation of shrinkage in reinforced concrete, internal forces and shrinkage stresses in a structure imposed by external and internal constraints, residual shrinkage stresses, massive elements.

1. Introduction

The publication is devoted to the role of concrete shrinkage in the assessment of limit states of load capacity and serviceability of reinforced concrete structural members and structures. There are many research works worldwide on the problems of concrete shrinkage, its causes, development in time, thermal, moisture and geometry conditions [1–4]. Fewer works, although also numerous, have been devoted to the problem of the influence of concrete shrinkage on reinforced members and structures [5–9]. These works were a basis for methods of allowing for this effect in the analysis of the internal forces in a structure as well as cracking and deflections affecting the durability.

It should be stated that these methods are not very commonly followed in civil engineering and there are many cases of unintended cracking resulting from insufficient knowledge in this area. The present article is an example of a synthetic approach [10] to this problem, aimed at designers who need to better understand it in order to deal with the practical aspects of the phenomenon of shrinkage in concrete.

The aim of the paper is therefore an overview of shrinkage modelling and its importance in design of concrete elements used in civil structures.

2. Unrestrained shrinkage of concrete ε_{cs} (t)

Concrete shrinkage is a physico-chemical phenomenon strictly connected with the process of concrete setting and maturing as well as moisture exchange between maturing or mature concrete and the surroundings. The process of concrete curing (setting plus maturing) involves the phenomenon of so-called contraction which affects the cement minerals binding

the water of crystallisation. This is manifested by the so-called autogenous shrinkage ε_{ca}, which can be expressed by formula (after Eurocode 2) [11]:

$$\varepsilon_{ca}(t) = \beta_{as}(t) \cdot \varepsilon_{ca}(\infty) \tag{1}$$

where

$$\varepsilon_{ca}(\infty) = -2.5(f_{ck} - 10) \cdot 10^{-4}, \tag{2}$$

$$\beta_{as}(t) = 1 - \exp(-0.2t^{0.5}), \tag{3}$$

t – age of concrete in days (from the moment of casting).

The values of $\varepsilon_{ca}(\infty)$ increase with the increase of concrete class, from 0.01‰ for class C 12/15 to 0.20‰ for class C 90/105. For concrete class C40/50 the value of $\varepsilon_{ca}(\infty)$ is 0.08‰ $(0.8 \cdot 10^{-4})$. The value of coefficient $\beta_{as}(t)$ quickly increases and after $t = 7$ days is 0.41 while after 28 days of curing – 0.65, which is demonstrated by the fact that with time the processes connected with cement hydration in concrete gradually decay.

After the formwork of the structure has been removed another component of concrete shrinkage ε_{cd}, connected with concrete drying (the so-called physical shrinkage) soon becomes evident. The water-cement ratio in the concrete mix, due to its workability, usually is W/C = 0.3÷0.5. However, for cement complete hydration only $\text{W/C}\big|_{hydr}^{theor} = 0.20{\div}0.25$ is sufficient, but complete hydration generally never happens and then $\text{W/C}\big|_{hydr}^{real} = 0.12{\div}0.15$ [2]. This indicates that the excess of water from the concrete mix must evaporate, the value increasing with higher W/C, i.e. the lower the concrete class the more water should evaporate. There is also a lower limit of excess water evaporation, which is when the vapour partial pressure in concrete pores is equal to the vapour pressure

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in the atmospheric air (the so-called hygrothermal balance). The pressure in pores depends on the capillary diameter, the atmospheric pressure on weather conditions (temperature and air humidity). It follows that after the hygrothermal balance has been reached, water vapour particles can move both from concrete to the outside and from the outside air into concrete. The resulting changes in the moisture state generally take place only in the concrete layer near the surface.

The value of shrinkage due to drying is described in EC2 by formula:

$$\varepsilon_{cd}(t) = \beta_{ds}(t - t_s) \cdot k_h \cdot \varepsilon_{cd,0}, \qquad (4)$$

where

$$\beta_{ds}(t - t_s) = \frac{t - t_s}{(t - t_s) + 0.04\sqrt{h_0^3}}, \qquad (5)$$

t – age of concrete at the given time, in days, t_s – age of concrete (in days) at the start of shrinkage due to drying; generally corresponding to concrete curing termination, $h_0 = 2A_c/u$ – notional size (mm) of element's cross-section, A_c – area of element's cross-section, u – part of section perimeter exposed to air operation, k_h – factor dependent on notional size of element h_0 (for $h_0 = 100$ mm – $k_h = 1.0$; for 200 mm – 0.85, for 300 mm – 0.75; for $h_0 \geq 500$ mm – $k_h = 0.70$),

$$\varepsilon_{cd,0} = 0.85 \left[(220 + 110 \cdot \alpha_{ds1}) \cdot \exp\left(-\alpha_{ds2} \cdot \frac{f_{cm}}{f_{cm0}}\right) \right] \\ \cdot 10^{-4} \cdot \beta_{RH}, \qquad (6)$$

$$\beta_{RH} = 1.55 \left[1 - \left(\frac{RH}{RH_0}\right)^3 \right], \qquad (7)$$

$\alpha_{ds1}, \alpha_{ds2}$ – factors dependent on type of cement, f_{cm} – mean compressive strength of concrete (MPa), $f_{cm0} = 10$ MPa, RH – relative humidity of surroundings (%), $RH_0 - 100\%$.

In the Polish environmental conditions RH = 40÷90%, so the range of the value of $\varepsilon_{cd,0}$ is very wide: (0.64÷0.25)‰ for RH = 40% and concrete classes from C 12/15 to C 90/105 and (0.19÷0.07)‰ for RH = 80% and concrete classes as above.

For the humidity values most frequent in Poland RH = 60÷80% these ranges are narrower, (0.54÷0.21)‰ for RH = 60 % and concrete classes from C 12/15 to C 90/105 and (0.33÷0.13)‰ for RH = 80% and concrete as above. It should

be mentioned here that the limit deformability of plain concrete in tension is only $\varepsilon'_{lim} = 0.10\%$ ($1 \cdot 10^{-4}$), which sometimes leads to shrinkage cracking.

The total effect of concrete shrinkage on concrete elements and structures is then:

$$\varepsilon_{cs}(t) = \varepsilon_{cd}(t) + \varepsilon_{ca}(t). \qquad (8)$$

For example, for concrete class C 30/37, the value of $\varepsilon_{cs}(t)$ after 180 days of maturing, $t_s = 7$ days, $h_0 = 200$ mm, RH = 70%, after the above formulae is:

$$\varepsilon_{cs}(180) = 0.66 \cdot 0.85 \cdot 0.35 + 0.93 \cdot 0.06$$

$$= 0.196 + 0.056 = 0.252\% = \underline{2.52 \cdot 10^{-4}}.$$

3. Design values of concrete shrinkage in reinforced concrete ε_{cs}^{RC}

In RC elements the design values of concrete shrinkage depend on the degree of element's reinforcement. Even the old Polish standard PN-76/B-03264 stipulated that the values of concrete shrinkage strain should be adopted as follows:

- in concrete structures $\varepsilon_{cs} = 0.0003$,
- in reinforced concrete structures $\varepsilon_{cs} = 0.0002''$,

while after standard PN-84/B-03264 the corresponding values were $\varepsilon_{cs} = 0.0003$ and $\varepsilon_{cs} = 0.00015$, respectively. This indicates that the standard values included the effect of the averaged degree of element's reinforcement on its shrinkage strain. In both standards it was also stated that "the effect of shrinkage in RC structures can be considered equivalent with temperature drop by 15°C".

An approximate dependence between concrete shrinkage in reinforced concrete ε_{cs}^{RC} and the unrestrained shrinkage of concrete, introduced by the present author in [12] for a prismatic element of symmetrical section, reinforced symmetrically (Fig. 1) takes the form:

$$\varepsilon_{cs}^{RC} = \varepsilon_{cs}(1 - \alpha_3) \cdot k_3, \qquad (9)$$

where

$$\alpha_3 = \frac{n_0\mu_0}{1 + \eta_0\mu_0}, \qquad n_0 = \frac{E_s}{E_{cm}(t_s)},$$

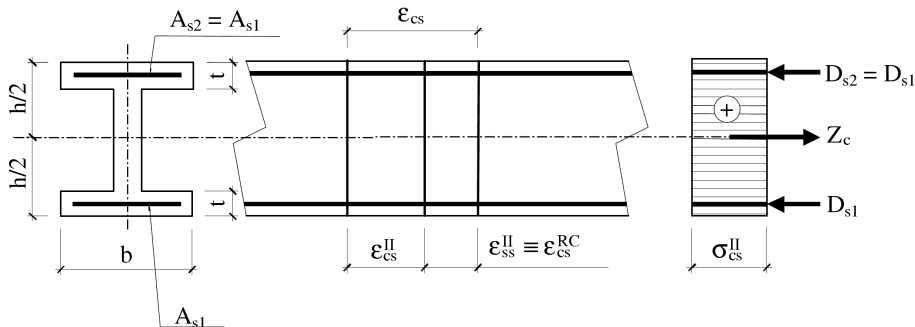$$\mu_0 = \frac{A_{s1} + A_{s2}}{A_c} = \rho_1 + \rho_2,$$

Fig. 1. Shrinkage strain and stress forced by reinforcement resistance in the section of RC symmetrical prismatic element, reinforced symmetrically after Ref. 12

k_3 – coefficient of concrete creep induced shrinkage stress relaxation; for modified ageing theory is [14]:

$$k_3 = \frac{1}{1 + \beta \alpha_3 \varphi(\infty, t_s)}, \qquad (10)$$

$E_{cm}(t_s)$ – modulus of elasticity of concrete at the start of shrinkage; can be adopted as $E_{cm}(t_s) \cong E_{cm}$, β – ageing function; for shrinkage load can be adopted as $\beta = 0.8$, $\varphi(\infty, t_s)$ – creep coefficient.

Consequently, for a RC beam of section of 300/600 mm ($h_0 = 200$ mm) from concrete class C 30/37 ($E_{cm} = 32.0$ GPa) of reinforcement degree $\rho_1 = \rho_2 = 0.01$ ($\mu_0 = 0.02$),

$$n_0 = \frac{200}{32} = 6.25,$$

$\beta = 0.8$, $t_s = 14$ days, RH = 70%, $\varphi(\infty, t_s) = 2.3$ [14] we obtain:

$$\alpha_3 = \frac{0.02 \cdot 6.25}{1 + 0.02 \cdot 6.25} = 0.111,$$

$$k_3 = \frac{1}{1 + 0.8 \cdot 0.111 \cdot 2.3} = 0.830,$$

$$\varepsilon_{cs}^{RC} = \varepsilon_{cs}(1 - 0.111) \cdot 0.830 = \underline{0.738\varepsilon_{cs}},$$

that is, for instance, for $\varepsilon_{cs}(\infty) = 0.00030 \rightarrow \varepsilon_{cs}^{RC} = \underline{0.000221}$.

The difference in the values of shrinkage ε_{cs}^{RC} and ε_{cs} induces in the element (due to the resistance of reinforcing bars) shrinkage stresses, compressive in the steel σ_{ss} and tensile in the concrete σ_{cs} of the values [12]:

$$\sigma_{ss} = -\varepsilon_{cs} E_s (1 - \alpha_3) k_3, \qquad (11)$$

$$\sigma_{cs} = \varepsilon_{cs} E_{cm} \alpha_3 k_3, \qquad (12)$$

which for $\varepsilon_{cs} = 0.00030$, $E_{cm} = 32.0$ GPa, $E_s = 200$ GPa, $\alpha_1 = 0.111$, $k_3 = 0.830$ leads to the value $\sigma_{ss} = -44.3$ MPa (compression of the reinforcing steel) and $\sigma_{cs} = 0.88$ MPa (concrete tension).

The compressive stress in the steel increases the theoretical breaking moment in the section (that is has a favourable effect). On the other hand, the tensile stress in the steel of the value of $\sigma_{cs} = 0.88$ MPa = 0.44 f_{ctk} is a significant stress decreasing the cracking moment of the section and should be taken into account in the analysis of serviceability limit states.

4. Internal forces and stresses in a structure induced by concrete shrinkage

Additional stresses induced in elements or structures by concrete shrinkage can be divided into imposed or internal. The imposed stresses appear when concrete shrinkage (shortening of structural elements) is opposed by external or internal constraints. The external constraints include flexible or rigid supports and joints of a structure. The internal constraints include reinforcing bars resisting the shrinking concrete. The imposed stresses are significant mainly in non-massive elements and those of $h_0 < 0.30$ m. For these elements a uniform, linear course of shrinkage along the section height can be assumed.

On the other hand, for elements of $h_0 > 0.40$ m the internal shrinkage stresses are predominant, self-counterbalancing in section. These stresses appear between adjacent fibres of the element due to different shrinkage caused by non-linear and non-stationary moisture areas in the section. As a rule, they are tensile for the outer fibres and compressive for the inner fibres of the concrete.

4.1. Internal forces and stresses σ_{cs}^{I} induced in a structure by external constraints.
The forces included in structural-strength calculations of concrete structures most frequently are generalised forces, induced in the structure by external constraints. These constraints can be rigid (e.g. floors connected with stair and/or lift shafts) or flexible (e.g. in the connections of spandrel beams with flexible columns in frames).

In both cases an additional analysis should be made (cf. Fig. 2.), using the mean value of shrinkage effect in reinforced concrete section, i.e. ε_{cs}^{RC}.
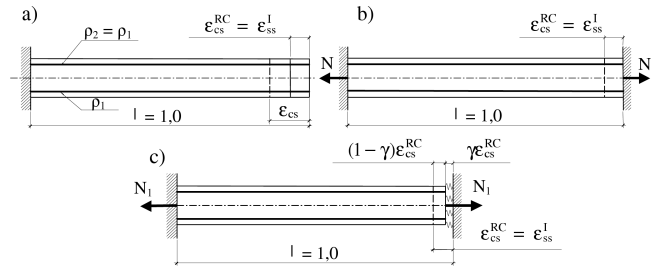

Fig. 2. Examples of shrinkage effect on weightless symmetrical RC element reinforced symmetrically: a) element with no external constraints, b) element with external rigid constraints, c) element with external flexible constraints after Ref. 12

In two extreme cases (for weightless beam) we have:

a) when there are no external constraints (Fig. 2a) – only a displacement of ε_{cs}^{RC}, with no additional external tensile force N,

b) in case of rigid constraints (Fig. 2b) – tension in the element caused by external force:

$$N = \varepsilon_{cs}^{RC} A_c E_{cm} (1 + n_0 \mu_0) = \varepsilon_{cs} A_c E_{cm} k_3,$$

with no displacement, where:

$$n_0 = \frac{E_s}{E_{cm}}, \quad \mu_0 = \rho_1 + \rho_2.$$

c) in the intermediate case (Fig. 2c), with flexible constraints, shrinkage strain ε_{cs}^{RC} is divided into two parts:

- displacement – $\gamma \varepsilon_{cs}^{RC}$, ($\gamma \leq 1.0$),
- strain – $(1 - \gamma)\varepsilon_{cs}^{RC}$, an external force causing tension:

$$N_1 = (1 - \gamma)\varepsilon_{cs}^{RC} A_c E_{cm} (1 + n_0 \mu_0). \qquad (13)$$

In actual practice instead of design average shrinkage ε_{cs}^{RC} structural diagrams are deployed, which include a structure's temperature drop Δt equivalent to shrinkage. As shown in [13], the value $\Delta t = 15°$C adopted most frequently may be inappropriate since Δt depends on a number of factors (material, geometric, climatic) and may be different for different elements of the same structural system. As structural analysis

shows, concrete shrinkage causes in the discussed structures additional internal forces M, N, Q that affect the structure's overall effort, i.e. its design load capacity limit state. In Eurocode 2 – 2.3.2.2. this issue is treated as follows: "In general, the effect of shrinkage and creep should be included in monitoring serviceability limit states". And further on: "The effect of shrinkage and creep on load capacity limit states should be taken into account only when it is substantial, e.g. in monitoring the stability limit state of the elements for which the effects of second rank are of significant importance. In other cases this effect need not be taken into account on condition the ductility of elements and their ability to rotate are sufficient".

In structures, e.g. bridges, that permit it the tendency is to eliminate the effect of strain part of the deformation $(1 - \gamma)\varepsilon_{cs}^{RC}$ on the structure's internal forces by deploying expansion bearings, expansion joints, etc. Then the role of this part of shrinkage deformation may become of minor importance. A similar case may occur for a RC slab foundation if relatively free shrinkage displacements are predicted on the substrate or by insulating it on a layer of lean concrete.

In case of rigid constraints, the internal tensile forces generated in a structure are usually so great that they cause section cracking – through cracks when the axial force N prevails (eccentric tension at low eccentricity), or unilateral with heavy contribution from the bending moment M (eccentric tension at high eccentricity).

Shrinkage induced cracking caused by external constraints need not always be harmful to the structure. This is because it relieves the undesirable stress state such as tensile stress in the concrete and compressive stress in the steel (in case of flexible constraints).

When the cracking of an element with external constraints is caused only by shrinkage, the mean stress in the steel along the element length, caused by shrinkage, is reduced to zero and in this sense the recommendations of Eurocode 2 in this aspect are correct. However, shrinkage tensile stress σ_{cs}^{I} accelerate structure cracking induced by other "massive" external loads, which means that the element will reach the design serviceability limit states much sooner and structure durability will be reduced. Moreover, earlier cracking may change the operation phase of the RC section, which also affects load capacity limit state.

4.2. Shrinkage stress in a structure σ_{cs}^{II} induced by internal constraints (counteraction of reinforcement). The stresses in the sections of the structure's elements imposed by the internal constraints such as the counteraction of reinforcement bars against unrestrained shrinkage ε_{cs} is the other part of the effect of shrinkage on concrete structures. These constraints usually induce tensile stresses in concrete and compressive stresses in the reinforcing steel. Owing to these constraints the shrinkage strains in RC elements ε_{cs}^{RC} are smaller than in concrete elements ε_{cs}, but in the section of each element there appears an additional stress field σ_{cs}^{II} which is usually disregarded in strength analyses.

In a symmetric RC section reinforced symmetrically and linear shrinkage as well as homogeneous in section (Fig. 1) the values of such stresses in an element with no external constraints are [12] (cf. formulae (11) and (12)):

$\sigma_{ss}^{II} = -\varepsilon_{cs}E_s(1 - \alpha_3)k_3$ – compression,

$\sigma_{cs}^{II} = \varepsilon_{cs}E_{cm}\alpha_3 k_3$ – tension.

In the scheme shown in Fig. 1 the lowest tensile stresses σ_{cs}^{II} are obtained. The highest values on the edge in tension are obtained for a T-beam one-way reinforced. For a rectangular beam one-way reinforced (Fig. 3) the following formulae are obtained [12]:

$$\sigma_{ss}^{II} = -\varepsilon_{cs}E_s \left(1 - \alpha'\right) k_3, \tag{14}$$

$$\sigma_{cs}^{II} = \varepsilon_{cs}E_{cm}\alpha'' k_3, \tag{15}$$

$$\sigma_{cs'}^{II} = \varepsilon_{cs}E_{cm}\alpha''' k_3, \tag{16}$$

where

$$\alpha' = \frac{n_0\mu_0\eta}{1 + n_0\mu_0\eta}, \qquad \alpha'' = a' \cdot \frac{1 + 6e/h}{\eta},$$

$$\alpha''' = a' \cdot \frac{1 - 6e/h}{\eta}, \qquad \eta = 1 + 12\left(\frac{e}{h}\right)^2, \tag{17}$$

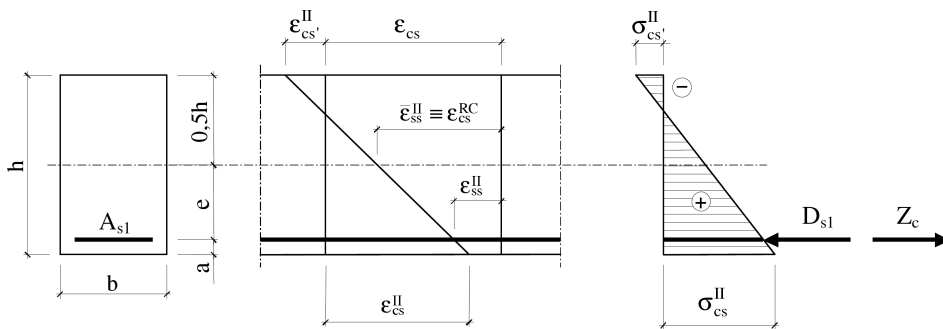$$k_3 = \frac{1}{1 + \beta\alpha'\varphi\left(\infty, t_s\right)}.$$

Fig. 3. Shrinkage strains and stresses imposed by counteraction of reinforcement in the section of a symmetric RC element reinforced unilaterally (element with no external constraints) after Ref. 12

Adopting in formulae (11), (12), (14), (15) and (16) the value $\varepsilon_{cs}(\infty)$ we obtain maximum values of shrinkage stresses that may occur in RC structural elements after the whole shrinkage process of concrete has terminated, i.e. after ca. $3 \div 30$ years, depending on the design size of the element $h_0 = 2A_c/u$ (cf. Fig. 4). In the preceding periods shrinkage $\varepsilon_{cs}(t_s)$ is less extensive and the stresses indicated in above formulae are also lower.
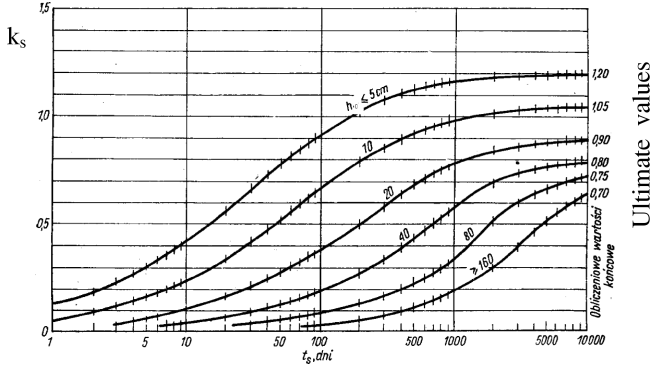

Fig. 4. Coefficient k_s dependent on concrete design age t_s, for calculating the shrinkage measure variability in time after Ref. 8

Let us consider a case of an element from concrete class C 20/25, $E_s = 200$ GPa, $E_{cm} = 30.0$ GPa, $n_0 = 6.67$, $\mu_0 = \rho_1 = 0.015$, $\varepsilon_{cs}(\infty) = 3.0 \cdot 10^{-4}$, $\beta = 0.8$, $\varphi(\infty, t_s) = 2.4$, $e/h = 0.4$, $\eta = 2.92$, $a' = 0.226$, $a'' = 0.263$, $a''' = -0.108$, $k_3 = 0.697$, we obtain:

$$\sigma_{ss}^{\mathrm{II}} = -3.0 \cdot 10^{-4} \cdot 200 \cdot 10^3 (1 - 0.226) \cdot 0.697$$
$$= \underline{-32.4 \text{ MPa}},$$

$$\sigma_{cs}^{\mathrm{II}} = 3.0 \cdot 10^{-4} \cdot 30.0 \cdot 10^3 \cdot 0.263 \cdot 0.697$$
$$= \underline{1.65 \text{ MPa}} = 0.75 f_{ctm} > f_{ctk} = \underline{1.50 \text{ MPa}},$$

$$\sigma_{cs'}^{\mathrm{II}} = -3.0 \cdot 10^{-4} \cdot 30.0 \cdot 10^3 \cdot 0.108 \cdot 0.697 = \underline{-0.68 \text{ MPa}}.$$

It follows from the example the imposed stresses $\sigma_{cs}^{\mathrm{II}}$ can reach considerable values. These stresses are particularly high in tall T-sections commonly used in bridge engineering. In [14, 15] it was estimated that such stresses in two actual bridges were (Fig. 5):
1) $\sigma_{cs}^{\mathrm{II}} = 1.31$ MPa $= 0.82\ f_{ctm} > f_{ctk} = 1.10$ MPa,
2) $\sigma_{cs}^{\mathrm{II}} = 1.25$ MPa $= 0.66\ f_{ctm} < f_{ctk} = 1.30$ MPa.
In Fig. 5 the broken lines indicate the values of shrinkage stresses $\sigma_{cs}^{\mathrm{II}}$ disregarding relaxation of these stresses, caused by concrete creep.

The values of shrinkage stresses $\sigma_{cs}^{\mathrm{II}}$ increase with the increase of concrete shrinkage in time, similarly as stresses σ_{cs}^{I}. Coefficient k_3 is a measure relaxation of these stresses due to concrete creep, which for reinforced concrete can be calculated from formulae (10) or (17), and from the approximate dependence after [8]:
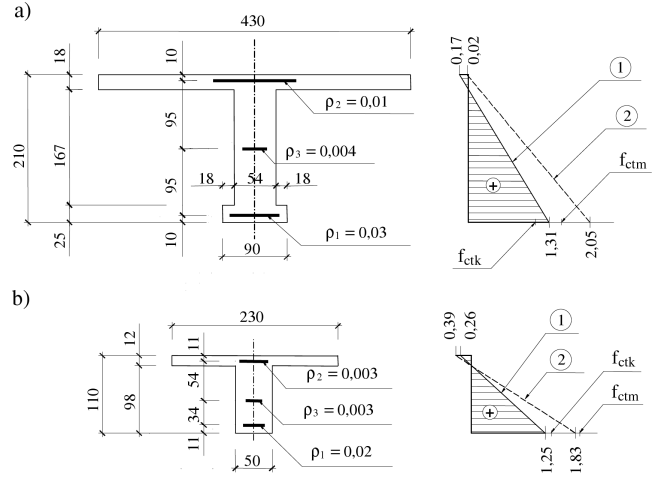
$$k_3 = \frac{1}{1 + 0.2\varphi(\infty,\ t_s)}. \tag{18}$$

Fig. 5. Shrinkage stresses $\sigma_{cs}^{\mathrm{II}}$ (in MPa) caused by reinforcement resisting force in the discussed beams (an element with no external constraints) after Ref. 15

From formula (18) it follows that if $\varphi(\infty, t_s) = 2.4$, then $k_3 = 0.68$. As mentioned earlier, the stresses discussed $\sigma_{cs}^{\mathrm{II}}$ are not included in structural analysis, which in some cases, e.g. T-beams one-way reinforced, is a mistake. It should be emphasised that tensile stresses in concrete $\sigma_{cs}^{\mathrm{II}}$ clearly reduce the cracking moment M_{cr} or the cracking force N_{cr} in an element, accelerating the cracking process. This in turn affects the reduced stiffness of the element and its reduced corrosion resistance.

On the other hand, the advantages from compressive stresses $\sigma_{ss}^{\mathrm{II}}$ in reinforcing steel are illusory since the moment an element is cracked due to shrinkage these stresses decay and their influence on increasing the section load capacity is but minor.

4.3. Shrinkage residual stresses $\sigma_{cs}^{\mathrm{III}}$ in section. The two components of shrinkage stresses σ_{cs} in a RC element cross-section, described in Subsecs. 4.1 and 4.2, are to some extent independent of the designer who is restricted by the structure model and the resulting distribution of reinforcement in elements. He can, however, have an influence on the reduction of the value of ε_{cs} in the concrete used by an appropriate technology of the concrete mix. The factors that matter include low W/C, reduced consumption of cement, application of superplasticisers and an appropriate consolidation technology, e.g. the vacuum process.

There is, however, a third component of shrinkage stresses and strains in concrete elements section, that is shrinkage residual stresses $\sigma_{cs}^{\mathrm{III}}$, induced in the section by non-linear and nonstationery (time variable) moisture fields in the element.

They are generated by moisture concentration gradient in a section (cf. Fig. 6) and usually stretch the near-surface fibres of the element and compress the fibres farther inside the section. These stresses, self-counterbalancing in section, are induced by the resistance of some layers of concrete against other layers, which is caused by their tendency to non-uniform shrinkage displacements.
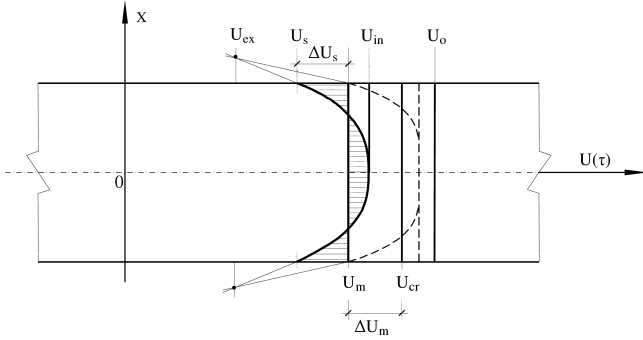
Fig. 6. Nonlinear and unsteady moisture fields in the section of a concrete wall after Ref. 7

The near-surface gradient of the moisture fields $\frac{dU(\tau)}{dx}\big|_s$ is the highest after the removal of formwork of the structure (time τ_r); with time the moisture of the element's interior tends to equalizing and stabilizes at the level of hygrothermal balance with the surrounding atmosphere.

Residual stresses σ_{cs}^{III} can be calculated from the formula:

$$\sigma_{cs}^{III} = k_3\beta_w \int_{\tau_r}^{\infty} E_{ct}(\tau)\frac{d\Delta U_s(\tau)}{d\tau}d\tau, \qquad (19)$$

where β_w – linear coefficient of concrete relative moisture deformability for a unitary relative moisture change by weight; it can be adopted as $\beta_w = 3.0\cdot10^{-2}\frac{mm/mm}{g/g}$, $E_{ct}(\tau)$ – modulus of elasticity in tension, usually adopted as $E_{ct}(\tau) = E_{cm}(\tau)$.
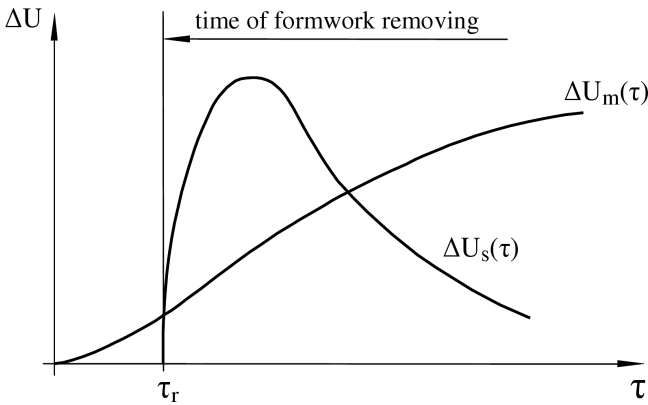

Fig. 7. Time dependent development of design increment of mean moisture $\Delta U_m(\tau)$ and near-surface moisture $\Delta U_s(\tau)$ in a concrete slab (wall) after Ref. 12

Shrinkage strains ε_{cs}, responsible for shrinkage stresses σ_{cs}^{I} and σ_{cs}^{II} are a function of moisture mean change $\Delta U_m(\tau)$ in section (Fig. 7):

$$\varepsilon_{cs}^{I} \equiv \varepsilon_{cs}^{II} = \beta_w\left[U_{cr} - U_m(\tau)\right] = \beta_w\Delta U_m(\tau), \qquad (20)$$

while

$$\varepsilon_{cs}^{III} = \beta_w\left[U_m(\tau) - U_s(\tau)\right] = \beta_w\Delta U_s(\tau). \qquad (21)$$

Dependencies $\Delta U_m(\tau)$ and $\Delta U_s(\tau)$ have been shown in Fig. 7. It is clear that while shrinkage strains $\varepsilon_{cs}^{I,II}$ tend to in-

crease with time, shrinkage strains ε_{cs}^{III}, responsible for stresses σ_{cs}^{III}, on reaching the maximum in a short period after the removal of formwork show a decreasing tendency.

The estimation of the values of stresses σ_{cs}^{III} poses a lot of difficulties. Usually, the designer does not know when the formwork will be removed, he cannot predict the weather conditions when the concrete matures, he does not know the deployed curing technology. It is important to perform the formwork removal process in such a way that the extreme value of σ_{cs}^{III} was below the concrete tensile strength curve σ_{cs}^{III} – cf. Fig. 8. Otherwise crack will form in the near-surface area of the element at an early stage of its "life". On the one hand, it will "relieve" the undesirable state of residual stresses, on the other hand, however, the cracks will remain in the element permanently, which means partial degradation of the section in the aspect of its strength and stiffness, and consequently its durability.
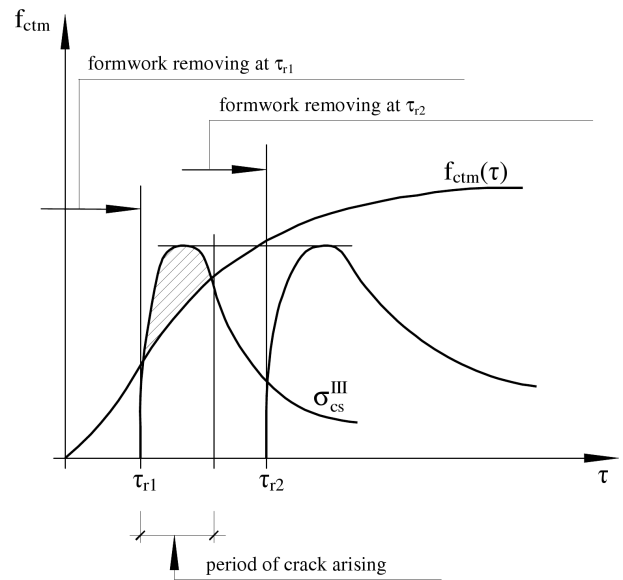

Fig. 8. Criterion of shrinkage crack formation in the near-surface zone of concrete elements, in the function of formwork removal time τ_r after Ref. 12

An attempt to estimate stresses σ_{cs}^{III} was taken in [15, 16], and strains ε_{cs}^{III} in [17]. The highest stress σ_{cs}^{III} calculated on the basis of [16], at the assumption of concrete class B25 and $k_3 = 0.7$, would be 1.28 MPa = 0.58 $f_{ctm} < f_{ctk}$ = 1.50 MPa, while on the basis of [17] – at concrete class B20 and $k_3 = 0.7 - \sigma_{cs}^{III}$ = 1.87 MPa = 0.98 $f_{ctm} > f_{ctk}$ = 1.30 MPa. This indicates great importance of the stresses discussed, particularly so as these stresses generally sum up with shrinkage stresses σ_{cs}^{I} and σ_{cs}^{II} (cf. Fig. 9), so that the probability of $\Sigma\sigma_{cs}^{(I,II,III)} \geq f_{ctm}(\tau)$ is very high. However, it should be noted here that the extremes of particular components $f_{ctm}(\tau)$ of this sum do not occur simultaneously, which mitigates the severity of the notation.
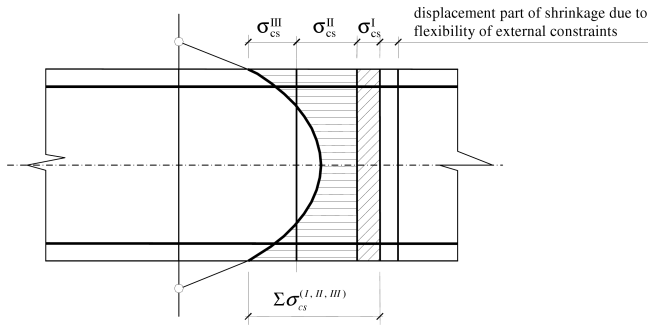
Fig. 9. Shrinkage stresses σ_{cs}^{I}, $\sigma_{cs}^{\mathrm{II}}$, $\sigma_{cs}^{\mathrm{III}}$ in the section of a concrete slab (wall) after Ref. 15

Table 1
Maximum bar diameter vs σ_{cs}^{real} for crack control

Steel stress [Mpa]	Maximum bar diameter [mm]	
	$w_k = 0.3$ mm	$w_k = 0.2$ mm
160	32	25
200	25	16
240	16	12
280	12	8
320	10	6
360	8	5
400	6	4
450	5	–

Stresses $\sigma_{cs}^{\mathrm{III}}$ decrease the faster the smaller is the equivalent thickness of the element. This is because the phenomenon that occurs here is reverse to stresses $\sigma_{cs}^{\mathrm{II}}$ which increase the faster the smaller is the equivalent thickness of the element. In non-massive elements the predominant role is played by stresses $\sigma_{cs}^{\mathrm{II}}$, while in massive ones – $\sigma_{cs}^{\mathrm{III}}$.

Relaxation of stresses $\sigma_{cs}^{\mathrm{III}}$ due to concrete creep is more pronounced in massive elements than in non-massive ones. According to [8], relaxation coefficient k_3 for residual shrinkage stresses can be defined from the approximate dependence:

$$k_3 = \frac{1}{2\varphi(\infty,\,t_s)} + 0.2. \tag{22}$$

So, at $\varphi(\infty,\,t_s) = 2.0$ (long-term effect of stresses $\sigma_{cs}^{\mathrm{III}}$ in a massive element) – $k_3 = 0.45$, at $\varphi(\infty,\,t_s) = 1.0$ (decaying during the operation of stresses $\sigma_{cs}^{\mathrm{III}}$ in a non-massive element) – $k_3 = 0.70$.

The formation of residual tensile stresses $\sigma_{cs}^{\mathrm{III}}$ in the near-surface zones of RC elements often leads to cracks of these zones unforeseen in design. This is certainly true for massive elements ($h_0 \geq 1.0$ m) and those of medium massiveness (0.40 m $< h_0 < 1.0$ m). In such cases it should be assumed that in the extreme fibres of the section, due to uneven shrinkage in the section, stresses $\sigma_{cs}^{real} = f_{ctm}$ appear. The thickness of concrete zone in tension can be assessed after monograph [12], the amount of near-surface reinforcement can be defined from the EC2 formula for the minimal cross-section of reinforcement $A_{s,\min}$, necessary to restrict cracks width to w_{lim} in case they form:

$$A_{s,\min} = k_c k f_{ct,eff} \frac{A_{ct}}{\sigma_{s,\lim}}, \tag{23}$$

where k_c – coefficient including stress distribution in section at the moment preceding cracking ($k_c = 1.0$ for axial tension and $k_c = 0.4$ – for bending), k_c – coefficient including the effect of non-uniform self-counterbalancing stresses in section ($k = 1.0$ for $b \leq 300$ mm and $k = 0.65$ for $b \geq 800$ mm), $f_{ct,\,eff}$ – concrete mean tensile strength at the moment of expected cracking; can be adopted as $f_{ct,\,eff} = f_{ctm}$, A_{ct} – section area in tension at the moment preceding cracking, $\sigma_{s,\,lim}$ – maximal stress adopted in reinforcement in tension immediately after cracking, dependent on crack limit width and RC bars diameter (cf. Table 1).

5. Conclusions

The aim of this article was to present an outline of the knowledge indispensable for correct assessment of the effect of concrete shrinkage on additional internal forces and stresses in reinforced concrete structural elements. These forces and stresses, variable in time, can affect both the structure's ultimate load capacity (stresses σ_{cs}^{I}) and – primarily – the serviceability limit states" cracking and deflections.

A distinction has been made between shrinkage stresses imposed by member's external constraints and internal constraints (due to member's reinforcement), and internal self-stresses induced by moisture fields in members' cross sections. It has been shown that stresses and have a decisive role in non-massive elements and those of medium massiveness ($h_0 \leq 0.30$ m), while stresses in massive elements and those of medium massiveness, by $h_0 \geq 0.40$ m.

In the former case cracking occurs earlier than would result from the internal forces of gravitational actions on the element (particularly in single reinforcement T-beams, – cf. Subsec. 4.2), while in the latter case tension in near-surface zones of concrete takes place only. Such zones usually require special anti-shrinkage reinforcement which protect the structures from excessive cracking that greatly affects their durability and rigidity. In the present article an outline of such calculations has been given, for detailed examples see [12] by the author.

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