

## Modelling of a Rotor with Gyroscopic Coupling in a Rotating Reference Frame Using Ansys and Matlab

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### Abstract

The work presents a method that expands Ansys capabilities and allows for studying the dynamics of rotors with significant gyroscopic coupling in reference to rotating coordinates. The analyses of rotors in the rotating reference frame are less common, and in FEM software some limitations in such cases can be observed. There are specialised rotors for which the use of a non-inertial reference frame is inevitable. The current work proposes a solution. In the presented method, Ansys is used to generate and export systems matrices so that the manual creation of an FE model is unnecessary. The rest of the operations are realised in Matlab. A simple numerical example is presented to clarify all the steps and a more complex example concludes the paper.

**Keywords:** rotor dynamics, FEM, Ansys, Matlab, gyroscopic coupling, rotating reference frame

### 1. Introduction

The dynamics of rotating machines incorporates a wide range of knowledge with many phenomena that often have a non-intuitive nature. There are many programs that support engineers in the design of such machines. Besides the general-purpose FEM software (Ansys, Nastran), one can find many specialised programs that are designed to cope only with rotor problems (XLRotor, DynRot, MESWIR). In general, in numerical models of rotors, the approach that the system motion is analysed with reference to the inertial frame is dominant. Such results are consistent with real-life observations. However, there are less common cases where the analysis in a rotating reference frame is inevitable.

The current work is motivated by an attempt to analyse the structure that has rotor-mounted sensors and actuators. For such a structure, the use of the rotating reference frame is necessary and in Ansys (as available option), when one performs such an analysis, the gyroscopic coupling is neglected (a short summary of all capabilities can be found in the Ansys manual [1] or in ref. [2]). In the following work, the author proposes a method that completes the FE model so that the analysis can be correctly performed. A similar approach can be found in the author's earlier works [3, 4], however the effect of gyroscopic coupling was not considered.

In general, the presented models use a linear approximation of dynamics of gyroscopic systems and are valid for small deflections.

### 2. The studied geometry and analytical considerations

The work analyses the dynamics of a Stodola-Green rotor, as shown in Fig. 1. The structure consists of an elastic shaft, which at its end has a symmetric and rigid disc.

The bearing is idealised. The rotor rotates with the constant rotational speed  $\Omega$ . In particular, the work considers a shaft with a square cross-section, which can be treated as of circular type [5]. Two sets of coordinates are introduced. The  $xyz$  axes represent an inertial or stationary reference frame (SRF). The  $\xi\eta\zeta$  axes rotate with the shaft and will be referred to as a rotating reference frame (RRF). The equations of motion of the structure with respect to stationary reference frame (SRF) are as follows [5]:

$$\begin{bmatrix} m & 0 & 0 & 0 \\ 0 & m & 0 & 0 \\ 0 & 0 & J_t & 0 \\ 0 & 0 & 0 & J_p \end{bmatrix} \begin{Bmatrix} \ddot{x}_c \\ \ddot{y}_c \\ \ddot{\varphi}_x \\ \ddot{\varphi}_y \end{Bmatrix} + \Omega \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & J_p \\ 0 & 0 & -J_p & 0 \end{bmatrix} \begin{Bmatrix} \dot{x}_c \\ \dot{y}_c \\ \dot{\varphi}_x \\ \dot{\varphi}_y \end{Bmatrix} + \begin{bmatrix} K_{11} & 0 & 0 & -K_{12} \\ 0 & K_{11} & K_{12} & 0 \\ 0 & K_{21} & K_{22} & 0 \\ -K_{21} & 0 & 0 & K_{22} \end{bmatrix} \begin{Bmatrix} x_c \\ y_c \\ \varphi_x \\ \varphi_y \end{Bmatrix} = \begin{Bmatrix} F_x \\ F_y \\ M_x \\ M_y \end{Bmatrix} \quad (1)$$

$$\mathbf{Mu} + \mathbf{Gu} + \mathbf{Ku} = \mathbf{w} \quad (2)$$

where:  $x_c, y_c$  are displacements and  $\varphi_x, \varphi_y$  are rotations of the disc in SRF,  $m$  – reduced mass,  $J_t$  – transverse mass moment of inertia,  $J_p$  – polar mass moment of inertia,  $K_{ij}$  – the stiffness of a shaft with respect to transversal and rotational deflections [5].

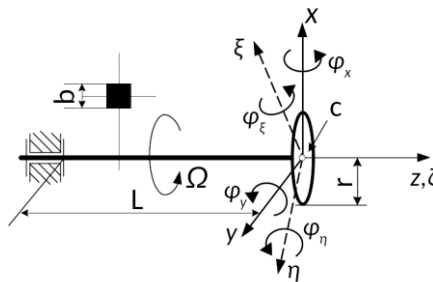


Figure 1. The Stodola-Green rotor with coordinate systems

In the literature one can find different forms of eq. (1), where the assumed direction of rotation about the  $x$  axis is the opposite [6] or where the equations are interlaced and the stiffness matrix has the opposite cross-coupling elements [7]. The opposite elements in the stiffness matrix are the result of a different coordinate system. The form of eq. 1 has been chosen because the directions of the rotations are adopted in the same way as in Ansys, using the right-hand screw rule (positive rotation  $\varphi_x$  will cause a right-hand screw to advance in  $x$  direction).

In a linear approximation of a motion of the disc (eqs. (1) and (2)), it is assumed that the inclination angles ( $\varphi_x, \varphi_y$ ) are small. Using this assumption the rotations can be treated as a generalised coordinates, and therefore a transformation to the rotating coordinate system is possible. Between the variables from stationary and rotating reference frames there is the following relation:

$$\begin{Bmatrix} x_c \\ y_c \\ \varphi_x \\ \varphi_y \end{Bmatrix} = \begin{bmatrix} \cos(\Omega t) & -\sin(\Omega t) & 0 & 0 \\ \sin(\Omega t) & \cos(\Omega t) & 0 & 0 \\ 0 & 0 & \cos(\Omega t) & -\sin(\Omega t) \\ 0 & 0 & \sin(\Omega t) & \cos(\Omega t) \end{bmatrix} \begin{Bmatrix} \xi_c \\ \eta_c \\ \varphi_\xi \\ \varphi_\eta \end{Bmatrix} \quad (3)$$

where:  $\xi_c, \eta_c$  are displacements and  $\varphi_\xi, \varphi_\eta$  are rotations in RRF.

The equations in a rotating frame of reference can be obtained by differentiating of the eq. (3) so as to get velocities and accelerations. The additional terms come from the fact that the time derivative with respect to the rotating coordinate system is calculated. The equations of motion in RRF are:

$$\begin{aligned} & \begin{bmatrix} m & 0 & 0 & 0 \\ 0 & m & 0 & 0 \\ 0 & 0 & J_t & 0 \\ 0 & 0 & 0 & J_p \end{bmatrix} \begin{Bmatrix} \ddot{\xi}_c \\ \ddot{\eta}_c \\ \ddot{\varphi}_\xi \\ \ddot{\varphi}_\eta \end{Bmatrix} + \left( \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & J_p \\ 0 & 0 & -J_p & 0 \end{bmatrix} + 2\Omega \begin{bmatrix} 0 & -m & 0 & 0 \\ m & 0 & 0 & 0 \\ 0 & 0 & 0 & -J_t \\ 0 & 0 & J_t & 0 \end{bmatrix} \right) \begin{Bmatrix} \dot{\xi}_c \\ \dot{\eta}_c \\ \dot{\varphi}_\xi \\ \dot{\varphi}_\eta \end{Bmatrix} \\ & + \left( \begin{bmatrix} K_{11} & 0 & 0 & -K_{12} \\ 0 & K_{11} & K_{12} & 0 \\ 0 & K_{21} & K_{22} & 0 \\ -K_{21} & 0 & 0 & K_{22} \end{bmatrix} - \Omega^2 \begin{bmatrix} m & 0 & 0 & 0 \\ 0 & m & 0 & 0 \\ 0 & 0 & J_t & 0 \\ 0 & 0 & 0 & J_p \end{bmatrix} + \Omega^2 \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & J_p & 0 \\ 0 & 0 & 0 & J_p \end{bmatrix} \right) \begin{Bmatrix} \xi_c \\ \eta_c \\ \varphi_\xi \\ \varphi_\eta \end{Bmatrix} = \begin{Bmatrix} F_\xi \\ F_\eta \\ M_\xi \\ M_\eta \end{Bmatrix} \end{aligned} \tag{4}$$

$$\mathbf{M}\ddot{\mathbf{v}} + (\mathbf{G} + 2\mathbf{MH})\dot{\mathbf{v}} + (\mathbf{K} - \Omega^2\mathbf{M} + \mathbf{GH})\mathbf{v} = \boldsymbol{\tau} \tag{5}$$

Comparing the eqs. (2) and (5) one can observe that only one additional matrix is needed to obtain equations in RRF. This is the so-called circulatory matrix  $\mathbf{H}$ , which is skew-symmetric and proportional to rotational speed (the same as the gyroscopic matrix  $\mathbf{G}$ ). The presented method proposes to determine the matrix  $\mathbf{H}$  utilising the Coriolis effect matrix ( $2\mathbf{MH}$ ). In Ansys, the Coriolis matrix is generated during analyses in RRF and its elements are added to the damping matrix.

In order to create a model in Ansys, it is assumed that shaft is made from aluminium, has a length of  $L = 0.7$  m and side length  $b = 15$  mm. The disc has a radius  $r = 70$  mm and mass  $m_d = 604.2$  g (which is equivalent to a steel disc with thickness equal to  $t = 5$  mm). The proposed dimensions are considered to be used in future experiments. The assumed constant rotational speed  $\Omega$  is 45 rad/s.

### 3. Ansys – modelling and exporting of system matrices

As a numerical example, the simplest possible FEM realisation of the proposed geometry is created. The shaft is modelled by one element of beam type and, as a disc, a point mass element with rotary inertia is used. When the free side of the shaft is fixed, the model has 6 DOFs. To obtain the same size of matrices as in the presented equations, two additional DOFs have been removed, i.e. displacement in  $z$  direction and rotation about  $z$  axis.

Before the exporting of system matrices in Ansys, all necessary steps that are typical for modal analysis should be performed (the assignment of elements, materials and constants, definition of boundary conditions). In order to export damping related matrices, the damped modal solver should be pre-set. As mentioned earlier, in Ansys the analysis of rotating systems can be done with respect to the stationary and rotating reference frames (*coriolis* command). A model of the gyroscopic system in the rotating reference frame can be created by making both analyses – the first one is realised to obtain mass, stiffness and gyroscopic matrices, and the second one focuses on a Coriolis

matrix. All needed matrices can be exported by using an assembled global matrix file (generated during solution) and *hbm* command. The generated files utilise the Harwell-Boeing format, which is efficient with sparse matrices. The Ansys manual [1] informs that in the case of a system with non-symmetric matrices (which includes all rotor dynamics problems), then the transpose of the matrices is written to files.

The physical meaning of the particular elements in the matrices can be found by using mapping files, which are optionally generated together with the matrix files. The mapping file links the row/column number with the node number and with one of the degrees of freedom that is available for a particular node. In other words, the mapping file shows for each DOF the corresponding row/column and node number.

In the analyses that prepare system matrices, it is advisable to avoid modelling of damping because it will add further elements to the gyroscopic or Coriolis matrices.

#### 4. Matlab – model preparation and solution

By default, Matlab does not recognise the Harwell-Boeing format, however software tools can be found that import the model data [8, 9]. Next, the mapping file is utilised to read the order of DOFs:  $[UX, UY, ROTX, ROTY]^T$  (the same as in eqs. (1), (3) and (4)). Six matrices are obtained because two types of analyses have been solved. The obtained matrices (after the transposition that restores its original shape) are presented below, where the first three are from SRF and the additional three from RRF (subscript R):

$$\mathbf{M} = \text{diag}[0.746, 0.746, 7.428e-4, 7.428e-4]$$

$$\mathbf{G} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0669 \\ 0 & 0 & -0.0669 & 0 \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} 5.35e4 & 0 & 0 & -1.87e4 \\ 0 & 5.35e4 & 1.87e4 & 0 \\ 0 & 1.87e4 & 6.976e3 & 0 \\ -1.87e4 & 0 & 0 & 6.976e3 \end{bmatrix}$$

$$\mathbf{M}_R = \mathbf{M}$$

$$\mathbf{Cor}_R = \begin{bmatrix} 0 & -67.14 & 0 & 0 \\ 67.14 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0669 \\ 0 & 0 & -0.0669 & 0 \end{bmatrix}, \quad \mathbf{K}_R = \begin{bmatrix} 5.20e4 & 0 & 0 & -1.87e4 \\ 0 & 5.20e4 & 1.87e4 & 0 \\ 0 & 1.87e4 & 6.976e3 & 0 \\ -1.87e4 & 0 & 0 & 6.976e3 \end{bmatrix}$$

The mass of the node in the mass matrix  $\mathbf{M}$  (and  $\mathbf{M}_R$ ) is bigger than the mass of the disc. The difference is due to the non-zero mass of the shaft, which to some extent adds to the mass of the disc. Comparing the matrix  $\mathbf{Cor}_R$  with the corresponding one in eq. (4), it can be observed that two elements have the opposite sign. The same effect was consequently observed for other models and always just the rotational degrees-of-freedom was involved. By transposing only the rows and columns that are linked with the rotational DOFs, the Coriolis matrix gets the same form as in eq. (4).

The circulatory matrix can be determined by using the following relation:

$$\mathbf{H} = \frac{1}{2} \mathbf{M}^{-1} \mathbf{Cor}_{RM} \tag{6}$$

where the  $\mathbf{Cor}_{RM}$  indicates that eventual partial transformation of  $\mathbf{Cor}_R$  was realised (as mentioned above). Substituting numeric values one can obtain:

$$\mathbf{H} = \begin{bmatrix} 0 & -45 & 0 & 0 \\ 45 & 0 & 0 & 0 \\ 0 & 0 & 0 & -45 \\ 0 & 0 & 45 & 0 \end{bmatrix}$$

which clearly shows its dependence on rotational speed.

Knowing all the necessary matrices, the rotor model can be recreated in Matlab using, for example, a space-state approach. The descriptor space-state model (*dss* command) can assume the following form:

$$\begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{G} + 2\mathbf{MH} & \mathbf{M} \end{bmatrix} \begin{Bmatrix} \dot{\chi}_1 \\ \dot{\chi}_2 \end{Bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{1} \\ -(\mathbf{K} - \Omega^2\mathbf{M} + \mathbf{GH}) & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \chi_1 \\ \chi_2 \end{Bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{1} \end{bmatrix} \boldsymbol{\tau} \tag{7}$$

$$\mathbf{v} = \begin{bmatrix} \mathbf{1} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \chi_1 \\ \chi_2 \end{Bmatrix} + \mathbf{0}\boldsymbol{\tau}$$

where  $\mathbf{1}$  and  $\mathbf{0}$  are, respectively, the identity and zeros matrices.

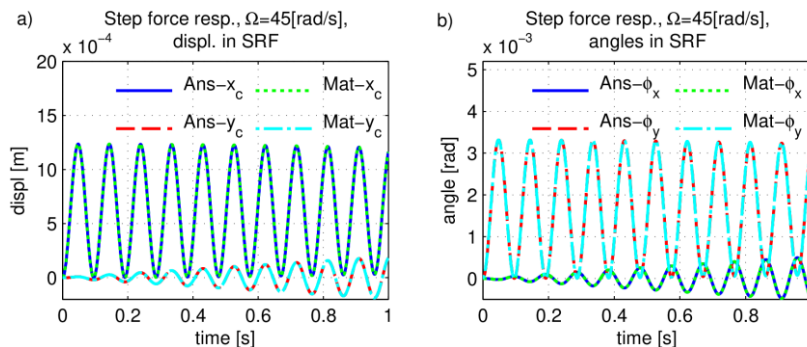


Figure 2. Comparison of results obtained in Ansys and in Matlab (w/o damping):  
a) displacements in SRF, b) angles in SRF

The method has been verified by comparing Matlab results with those obtained in Ansys. As excitation, a step force that acts in x direction (in SRF) was chosen. To excite the RRF referenced model in the same manner, the force has to be appropriately transformed. Two models have been simulated – the first one in Ansys in SRF (prefix Ans- in Fig. 2) and the second one in Matlab, with respect to RRF (prefix Mat- in Fig. 2). The results obtained in Matlab are transformed using eq. (3) to allow direct comparison. The results from both models are exact (Fig. 2). The gyroscopic coupling causes vibrations to arise in a y direction.

In the presented method damping can be also introduced. In rotating systems the overall damping is distinguished between internal and external damping, and both mechanisms should be included. In Ansys, the internal damping can be modelled as

proportional to stiffness. In the case of the external damping, Ansys (in SRF) offers more sophisticated models, e.g. damping related to anisotropic bearings.

To allow the comparison between models simulated in Ansys and Matlab, the Rayleigh damping has been utilised. The same damping conditions can be realized using the  $\alpha$  part (proportional to a mass matrix) as a model of the external damping, and  $\beta$  part (proportional to the stiffness matrix) as a model of the internal one. The eq. (5) with such defined damping takes the form:

$$\mathbf{M}\ddot{\mathbf{v}} + (\mathbf{G} + 2\mathbf{MH} + \alpha\mathbf{M} + \beta\mathbf{K})\dot{\mathbf{v}} + (\mathbf{K} - \Omega^2\mathbf{M} + \mathbf{GH} + \alpha\mathbf{MH})\mathbf{v} = \boldsymbol{\tau} \quad (8)$$

where  $\alpha$  and  $\beta$  are Rayleigh damping coefficients. The term  $\alpha\mathbf{MH}$  results from a transformation of forces related to external damping into RRF. The details can be found in [7].

Two solutions have been obtained using the same Rayleigh damping coefficients (i.e.  $\alpha = 0.5 \text{ s}^{-1}$ ,  $\beta = 1 \cdot 10^{-4} \text{ s}$ ) and the same as before excitation. The first solution comes from the Ansys solver and the model referenced to the inertial frame (prefix Ans-). The second one comes from Matlab by solving the eq. (8), i.e. the model referenced to the rotating frame (prefix Mat-). The results (after transformation) are presented in Fig. 3. These results are the same, proving the correctness of the presented approach. Using this method other damping models can be also introduced, however it is beyond the scope of this article.

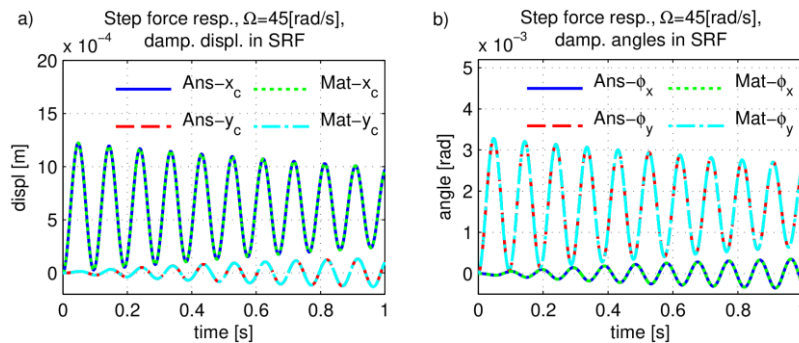


Figure 3. Comparison of results obtained in Ansys and Matlab (with damping): a) displacements in SRF, b) angles in SRF

It should be noted that both the gyroscopic and circulatory matrices depend not only on the value of rotational speed but also on the direction of rotation. In the analysed example, the system could rotate in the opposite direction and it would result in transposed matrices  $\mathbf{Corr}$  and  $\mathbf{G}$ . To avoid confusion, the system matrices should be exported using the same direction of rotation.

### 5. Application example

In the case of simple rotors, the usefulness of the described method is questionable. However, for more complex systems the proposed method can be unavoidable.

As a sample, an elastic rotor with piezoelectric patches on the surface of the shaft will be considered.

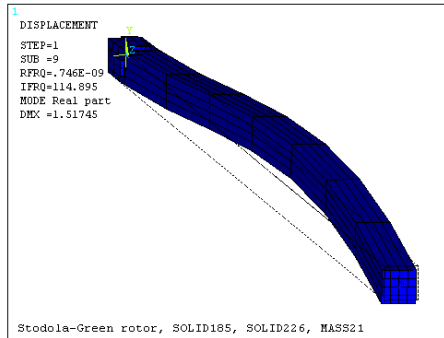


Figure 4. The second lateral mode of the analysed rotor

The deflected rotor is shown in Fig. 4. The model consists mainly of brick elements and, as a disc, a point mass element is used (the same as before, symbolised by an asterisk). Between the shaft and the disc the rigid connection is set. The four piezoelectric patches have pre-set polarisation and their electrodes are modelled by coupling electrical DOF of proper nodes. The dimensions of the rotor are the same as earlier, thus significant gyroscopic coupling exists. To obtain the voltages generated by the piezoelectric patches, the rotating reference frame has to be used, consequently this method is utilised. Figure 5 shows the final results obtained in Matlab. When the model contains the electrical DOFs, the imported mass matrix is rank deficient. To solve eq. (6), only the mechanical part of the system should be used. Further information can be found in refs. [3, 4].

When comparing Fig. 5a and Fig. 3a, one can notice that qualitatively the displacements are very similar. The quantitative differences result from other degrees of models complexity. Fig. 5b shows voltages generated by piezoelectric elements that are bonded to the shaft, thus they rotate with it. These elements can be also used as actuators, however it requires additional steps when the model is being prepared in Matlab. Some details can be found in refs. [3, 4].

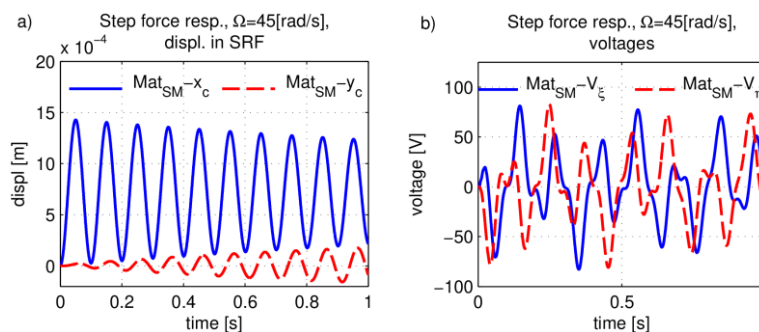


Figure 5. The results for an elastic rotor with piezoelectric patches: a) displacements transformed to SRF, b) voltages generated by piezoelectric patches

## 6. Conclusions

The article presents a combined method which expands Ansys capabilities. In particular, this method concerns the dynamics of rotors referenced to a rotating frame. Less popular analyses in RRF are limited because of lack of gyroscopic coupling in Ansys. The author proposes a solution whereby Ansys is used to create an FE model of a structure and then to move the system matrices into Matlab. In Matlab, by using imported system matrices, one can obtain a so-called circulatory matrix that allows for recreating a system's dynamics in RRF. A simple numerical example is provided to clarify all the needed steps. More complex systems such as smart rotors can be also analysed.

The described method has some limitations. The overall number of DOFs cannot be too large, otherwise it causes memory problems in Matlab. Another limiting factor is the characteristic of the bearings, which currently can be only isotropic.

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