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Finite Element Analysis of Dynamic Properties of Thermally Optimal Two-phase Composite Structure

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Abstract

This paper presents modelling and a FEM analysis of dynamic properties of a thermally optimal two-phase composite structure. Simulations were provided for 2D models. At the first step, topology optimization was performed, where an internal energy was minimized. At the second step, analysis of dynamic properties was executed. Calculations allowed to determine eigenfrequencies and the mode shape of the structure. Solid isotropic material with penalization (SIMP) model was used to find the optimal solution. The optimization algorithm was based on SNOPT method and Finite Element Method.

Keywords: topology optimization, SIMP model, SNOPT, internal energy

1. Introduction

Determination of natural frequencies and mode shapes of the structure are usual first steps in performing a dynamic analysis. It is caused by the fact that these factors show how the structure will respond to dynamic load.

Natural frequencies of a structure are the frequencies at which system tends to oscillate without the absence of damping or driving force, whereas the mode shape (normal mode of vibration) is a deformed shape of the structure which appears at a specific natural frequency of vibration. Natural frequencies and mode shapes are functions of boundary conditions and structural properties.

There are many reasons why the analysis of dynamic properties is executed. One of them is to determine the dynamic interaction between a component and a structure to which it is attached, like for example, an air conditioner installed on the roof of a building. In this case, it is essential to check if the operating frequency of the rotating fan is not too close to the eigenfrequency of the building. Another example is comparing the results of the analysis with results obtained in real tests. Thereby, the results of the analysis can support the experiment, e.g. showing areas where accelerometers should be placed. Determination of eigenfrequencies and mode shapes is also used in the design process. It is necessary to check the influence of particular design changes of the structure on the dynamic parameters.

There are many examples of analysis of dynamic properties in literature. Paper [1] presents an investigation into the frequency dependant viscoelastic dynamics of a multifunctional composite structure from a finite element analysis and experimental validation. After model parameter identification, a numerical simulations were carried out. Thereby, the damping behaviour of first two vibrations modes was predicted. At the next step, the numerical results were validated by the experimental tests on the layered composite beam.

The dynamic problem of reinforced concrete slabs stiffened by steel beams with deformable connection including creep and shrinkage effect is considered in [2]. In the papers, authors took into consideration the in-plane forces and deformations of the plate in the adopted models and also the axial forces and deformations of the beam. The mode shapes and eigenfrequencies of the stiffened concrete slab were determined.

In papers [3], a four types of integral finite elements were developed and used to estimate the dynamical characteristics of elastic-viscoelastic composite structures. The composite structures were sandwich beam, plate and shell structures with viscoelastic materials as core layers. The results from the direct frequency response method and experiment were compared to the results of the integral finite element prediction, which revealed that integral finite elements are passable regarding to engineering applications.

An analysis of the dynamic properties of multiple damping layer, laminated composite beams with anisotropic stiffness layers was investigated in [4]. For this purpose the finite-element-based modal strain energy method was used. In this study the variation of resonance frequencies and modal loss factors of various beam samples with temperature were analyzed.

The dynamic behaviour of fibre reinforced plastic sandwich plates with PCV foam core was considered in [5]. The equations of motion, which were obtained by authors, are used to perform steady state analysis and to determine the natural frequencies and modal loss factors of specific composite sandwich plates.

Study [6] is intended to analyze the damping of PVC foams under flexural vibrations of clamped free beams. A finite element analysis based on the sandwich theory was used to model the natural frequencies and the damping of the beams. Authors took into account the numerical and experimental results to derive the shear modulus and the damping of PVC foams as functions of the frequency.

Papers [7] were devoted to examination of the viscoelastic damping model of the cylindrical hybrid panels with co-cured, free and constrained layers. For this purpose, the refined finite element method based on the layerwise shell theory was used. In this study, the damped natural frequencies, modal loss factors and frequency response functions of cylindrical viscoelastic hybrid panels were determined and compared with those of the base composite panel without a viscoelastic layer.

In [13] authors present computational analysis of sandwich-structured composites with an auxetic phase. The total energy strain is analyzed. In papers the application of SIMP model was used to find the optimal distribution of a given amount of materials in sandwich-structured composite. Authors also propose a multilayered composite structure in which internal layers surfaces are wavy.

2. Optimization of the average internal energy

The first step in presented effort was to optimize the average internal energy in considered two-phase structure. The average internal energy $\mathrm{Ei_{avg}}$ is calculated using equation 1:

$$Ei_{avg} = \frac{1}{A}\int_{\Omega} Ei\, d\Omega \tag{1}$$

Here Ω refers to the design domain, Ei is the internal energy and A is the area of the domain.
The objective function of the considered design optimization problem depends on the design variable $r = r(x)$ as follows:

$$Ei_{avg}(r) = \frac{1}{A}\int_{\Omega} Ei\,(r) d\Omega \tag{2}$$

The internal energy of the solid was calculated by the following equation:

$$Ei_{avg}(r) = c_p T(r) \tag{3}$$

where $\mathrm{c_p}$ is the heat capacity and $\mathrm{T(r)}$ is the temperature.
The temperature is calculated using Fourier's equation 4 (Fourier's law of steady state pure conduction) [12]:

$$-\nabla \cdot (k(r)\nabla T) = Q, \tag{4}$$

where $\mathrm{k(r)}$ is the thermal conductivity and Q is the heat source.

Using the Solid Isotropic Material with Penalization (SIMP) model in topology optimization in a two-phase structure [8], one can write the generalized thermal conductivity in the form of:

$$k(r) = k_1 + (k_2 - k_1) \cdot r^p,\ p > 1,\ k_1 < k_2 \tag{5}$$

Here r is a control variable (design variable), p is a penalty parameter, k_1 and k_2 are thermal conductivity values of the first and the second material respectively.

In the considered case, the control variable is related to thermal conductivity parameter of the isotropic material and is interpolated from 0 to 1, which corresponds to the first and the second material respectively, using penalty scheme which affects the material distribution. The value of the penalty parameter above 1 ensures that density values of 0 (first material) or 1 (second material) are favoured ahead of the intermediate values.

One can interpret a control (design) variable $r = r(x)$ as a generalized material density which satisfies the following constraints:

$$0 \le \int_{\Omega} r(x)\, d\Omega \le V,\ 0 \le r(x) \le 1, \tag{6}$$

where V is the second material's volume available for distribution.

In the considered structure, the optimal material distribution is found for a given objective and constraints by assigning each element an individual control variable value. For the purposes of this research, Sparse Nonlinear Optimizer (SNOPT) code was used. This gradient optimization algorithm was developed by P. E. Gill, W. Murray and M. A. Sunders [9]. In this method, the objective function can have any form and any constraints can be applied. SNOPT is suitable for large-scale linear and quadratic programming and for linearly constrained optimization, as well as for general nonlinear programs. This algorithm minimizes a linear or nonlinear function subject to bounds on the variables and sparse linear or nonlinear constraints.

3. Equation of motion of the solid

For the purpose of calculation the Navier's equation of motion was used which takes a form of [10]:

$$\rho\frac{\partial^2 \boldsymbol{u}}{\partial t^2} - \nabla\cdot\boldsymbol{\sigma} = 0 \tag{7}$$

where the force has been omitted, u is the displacement vector, ρ is the density, σ is the stress tensor and can be written as [11]:

$$\boldsymbol{\sigma} = \boldsymbol{D\varepsilon} = \lambda(\nabla\cdot\boldsymbol{u})\boldsymbol{I} + 2\mu\boldsymbol{\varepsilon}. \tag{8}$$

Here I is the identity matrix, $\varepsilon = \frac{1}{2}(\nabla\cdot \mathrm{u} + (\nabla\cdot \mathrm{u})^{\mathrm{T}})$ is the strain tensor, λ i μ are Lame parameters presented in the equation:

$$\mu = G = \frac{E}{2(1+\nu)}, \lambda = \frac{E\cdot\nu}{(1-2\nu)(1+\nu)} \tag{9}$$

where E – Young's modulus, G – shear modulus, ν – Poisson's ratio.

Using the aforementioned equation one can write Navier's equation of motion for isotropic solid for the linear constitutive relation between stresses and deformations [10] as:

$$\rho\frac{\partial^2 \boldsymbol{u}}{\partial t^2} - (\mu\nabla^2\boldsymbol{u} + (\lambda+\mu)\nabla\nabla\cdot\boldsymbol{u}) = \boldsymbol{0}. \tag{10}$$

A real harmonic displacement satisfies the equation:

$$\frac{\partial^2 \boldsymbol{u}}{\partial t^2} = -\omega^2\boldsymbol{u}. \tag{11}$$

where ω is the circular frequency with period $2\pi/\omega$.
The displacement vector can be written in the complex form of:

$$\boldsymbol{u}(x) = \boldsymbol{u}_1(x) + i\boldsymbol{u}_2(x). \tag{12}$$

Here the harmonic displacement became a real part of complex field:

$$\boldsymbol{u}(\boldsymbol{x},t) = \mathcal{R}e[\boldsymbol{u}(\boldsymbol{x})e^{-i\omega t}]. \tag{13}$$

Pursuant to the above equations the harmonic equation of motion satisfies formula:

$$-\rho\omega^2\boldsymbol{u} - (\mu\nabla^2\boldsymbol{u} + (\lambda+\mu)\nabla\nabla\cdot\boldsymbol{u}) = 0. \tag{14}$$

The foregoing equation can be viewed as an eigenvalue equation for the operator $\mu\delta_{ij}\nabla^2 + (\lambda+\mu)\nabla_i\nabla_j$ with eigenfunction u(x) and eigenvalue in the form of $-\rho\omega^2$ [10].

4. Numerical results

This section presents an analysis of dynamic properties for a two-phase structure whose topology was optimized. The considered model consists of steel and polyurethane foam. The thermal and mechanical properties are presented in Table 1.

As it was mentioned above, the first step was to minimize the average internal energy. For this purpose, a 2D model, with applied boundary conditions, was prepared (see

Figure 1). A fraction of the domain to use for the distribution of the second material is equal to A_{frac} and took the value of 0.4.

Table 1. Thermal and mechanical properties of the model

Property	Material 1– Polyurethane foam	Material 2 – Steel
E [Pa]	4e9	2e11
ν	0.4	0.25
ρ [kg/m^3]	50	8000
k [W/mK]	0.03	58

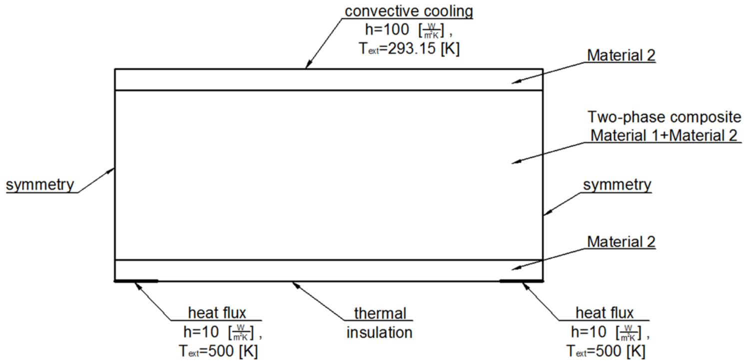

Figure 1. Boundary conditions for topology optimization

During the optimization process, a distribution of the control variable was obtained, as it is presented in Figure 2. In the figure below, value 1 is assigned to material 2 (white colour) with higher thermal conductivity, and value 0 is assigned to material 1 (black colour) [14].

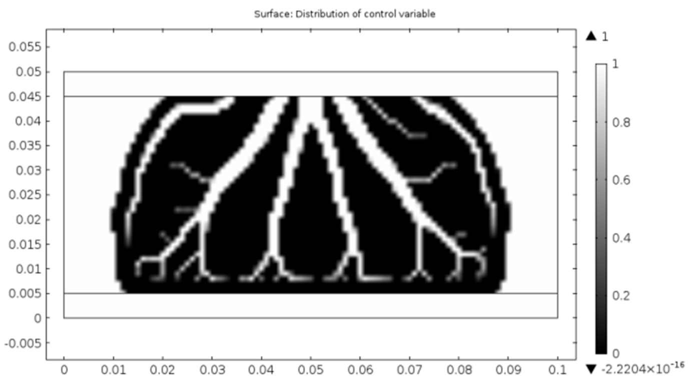

Figure 2. Distribution of control variable for A_{frac}=0.4

In Figure 3, boundary conditions for analysis of dynamic properties are presented. At the top of the model, boundary load $F(t)$ was applied, additionally the model was fixed on two sections at the bottom boundary.

At the second step of calculations, six eigenfrequencies were determined. The values of the eigenfrequencies are presented in Table 2. In Figures 4–9, the amplitude of the forced vibration and mode shapes are presented for each eigenfrequency.

Table 2. Determined eigenfreqiencies

No.	1	2	3	4	5	6
Value of eigenfrequency [Hz]	5502.93	7712.40	14618.93	15047.05	17713.10	19548.60

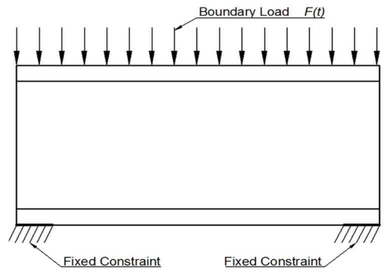

Figure 3. Boundary conditions for analysis of dynamic properties

The boundary load can be written as:

$$F(t) = F_{load}\sin(\omega t), \tag{15}$$

where $F_{load} = 10000\ [N/m^2]$.

For the purpose of the analysis of dynamic properties Young modulus, Poisson's ratio and material density were written in the form of interpolation scheme SIMP for isotropic materials, as it is presented in formulas (16), (17) and (18).

$$E(r) = E_1 + (E_2 - E_1) \cdot r^p,\ p > 1\,,\ \ E_1 < E_2 \tag{16}$$

$$\nu(r) = \nu_1 + (\nu_2 - \nu_1) \cdot r^p,\ p > 1\,,\ \ \nu_1 > \nu_2 \tag{17}$$

$$\rho(r) = \rho_1 + (\rho_2 - \rho_1) \cdot r^p,\ p > 1\,,\ \ \rho_1 < \rho_2 \tag{18}$$

where: E_1 and E_2 are Young's moduli, ν_1 and ν_2 are Poisson's ratios, ρ_1 and ρ_2 are densities for the first and the second material respectively.

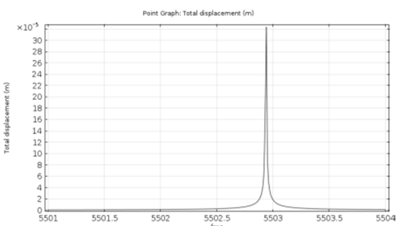

Figure 4. a)The amplitude of the forced vibration

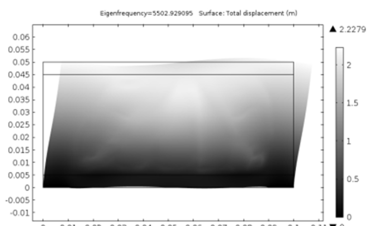

Figure 4. b) Mode shape for the first eigenvalue

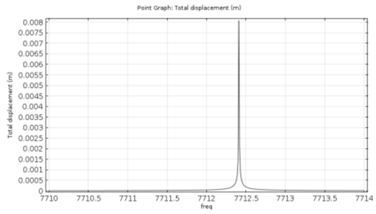

Figure 5. a) The amplitude of the forced vibration

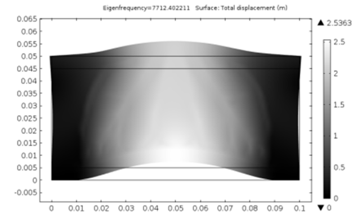

Figure 5. b) Mode shape for the second eigenvalue

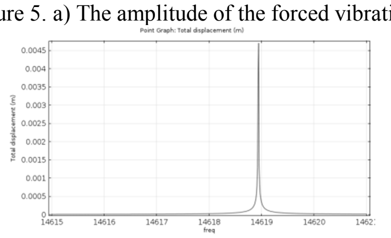

Figure 6. a) The amplitude of the forced vibration

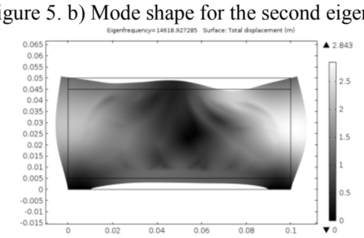

Figure 6. b) Mode shape for the third eigenvalue

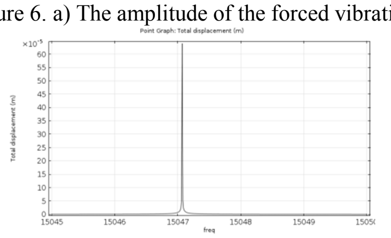

Figure 7. a) The amplitude of the forced vibration

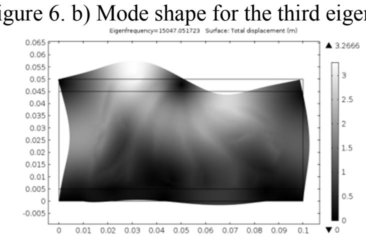

Figure 7. b) Mode shape for the fourth eigenvalue

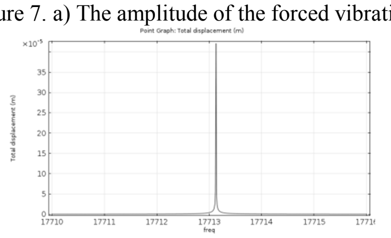

Figure 8. a) The amplitude of the forced vibration

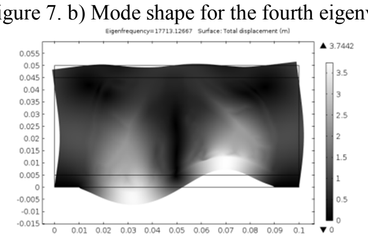

Figure 8. b) Mode shape for the fifth eigenvalue

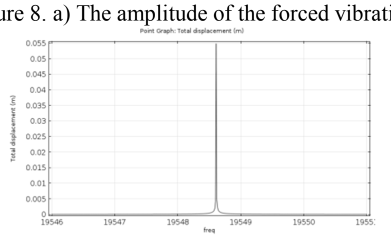

Figure 9. a) The amplitude of the forced vibration

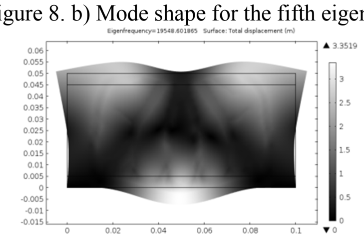

Figure 9. b) mode shape for the sixth eigenvalue

5. Conclusions

The paper presents an analysis of the dynamic properties of a thermally optimal two-phase structure. In the first stage of the calculations, a 2D model of a two-phase structure was optimized. Thereby, the minimum average internal energy was achieved. At the second step, six eigenfrequencies were determined for the model with the optimal topology. Subsequently, the amplitude of the forced vibration and mode shapes are presented for each eigenfrequency.

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