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Fuzzy trading system on the forex market for deriving the portfolio of instruments*

by

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Abstract: Decision support and trading systems for the forex market mostly derive a single signal for the decision-maker. This is so, because instruments are evaluated based on a single criterion, which creates a ranking of instruments, from which the best one is selected. At the same time, one can observe a lack of tools allowing one to derive the set of non-dominated trading opportunities considered in the multicriteria space.

This article focuses on multicriteria analysis, in which several different market indicators describe a single instrument on the forex market (currency pair), leading to definite criteria. Thus, for a given time horizon, we consider a set of currency pairs described by a group of technical market indicators in every trading session. However, instead of deriving crisp information, based on the buy-no buy binary logic, we use concepts from the fuzzy sets theory, in which each criterion for a single variant takes a value from the $\langle 0, 1\rangle$ interval. We select only the non-dominated variants from such a set, which will be used as elements of the portfolio of currency pairs on the forex market.

We test our idea on the real-world data covering more than ten years, several technical market indicators, and over twenty different currency pairs. The preliminary results show that the proposed idea can be treated as a promising concept for deriving a portfolio of currency pairs instead of focusing on only a single currency pair.

Keywords: trading systems, forex market, fuzzy sets, multicriteria optimization

1. Introduction

Nowadays, we are facing many investment opportunities available for decision-makers. We can find numerous examples of trading systems. The typical trading

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system sequentially reads information from the market. Different market indicators are formulated and calculated based on historical data. The system using the indicators generates "buy" or "sell" signals for the decision-maker in respective specific situations on the market. These systems are built using ideas like pattern recognition (Naranjo and Santos, 2019), neural networks (Banik et al., 2022), fundamental information (Brzeszczynski and Ibrahim, 2019), and several more. However, there are also plenty of papers related to the forex markets, where simple ideas based on rule-based systems like Ozturk, Toroslu and Fidan (2016), or Chmielewski, Janowicz and Kaleta (2015) are found along with more complex black-box approaches related to machine learning (Petropoulos et al., 2017), neural networks (Thawornwong, Enke and Dagli, 2003), fuzzy systems (Dymova, Sevastjanov and Kaczmarek, 2016) and yet other ones.

On the other hand, papers dealing with building a portfolio on the forex market are rare. Works focused on the stock market have been present since the Markowitz breakthrough paper about the bicriteria financial model, published in 1952 (Markowitz, 1952). Later models, extending this idea into a higher number of criteria, are presented in papers from Briec, Kerstens and de Woestyne (2013) and Utz, Wimmer and Steuer (2015). On the other hand, Merton (1974) presented a model describing the possibility of short selling.

Chronologically one of the first concepts which extended the original Markowitz model by a third criterion was presented in the year 1972 (Lee, 1972). In this approach, maximization of the dividend was added to the original model. The extensive list of all articles connected with the portfolio selection before the year 2003 may be found in Steuer and Na (2003). A survey article, summarizing the state as of before the year 2007, concerned with the original Markowitz model is the one by Steuer, Qi and Hirschberger (2007). In the same article, a generalized division of articles connected with the portfolio selection may be found as well, having the following form:

- Portfolio selection framework – where the classical mean-variance model and its modifications were used as an element of the more general framework. Examples of such approaches may be found in Angelelli, Mansini and Speranza (2012), where the kernel search algorithm was proposed. The main concept of this article is to generate a small promising set of securities that are later expanded by the new portfolio elements. Basically, this idea may be considered to constitute an incremental model. A newer concept of the framework was proposed in Yaoa, Lib and Lai (2013), where the model with the conditional Value-at-Risk was experimentally verified by the Monte Carlo method.

- Portfolio ranking – the second area of interest includes articles, in which the portfolio is built on the basis of asset rating. One of the main algorithms included in this approach is the PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluations) method. An example of studies, concerned with this approach is constituted by the paper of Vetschera and de Almeida (2012).

- Decision support – within this area, the portfolio selection problem was presented as a support decision mechanism (Zopounidis and Doumpos, 2013). One of the newest articles in this area is a work published in 2016, where the decision support system based on the value of the information and the value of the disappointment in selecting portfolio elements was introduced (Kaoa and Steuer, 2016).

The portfolio investment problem is still at the center of attention in modern economics and computer science. However, the essential problems related to the portfolio on the forex market are mostly related to its high volatility and correlations between instruments. For the vast majority of approaches, signals are generated separately for the instruments. Thus, it is difficult to derive a formal description of the portfolio investment process.

Studies, investigating the opportunity of building a portfolio were presented even for the case of the cryptocurrency market and bitcoin (Anyfantaki, Arvanitis and Topaloglou, 2021). However, in the forex market, there is a visible gap, related to the possibilities of extending the traditional trading systems into portfolio-generating mechanisms.

The forex market is a decentralized global market with currency pairs as instruments. A single currency pair is represented by the ratio of two different currencies (for example, EUR/USD equal to 1.1 means that we have to pay 1.1 US Dollars for a single Euro). The forex market is highly volatile, and its daily turnovers reach trillions of dollars. Moreover, the potential forex decision-makers face problems, related, for instance, to high correlation between the currency pairs. This makes the possible derivation of the portfolio on the forex market a challenging problem.

At the same time, the existing crisp trading systems generate potential trading signals, valid only for a limited time. The situations, in which several signals are generated simultaneously for different indicators are scarce. Therefore, if the decision-maker considers only the best investment opportunities, he/she has a relatively low number of possibilities. Thus, a decision-maker must introduce new tools that effectively increase the number of possible investment options.

In this paper, we shortly remind our idea of a fuzzy trading system operating in the multicriteria space of market indicators, presented in Juszczuk and Kruś (2020), where time for the decision (opening the trade on the given currency pairs) is extended. The general idea, presented in the paper, is based on the fuzzy sets theory. For comparison, the signals for selected market indicators are presented traditionally as binary functions, where the signal is observed only in a very narrow time window. In this case, it is difficult to include multiple signals derived from different instruments simultaneously. To solve the problem, the proposed idea focuses on the signals based on the fuzzy membership functions, where the time for opening the position is extended. Thus, a single instrument for a given time is described by a number of fuzzy membership functions.

In classical trading systems, the trading signals, originating from technical analysis are generated using crisp rules. In the proposed system, the signals are generated using the proposed fuzzy relations. These signals, treated as initial, are analyzed in multicriteria space in which the criteria are defined by the membership functions referring to particular trading indicators. An algorithm, deriving non-dominated variants of instruments, has been proposed and tested.

In this paper, the above idea has been used to propose a new trading system, generating fuzzy signals. The novelties of the present paper include a general concept of the system acting on-line on real data from the market. Theoretical extensions include analysis of fuzzy signals, generated not only for independent instruments, but also for portfolios of the instruments, and the construction of the Pareto set. This Pareto set is presented to the decision-maker in the considered multicriteria space. The information presented to the decision maker can be considered as consisting of fuzzy signals for further analysis and possible decisions. An experimental system, generating such fuzzy signals, has been implemented and tested on real data from the market.

The problem of portfolio construction is considered from a different point of view than in the standard Markovitz scheme. In the latter case, the portfolio is constructed on the basis of different instruments taking into account such criteria as the expected profit and a risk measure. Typical applications relate to investments in the stock market in a respectively longer time horizon. This paper deals with signals generated with the use of indicators of technical analysis in the case of the forex market. In this case, the trading system generates buy or sell signals when the market is, respectively, in an oversold or overbought state. The investments undertaken on this basis have a short-term horizon. The proposed trading system, in comparison to the standard traditional system, generates fuzzy signals, including information not only concerning one but a number of selected indicators. The signals relate not only to different instruments independently but also to their possible combination, representing possible portfolios. This makes it possible to derive the Pareto set in the multicriteria space. The particular criteria represent the membership functions defined for different indicators.

The proposed system may generate several fuzzy signals for different indicators. The variants of market instruments for which the signals exist are considered in the multicriteria space of the indicators. An inbuilt algorithm derives the set of non-dominated variants. These variants are used to construct a portfolio by the decision-maker. The decision-maker can build the portfolio according to his preferences as he wants to diversify his investment. Finally, we discuss a few visual examples of the situation, in which one could derive the portfolio on the forex market.

The rest of the paper is organized as follows. In the next section, we briefly describe the traditional (called crisp) trading systems and the proposed fuzzy system, including the information about the form of the used membership functions. Next, in Section 3, generation of the non-dominated variants is described, including some formal definitions. Section 4 presents a new idea of portfolio construction, expressing the preferences of the decision-maker. The preferences can be expressed by an aspiration point, assumed by the decision-maker in the multicriteria space. The portfolio is constructed using the Minkowski distance measures, related to the non-dominated variants to the aspiration point. An exemplary case analysis, including examples of portfolio construction using real data from the market, is presented in Section 5. Finally, conclusions and references close the paper.

2. Fuzzy trading systems and membership functions

This section, in general, will be used as a starting point for the idea of deriving the portfolio of instruments for the decision-maker. Please note that the concepts presented in this section were initially described in Juszczuk and Kruś (2020).

We introduce below some initial concepts used further in this paper:

- reading t – a discrete moment of time within the whole investing time period (denoted as T), for which signals for different currency pairs could be observed;
- market indicator – technical indicator of the market; the value of the indicator is calculated on the basis of historical instrument values; this indicator is used to highlight the investing opportunity related to the given instrument on the market at a given reading t;
- trading rule (function generating signal) – binary function, indicating the occurrence of the signal at a specific reading t for the instrument;
- fuzzy trading rule – membership function used to estimate the quality of the signal for a given instrument and market indicator at the reading t;
- time window – the discrete period of time that must pass between two successive readings.

Considering the above definitions, we propose to develop the fuzzy trading system, allowing us to derive several signals for a selected set of instruments I and a set of market indicators M. Every considered market indicator is associated with the trading rule used to determine the possible signal occurrence on the market at a specific reading t. The sequence of actions for the proposed trading system is presented in Fig. 1.

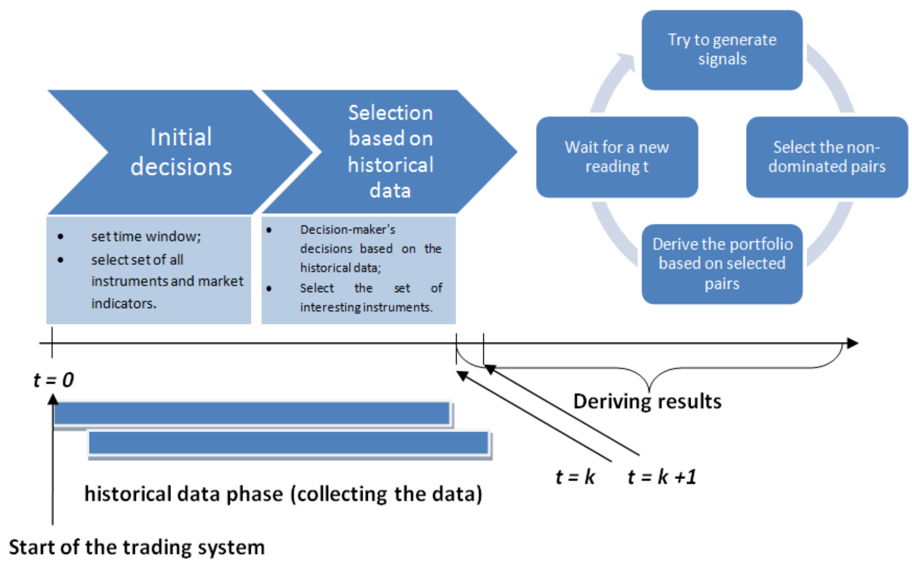

Figure 1. Sequence of actions in the proposed trading system

The sequence of actions is, namely, as follows:

- At the time $t = 0$, initial decisions related to setting the time window and selecting all available instruments and market indicators are made. In addition, the size of the historical data used to estimate the interesting instruments is also indicated.
- At the time $t = k$ initial quality (in the sense of signals) for both: instruments and market indicators is presented to the decision-maker. Decision-maker's preferences are taken into account, and he/she indicates the set of the most interesting instruments and market indicators selected from the initially assumed in the system.
- Starting at time $t = k$, the process of deriving the possible signals and building the portfolio is performed.

The above sequence of actions is repeated in time as the sytem is working on-line using real data. Therefore, the set of historical data used to calculate the market indicator values is updatet in successive time readings. The action, in which the system tries to build a portfolio at a given time reading is divided into three separate stages. At the stage named "Try to generate signals" the membership functions are used to evaluate the quality of signals for a given instrument and market indicator. In the case where the membership function is equal 0, this pair: instrument–market indicator is not taken into account in further stages of the process. In the next stage, named "Select the non-dominated pairs", the non-dominated elements with non-zero membership function values are selected. Finally, the non-dominated elements are used to build the portfolio for the decision-maker at a given reading t. The detailed process of deriving a portfolio based on a set of instruments will be presented further in this article.

Typically, the result of a decision taken on the basis of a market signal is measured in dollars, but a unit called pips is often used on the forex market. Pips stand for a value change equal to 0.0001 for currency pairs without Japanese Yen and 0.01 for the instruments including Japanese Yen. For the classical trading system settings, a single pip is worth 10$.

The trading system generates a signal when a given market indicator reaches a specific value or crosses a certain indicator level. For example, the CCI indicator is defined as follows:

$$CCI = \frac{1}{0.015} \cdot \frac{price_{typical} - MA(price_{typical})}{\sigma(price_{typical})}, \qquad (1)$$

where $price_{typical}$ is the average value of open, close, and minimum price, and σ is the mean absolute deviation. We can define the function generating a signal as:

$$f(CCI, I_l, t) = true\ if\ CCI^l_{nt}(t-1) < tr_{crisp}\ AND\ CCI^l_{nt}(t) > tr_{crisp}, \qquad (2)$$

where tr_{crisp} is the predefined oversold level related to this particular indicator; thus rule can be understood as crossing the oversold level tr_{crisp} by an indicator in two successive readings. The time of existence of such a signal is short. There are many independent signals generated for many different indicators. The time moments, in which there are signals for various indicators are scarce.

Let us assume that the market indicator value for a given time horizon is as presented in the upper part of Fig. 2. First, it is essential to define the minimal tr_{min} and maximal tr_{max} market indicator values, for which the membership function can be calculated.

Instead of the crisp condition (2), we propose a fuzzy condition using a fuzzy membership function for each market indicator. To calculate the fuzzy value in reading t, its previous value in reading $t-1$ is also needed. Thus, let us assume that to calculate the value of the membership function, we need information about the change in market indicator observed in two successive

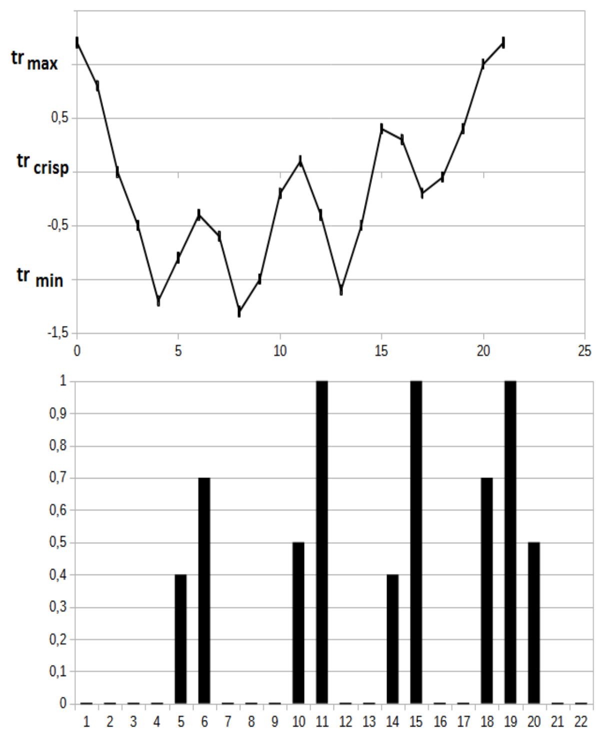

Figure 2. Market indicator values (upper part of the figure) and corresponding fuzzy membership function values (lower part of the figure)

readings: $\Delta ind = ind(t) - ind(t-1)$. Moreover, the original signal from the crisp system will also be included in these calculations when the indicator value crosses tr_{crisp}. In general, we can distinguish three different cases:

- $(ind(t-1) < tr_{crisp}) \wedge (ind(t) > tr_{crisp}) \wedge (ind(t) > ind(t-1))$ – this is the classical crisp signal, for which the membership function takes the value equal to 1;

- $(ind(t-1) < tr_{crisp}) \wedge (ind(t) \leq tr_{crisp}) \wedge (ind(t) \geq ind(t-1) + \Delta) \wedge (ind(t) > tr_{min})$, in which the membership function value depends on the distance between the respective $ind(t-1)$ and $ind(t)$ readings as well as the distance to the tr_{crisp} value. The respective membership value is equal to:
$$\mu(ind)(t) = \frac{ind(t) - (tr_{min} + \Delta)}{tr_{crisp} - tr_{min}}, \tag{3}$$

- $(ind(t-1) \geq tr_{crisp}) \wedge (ind(t) > tr_{crisp}) \wedge (ind(t) > ind(t-1)) \wedge (ind(t) < tr_{max})$ is the last case, for which both $ind(t-1)$ and $ind(t)$ are above the tr_{crisp} value. In this particular situation, the most important element indicating the membership function value is the distance between $ind(t)$ and tr_{max}, while the overall formula can be written down as follows:
$$\mu(ind)(t) = \frac{(tr_{max} - ind(t)}{tr_{max} - tr_{crisp}}. \tag{4}$$

An essential observation, derived from the above fuzzy trading system definition is that by the signal, we understand the situation, for which a given variant (currency pair) is described by a set of criteria (fuzzy membership functions related to different market indicators) presented to the decision-maker. This can be repeated for any time t within the investing time horizon T. Our goal is to reach the set of non-dominated variants, which can be used further in this article to derive the portfolios. For a more detailed explanation, please refer to Juszczuk and Kruś (2020).

3. Multicriteria optimization problem and generation of non-dominated variants

Each currency pair c is treated as a variant in decision analysis. It is evaluated by a vector y of n criteria $y_c = (y_{c_1}, \ldots y_{c_n})$. This vector is analyzed in the multicriteria space $\mathbb{R}^n$. The criteria refer to the particular indicators, represented by the values of membership functions. The decision-maker's preferences must be defined in the set of analyzed variants. The decision-maker tries to find a variant with possibly maximal values of all the criteria; therefore, we can define the following relations between variants in $\mathbb{R}^n$ space:

Definition 1 *Variant y is at least as preferred as variant z if each criterion of y is not worse than the respective criterion of z.*

$$y \succeq z \Leftrightarrow (y_1 \geq z_1) \wedge (y_2 \geq z_2) \wedge \ldots \wedge (y_n \geq z_n). \tag{5}$$

Definition 2 *Variant y is more preferred (better) than variant z according to the logical formula:*

$$y \succ z \Leftrightarrow (y \succeq z) \wedge \neg(z \succeq y). \tag{6}$$

Definition 3 *Variant y is incomparable with variant z if*

$$\neg(y \succeq z) \wedge \neg(z \succeq y). \tag{7}$$

The strict domination relation $\succ$ defines a preorder in the n dimensional space of criteria. This domination relation can be formulated and represented with the use of domination cones. The cones are formulated in the n dimensional criteria space $\mathbb{R}^n$, defined by the membership functions for all n indicators. The maximal attainable value of each criterion in this space is equal to 1 according to the definitions of the membership functions. A point $u = \{u_1, u_2, ..., u_n\}$ with $u_i = 1$ for all $i = 1, \ldots, n$, is called the utopia point. It relates to the best theoretical ideal variant. In general, such a variant may not exist. The definition used here differs from the definition typically most frequently used in the multicriteria analysis, where the maximal values of the criteria of existing variants constitute the ideal point.

Definition 4 *We say that an element y **dominates** an element z where $y, z \in \mathbb{R}^n$ and write $y \succ z$ if $y \in (z + D_+)$ where D_+ is the domination cone defined as $D_+ = \{y : y_1 \geq 0, \ldots, y_n \geq 0,$ and $y \neq 0\}$.*

Definition 5 *The **set of points dominating a given point** y is defined by $(y + D_+)$, i.e. by cone D_+ moved to point y.*

Definition 6 *The **set of points dominated by a given point** y is defined by $(y + D_-)$ where $D_- = \{y : y_1 \leq 0, \ldots, y_n \leq 0,$ and $y \neq 0\}$.*

Definition 7 *An element $y \in Y \subset \mathbb{R}^n$ is **non-dominated** in the set Y if there does not exist any element $z \in Y$ such that $z \in (y + D_+)$.*

We assume that the decision-maker first proposes a point $y^r \in \mathbb{R}^n$, called the reservation point, defining the non-accepted variants, which should be removed from further analysis:

$$y^r = \{y_1^r, y_2^r, ..., y_n^r\}, \quad \forall_i \quad 0 < y_i^r < u_i, \tag{8}$$

where n is the number of criteria, and y_i^r is the minimal accepted value of criterion i.

Point y^r is, of course, dominated by the utopia point, i.e., $y^r \in (u + D_-)$. All variants dominated by point y^r are removed, i.e., each variant y such that $y \in (x + D_-)$ is removed from further analysis. Let Y denote the set of all fuzzy variants the trading system generates. Having point y^r, we can define the set of non-accepted variants $Y_- = (x + D_-)$ and the set $Y_+ = Y \setminus Y_-$ of variants accepted for further analysis, in which the non-dominated variants are looked for. Point y^r relates to the risk aversion of the decision-maker with respect to particular criteria. It defines an extension of the set of accepted variants in the fuzzy approach compared to the crisp case.

In general, a definite number of non-dominated variants may exist. Therefore, a respective algorithm has been constructed and implemented to derive all such variants. In the algorithm, all variants generated by the trading system are sequentially compared according to the domination relation formulated in Definition 4. All of the dominated variants are removed from further analysis. This is made similarly to the way, in which the algorithm presented in Juszczuk and Kruś (2020) functions. Finally, only the non-dominated variants are selected and presented for analysis by the decision-maker.

4. Construction of portfolio expressing preferences of the decision-maker

The decision-maker has some number of non-dominated variants $y^1, \ldots, y^k$, generated by the system in the given time window. The variants are presented and considered in the multicriteria space R^n. Each criterion value is calculated as the membership function of the signal generated for the indicator considered by the decision-maker. Therefore, the number n of the criteria is defined as the number of indicators that the decision-maker considers. We assume that the decision-maker expresses his/her preferences in this criteria space, by proposing a so-called aspiration point in it.

We propose the construction of a portfolio including only the instruments indicated by the non-dominated variants. The problem is to properly allocate a given account balance in time t among the instruments. We propose to make this allocation using the aspiration point indicated by the decision-maker and the distance from the aspiration point to the non-dominated variants in the criteria space. The aspiration point should not be dominated by any of the non-dominated points. Let us note that the aspiration point is proposed when the set of non-dominated variants is known. In general, it can be the same as or different from the utopia point. The distances can be defined based on the Minkowski metric:

$$d_p(y, y^a) = [\Sigma_{j=1}^{n} |y_j - y_j^a|^p]^{1/p}, \tag{9}$$

where y is a non-dominated variant for which the distance d_p is calculated, y^a - aspiration point, n is the number of criteria. The metric expresses the different measures of distance depending on the parameter $p = 1, 2, 3, \ldots, \propto$. The following figures illustrate these measures of distance for $p = 1, p = 2, p = \propto$ in the space of two criteria y_1, y_2, representing the values of membership functions μ_1 and μ_2. The contours for distances $d_1, d_2, d_\propto$ refer to these three cases of measure. Each contour includes points being at the same distance from the utopia point Fig. 3 and from the aspiration point in Fig. 4.

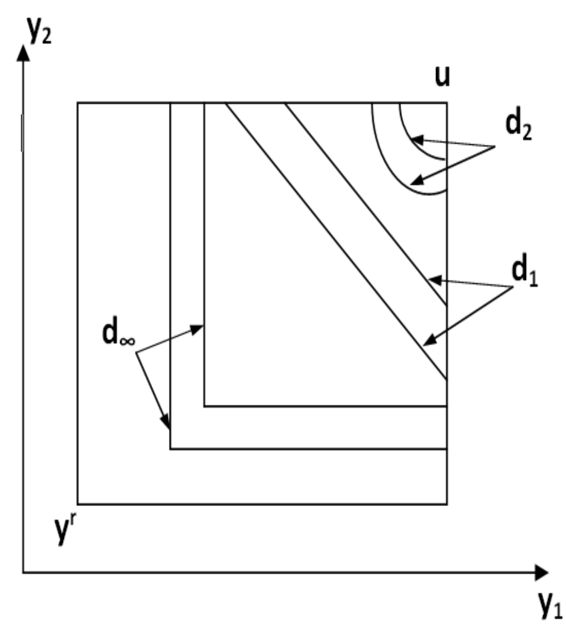

Figure 3. The proposed idea with contours for different distances from the utopia point

It is easy to see that for $p = 1$ the distance is calculated as the sum of the components' distances, for $p = 2$ it is the Euclidean distance, for $p = \propto$ - it is the maximal value of the components' distances. Different distance measures are illustrated in Fig. 5 for two variants y^1, y^2, and the aspiration point y^a in two criteria space.

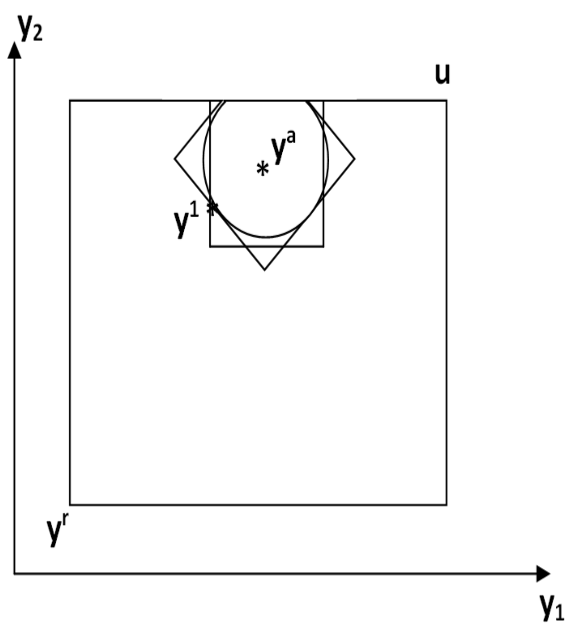

Figure 4. The proposed idea with contours for different distances from the aspiration point

The figure illustrates the distances:

$d_1(v^1, v^a) = (a^1 + b^1), d_1(v^2, v^a) = (a^2 + b^2);$
$d_2(v^1, v^a) = (c^1), d_2(v^2, v^a) = (c^2);$
$d_\propto(v!) = (a^1), d_\propto(v^2, v^a) = (b^2),$
with, as we can see, $a^1 > b^1$, and $b^2 > a^2$.

We assume that the decision-maker indicates the parameter p according to his/her preferences regarding distance measurement. It means that the decision-maker expresses preference not only by specifying the aspiration point but also by indicating the form of distance measurment.

Assume to be given the aspiration point y^a and the non-dominated variants $y^1, \ldots, y^k$. We calculate the distances $d_p(y^i, y^a)$ for each non-dominated point y^i, $i - 1, ..., k$, and the sum S^d of all these distances. The part of the account balance in time t allocated to a particular variant y^i in the portfolio will be: $x_i = 1 - d_p(y^i, y^a)/Sd, i = 1, ..., k.$

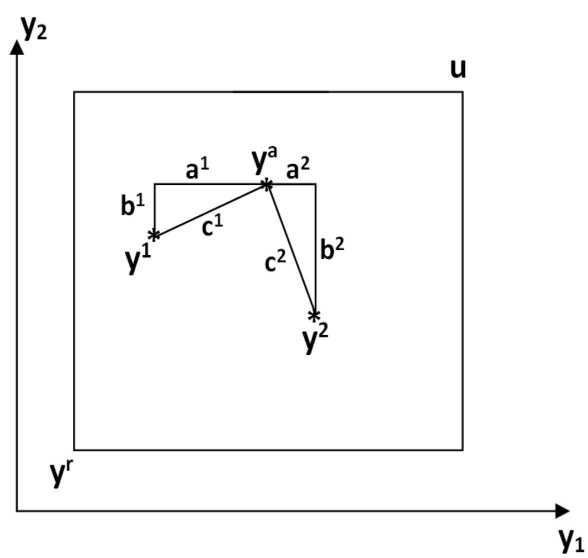

Figure 5. Illustration of different distance measures

5. Visual example – case analysis

In this section, we assume that preliminary calculations, leading to the non-dominated set of variants, were made. We aim to estimate the fraction of non-dominated variants in the portfolio, assuming that the initial account balance available for the decision-maker is constant for every single reading t during the investment process. These calculations will be performed for different locations of aspiration point y^a, selected by the decision-maker, and different p values $(1, 2, 3)$ used in the Minkowski distance calculation. We selected three test cases for the experiments for two criteria (membership functions generated based on the CCI and RSI indicators). We selected cases with two, three, and four non-dominated variants. The summary for the two criteria can be seen in Table 1. For all cases, we analyzed three different locations of the aspiration point y^a:

- $y^a = \{0.9, 0.9\}$;
- $y^a = \{0.95, 0.9\}$;
- $y^a = \{0.9, 0.95\}$.

The first location for the aspiration point was set to indicate the equal importance of both membership functions. In contrast, in the remaining cases, we put a greater emphasis on the CCI-based membership function and the RSI-based

Table 1. Real-world test cases used in the experiments, non-dominated variants and values of membership functions – criteria for the CCI and RSI indicators

	Two Criteria	
	CCI	RSI
	Case 1	
EUR/USD	0.83523	0.97362
GBP/USD	0.94721	0.89363
	Case 2	
AUD/USD	0.80808	0.98989
EUR/USD	0.87891	0.90414
GBP/USD	0.96073	0.83653
	Case 3	
EUR/GBP	0.97432	0.81323
EUR/USD	0.96561	0.8622
NZD/USD	0.88902	0.89212
AUD/USD	0.8309	0.98432

membership function. The the reservation point was set to $y^r = \{0.8, 0.8\}$ for all cases.

In our initial tests, we investigated three problems:

- what is the impact of the aspiration point location, representing the preferences of the decision-maker, on the share of nondominated variants;
- how the p value, used in the Minkowski distance calculation, affects the importance of non-dominated variants for different aspiration points;
- what is the actual share of non-dominated variants in the cases where different numbers of these variants are included?

Thus, we investigated the aspiration point location and impact of some parameters (p parameter in Minkowski distance) on the results and, more importantly, the share of different non-dominated variants in the portfolio. The second question is related to a more general aspect of the forex market: is it possible to build a portfolio including only the currency pairs, and what is the efficiency (in the measurement of profits) achieved by such a portfolio?

In Table 2, we can see the distance between the aspiration point and the non-dominated variants. In each of the examples with three and four variants, there are elements close to the aspiration point (like variant C_2 for case 2 and C_3 for case 3). The most important observation is that non-dominated variants, which at the same time dominate the reservation point, have a relatively high share in the portfolio.

Table 2. Distance calculation in Minkowski space for different p values and different aspiration points y^a

	$y^a = (0.9; 0.9)$			$y^a = (0.95; 0.9)$			$y^a = (0.9; 0.95)$		
	$p = 1$	$p = 2$	$p = 3$	$p = 1$	$p = 2$	$p = 3$	$p = 1$	$p = 2$	$p = 3$
	Case 1			Case 1			Case 1		
C_1	0.1384	0.0981	0.0875	0.1884	0.1364	0.1241	0.0884	0.0689	0.0658
C_2	0.0536	0.0476	0.0472	0.0092	0.0070	0.0065	0.1036	0.0735	0.0658
	Case 2			Case 2			Case 2		
C_1	0.1818	0.1286	0.1145	0.2318	0.1680	0.1530	0.1318	0.1002	0.0944
C_2	0.0252	0.0215	0.0212	0.0752	0.0712	0.0711	0.0670	0.0505	0.0473
C_3	0.1242	0.0878	0.0783	0.0742	0.0644	0.0636	0.1742	0.1287	0.1190
	Case 3			Case 3			Case 3		
C_1	0.1611	0.1142	0.1021	0.1111	0.0901	0.0874	0.2111	0.1557	0.1021
C_2	0.1034	0.0757	0.0696	0.0534	0.0409	0.0387	0.1534	0.1096	0.0696
C_3	0.0189	0.0135	0.0122	0.0689	0.0615	0.0610	0.0689	0.0589	0.0122
C_4	0.1534	0.1090	0.0976	0.2034	0.1459	0.1318	0.1034	0.0772	0.0976

The results with respect to the exact shares of assets in the portfolio are presented in Table 3. Let us compare two aspiration points: (0.95; 0.9) and (0.9; 0.95). The first one means that the decision-maker prefers more the CCI indicator than RSI, and the second – vice-versa. Analyzing the shares of non-dominated variants, we observe that the highest share has the variant having the highest membership function value for the CCI indicator, and the variant with the lowest value for this indicator has the lowest share. The second aspiration point means that the decision-maker prefers - has greater trust in–the RSI indicator. In this case, in all three examples (with 2, 3, and 4 variants), the variants with the higher criterion value for the RSI indicator have a relatively greater share in the portfolio.

The following main observations can be seen:

- the location of the aspiration point can be used to express the decision-maker's preferences in the portfolio construction;
- all non-dominated variants are included in the portfolio. The overall number of non-dominated variants does not significantly affect the share in the portfolio;
- distance of particular non-dominated variants from the aspiration point y^a have an important (visible) impact on the share.

Moreover, the choice of p for the distance calculation has only a limited impact on the results. However, one should note that the non-dominated variants are relatively close to each other, and the aspiration point is close to the utopia point in these experimental examples. The impact could be more significant for the fuzzy system, where these values are not so close. Further experiments are planned.

6. Conclusions and summary

In this article, we proposed the initial concepts directed toward deriving a portfolio of instruments on the forex market. Our goal was to present a methodology that could be easily implemented in the existing trading systems so that no additional modifications would be necessary. The idea behind this approach is to focus on deriving the set of non-dominated variants for the decision-maker and further use all these options rather than selecting a single, best-fitting variant. We used the idea of a trading system based on fuzzy sets, in which the number of non-dominated variants is generated using fuzzy membership functions based on the frequently used market indicators. The authors initially presented the idea behind the fuzzy trading system in Juszczuk and Kruś (2020), and now it was extended towards building the portfolio.

All pairs of market indicators and currency pairs were treated as variants in the multicriteria space. Thus, we used the concepts like the domination relation or the incomparability among variants. First, it is necessary to assume that decision-maker's preferences are considered in the decision process. It is done based on aspiration point y^a location, which indicates the general preferences of the decision-maker. Next, we used the Minkowski measure to calculate distances between the aspiration point and every non-dominated variant available in the test cases. Finally, we focused on the problem of estimating the relative importance of different variants in a situation where the number of non-dominated variants is greater than 2.

In the proposed approach, the trading system plays a role of a tool supporting the analysis carried out by the decision maker before the investment decision. During the analysis, the decision maker can assume specific initial information and parameters and introduce them into the system. They include, among others, a set of considered instruments, a set of market indicators, the period of historical data used to calculate the indicators, and the horizon analysis. He can limit the fuzziness threshold. He can have a different level of trust in different market indicators. The last can be expressed by the respective location of the aspiration point and the distance measure, described in the presented idea of the portfolio construction. The proposed approach is illustrated by an example - a case study on the real data from the forex market. The preliminary results confirm the proposed approach, showing mainly that the location of the

Table 3. Shares of a single, non-dominated variant in the portfolio for different p values and different aspiration points y^a

	p=1	p=2	p=3	p=1	p=2	p=3	p=1	p=2	p=3
	Case 1			Case 1			Case 1		
C_1	47.65%	48.64%	48.92%	45.03%	46.52%	46.86%	50.42%	50.12%	50.00%
C_2	52.35%	51.36%	51.08%	54.97%	53.48%	53.14%	49.58%	49.88%	50.00%
	Case 2			Case 2			Case 2		
C_1	30.66%	31.55%	31.78%	29.33%	30.86%	31.23%	33.05%	33.07%	33.06%
C_2	36.53%	35.43%	35.13%	35.31%	34.45%	34.25%	35.52%	34.90%	34.78%
C_3	32.82%	33.02%	33.08%	35.35%	34.70%	34.53%	31.44%	32.03%	32.16%
	Case 3			Case 3			Case 3		
C_1	23.54%	24.02%	24.15%	24.95%	24.85%	24.79%	22.78%	23.46%	24.15%
C_2	25.16%	25.07%	25.02%	26.57%	26.19%	26.12%	24.45%	24.74%	25.02%
C_3	27.54%	26.75%	26.56%	26.13%	25.63%	25.51%	26.89%	26.15%	26.56%
C_4	23.76%	24.16%	24.27%	22.36%	23.33%	23.59%	25.89%	25.64%	24.27%

aspiration point can be used to express the decision-maker's preferences in the portfolio construction regarding market indicators and instruments.

In this article, we mainly focused on presenting the general idea. However, additional tests, including multiple fuzzy membership functions generated based on market indicators, are necessary. At the same time, a method of estimating the actual value of such portfolio (in dollars) would be an interesting possibility.

References

Angelelli, E., Mansini, R. and Grazia Speranza, M. (2012) Kernel Search: a new heuristic framework for portfolio selection. *Computational Optimization and Applications*, **51**(1), 345–361.

Anyfantaki, S., Arvanitis, S. and Topaloglou, N. (2021) Diversification benefits in the cryptocurrency market under mild explosivity. *European Journal of Operational Research*, **295**, 1, 378–393.

Arévalo, R., Garcia, J., Guijarro, F. and Peris, A. (2017) A dynamic trading rule based on filtered flag pattern recognition for stock market price forecasting. *Expert Systems with Applications*, 81, 177–192.

Bagheri, A., Peyhani, H. M. and Akbari, M. (2014) Financial forecasting using ANFIS networks with Quantum-behaved Particle Swarm Optimization. *Expert Systems with Applications*, **41**, 14, 6235–6250.

Banik, S., Sharma, N., Mangla, M., Mohanty, S. and Shitharth, S. (2022) LSTM based decision support system for swing trading in stock market. *Knowledge-Based Systems*, 239, 107944.

Briec, W., Kerstens, K. and de Woestyne, I.V. (2013) Portfolio selection with skewness: a comparison of methods and a generalized one fund result. *European Journal of Operational Research*, **230**, 2, 412–421.

Brzeszczyński, J. and Ibrahim, B. M. (2019) A stock market trading system based on foreign and domestic information. *Expert Systems with Applications*, 118, 381–399.

Chmielewski, L., Janowicz, M. and Kaleta, J. (2015) Pattern recognition in the Japanese candlesticks. *Soft Computing Computer Information Sciences*, 342, 227–234.

Dymova, L., Sevastjanov, P. and Kaczmarek, K. (2016) A forex trading expert system based on a new approach to the rule-base evidential reasoning. *Expert Systems with Applications*, 51, 1–13.

Juszczuk, P. and Kruś, L. (2020) Soft multicriteria computing supporting decisions on the Forex market. *Applied Soft Computing*, 96, 106654.

Kaoa, C and Steuer, R. E. (2016) Value of information in portfolio selection, with a Taiwan stock market application illustration. *European Journal of Operational Research*, **253**(2), 418–427

Lee, S. M. (1972) *Goal Programming for Decision Analysis*. Auerbach Publishers, Philadelphia.

Markowitz, H. (1952) Portfolio selection. *Journal of Finance*. **7**, 1, 77–91.

Merton, R. C. (1974) On the Pricing of Corporate Debt: The Risk Structure of Interest Rates. *Journal of Finance*, **29**, 2, 449–470.

Naranjo, R. and Santos, M. (2019) A fuzzy decision system for money investment in stock markets based on fuzzy candlesticks pattern recognition. *Expert Systems with Applications*, 133, 34–48.

Ozturk, M., Toroslu, I. H. and Fidan, G. (2016) Heuristic based trading system on forex data using technical indicator rules. *Applied Soft Computing*, 43, 170–186.

Petropoulos, A., Chatzis, S.P., Siakoulis, V. and Vlachogiannakis, N. (2017) A stacked generalization system for automated FOREX portfolio trading. *Expert Systems with Applications*, 90, 290–302.

Steuer, R. E. and Na, P. (2003) Multiple criteria decision making combined with finance: A categorized bibliography. *European Journal of Operational Research*, **150**(3):496–515.

Steuer, R. E., Qi, Yue and Hirschberger, M. (2007) Suitable-portfolio investors, nondominated frontier sensitivity, and the effect of multiple objectives on standard portfolio selection. *Annals of Operations Research*, **152**(1):297–317.

Thawornwong, S., Enke, D. and Dagli, C. (2010) Neural Pattern recognition with self-organizing maps for efficient processing of forex market data streams. *Artificial Intelligence and Soft Computing*, **LNCS, 6113**, 307–314.

Utz, S., Wimmer, M. and Steuer, R.E. (2015) Tri-criterion modeling for constructing more-sustainable mutual funds. *European Journal of Operational Research*, **246**, 1, 331–338.

Yaoa, H., Lib, Z. and Lai, Y. (2013) Mean–CVaR portfolio selection: A nonparametric estimation framework. *Computers & Operations Research*, **40**(4),1014–1022.

Vetschera, R. and de Almeida, A. T. (2012) A PROMETHEE-based approach to portfolio selection problems. *Computers & Operations Research*, 39, 1010–1020.

Zopounidis, C. and Doumpos, M. (2013) Multicriteria decision systems for financial problems. *TOP*, **21**(2), 241–261.