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Integrated model of critical infrastructure accident consequences

Keywords

critical infrastructure, accident, initiating events, environment threats, environment degradation, losses

Abstract

An integrated general model of critical infrastructure accident consequences including the process of initiating events, the process of environment threats and the process of environment degradation models is presented. The model is proposed to the evaluation of losses associated with the environment degradation caused by the critical infrastructure accident.

1. Introduction

The critical infrastructure accident is understand as an event that causes changing the critical infrastructure safety state into the safety state worse than the critical safety state that is dangerous for the critical infrastructure itself and its operating environment as well [Bogalecka, Kołowrocki, 2006], [Grabski, 2015], [IMO, 2008], [Jakusik et al., 2012b], [Klabjan, Adelman, 2006], [Kołowrocki, 2013a], [Kołowrocki, 2013b], [Kołowrocki, 2014a], [Kołowrocki, 2014b], [Kołowrocki, Soszyńska-Budny, 2011], [Kołowrocki, Soszyńska-Budny, 2012a], [Kołowrocki, Soszyńska-Budny, 2012b], [Kołowrocki, Soszyńska-Budny, 2012c], [Kołowrocki, Soszyńska-Budny, 2012d], [Kołowrocki, Soszyńska-Budny, 2013], [Kołowrocki, Soszyńska-Budny, 2014a], [Kołowrocki, Soszyńska-Budny, 2014b], [Tang et al., 2007]. Each critical infrastructure accident can generate the initiating event causing dangerous situations in the critical infrastructure operating surroundings. The process of those initiating events can result in this environment threats and lead to the environment dangerous degradations (*Figure 1*) [Bogalecka, 2010], [Bogalecka, Kołowrocki, 2015a], [Bogalecka, Kołowrocki, 2015b].

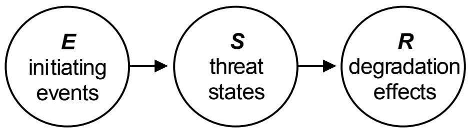


Figure 1. Interrelations of the critical infrastructure accident consequences general model.

Thus, the general model of a critical infrastructure accident consequences is constructed as a joint probabilistic model including the process of initiating events generated either by its accident or by its loss of safety critical level, the process of environment threats and the process of environment degradation.
To construct this general model of critical infrastructure accident consequences and to apply it practically, the basic notions concerned with those three particular processes it is composed of should be defined and the methods and procedures of estimating those processes unknown parameters should be developed. Under those all assumptions from the constructed model after its unknown parameters identification, the main characteristics of the process of environment degradation can be predicted. Finally, the proposed model can be applied to modelling, identification and prediction of the critical infrastructure accident consequences generated by real critical infrastructures.
The proposed approach and the methods developed will be applied in the Project Case Study 2, Scenario

2 [EU-CIRCLE Report D6.4, 2018] to modelling, identification and prediction of the critical infrastructure accident consequences generated by a ship operating in the Baltic Sea area, the member of Baltic Shipping Critical Infrastructure Network (BSCIN) defined in [EU-CIRCLE Report D1.2-GMU1, 2016].

2. Process of initiating events modelling

We call a particular consequence of the critical infrastructure accident caused by the loss of its required safety critical level the initiating event that is an event initiating dangerous threats for the critical infrastructure operating environment. Next, we can define the process of all initiating events caused by the critical infrastructure accident placed in the critical infrastructure operating environment, interacting with that environment and changing in time its states.

To model the process of initiating events, we fix the time interval $t \in <0,+\infty)$, as the time of a critical infrastructure operation and we distinguish n_1, $n_1 \in N$, events initiating the dangerous situation for the critical infrastructure operating environment and mark them by $E_1, E_2, ..., E_{n_1}$. Further, we introduce the set of vectors

$$E = \{e: \quad e = [e_1, e_2, ..., e_{n_1}], e_i \in \{0,1\}\},$$

where

$$e_i = \begin{cases} 1, \text{if the initiating event } E_i \text{ occurs,} \\ 0, \text{if the initiating event } E_i \text{ does not occur,} \end{cases}$$

for $i = 1,2,\ldots,n_1$.

We may eliminate vectors that cannot occur and we number the remaining states of the set E from $l = 1$ up to ω, $\omega \in N$, where ω is the number of different elements of the set

$$E = \{e^1, e^2, ..., e^{\omega}\},$$

where

$$e^l = [e_1^l, e_2^l, ..., e_{n_1}^l], \ l = 1,2,\ldots,\omega,$$

and

$$e_i^l \in \{0,1\}, \ i = 1,2,\ldots,n_1.$$

Next, we can define the process of initiating events $E(t)$ on the time interval $t \in <0,+\infty)$, with its discrete states from the set

$$E = \{e^1, e^2, ..., e^{\omega}\}.$$

After that, we assume a semi-Markov model [Dziula et al., 2014], [EU-CIRCLE Report D3.3-GMU22, 2016], [Kołowrocki, 2014a], [Kołowrocki, 2014b] [Soszyńska, 2007], [Soszyńska-Budny, 2014] of the process of initiating events $E(t)$ that may be described by the following parameters:

– the number of states ω, $\omega \in N$,
– the initial probabilities $p^l(0) = P(E(0) = e^l)$, $l = 1,2,\ldots,\omega$, of the process of initiating events $E(t)$ staying at the states e^l at the moment $t = 0$,
– the probabilities of transitions p^{lj}, $l, j = 1,2,\ldots,\omega$, between the states e^l and e^j,
– the conditional distribution functions $H^{lj}(t) = P(\theta^{lj} < t)$, $t \in <0,+\infty)$, $l, j = 1,2,\ldots,\omega$, $l \neq j$, of the process of initiating events $E(t)$ conditional sojourn times θ^{lj} at the states e^l while its next transition will be done to the state e^j, $l, j = 1,2,\ldots,\omega$, $l \neq j$, and their mean values $M^{lj} = E[\theta^{lj}]$, $l, j = 1,2,\ldots,\omega$, $l \neq j$.

The statistical identification of the unknown parameters of the process of initiating events, i.e. estimating the probabilities of this process staying at the states at the initial moment, the probabilities of this processes transitions between its states and the parameters and forms of the distributions fixed for the description of this process conditional sojourn times at their states can be performed according to the way presented in [Bogalecka, Kołowrocki, 2017a].

After identification of the process of initiating events, its main characteristics can by predicted [EU-CIRCLE Report D3.3-GMU23, 2016].

From the formula for total probability, it follows that the unconditional distribution functions of the sojourn times θ^l, $l = 1,2,\ldots,\omega$, of the process of initiating events $E(t)$ at the states e^l, $l = 1,2,\ldots,\omega$, are determined by [Kołowrocki , Soszyńska-Budny, 2011]

$$H^l(t) = \sum_{j=1}^{\omega} p^{lj} H^{lj}(t), \ l = 1,2,\ldots,\omega.$$

Hence, the mean values $E[\theta^{lj}]$ of the process of initiating events $E(t)$ unconditional sojourn times θ^l, $l = 1,2,\ldots,\omega$, at the states are given by

$$M^l = E[\theta^l] = \sum_{l=1}^{\omega} p^{lj} M^{lj},\ l = 1,2,\ldots,\omega. \quad (1)$$

The limit values of the process of initiating events $E(t)$ transient probabilities at the particular states

$$p^l(t) = P(E(t) = e^l),\ t \in <0,+\infty),\ l = 1,2,\ldots,\omega, \quad (2)$$

are given by [Kołowrocki, Soszyńska-Budny, 2011]

$$p^l = \lim_{t\to\infty} p^l(t) = \frac{\pi^l M^l}{\sum_{j=1}^{\omega} \pi^j M^j},\ l = 1,2,\ldots,\omega, \quad (3)$$

where M^l are given by (1), while the steady probabilities π^l of the vector $[\pi^l]_{1\mathrm{x}\omega}$ satisfy the system of equations

$$\begin{cases} [\pi^l] = [\pi^l][p^{lj}] \\ \sum_{j=1}^{\omega} \pi^j = 1, \end{cases}$$

$l = 1,2,\ldots,\omega.$

The asymptotic distribution of the sojourn total time $\hat{\theta}^l$ of the process of initiating events $E(t)$ in the time interval $\langle 0,\theta \rangle,\ \theta > 0$, at the state e^l is normal with the expected value

$$\hat{M}^l = E[\hat{\theta}^l] \cong p^l \theta,$$

where p^l are given by (3).

3. Process of environment threats modelling

To construct the general model of the environment threats caused by the process of the initiating events generated by critical infrastructure loss of required safety critical level, we distinguish the set of n_2, $n_2 \in N$, kinds of threats as the consequences of initiating events that may cause the sea environment degradation and denote them by $H_1, H_2, \ldots, H_{n_2}$ [Bogalecka, Kołowrocki, 2015b].

We also distinguish n_3, $n_3 \in N$, environment sub-regions $D_1, D_2, \ldots, D_{n_3}$ of the considered critical infrastructure operating environment region $D = D_1 \cup D_2 \cup \ldots \cup D_{n_3}$, that may be degraded by the environment threats H_i, $i = 1,2,\ldots,n_2$. The environment threats possibility of influence on the distinguished its operating environment sub-regions is presented in *Figure 2*.

We assume that the operating environment region D can be affected by some of threats H_i, $i = 1,2,\ldots,n_2$, and that a particular environment threat H_i, $i = 1,2,\ldots,n_2$, can be characterised by the parameter f^i, $i = 1,2,\ldots,n_2$. Moreover, we assume that the scale of the threat H_i, $i = 1,2,\ldots,n_2$, influence on region D depends on the range of its parameter value and for particular parameter f^i, $i = 1,2,\ldots,n_2$, we distinguish l_i ranges $f^{i1}, f^{i2}, \ldots, f^{il_i}$ of its values.

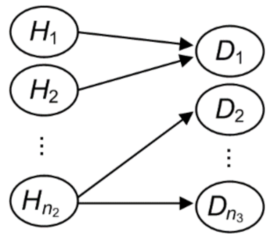


Figure 2. Illustration of environment threats possibility of influence on the critical infrastructure operating environment sub-regions.

After that, we introduce the set of vectors

$$s_{(k)} = [f^1_{(k)}, f^1_{(k)}, \ldots, f^{n_2}_{(k)}],\ k = 1,2,\ldots,n_3, \quad (4)$$

where

$$f^i_{(k)} = \begin{cases} 0, & \text{if a threat } H_i \text{ does not appear at the sub-region } D_k, \\ f^{ij}_{(k)}, & \text{if a threat } H_i \text{ appears at the sub-region } D_k \text{ and its parameter is in the range } f^{ij}_{(k)},\ j = 1,2,\ldots,l_i, \end{cases} \quad (5)$$

for $i = 1,2,\ldots,n_2$, $k = 1,2,\ldots,n_3$,

is called the environment threat state of the sub-region D_k. From the above definition, the maximum number of the environment threat states for the sub-region D_k, $k = 1,2,\ldots,n_3$, is equalled to

$$\upsilon_k = (l^1_{(k)} + 1), (l^2_{(k)} + 1), \ldots, (l^{n_2}_{(k)} + 1),\ k = 1,2,\ldots,n_3.$$

Further, we number the sub-region environment threat states defined by (4) and (5) and mark them by

$s^{\upsilon}_{(k)}$ for $\upsilon = 1,2,\ldots,\upsilon_k$, $k = 1,2,\ldots,n_3$, and form the set

$$S_{(k)} = \{s^{\upsilon}_{(k)},\ \ \upsilon = 1,2,...,\upsilon_k\},\ k = 1,2,\ldots,n_3,$$

where

$$s^{i}_{(k)} \neq s^{j}_{(k)} \text{ for } i \neq j,\ i,j \in \{1,2,...,\upsilon_k\}.$$

The set $S_{(k)}$, $k = 1,2,\ldots,n_3$, is called the set of the environment threat states of the sub-region D_k, $k = 1,2,\ldots,n_3$, while a number υ_k is called the number of the environment threat states of this sub-region.
A function

$$S_{(k)}(t),\ k = 1,2,\ldots,n_3,$$

defined on the time interval $t \in <0,+\infty)$, and having values in the environment threat states set

$$S_{(k)},\ k = 1,2,\ldots,n_3,$$

is called the sub-process of the environment threats of the sub-region D_k, $k = 1,2,\ldots,n_3$.
Next, to involve the sub-process of environment threats of the sub-region with the process of initiating events, we introduced the function

$$S_{(k/l)}(t),\ k = 1,2,\ldots,n_3,\ l = 1,2,\ldots,\omega,$$

defined on the time interval $t \in <0,+\infty)$, depending on the states of the process of initiating events $E(t)$ and taking its values in the set of the environment threat states set $S_{(k)}$, $k = 1,2,\ldots,n_3$. This function is called the conditional sub-process of the environment threats in the sub-region D_k, $k = 1,2,\ldots,n_3$, while the process of initiating events $E(t)$ is at the state e^l, $l = 1,2,\ldots,\omega$.
We assume a semi-Markov model of the sub-process $S_{(k/l)}(t)$, $k = 1,2,\ldots,n_3$, $l = 1,2,\ldots,\omega$, that may be described by the following parameters:
– the number of states υ_k, $\upsilon_k \in N$,
– the initial probabilities $p^{i}_{(k/l)}(0) = P(S_{(k/l)}(0) = s^{i}_{(k)})$, $i = 1,2,\ldots,\upsilon_k$, $k = 1,2,\ldots,n_3$, $l = 1,2,\ldots,\omega$, of the conditional sub-process of environment threats $S_{(k/l)}(t)$, staying at the states $s^{i}_{(k)}$ at the moment $t = 0$,
– the probabilities of transitions $p^{ij}_{(k/l)}$, $i,j = 1,2,\ldots,\upsilon_k$, $k = 1,2,\ldots,n_3$, $l = 1,2,\ldots,\omega$, of the conditional sub-process of environment threats $S_{(k/l)}(t)$ between the states $s^{i}_{(k)}$ and $s^{j}_{(k)}$,
– the conditional distribution functions $H^{ij}_{(k/l)}(t) = P(\eta^{ij}_{(k/l)} < t),\ t \in <0,+\infty),\ i,j = 1,2,\ldots,\upsilon_k$, $k = 1,2,\ldots,n_3$, $l = 1,2,\ldots,\omega$, of the conditional sub-process of environment threats $S_{(k/l)}(t)$, conditional sojourn times $\eta^{ij}_{(k/l)}$ at the states $s^{i}_{(k)}$, while its next transition will be done to the state $s^{j}_{(k)}$, $i, j = 1,2,\ldots,\upsilon_k$, $i \neq j$, and their mean values $M^{ij}_{(k/l)} = E[\eta^{ij}_{(k/l)}]$, $i, j = 1,2,\ldots,\upsilon_k$, $i \neq j$, $k = 1,2,\ldots,n_3$, $l = 1,2,\ldots,\omega$.
The statistical identification of the unknown parameters of the process of environment threats i.e. estimating the probabilities of this process of staying at the states at the initial moment, the probabilities of this processes transitions between its states and the parameters and forms of the distributions fixed for the description of this process conditional sojourn times at their states can be performed according to the way presented in [Bogalecka, Kołowrocki, 2017b].
After identification of the process of environment treats, it can be predicted by finding its main characteristics like ones listed below and other [EU-CIRCLE Report D3.3-GMU23, 2016].
From the formula for total probability, it follows that the unconditional distribution functions of the sojourn times $\eta^{i}_{(k/l)}$, $i = 1,2,\ldots,\upsilon_k$, $k = 1,2,\ldots,n_3$, $l = 1,2,\ldots,\omega$, of the sub-process of environment threats $S_{(k/l)}(t)$ at the states $s^{i}_{(k/l)}$ $i = 1,2,\ldots,\upsilon_k$, $k = 1,2,\ldots,n_3$, $l = 1,2,\ldots,\omega$, are determined by [Kołowrocki, Soszyńska-Budny, 2011]

$$H^{i}_{(k/l)}(t) = \sum_{j=1}^{\upsilon_k} p^{ij}_{(k/l)} H^{ij}_{(k/l)}(t),\ i = 1,2,\ldots,\upsilon_k,$$

$$k = 1,2,\ldots,n_3,\ l = 1,2,\ldots,\omega,$$

Hence, the mean values $E[\eta^{ij}_{(k/l)}]$ of the sub-process of environment threats $S_{(k/l)}(t)$ unconditional sojourn times $\eta^{i}_{(k/l)}$, $i = 1,2,\ldots,\upsilon_k$, $k = 1,2,\ldots,n_3$, $l = 1,2,\ldots,\omega$, at the states are given by

$$M^{i}_{(k/l)} = E[\eta^{i}_{(k/l)}] = \sum_{i=1}^{\upsilon_k} p^{ij}_{(k/l)} M^{ij}_{(k/l)},\ i = 1,2,\ldots,\upsilon_k,$$

$$k = 1,2,\ldots,n_3,\ l = 1,2,\ldots,\omega. \quad (6)$$

The limit values of the sub-process of environment threats $S_{(k/l)}(t)$ transient probabilities at the particular states

$$p^i_{(k/l)}(t) = P(S_{(k/l)}(t) = s^i_{(k/l)}),\ t \in <0,+\infty),$$

$$i = 1,2,\ldots,\upsilon_k,\ k = 1,2,\ldots,n_3,\ l = 1,2,\ldots,\omega, \tag{7}$$

are given by [Kołowrocki, Soszyńska-Budny, 2011]

$$p^i_{(k/l)} = \lim_{t\to\infty} p^i_{(k/l)}(t) = \frac{\pi^i_{(k/l)} M^i_{(k/l)}}{\sum_{j=1}^{\upsilon_k} \pi^j_{(k/l)} M^j_{(k/l)}},$$

$$i = 1,2,\ldots,\upsilon_k,\ k = 1,2,\ldots,n_3,\ l = 1,2,\ldots,\omega, \tag{8}$$

where $M^i_{(k/l)}$ are given by (6), while the steady probabilities $\pi^i_{(k/l)}$ of the vector $[\pi^i_{(k/l)}]_{1\mathrm{x}\upsilon_k}$ satisfy the system of equations

$$\begin{cases} [\pi^i_{(k/l)}] = [\pi^i_{(k/l)}][p^{ij}_{(k/l)}] \\ \sum_{j=1}^{\upsilon_k} \pi^j_{(k/l)} = 1, \end{cases}$$

$$i = 1,2,\ldots,\upsilon_k,\ k = 1,2,\ldots,n_3,\ l = 1,2,\ldots,\omega.$$

The asymptotic distribution of the sojourn total time $\hat{\eta}^i_{(k/l)}$ of the sub-process of environment threats $S_{(k/l)}(t)$ in the time interval $\langle 0,\eta\rangle$, $\eta > 0$, at the state $s^i_{(k/l)}$ is normal with the expected value

$$\hat{M}^i_{(k/l)} = E[\hat{\eta}^i_{(k/l)}] \cong p^i_{(k/l)}\eta,$$

$$i = 1,2,\ldots,\upsilon_k,\ k = 1,2,\ldots,n_3,\ l = 1,2,\ldots,\omega.$$

Thus, according to the formula for total probability and (2) and (7), the probabilities

$$p^i_{(k)}(t) = P(S(t) = s^i_{(k)}),\ t \in <0,+\infty),\ i = 1,2,\ldots,\upsilon_k,$$

$$k = 1,2,\ldots,n_3, \tag{9}$$

are defined by

$$p^i_{(k)}(t) = \sum_{l=1}^{\omega} P\left(E(t) = e^l\right) \cdot P\left(S_{(k)}(t) = s^i_{(k)} \middle| E(t) = e^l\right)$$

$$= \sum_{l=1}^{\omega} p^l(t) \cdot p^i_{(k/l)}(t),$$

$$i = 1,2,\ldots,\upsilon_k,\ k = 1,2,\ldots,n_3,$$

and according to (3) and (8) their limit forms are

$$p^i_{(k)} = \sum_{l=1}^{\omega} p^l \cdot p^i_{(k/l)},$$

$$i = 1,2,\ldots,\upsilon_k,\ k = 1,2,\ldots,n_3. \tag{10}$$

4. Process of environment degradation modelling

The particular states of the process of the environment threats $S_{(k)}(t)$ of the sub-region D_k, $k = 1,2,\ldots,n_3$, may lead to dangerous effects degrading the environment at this sub-region. Thus, we assume that there are m_k different dangerous degradation effects for the environment sub-region D_k, $k = 1,2,\ldots,n_3$, and we mark them by

$$R^1_{(k)}, R^2_{(k)}, \ldots, R^{m_k}_{(k)}.$$

This way the set

$$R_{(k)} = \{R^1_{(k)}, R^2_{(k)}, \ldots, R^{m_k}_{(k)}\},\ k = 1,2,\ldots,n_3,$$

is the set of degradation effects for the environment of the sub-region D_k.
These degradation effects may attain different levels. Namely, the degradation effect

$$R^m_{(k)},\ m = 1,2,\ldots,m_k,$$

may reach $\nu^m_{(k)}$ levels

$$R^{m1}_{(k)}, R^{m2}_{(k)}, \ldots, R^{m\nu^m_{(k)}}_{(k)},\ m = 1,2,\ldots,m_k,$$

that are called the states of this degradation effect.
The set

$$R^m_{(k)} = \{R^{m1}_{(k)}, R^{m2}_{(k)}, \ldots, R^{m\nu^m_{(k)}}_{(k)}\},\ m = 1,2,\ldots,m_k,$$

is called the set of states of the degradation effect $R^m_{(k)}$, $m = 1,2,\ldots,m_k$, $k = 1,2,\ldots,n_3$ for the environment of the sub-region D_k, $k = 1,2,\ldots,n_3$.
Under the above assumptions, we can introduce the environment sub-region degradation process as a vector

$$R_{(k)}(t) = [R^1_{(k)}(t), R^2_{(k)}(t), \ldots, R^{m_k}_{(k)}(t)],$$

$t \in <0,+\infty)$,

where

$R^m_{(k)}(t)$, $t \in <0,+\infty)$, $m = 1,2,\ldots,m_k$, $k = 1,2,\ldots,n_3$,

are the processes of degradation effects for the environment of the sub-region D_k, defined on the time interval $t \in <0,+\infty)$, and having their values in the degradation effect state sets

$m = 1,2,\ldots,m_k$, $k = 1,2,\ldots,n_3$,

is called the degradation process of the environment of the sub-region D_k.
The vector

$$r^m_{(k)} = [d^1_{(k)}, d^2_{(k)}, \ldots, d^{m_k}_{(k)}],\ k = 1,2,\ldots,n_3, \quad (11)$$

where

$$d^m_{(k)} = \begin{cases} 0, & \text{if a degradation effect } R^m_{(k)} \text{ does not appear at the sub-region } D_k, \\ R^{mj}_{(k)}, & \text{if a degradation effect } R^m_{(k)} \text{ appears at the sub-region } D_k \text{ and its level is equal to } R^{mj}_{(k)},\ j = 1,2,\ldots,\nu^m_{(k)}, \end{cases} \quad (12)$$

for $m = 1,2,\ldots,m_k$, $k = 1,2,\ldots,n_3$,

is called the degredation state of the sub-region D_k.
From the above definition, the maximum number of the environment degradation states for the sub-region D_k, $k = 1,2,\ldots,n_3$, is equalled to

$$\ell_k = (\nu^1_{(k)} + 1), (\nu^2_{(k)} + 1), \ldots, (\nu^{m_k}_{(k)} + 1),\ k = 1,2,\ldots,n_3.$$

Further, we number the sub-region D_k, $k = 1,2,\ldots,n_3$, degradation states defined by (11) and (12) and mark them by

$r^\ell_{(k)}$ for $\ell = 1,2,\ldots,\ell_k$, $k = 1,2,\ldots,n_3$,

and form the set of degradation states

$$R_{(k)} = \{r^\ell_{(k)},\ \ell = 1,2,\ldots,\ell_k\},\ k = 1,2,\ldots,n_3,$$

where

$r^i_{(k)} \neq r^j_{(k)}$ for $i \neq j$, $i, j \in \{1,2,\ldots,\ell_k\}$.

The set $R_{(k)}$, $k = 1,2,\ldots,n_3$, is called the set of the environment degradation states of the sub-region D_k, $k = 1,2,\ldots,n_3$, while a number ℓ_k is called the number of the environment degradation states of this sub-region.
A function

$R_{(k)}(t)$, $k = 1,2,\ldots,n_3$,

defined on the time interval $t \in <0,+\infty)$, and having values in the environment degradation states set

$R_{(k)}$, $k = 1,2,\ldots,n_3$,

is called the sub-process of the environment degradation of the sub-region D_k, $k = 1,2,\ldots,n_3$.
Next, to involve the environment sub-region D_k, $k = 1,2,\ldots,n_3$, degradation process with the process of the environment threats, we define the conditional environment sub-region degradation process, while the process of the environment threats $S_{(k)}(t)$ of the sub-region D_k, is at the state $s^\upsilon_{(k)}$, $\upsilon = 1,2,\ldots,\upsilon_k$, as a vector

$$R_{(k/\upsilon)}(t) = [R^1_{(k/\upsilon)}(t), R^2_{(k/\upsilon)}(t), \ldots, R^{m_k}_{(k/\upsilon)}(t)], \quad (13)$$

$t \in <0,+\infty)$,

where

$R^m_{(k/\upsilon)}(t)$, $t \in <0,+\infty)$, $m = 1,2,\ldots,m_k$, $k = 1,2,\ldots,n_3$,

$\upsilon = 1,2,\ldots,\upsilon_k$,

defined on the time interval $t \in <0,+\infty)$, and having values in the degradation effect states set $R^m_{(k)}$, $m = 1,2,\ldots,m_k$, $k = 1,2,\ldots,n_3$.
The above definition means that the conditional environment sub-region degradation process $R_{(k/\upsilon)}(t)$, $t \in <0,+\infty)$, also takes the degradation states from the set $R_{(k)}$ of the unconditional sub-region degradation process $R_{(k)}(t)$, $t \in <0,+\infty)$, defined by (13).
We assume a semi-Markov model of the sub-process $R_{(k/\upsilon)}(t)$, $k = 1,2,\ldots,n_3$, $\upsilon = 1,2,\ldots,\upsilon_k$, that may be described by the following parameters:
– the number of states ℓ_k, $\ell_k \in N$,
– the initial probabilities $q^i_{(k/\upsilon)}(0) = P(R_{(k/\upsilon)}(0) = r^i_{(k)})$, $i = 1,2,\ldots,\ell_k$, $k = 1,2,\ldots,n_3$, $\upsilon = 1,2,\ldots,\upsilon_k$, of the conditional sub-process of environment

degradation $R_{(k/\upsilon)}(t)$, staying at the states $r^i_{(k)}$ at the moment $t = 0$,

– the probabilities of transitions $q^{ij}_{(k/\upsilon)}$, $i,j = 1,2,...,\ell_k$, $k = 1,2,\ldots,n_3$, $\upsilon = 1,2,\ldots,\upsilon_k$, of the conditional sub-process of environment degradation $R_{(k/\upsilon)}(t)$ between the states $r^i_{(k)}$ and $r^j_{(k)}$,

– the conditional distribution functions $G^{ij}_{(k/\upsilon)}(t) = P(\zeta^{ij}_{(k/\upsilon)} < t)$, $t \in <0,+\infty)$, $i,j = 1,2,...,\ell_k$, $k = 1,2,\ldots,n_3$, $\upsilon = 1,2,\ldots,\upsilon_k$. of the conditional sub-process of environment degradation $R_{(k/\upsilon)}(t)$, conditional sojourn times $\zeta^{ij}_{(k/\upsilon)}$ at the states $r^i_{(k)}$ while its next transition will be done to the state $r^j_{(k)}$, $i,j = 1,2,...,\ell_k$, $i \neq j$, and their mean values $M^{ij}_{(k/\upsilon)} = E[\zeta^{ij}_{(k/\upsilon)}]$, $i,j = 1,2,...,\ell_k$, $i \neq j$, $k = 1,2,\ldots,n_3$, $\upsilon = 1,2,\ldots,\upsilon_k$.

The statistical identification of the unknown parameters of the process of environment degradation i.e. estimating the probabilities of this process of staying at the states at the initial moment, the probabilities of this processes transitions between its states and the parameters and forms of the distributions fixed for the description of this process conditional sojourn times at their states can be performed according to the way presented in [Bogalecka, Kołowrocki, 2017c].

After identification of the process of environment degradation, it can be predicted by finding its main characteristics like ones listed below and other [EU-CIRCLE Report D3.3-GMU23, 2016].

From the formula for total probability, it follows that the unconditional distribution functions of the sojourn times $\zeta^i_{(k/\upsilon)}$, $i = 1,2,...,\ell_k$, $k = 1,2,\ldots,n_3$, $\upsilon = 1,2,\ldots,\upsilon_k$, of the sub-process of environment degradation $R_{(k/\upsilon)}(t)$ at the states $r^i_{(k)}$, $i = 1,2,...,\ell_k$, $k = 1,2,\ldots,n_3$, $\upsilon = 1,2,\ldots,\upsilon_k$, are determined by [Kołowrocki, Soszyńska-Budny, 2011]

$$H^i_{(k/\upsilon)}(t) = \sum_{j=1}^{\ell_k} q^{ij}_{(k/\upsilon)} H^{ij}_{(k/\upsilon)}(t), \ i = 1,2,...,\ell_k,$$

$$k = 1,2,\ldots,n_3, \ \upsilon = 1,2,\ldots,\upsilon_k,$$

Hence, the mean values $E[\zeta^{ij}_{(k/\upsilon)}]$ of the sub-process of environment degradation $R_{(k/\upsilon)}(t)$ unconditional sojourn times $\zeta^i_{(k/\upsilon)}$, $i = 1,2,...,\ell_k$, $k = 1,2,\ldots,n_3$, $\upsilon = 1,2,\ldots,\upsilon_k$, at the states are given by

$$M^i_{(k/\upsilon)} = E[\zeta^i_{(k/\upsilon)}] = \sum_{i=1}^{\ell_k} q^{ij}_{(k/\upsilon)} M^{ij}_{(k/\upsilon)}, \ i = 1,2,...,\ell_k,$$

$$k = 1,2,\ldots,n_3, \ \upsilon = 1,2,\ldots,\upsilon_k. \quad (14)$$

The limit values of the sub-process of environment degradation $R_{(k/\upsilon)}(t)$ transient probabilities at the particular states

$$q^i_{(k/\upsilon)}(t) = P(R_{(k/\upsilon)}(t) = r^i_{(k/\upsilon)}), \ t \in <0,+\infty),$$

$$i = 1,2,...,\ell_k, k = 1,2,\ldots,n_3, \ \upsilon = 1,2,\ldots,\upsilon_k, \quad (15)$$

are given by [Kołowrocki, Soszyńska-Budny, 2011]

$$q^i_{(k/\upsilon)} = \lim_{t\to\infty} q^i_{(k/\upsilon)}(t) = \frac{\pi^i_{(k/\upsilon)} M^i_{(k/\upsilon)}}{\sum_{j=1}^{\ell_k} \pi^j_{(k/\upsilon)} M^j_{(k/\upsilon)}},$$

$$i = 1,2,...,\ell_k, k = 1,2,\ldots,n_3, \ \upsilon = 1,2,\ldots,\upsilon_k, \quad (16)$$

where $M^i_{(k/\upsilon)}$ are given by (14), while the steady probabilities $\pi^i_{(k/\upsilon)}$ of the vector $[\pi^i_{(k/\upsilon)}]_{1\mathrm{x}\ell_k}$ satisfy the system of equations

$$\begin{cases} [\pi^i_{(k/\upsilon)}] = [\pi^i_{(k/\upsilon)}][q^{ij}_{(k/\upsilon)}] \\ \sum_{j=1}^{\ell_k} \pi^j_{(k/\upsilon)} = 1, \end{cases}$$

$$i = 1,2,...,\ell_k, k = 1,2,\ldots,n_3, \ \upsilon = 1,2,\ldots,\upsilon_k.$$

The asymptotic distribution of the sojourn total time $\hat{\zeta}^i_{(k/\upsilon)}$ of the sub-process of environment degradation $R_{(k/\upsilon)}(t)$ in the time interval $\langle 0, \zeta \rangle$, $\zeta > 0$, at the state $r^i_{(k/l)}$ is normal with the expected value

$$\hat{M}^i_{(k/\upsilon)} = E[\hat{\zeta}^i_{(k/\upsilon)}] \cong q^i_{(k/\upsilon)} \zeta,$$

where $q^i_{(k/\upsilon)}$ are given by (16).

Thus, according to the formula for total probability and (9) and (15), the probabilities

$$q^i_{(k)}(t) = P(R(t) = r^i_{(k)}), \ t \in <0,+\infty), \ i = 1,2,...,\ell_k,$$

$k = 1,2,\ldots,n_3$,

are defined by

$$q^i_{(k)}(t) = \sum_{\upsilon=1}^{\upsilon_k} P\left(S(t) = s^{\upsilon}_{(k)}\right) \cdot P\left(R_{(k)}(t) = r^i_{(k)} \middle| S(t) = s^{\upsilon}_{(k)}\right)$$

$$= \sum_{\upsilon=1}^{\upsilon_k} p^{\upsilon}_{(k)}(t) \cdot q^i_{(k/\upsilon)}(t),$$

$i = 1,2,\ldots,\ell_k$, $k = 1,2,\ldots,n_3$.

Hence, according to (10) and (16), for sufficiently large t, the boundary probabilities of the process of the environment degradation $R_{(k/\upsilon)}(t)$ at its particular states are given by

$$q^i_{(k)} \cong \sum_{\upsilon=1}^{\upsilon_k} p^{\upsilon}_{(k)} \cdot q^i_{(k/\upsilon)} = \sum_{\upsilon=1}^{\upsilon_k} [\sum_{l=1}^{\omega} p^l \cdot p^{\upsilon}_{(k/l)}] q^i_{(k/\upsilon)}$$

$$i = 1,2,\ldots,\ell_k, \; k = 1,2,\ldots,n_3, \qquad (17)$$

where p^l, $p^{\upsilon}_{(k/l)}$ and $q^i_{(k/\upsilon)}$ are defined respectively by (3), (8) and (16).

Hence, the sojourn total time $\zeta^i_{(k)}$ of the process of the environment degradation $R_{(k)}(t)$, $k = 1,2,\ldots,n_3$, in the time interval $\langle 0,\theta \rangle$, $\theta > 0$, at the state $r^i_{(k)}$ has normal distribution with the expected value

$$E[\zeta^i_{(k)}] \cong q^i_{(k)}\theta, \; i = 1,2,\ldots,\ell_k,$$

where $q^i_{(k)}$ are given by (17).

5. Critical infrastructure accident area losses

We denote by

$$C^i_{(k)}(t), \; i = 1,2,\ldots,\ell_k, \; k = 1,2,\ldots,n_3, \qquad (18)$$

the losses associated with the process of the environment degradation as a result of critical infrastructure accident

$$R_{(k)}(t), \; t \in \langle 0,+\infty \rangle, \; k = 1,2,\ldots,n_3,$$

in the sub-region D_k, $k = 1,2,\ldots,n_3$, at the environment degradation state $r^i_{(k)}$, $i = 1,2,\ldots,\ell_k$, $k = 1,2,\ldots,n_3$, in the time interval $\langle 0,t \rangle$.

Thus, the approximate expected value of the losses in the time interval $\langle 0,\theta \rangle$, associated with the process of the environment degradation $R_{(k)}(t)$, of the sub-region D_k can be defined by

$$C_{(k)}(\theta) \cong \sum_{i=1}^{\ell_k} q^i_{(k)} \cdot C^i_{(k)}(\theta) \text{ for } k = 1,2,\ldots,n_3, \qquad (19)$$

where $q^i_{(k)}$, $i = 1,2,\ldots,\ell_k$, are given by (17) and $C^i_{(k)}(\theta)$, $k = 1,2,\ldots,n_3$, are defined by (18).

The total expected value of the losses in the time interval $\langle 0,\theta \rangle$, associated with the process of the environment degradation $R(t)$, in all sub-regions of the considered critical infrastructure operating environment region D, can be evalueted by

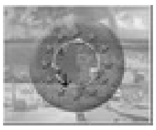
$$C(\theta) \cong \sum_{k=1}^{n_3} C_{(k)}(\theta),$$

where $C_{(k)}(\theta)$ are given by (19).

6. Conclusions

Modelling critical infrastructure accident consequences through designing the General Model of Critical Infrastructure Accident Consequences (GMCIAC) was performed in [Bogalecka, Kołowrocki, 2016], [EU-CIRCLE Report D3.3-GMU21, 2016]. The identification methods of its unknown parameters were proposed in [EU-CIRCLE Report D3.3-GMU22, 2016]. Moreover, the GMCIAC adaptation to the prediction of critical infrastructure accident consequences were done in [EU-CIRCLE Report D3.3-GMU23, 2016] and its practical applications is performed in [EU-CIRCLE Report D3.3-GMU24, 2017] to the chemical spill consequences generated by the accident of one of the ships of the shipping critical infrastructure network operating at the Baltic Sea waters as the preparatory approach to the Case Study 2: Sea Surge and Extreme Winds at Baltic Sea Area, Scenario 2, Chemical Spill due to Extreme Surges [EU-CIRCLE Report D6.4, 2018]. Having presented here the integrated models of critical infrastructure consequences and losses it is possible to optimize the losses by their minimization and to investigate the climate-weather influence on these lossess what is expected to be done in further steps of EU-CIRCLE project activity.

Acknowledgments



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European framework for strengthening Critical Infrastructure resilience to climate change" that has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 653824. http://www.eu-circle.eu/.

References

Bogalecka M., Analysis of sea accidents initial events. Polish Journal of Environmental Studies, 19 (4A), 5-8, 2010

Bogalecka M., Kołowrocki K., Probabilistic approach to risk analysis of chemical spills at sea. International Journal of Automation and Computing, 2, 117-124, 2006

Bogalecka M., Kołowrocki K., Modelling, identification and prediction of environment degradation initial events process generated by critical infrastructure accidents. Journal of Polish Safety and Reliability Association, Summer Safety and Reliability Seminars, 6(1), 47-66, 2015a

Bogalecka M., Kołowrocki K., The process of sea environment threats generated by hazardous chemicals release. Journal of Polish Safety and Reliability Association, Summer Safety and Reliability Seminars, 6(1), 67-74, 2015b

Bogalecka M., Kołowrocki K., Modelling critical infrastructure accident consequences – an overall approach. Journal of Polish Safety and Reliability Association, Summer Safety and Reliability Seminars, 7(1), 1-13, 2016

Bogalecka M., Kołowrocki K., Statistical identification of critical infrastructure accident consequences process. Part 1. Process of initiating events. Proc. 17th Conference of the Applied Stochastic Models and Data Analysis (ASMDA) International Society and the Demographics2017 Workshop, London, UK, (in press), 2017a

Bogalecka M., Kołowrocki K., Statistical identification of critical infrastructure accident consequences process. Part 2. Process of environment threats. Proc. 17th Conference of the Applied Stochastic Models and Data Analysis (ASMDA) International Society and the Demographics2017 Workshop, London, UK, (in press), 2017b

Bogalecka M., Kołowrocki K., Statistical identification of critical infrastructure accident consequences process. Part 3. Process of environment degradation. Proc. 17th Conference of the Applied Stochastic Models and Data Analysis (ASMDA) International Society and the Demographics2017 Workshop, London, UK, (in press), 2017c

Dziula P., Kołowrocki K., Siergiejczyk M., Critical infrastructure systems modeling. Journal of Polish Safety and Reliability Association, Summer Safety and Reliability Seminars, 5(1), 41-46, 2014

EU-CIRCLE Report D1.2-GMU1, Identification of existing critical infrastructures at the Baltic Sea area and its seaside, their scopes, parameters and accidents in terms of climate change impacts, 2016

EU-CIRCLE Report D3.3-GMU21, Modelling critical infrastructure accident consequences – designing the General Model of Critical Infrastructure Accident Consequences (GMCIAC), 2016

EU-CIRCLE Report D3.3-GMU22, Identification of unknown parameters of the General Model of Critical Infrastructure Accident Consequences (GMCIAC), 2016

EU-CIRCLE Report D3.3-GMU23, Adaptation of the general model of critical infrastructure accident consequences (GMCIAC) to the prediction of critical infrastructure accident consequences, 2016

EU-CIRCLE Report D3.3-GMU24, Practical application of the General Model of Critical Infrastructure Accident Consequences (GMCIAC) to the chemical spill consequences generated by the accident of one of the ships of the ship critical infrastructure network operating at the Baltic Sea waters, 2017

EU-CIRCLE Report D6.4., Case Study 2: sea surge and extreme winds at Baltic Sea area , Scenario 2, Chemical spill due to extreme surges, 2018

Grabski F., Semi-Markov processes: applications in system reliability and maintenance. Elsevier, 2015

IMO, Casualty-related matters reports on marine casualties and incidents. MSC-MEPC.3/Circ.3, London, 2008

Jakusik E., Czernecki B., Marosz M., Pilarski M., Miętus M., Changes of wave height in the Southern Baltic in the 21st century, [in:] J. Wibig and E. Jakusik (eds.). Climatological and ocean-graphical conditions in Poland and Southern Baltic. Climate change projections and guidelines for developing adaptation strategies. Monograph, 216-232, 2012a

Jakusik E., Wójcik R., Pilarski M., Biernacik D., Miętus M., Sea level in the Polish coastal zone – the current state and projected future changes, [in:] J. Wibig and E. Jakusik (eds.). Climatological and ocean-graphical conditions in Poland and Southern Baltic. Climate change projections and guidelines for developing adaptation strategies. Monograph, 146-169, 2012b

Klabjan D., Adelman D., Existence of optimal policies for semi-Markov decision processes using duality for infinite linear programming. *SIAM* Journal *on* Control and Optimization, 44(6), 2104-2122, 2006

Kołowrocki K., Safety of critical infrastructures. Journal of Polish Safety and Reliability Association, Summer Safety and Reliability Seminars, 4(1), 51-72, 2013a

Kołowrocki K., Safety of critical infrastructures – an overall approach, Keynote Speech. International Conference on Safety and Reliability - KONBiN 2013, 2013b

Kołowrocki K., Modeling reliability of critical infrastructures with application to port oil transportation system. Proc. 11th International Fatigue Congress – IFC 2014, Melbourne, Australia, 2014, Advances Materials research Vols. 891-892, 1565-1570, 2014a

Kołowrocki K., Reliability of large and complex systems. Amsterdam, Boston, Heidelberd, London, New York, Oxford, Paris, San Diego, San Francisco, Singapore, Sidney, Tokyo, Elsevier, 2014b

Kołowrocki K., Kuligowska E., Soszyńska-Budny J., Monte Carlo simulation application to reliability assessment of an exemplary system operating at variable conditions. Journal of Polish Safety and Reliability Association, Summer Safety and Reliability Seminars, 6(1), 137-144, 2015a

Kołowrocki K., Kuligowska E., Soszyńska-Budny J., Reliability assessment of an exemplary system operating at variable conditions. Journal of Polish Safety and Reliability Association, Summer Safety and Reliability Seminars, 6(1), 129-136, 2015b

Kołowrocki K., Soszyńska-Budny J., Reliability and safety of complex technical systems and processes: modeling – identification – prediction – optimization. London, Dordrecht, Heildeberg, New York, Springer, 2011

Kołowrocki K., Soszyńska-Budny J., Introduction to safety analysis of critical infrastructures. Proc. International Conference on Quality, Reliability, Risk, Maintenance and Safety Engineering - QR2MSE-2012, Chendgu, China, 1-6, 2012a

Kołowrocki K., Soszyńska-Budny J., Preliminary approach to safety analysis of critical infrastructures. Journal of Polish Safety and Reliability Association, Summer Safety and Reliability Seminars, 3(1), 73-88, 2012b

Kołowrocki K., Soszyńska-Budny J., *Safety of complex technical systems – an overall approach*. Keynote Speech, International Conference on Quality, Reliability, Risk, Maintenance and Safety Engineering – QR2MSE-2012, Chendgu, China, Final Program, 10, 2012c

Kołowrocki K., Soszyńska-Budny J., Safety of complex technical systems – an overall approach with application in maritime transport, Plenary Talk, Proc. International Symposium on Advanced Intelligent Maritime Safety Technology, Ai-MAST 2012, Mokpo, South Korea, 4-13, 2012d

Kołowrocki K., Soszyńska-Budny J., On safety of critical infrastructure modeling with application to port oil transportation system, Journal of Polish Safety and Reliability Association, Summer Safety and Reliability Seminars, 4(2), 189-204, 2013

Kołowrocki K., Soszyńska-Budny J., Optimization of critical infrastructures safety, Proc. 10th International Conference on Digital Technologies – DT 2014, Zilina, Slovakia, 150-156, 2014a

Kołowrocki K., Soszyńska-Budny J., Prediction of critical infrastructures safety, Proc. 10th International Conference on Digital Technologies – DT 2014, Zilina, Slovakia, 141-149, 2014b

Soszyńska J., Systems reliability analysis in variable operation conditions. International Journal of Reliability, Quality and Safety Engineering. Special Issue: System Reliability and Safety, 14(6), 617-634, 2007

Soszyńska-Budny J., Optimizing reliability of critical infrastructures with application to port oil piping transportation system. Proc. 11th International Fatigue Congress – ICF 2014, Melbourne, Australia, 2014, Advances Materials Research, 891-892, 2014

Tang H., Yin B.G., Xi H.S., Error bounds of optimization algorithms for semi-Markov decision processes, *International Journal of Systems Science,* 38(9), 725-736, 2007