JEE Journal of Ecological Engineering

Journal of Ecological Engineering 2022, 23(6), 164–171
https://doi.org/10.12911/22998993/147808
ISSN 2299–8993, License CC-BY 4.0

Received: 2022.03.17
Accepted: 2022.04.12
Published: 2022.04.20

Set of Suffosion Holes Occurring After a Water Supply Failure as a Structure with Fractal Features

Małgorzata Iwanek[1]

[1] Faculty of Environmental Engineering, Lublin University of Technology, ul. Nadbystrzycka 40B, 20-618 Lublin, Poland
e-mail: m.iwanek@pollub.pl

ABSTRACT
As a result of a buried water pipe unsealing, water often flows from the pipe to the soil surface, washing out the solid particles of soil and creating the so-called suffosion holes. It is a dangerous phenomenon, especially in urbanized areas, where it poses a threat to human safety and the stability of infrastructure. Uncontrolled outflows of water from water pipes belong to the main causes of suffosion in cities, occur in all water networks around the world and are difficult to predict. Therefore, it seems to be important to determine the size of the so-called the protection zone, which is the area around the potential leak where, in the event of a water pipe failure, it would be possible for water to flow in the soil. The analysis of the suffosion holes distribution around the place of leakage may be helpful in determining the size of the protection zone. Previous studies have shown that this distribution is random. Thus, the structure consisting of suffusion holes creates a certain geometric shape, which is difficult to describe using the classical concepts of Euclidean geometry. The study showed that this structure meets the conditions for probabilistic fractals, which means that elements of fractal geometry can be used to determine the size of the protection zone.

Keywords: water pipe failure, suffosion holes, fractal features.

INTRODUCTION

The most common failures of buried water pipes are their cracks, fractures, perforations and sealing blows (Kwietniewski and Rak, 2010). Regardless of the causes and type of these failures, they result in an uncontrolled outflow of water from the pipe. The velocity of the water flow can be high enough to wash out solid particles from the soil matrix, i.e. to cause the suffosion. Leakages of water from the water pipes are among the main causes of suffosion in urbanized areas (Khomenko, 2009). Moreover, they occur in all water networks worldwide throughout the entire period of their operation, and are difficult to predict (Kwietniewski and Rak, 2010). The consequence of suffosion is the formation of the empty spaces below the soil surface, which may lead to the collapse of the surface, as well as the formation of so-called of suffosion holes (Suchorab et al., 2016). Suffosion resulting from the water leakage from the buried water pipes is therefore a dangerous phenomenon, especially in urbanized areas, characterized by a relatively high density of infrastructure. Water supply systems are usually located in a road lane; therefore, in the event of a leak, mainly road users are at risk of the effects of suffosion. Leakages from water pipes also pose a threat to the stability of various types of infrastructure, which is due to the possibility of disturbing their subgrades by flowing water, e.g. the subgrade under the foundation of a building or sand subgrade under a pipeline. Therefore, it seems to be important to determine the size of the so-called protection zone, i.e. the area around a potential leak, within which, in the event of a water pipe failure, water flow through the ground would be possible (Iwanek et al., 2019). The protection zone determination requires an analysis of the location of the suffusion holes, taking into account their distance from the leak site and their location on the soil surface. The previous research has shown that the location of the suffosion holes around the leak is random

(Iwanek, 2021). The set made of suffosion holes creates a certain geometric figure that is difficult to describe using the classical concepts of Euclidean geometry. This is the feature of many naturally occurring structures (Mandelbrot, 1982). To describe these types of structures, both natural (e.g. soil medium – Ghanbarian et al., 2013, Xu, 2015) and those resulting from human activity (e.g. pipe systems in water supply networks – Kowalski et al., 2014, 2015, Iwanek et al. 2020), it is often possible to use the fractal geometry. According to the definition given by Mandelbrot (1982), a fractal is an object composed of parts similar at least approximately to the whole object. This definition describes the basic property of fractals, which is self-similarity. The Mandelbrot's method of describing fractals based on self-similarity, although relatively frequent in the literature (e.g. Oleschko, 2000, Khabbazi et al., 2015), is not a strict definition of fractals. One of the existing methods of defining fractals in the mathematical sense is the use of iterated function system (IFS) (Barnsley et al., 2005, Martyn, 2011). Fractals can be constructed using a system of iterated functions the form of the set $\{f_1,...,f_k\}$, where $f_i : X \rightarrow X$ is a contraction for $i = 1,..., k,$ while X – a closed subset of n-dimensional Euclidean space ^{n}. IFS in the above-mentioned form determines the Hutchinson operator (1981), defined on a compact and non-empty subset S of set X as (Barnsley, 2012):

$$H(S) = \bigcup_{i=1}^{n} f_i\,(S) \tag{1}$$

On the basis of the Banach fixed-point theorem (e.g. Palais, 2007), the iteration of the Hutchinson operator converges to the unique attractor AT (an attractor is a set to which the trajectories starting in different regions of the space go). The attractor AT can be described for the IFS system using a recursive sequence of the form:

$$\begin{cases} H_0(S) = S \\ H_k(S) = H\big(H_{k-1}(S)\big), k \geq 1 \end{cases} \tag{2}$$

The attractor AT is the limit of the sequence above, which can be written as:

$$AT = \lim_{k \to \infty} H_k\,(S) \tag{3}$$

The attractor described in this way is often a fractal, but it can also be an object without fractal features, called an IFS attractor (Barnsley et al., 2005, Gdawiec and Kotarski, 2008, Martyn, 2011). The fractals described as attractors are deterministic fractals, characterized by strict self-similarity. The process of their construction, consisting in repeating the same actions based on a strictly developed algorithm, is carried out infinitely (Falconer, 2014). Many fractals have some degree of self-similarity, i.e. they are made up of parts that resemble the whole, but the geometrical similarity is not strict. Such fractals (sometimes called probabilistic – e.g. Nowak, 1992, or random – e.g. Ratajczak, 1998) are characterized by approximate or statistical self-similarity (Falconer, 2014, Hassan and Kurths, 2002, Barnsley et al., 2005). Statistical self-similarity mainly concerns the fractals occurring in nature – e.g. mountain ranges, river systems, clouds, tree branches or blood vessel systems. The process of constructing a fractal reflecting a natural object cannot be carried out indefinitely, there is a lower and an upper limit (Pfeifer, 1984), and the addition of subsequent elements of the set is random (Nowak, 1992).

Due to the large variety of sets commonly considered as fractals, with the simultaneous lack of a single mathematical definition of fractals, some common features have been distinguished that can be used to assess the fractal nature of objects. In addition to the afore-mentioned self-similarity (feature No. 1), fractals are characterized by (Kudrewicz, 2007, Kowalski, 2010, Falconer, 2014):

- non-trivial (complicated) structure at every scale (feature no. 2),
- recursive construction procedure – the same steps are repeated in subsequent steps (feature no.3),
- necessity to use recursive relationships in the analytical description (feature No. 4),
- difficulty of description with the use of the terms of classical geometry (feature No. 5),
- difficulty of geometric description of the constituent parts of a set – almost every infinitely small element consists of a large number of other elements separated by spaces of variable dimensions (feature No. 6).

It should be emphasized that not every fractal has all of the above-mentioned features. It is enough for a set to meet most of the conditions to be considered as a fractal. This is especially true of probabilistic fractals with statistical self-similarity (Falconer, 2014). The aim of this article was to evaluate the possibility of using fractal geometry elements to describe the location of the suffusion holes formed after the failure of the buried water pipe. This description is needed to determine the dimensions of the previously defined protection zone, important for the safe use of water supply systems.

RESEARCH METHODOLOGY

To achieve the goal of the work, it was necessary to:

1) Create geometric structures consisting of suffosion holes formed on the soil surface after the failure of the buried water pipe, hereinafter referred to as the suffosion hole sets,
2) Carry out an analysis of the properties and the creation of the suffosion hole sets – checking whether these sets meet the requirements for fractal sets.

In order to collect the data for analysis – to create the suffosion hole sets – experimental studies were carried out involving the physical simulation of a water pipe failure. Simulations of the outflow of water from the water pipe to the soil were carried out on a laboratory setup in the scale of 1:10. The construction of the setup was preceded by a dimensional analysis. The main element of the laboratory setup was a water supply pipe with an internal diameter of 20 mm, consisting of two equal length parts connected by a bell-and-spigot joint, laid in the compacted sandy soil in the box. The pipe was supplied with water from a reservoir, the height of which governed the water pressure in the pipe. During the flow of water, the bell-and-spigot joint on the pipe was unsealed. This resulted in a leak between the spigot end and the socket end with an area of 9.42 cm^2, through which water flowed from the pipe into the soil and then onto the soil surface, creating suffosion holes. The remaining water that flowed through the pipe was disposed into the sewage system. The parameters of the soil used in the tests (degree of compaction, volumetric water content, saturated conductivity and particle-size distribution) were determined in the laboratory using standard procedures (PN-B-04481:1988).

Physical simulations of the water pipe failure were carried out for 7 hydraulic pressure heights *H* in the pipe: from 3.0 m H_2O to 6.0 m H_2O, every 0.5 m H_2O. The pressure was the only parameter that was varied during the tests. To determine the location of suffosion holes, the Cartesian coordinate system was used. It was assumed that the coordinate system is on the soil surface, its origin is directly above the leak in the pipe, and the horizontal axis is parallel to the pipe. After each experiment, the number of the suffosion holes (n), their belongingness to a quadrant of the coordinate system (q) and the distance between the farthest point of a suffosion hole from the origin (r) were determined. The suffusion holes obtained as a result of the laboratory tests created 7 sets ($A \div G$) – each for a different amount of hydraulic pressure in the pipe. Each of the 7 suffosion hole sets was created gradually. In consecutive steps, corresponding to successive repetitions of the experiment, next points were added to the set. The number of steps was limited – the experiment was repeated 7 times for each pressure level. The number of repetitions of the experiment for a given *H* value has been statistically determined as the minimum sample size for this experiment (Tucker, 2014). As a result, elements of each of the sets were suffosion holes created in all repetitions of the experiment for a given pressure height in the pipe. The scheme of the formation of the suffosion holes set is shown in Figure 1 on the example of the set *E* (H = 5 m H_2O).

The course of each replication of the experiment for all *H* values was documented (photos and video) using 2 cameras. One of them was placed 1.5 m above the laboratory setup, directly above the leakage site. The second one was not fixed; it was used to take pictures from various places and distances. Using the documentation, the process of formation and construction of each of the suffosion hole sets was analyzed frame by frame. Moreover, it was evaluated how many holes are formed in each repetition of the experiment and what number of holes occurs most often. Using the Student's t-test at the significance level of 0.05, the hypothesis that the number of created holes is always equal to the number of holes most frequently occurring in the conducted experimental tests was verified.

RESULTS AND DISCUSSION

The characteristics of the suffosion holes sets obtained in the laboratory tests are presented in Table 1.

The sets presented in Table 1 differ in the number of suffosion holes *n* (from 9 for sets *E* and *F* to 18 for set *C*) and the distance of the holes from the coordinate system origin *r* (from 7.47 cm to 48.90 cm). All the holes in all sets occurred

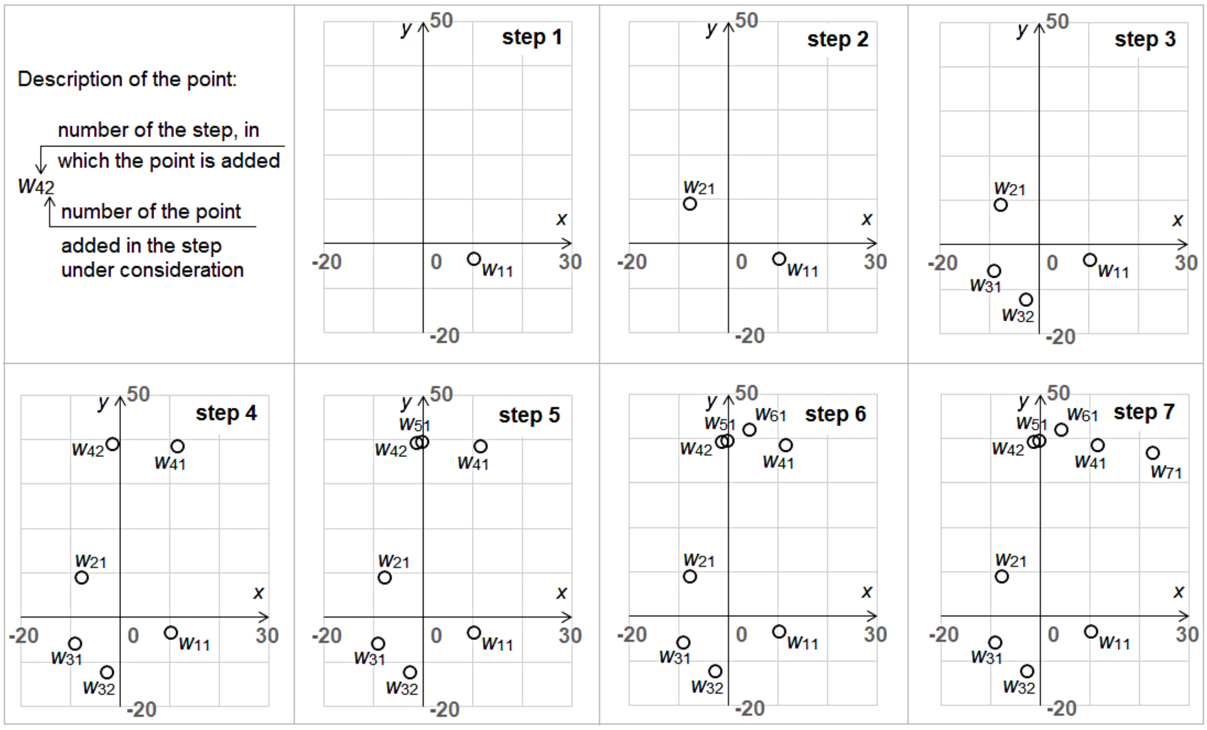

Figure 1. Scheme of the formation of the set E for $H = 5$ m H_2O; the suffosion holes are represented by the points belonging to the hole, located the farthest from the coordinate system origin

at a distance not exceeding 0.5 m from the place of leakage on the pipe (the coordinate system origin). The probability of water outflow onto the surface more than 0.5 m (i.e. under real conditions more than 5 m) from the leakage place can therefore be considered equal to zero for the test conditions. The highest difference in the distance r within one set exceeded 30 cm (from 32.63 cm to 39.57 cm) for sets $A \div F$ (Table 1). Only in the case of set G, the suffosion holes were located closer to the leakage place and the highest difference in the distance r was 5.17 cm. In each of the seven sets $A \div G$, the suffosion holes were located in all four quadrants of the coordinate system. It

Table 1. Basic data on the suffosion hole sets (n – number of created ceiling openings, r – distance of the suffosion holes from the coordinate system origin, q – quadrant of the coordinate system)

File number	H m H_2O	Data	Number of experiment repetition						
			1.	2.	3.	4.	5.	6.	7.
A	3.0	*n*	1	2	2	1	1	2	3
		r, cm	47.30	44.64; 48.90	16.43; 16.45	37.30	16.22	38.91; 32.53	36.27; 38.37; 34.10
		q	II	II; II	IV; IV	I	III	I; I	I; I; II
B	3.5	*n*	3	1	1	2	1	1	1
		r, cm	41.51; 43.32; 43.40	39.28	19.81	40.75; 41.52;	10.48	27.86	10.01
		q	I; IV; IV	III	III	IV; IV	III	II	II
C	4.0	*n*	1	1	2	3	4	5	2
		r, cm	31.39	8.27	11.72; 9.28	44.30; 46.30; 41.83	44.53; 46.48; 47.12; 45.09	38.42; 39.42; 39.34; 44.99; 41.08	33.18; 29.02
		q	III	II	I; IV	III; IV; IV	I; IV; IV; IV	II; II; III; IV; IV	III; III

Table 1. Cont. Basic data on the suffosion hole sets (n – number of created ceiling openings, r – distance of the suffosion holes from the coordinate system origin, q – quadrant of the coordinate system)

D	4.5	*n*	1	3	3	1	2	1	1
		r, cm	30.67	30.68; 28.00; 39.94	47.04; 45.53; 44.36	35.56	42.38; 41.08	38.99	7.47
		q	III	III; III; III	III; I; I	IV	III; I	II	IV
E	5.0	*n*	1	1	2	2	1	1	1
		r, cm	10.60	12.15	10.84; 12.56	40.20; 39.46	39.67	42.40	43.23
		q	IV	II	III; III	I; II	II	I	I
F	5.5	*n*	1	1	2	1	1	2	1
		r, cm	40.54	27.00	16.46; 15.75	22.31	31.98	46.83; 44.09	10.24
		q	I	IV	III; III	III	II	III; III	IV
G	6.0	*n*	2	2	1	1	3	1	1
		r, cm	8.47; 12.53	10.64; 11.54	10.59	10.60	11.89; 13.64; 8.62	9.74	10.30
		q	IV; II	I; IV	III	III	III; I; IV	II	III

follows that the location of suffosion holes is not obvious. The structure composed of the suffosion holes is therefore a non-trivial object, which is one of the characteristics of fractals (a feature marked in the Introduction section as feature No. 2). According to the research methodology, the suffosion hole sets were constructed gradually in subsequent steps (Fig. 1). The same actions were repeated in each step. Thus, the analyzed structure was subject to a recursive construction procedure, which is another feature of fractals (feature No. 3). The process of formation of the analyzed geometric structure, a special case of which is shown in Figure 1, can be written in a general form, using the formula (4):

$$\begin{cases} W_1 = \bigcup_{i=1}^{s_1} w_{1i}\,, s_1 \in N \\ W_n = W_{n-1} \cup \bigcup_{j=1}^{s_n} w_{nj}\,, s_n \in N, n \in N \text{ and } n \geq 2 \end{cases} \quad (4)$$

where: W_1, W_n– a set of suffosion holes obtained in the first and n -th step of the structure formation, respectively, w_{1i}– one of the points (i-th) forming the structure in the first step, w_{nj} – one of the points (j-th) added to the structure in the last n-th step, n – number of steps corresponding to the number of the experiments repetitions, s_1, s_n – number of all suffosion holes created in the first and n-th step of the structure formation, respectively.

The formula (4) is a recursive relationship. Therefore, the structure created by a suffosion hole set has another feature of fractals – feature No. 4. A geometric figure that is a subset of the Euclidean space, after adopting a coordinate system, can be described by a system of classical equations or inequalities connecting the coordinates of the points of the figure (Empacher et al., 1975). The analyzed structure – the suffosion hole set – cannot be described in the above way, due to the randomness of the location of the suffosion holes, improved previously (Iwanek, 2021), and also due to the fact that the number of holes obtained in each repetition of the experiment was not obvious. During the tests, 1 hole was obtained in 28 repetitions of the experiment, 2 holes in 13 repetitions, 3 holes in 6 repetitions, 4 holes in 1 repetition and 5 holes also in 1 repetition. Although the probability of obtaining 1 hole exceeds 50%, it is too low to assume at a significance level of 0.05 that 1 hole occurs in each repetition of the experiment (null hypothesis: $n = 1$). The calculations carried out with the use of the Student's ttest showed that the hypothesis $n = 1$ should be rejected (Table 2). Thus, the structure created by the suffusion hole set has one more feature of fractals – the difficulty of description using the concepts of classical mathematics (feature No. 5).

While analyzing the videos recorded in the laboratory showing the moment of water outflow on the soil surface, it was noticeable that

Table 2. Evaluation of the statistical hypothesis $n = 1$

Mean n	Expected n	t-value	Critical region CR	Result
1.65	1	4.89	<2.011, + ∞)	t-value $\in CR$

– contrary to what the human eye could see – the water did not flow out on the soil surface simultaneously through the entire suffosion hole, but in some places of the hole it appeared first, creating a "mini-hole". After a while equal to fractions of a second, another "mini-hole" appeared next to the previous one. The "mini-holes" became components of the final form of the suffosion hole (Fig. 2). In this way, after zooming in on the image, instead of one suffosion hole, usually several (always at least one) smaller component holes were visible. It would be possible to zoom in on the image until the water flowed out through the open soil pores and a moment later overcame the resistance of the solid particles, creating a "mini-hole". Thus, it can be said that almost every very small element of the analyzed structure consists of a number of other elements separated by spaces with variable dimensions. This is approximately the feature 6 of the fractals. The approximation is due to the fact that the mentioned elements are not smaller than the pore diameter of the soil, so they can be very small, but not infinitely small. The limitation of the size of the elements is the feature of probabilistic fractals. Moreover, after enlarging a fragment of a suffusion holes set, it is possible not only to see smaller and smaller elements of this set, but also to obtain an image similar to the whole. This means that the analyzed structure is characterized by self-similarity, which is considered to be the basic feature of fractals (feature No. 1). However, this is not a strict geometric similarity, but an approximate one – characteristic for probabilistic fractals (as in the case of the feature No. 6). The features of probabilistic fractals are revealed in the analyzed structure not only in the fact that its self-similarity is approximate and its components are not infinitely small. Characteristic for probabilistic fractals is also the fact that during the process of the structure construction, the number of added suffosion holes is not the same in each repetition of the experiment and the location of the added holes is not precisely determined. Moreover, the number of consecutive steps in the process of the structure construction corresponds to the number of repetitions of the experiment in the laboratory, and therefore is limited, which is also a feature of probabilistic fractals.

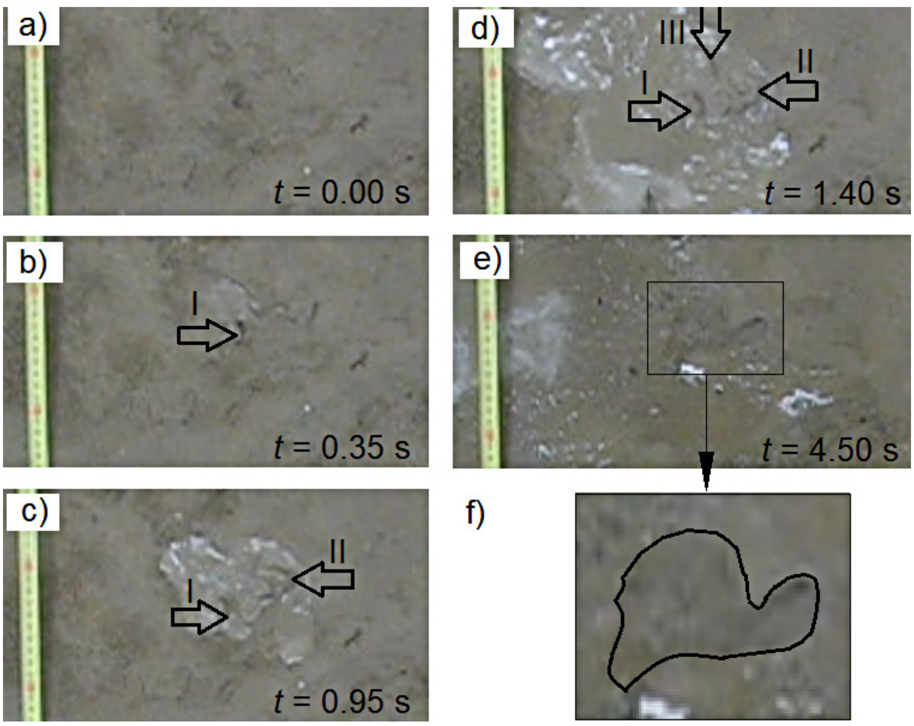

Figure 2. a)-e) Frames of the selected video of the outflow of water onto the soil surface recorded during laboratory tests; arrows show "mini-holes"; f) Zoom in on the formed suffosion hole (2×)

CONCLUSIONS

Conducting an analysis of the suffosion hole set formation process, determination of the significance of a random distribution of suffosion holes, as well as analysis of photographic and video documentation led to the conclusion that the geometric structure formed by a suffusion holes set created after a physical simulation of a water pipe failure meets the conditions for fractals. It has the basic characteristic of fractals, which is self-similarity, and moreover, it is created on the basis of a recursive construction procedure, requires the use of recursive relationships in the analytical description, has a non-trivial structure, cannot be described by the concepts of classical geometry and each of its elements consists of other smaller elements separated by spaces with variable dimensions. Since the addition of subsequent elements of the set is random, the construction process is not carried out indefinitely, the components are not infinitely small, and the self-similarity is approximate, the structure has the features of probabilistic fractals.

Demonstrating that a suffosion hole set creates a structure with fractal features allows the use of fractal properties to determine the protection zone – the area around a potential leak, where it would be possible for water to flow in the ground in the event of damage or failure of the water pipe. It is a very important issue in terms of the safe operation of water supply systems.

REFERENCES

1. Barnsley M.F. 2012. Fractals Everywhere. Dover Publications Inc., New York.
2. Barnsley M., Hutchinson J., Stenflo Ö. 2005. A fractal valued random iteration algorithm and fractal hierarchy. Fractals, 13(02), 111–146.
3. Empacher A.B., Sęp Z., Żakowska A., Żakowski W. 1975. Short Dictionary of Mathematics. Wiedza Powszechna, Warsaw. (in Polish)
4. Falconer K.J. 2014. Fractal Geometry: Mathematical Foundations and Applications. Wiley Academic.
5. Gdawiec K., Kotarski W. 2008. Fractal recognition of two-dimensional objects. In: Decision support systems. Institute of Computer Science of the University of Silesia, Katowice, 261–268. (in Polish)
6. Ghanbarian B., Hunt A.G., Ewing R.P., Sahimi M. 2013. Tortuosity in porous media: a critical review. Soil Science Society of America Journal, 77(5), 1461–1477.
7. Hassan M.K., Kurths J. 2002. Can randomness alone tune the fractal dimension? Physica A, 315, 342–352.
8. Hutchinson J.E. 1981. Fractals and self-similarity. Indiana Univ. Math. J., 30(5), 713–747.
9. Iwanek M. 2021. Application of Ripley's K-function in research on protection of underground infrastructure against selected effects of suffosion. International Journal of Conservation Science, 12, 827–834.
10. Iwanek M., Kowalski D., Kowalska B., Suchorab P. 2020. Fractal geometry in designing and operating water networks. Journal of Ecological Engineering, 21(6), 229–236.
11. Iwanek M., Suchorab P., Sidorowicz Ł. 2019. Analysis of the width of protection zone near a water supply network. Architecture - Civil Engineering – Environment, 12(1), 123–128.
12. Khabbazi A.E., Hinebaugh J., Bazylak A. 2015. Analytical tortuosity-porosity correlations for Sierpinski carpet fractal geometries. Chaos, Solitons & Fractals, 78, 124–133.
13. Khomenko V.P. 2009. Suffosion hazard: today's and tomorrow's problem for cities. In: Culshaw M.G., Reeves H.J., Jefferson I. Spink T.W. (eds) Engineering geology for tomorrow's cities. Geological Society, Engineering Geology Special Publication, London.
14. Kowalski D., Kowalska B., Kwietniewski M. 2015. Monitoring of water distribution system effectiveness using fractal geometry. Bulletin of the Polish Academy of Sciences: Technical Sciences, 63(1), 155–161.
15. Kowalski D., Kowalska B., Suchorab P. 2014. A proposal for the application of fractal geometry in describing the geometrical structures of water supply networks. In: Brebbia C.A., Mambretti S. (eds) WIT Transactions on The Built Environment, 139. Urban Water II, Southampton, Boston, UK: WIT Press, 75–90.
16. Kowalski, D. 2010. Is the water network a fractal? Gaz, Woda i Technika Sanitarna, 3, 29–33. (in Polish)
17. Kudrewicz J. 2015. Fractals and Chaos. WNT, Warsaw.
18. Kwietniewski M., Rak J.R. 2010. Reliability of water supply and sewage infrastructure in Poland. Committee of Civil and Water Engineering of the Polish Academy of Sciences. (in Polish)
19. Mandelbrot B.B. 1982. The Fractal Geometry of Nature. WH Freeman and Co., New York.
20. Martyn T. 2011. Geometric algorithms in the fractal visualization of iterated mapping systems. Scientific Works of the Warsaw University of Technology. Electronics, 178. (in Polish)
21. Nowak P. 1992. Fractal theory – a new way of describing geometrically irregular objects. Physicochemical problems of mineral processing, 25, 13–24. (in Polish)
22. Oleschko K., Figueroa B.S., Miranda M.E., Vuelvas M.A., Solleiro E.R. 2000. Mass fractal dimensions

and some selected physical properties of contrasting soils and sediments of Mexico. Soil and Tillage Research, 55(1–2), 43–61.

23. Palais R.S. 2007. A simple proof of the Banach contraction principle. J. Fixed Point Theory Appl., 2(2), 221–223.
24. Pfeifer P. 1984. Fractal dimension as working tool for surface-roughness problems Fractal dimensions as working tool for surface-roughness problems. Applications of Surface Science, 18(1–2), 146–164.
25. PN-B-04481:1988. Building soils. Laboratory tests (Polish Standard; in Polish).
26. Ratajczak W. 1998. Methodological aspects of fractal modeling of reality. Adam Mickiewicz University, Poznań. (in Polish)
27. Suchorab P., Kowalska B., Kowalski D. 2016. Numerical investigations of water outflow after the water pipe breakage. Annual Set The Environment Protection, 18(2), 416–427.
28. Tucker H.G. 2014. An introduction to probability and mathematical statistics. Academic Press.
29. Xu P. 2015. A discussion on fractal models for transport physics of porous media. Fractals, 23(3), 1530001.