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The Initial Velocities of Metal Plates Explosive-Driven from Asymmetric Sandwich

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Abstract. This paper presents modified Gurney formulas determining velocities of the explosive-driven metal plates which are parts of a flat asymmetric sandwich (AS). The modification takes into account a linear profile of the detonation products velocity in the Eulerian coordinates and a change of their density in terms of expansion time. Such variations of the parameters of the detonation products as the above are compatible with a conventional dynamic gas theory, and describes the dynamics of the detonation products behaviour during launching the plates from AS.
Keywords: Gurney velocity approximation, flat sandwiches, Gurney formulas

1. INTRODUCTION

The problem of launching the solids by the detonation products (DPs) is of an essential importance for estimating, among other, the initial velocity of fragments of different kinds of ammunition (bombs, projectiles, grenades), as well as for estimating the velocity of the driven plates of the reactive cassettes, after detonation of high explosive (HE) inside these cassettes [1-3].

This problem also occurs in the shaped charges. The interaction of the detonation products and the casing and a liner shaped charge determines the velocity of the liner elements and, as a result, the characteristics of a jet [4-7].

The problem of launching different solids by the DPs of the highly explosive materials was investigated by many researchers. The extensive literature connected with this problem is given in the fourth chapter of Walters and Zukas monograph [4].

One of the methods for estimation of velocity of the ammunition casings' fragments is the R. W. Gurney method [8-13].

Gurney considered, among other, the configuration of a so-called asymmetric sandwich (AS) (Fig. 1). It consists of a slab of explosive confined by two metal plates, one of them is a tamper. For this flat plate configuration, *C*, *M* and *N* are the explosive, metal plate, and tamper plate masses per unit area, respectively. Such kind of a configuration is often used in experiments to obtain constitutive properties of materials or when it is necessary to impact into a large surface of the target.

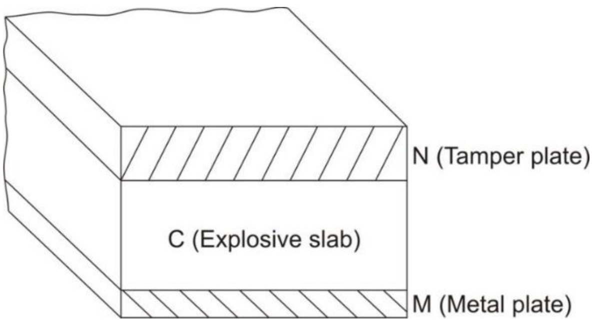

Fig. 1. Fragment of an asymmetric sandwich

The so-called Gurney approximation is based on the conservation of momentum and energy. By means of these balances have been defined the initial velocities of the explosive driven metal plates from asymmetric sandwich as:

$$\upsilon_{\mathrm{MG}} = \sqrt{2E}\left[\frac{1+A^3}{3(1+A)} + \frac{N}{C}A^2 + \frac{M}{C}\right]^{-\frac{1}{2}}$$
$$\upsilon_{\mathrm{NG}} = A\upsilon_{\mathrm{MG}}\,, \quad A = \frac{1+2\dfrac{M}{C}}{1+2\dfrac{N}{C}} \qquad (1.1)$$

where the quantity $\sqrt{2E}$ has the unit of the velocity and is termed the Gurney characteristic velocity for a given explosive.

The range of applicability of the Gurney formulas is restricted due to the simplifying assumptions in the derivation of these formulas, namely:

- distribution of the particle velocity of the expanding detonation products is linear function of the Lagrangian coordinate and persists this form for all time;
- gas products of detonation are assumed to expand uniformly with constant-density and it is equal to explosive density.

It can be observed, that the Gurney assumption of a linear velocity profile of constant density of the DPs considerably deviates from a conventional gas dynamics theory. Therefore, formulas (1.1) give inaccurate results.

Papers [14] and [15] prove that the particle velocity of the DPs expanding from the AS is a linear function in the Eulerian coordinate. On the contrary, this velocity is a non-linear function in the Lagrangian coordinate. Bearing in mind the above fact, an attempt to modify the Gurney formulas (1.1) is made. The modification is based on a linear profile of the DPs velocity in Eulerian coordinate and on the variable of the average value of the DPs density in terms of the time of their expansion. These modifications are conformable with a conventional gas dynamics theory. Accordingly, it improves an approximation of the true expansion of the DPs from the AS and, therefore, determines the velocities of the explosive-driven metal plates which are part of the AS.

2. FORMULATION OF THE SUBSTITUTE PROBLEM AND ASSUMPTIONS

In order to determine the initial velocities of the metal plates explosive-driven from AS, first of all the following substitute problem should be solved and analysed, namely: an HE charge between two rigid pistons is placed in the non-deformable infinite pipe with a unit of the cross-section area. The length and mass of the charge are represented by l and C, respectively. The pistons masses are denoted by M and N. At the moment t = 0, the HE charge is instantaneously detonated. The compressed gas products of detonation expand and launch the rigid pistons. The friction forces between the pipe wall and the pistons and DPs, as well as the resistance of air in the front of the pistons are neglected. The thermodynamics properties of the DPs are described by the ideal gas model. The problem formulated in such a way is adequate to the model of the AS. This problem has been solved at the following assumptions:

1) The hypothesis of instantaneous detonation [7, 9, 16] is used. In this context, the initial density of the DPs is equal to density of the HE.
2) Larger fraction (about 70 percent) energy of the explosive charge before detonation is converted directly to the kinetic energy of the driven rigid pistons and the expanding DPs after detonation.

The internal energy of the detonation products is neglected in the balance of energy.

3) The velocity of the DPs is linear function of the Eulerian coordinate, to be compatible with a convention gas dynamics theory.
4) The wave motion of the DPs during decompression is neglected. The DPs expand adiabatically. Distribution of the DPs density is uniform at the given moment.

3. SOLUTION OF THE PROBLEM

The graphic scheme of the under consideration problem is depicted in Fig. 2.

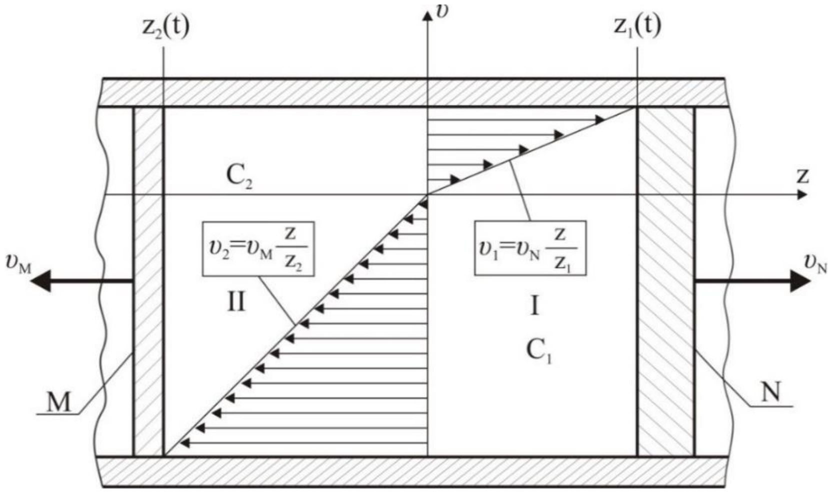

Fig. 2. Model of the problem under investigation

The symbols listed in Fig. 2 denote:

υ_M, υ_N – velocities of the piston M (AS metal plates) and the piston N (AS tamper plate);

$\upsilon_1(z)$, $\upsilon_2(z)$ – linear distribution of the velocities of the backward DPs versus the Eulerian coordinate in the Region I and Region II (Fig. 2) respectively;

C_1, C_2 – masses per unit area of the backward DPs in the Region I and Region II, respectively;

M, N – masses of the piston M (AS metal plate) and the piston N (AS tamper plate) per unit area respectively;

$z_1(t)$, $z_2(t)$ – Eulerian coordinates of the piston N (AS tamper plate), and the piston M (AS metal plate).

After the instantaneously detonation of the HE charge, the compressed DPs backward expand and displace the rigid piston in the opposite directions.

The average value of the density of the DPs in the space between the movable pistons can be expressed by the formula:

$$\rho = \frac{C_1 + C_2}{z_1 + z_2} = \frac{C}{z_1 + z_2}, \quad C_1 + C_2 = C \tag{3.1}$$

From the momentum conservation in the system considered, the following equations is obtained:

$$\frac{C_1 \upsilon_{\mathrm{N}}}{2} + N\upsilon_{\mathrm{N}} - \frac{C_2 |\upsilon_{\mathrm{M}}|}{2} - m\upsilon_{\mathrm{M}} = 0 \tag{3.2}$$

where the following relationships was applied:

$$\begin{gathered} \int_0^{z_1} \frac{\upsilon_{\mathrm{N}}}{z_1} z\rho dz = \frac{C_1 \upsilon_{\mathrm{N}}}{2}, \quad \int_0^{z_2} \frac{|\upsilon_{\mathrm{M}}|}{z_2} z\rho dz = \frac{C_2 |\upsilon_{\mathrm{M}}|}{2} \\ \rho z_1 = C_1 = \rho \upsilon_{\mathrm{N}} t, \quad \rho z_2 = C_2 = \rho |\upsilon_{\mathrm{M}}| t. \end{gathered} \tag{3.3}$$

The kinetic energy of the DPs is determined by the expression:

$$\begin{gathered} \frac{1}{2}\rho\left[\int_0^{z_1} \left(\frac{\upsilon_{\mathrm{N}}}{z_1} z\right)^2 dz + \int_0^{z_2} \left(\frac{|\upsilon_{\mathrm{M}}|}{z_2} z\right)^2 dz\right] = \\ \frac{1}{6}\left(\rho z_1 \upsilon_{\mathrm{N}}^2 + \rho z_2 \upsilon_{\mathrm{M}}^2\right) = \frac{C_1 \upsilon_{\mathrm{N}}^2}{6} + \frac{C_2 \upsilon_{\mathrm{M}}^2}{6} \end{gathered} \tag{3.4}$$

In turn, the principle of the energy conversion into the under investigation configuration yields:

$$\frac{C_1 \upsilon_{\mathrm{N}}^2}{6} + \frac{N \upsilon_{\mathrm{N}}^2}{2} + \frac{C_2 \upsilon_{\mathrm{M}}^2}{6} + \frac{M \upsilon_{\mathrm{M}}^2}{2} = CQ^* \tag{3.5}$$

where $Q^*=0.7Q$ – about 70% of total chemical energy realised during detonation of most explosives is converted directly into kinetic energy of the metal plates and into expansion of the explosive products [13].

According to the hypothesis of the instantaneous detonation and a hydrodynamics detonation theory [14, 16,17],

$$c_{\mathrm{e}}^2 = \frac{\gamma}{2}\frac{p_{\mathrm{cj}}}{\rho_{\mathrm{e}}} = \frac{\gamma}{2(\gamma+1)} D^2 = \frac{\gamma(\gamma^2-1)}{\gamma+1} Q = \gamma(\gamma-1)Q \tag{3.6}$$

where the symbols c_e, ρ_e, p_{cj}, D, Q and γ denote: the initial speed of sound into DPs, the density of the HE, the Chapman-Joguet pressure, the velocity of detonation of the HE, the heat change of HE, and isentropic exponent of the DPs, respectively.

From Eqs. (3.1), (3.2) and $(3.3)_2$ it follows that:

$$C_1 = \frac{C}{2}\frac{C+2M}{C+N+M}$$
$$C_2 = \frac{C}{2}\frac{C+2N}{C+N+M} \tag{3.7}$$

In turn, the solutions of the Eqs. $(3.3)_2$, (3.5) and (3.7), give:

$$\upsilon_M = D\left(\frac{6}{\gamma^2-1}\right)^{\frac{1}{2}}\sqrt{\frac{A_2}{B}}, \quad \upsilon_N = D\left(\frac{6}{\gamma^2-1}\right)^{\frac{1}{2}}\sqrt{\frac{A_1}{B}} \tag{3.8}$$

where:

$$\begin{cases} A_1 = C(C+M+N)(C+2M)^2, \\ A_2 = C(C+M+N)(C+2N)^2, \\ B = (C+2N)^2\left[C(C+2N)+6M(C+M+N)\right], \\ \quad +(C+2M)^2\left[C(C+2M)+6N(C+M+N)\right] \end{cases} \tag{3.9}$$

It follows from expressions (3.8) and (3.9) that:

$$\upsilon_M = 0{,}84D\left(\frac{6}{\gamma^2-1}\right)^{\frac{1}{2}}\left[\frac{(1+2y)^2(1+x+y)}{F(x,y)}\right]^{\frac{1}{2}} \tag{3.10}$$

$$\upsilon_N = 0.84D\left(\frac{6}{\gamma^2-1}\right)^{\frac{1}{2}}\left[\frac{(1+2x)^2(1+x+y)}{F(x,y)}\right]^{\frac{1}{2}} \tag{3.11}$$

$$F(x,y) = (1+2y)^2\left[1+2y+6x(1+x+y)\right]$$
$$+(1+2x)^2\left[1+2x+6y(1+x+y)\right] \tag{3.12}$$
$$x = \frac{M}{C}, \qquad y = \frac{N}{C}.$$

As it is observed, the final velocity of the flight plates depends on M/C and N/C as well as on constant $0.84D\sqrt{6/(\gamma^2-1)}$ that characterizes a given explosive by means of the readily available parameters D and γ.

If y=(N/C)=0 (a tamper vanishes), then from the expressions (3.10-3.12) there are obtained the formulas for open-faced sandwich, namely:

$$\upsilon_{\mathrm{M}} = 0.84D\left(\frac{3}{\gamma^2 - 1}\right)^{\frac{1}{2}}\left(\frac{1+x}{4x^3 + 9x^2 + 6x + 1}\right)^{\frac{1}{2}} \tag{3.13}$$

$$\upsilon_{\mathrm{N}} = \upsilon_0 = 0.84D\left(\frac{3}{\gamma^2 - 1}\right)^{\frac{1}{2}}\left(\frac{4x^3 + 8x^2 + 5x + 1}{4x^3 + 9x^2 + 6x + 1}\right)^{\frac{1}{2}} \tag{3.14}$$

where υ_0 denotes the velocity of the particles of the DPs that are on the free surface.

In turn, if $x = y$, i.e. (M/C) = (N/C), then we obtain simple formula for the symmetric sandwich as:

$$\upsilon_{\mathrm{M}} = \upsilon_{\mathrm{N}} = 0.84D\left(\frac{3}{\gamma^2 - 1}\right)^{\frac{1}{2}}\left(\frac{1}{1+6x}\right)^{\frac{1}{2}}. \tag{3.15}$$

The velocities of the explosive-driven plates from every flat sandwich have been determined in this way.

4. EXAMPLE

In order to compare the results obtained by means of Gurney formulas (1.1) and of the expressions (3.10-3.12), there have been performed calculations with application of the TNT and HMX explosives. These explosives are characterized by the following parameters [4,17] and [19]:

TNT: p_{cj} = 21 GPa, ρ_{e} = 1630 kg/m^3, γ = 3, D = 6930 m/s,

$$\sqrt{2E} = 3000\,\mathrm{m/s},\ c_e = \sqrt{\frac{3}{8}}D = 4240\,\mathrm{m/s}$$

HMX: p_{cj} = 42 GPa, ρ_{e} = 1891 kg/m^3, γ = 3, D = 9110 m/s,

$$\sqrt{2E} = 3370\,\mathrm{m/s},\ c_e = \sqrt{\frac{3}{8}}D = 5580\,\mathrm{m/s}.$$

The obtained results are presented in Figures 3, 4 and 5 in the form of graphs.

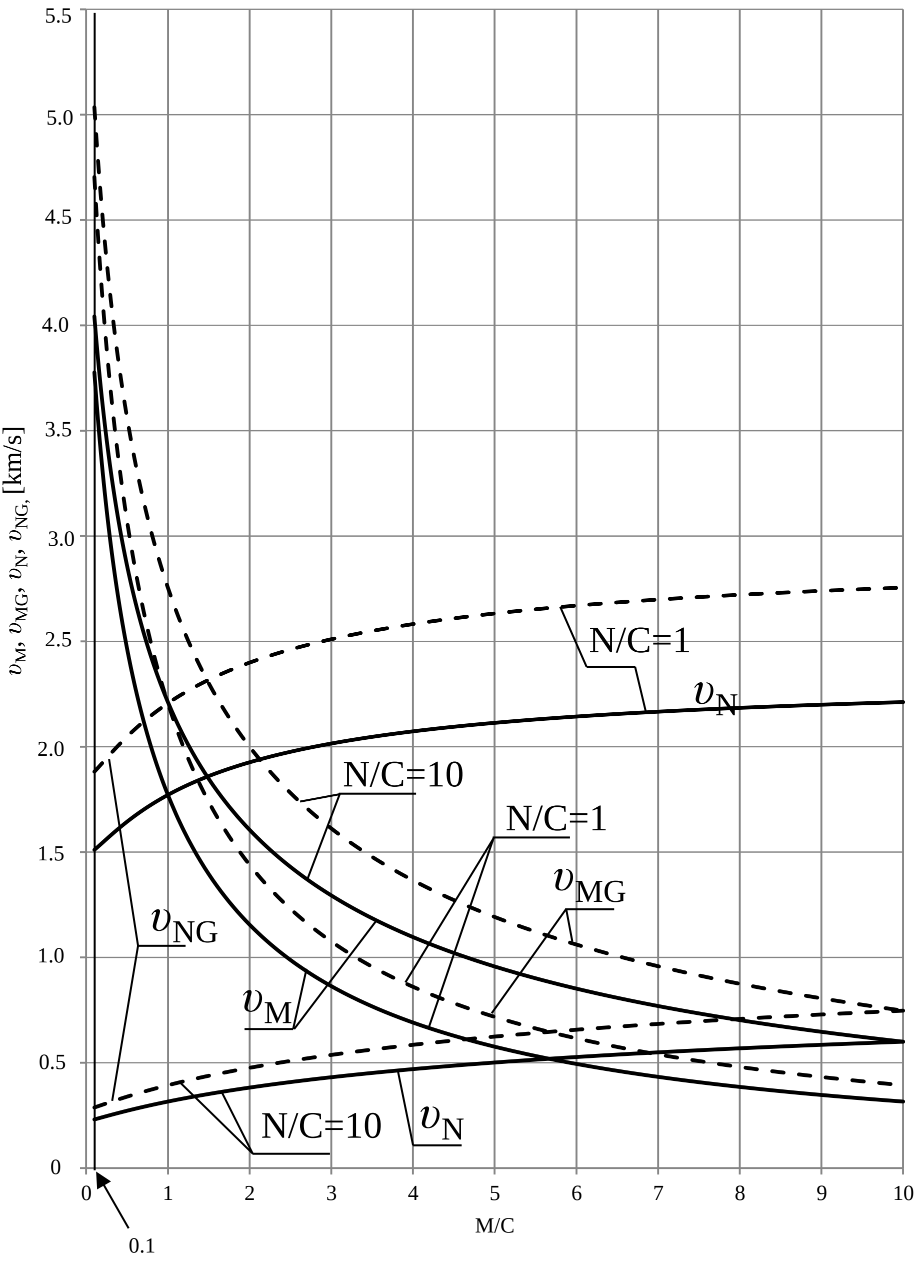

Fig. 3. υ_M, υ_{MG}, υ_N and υ_{NG} versus *M*/*C* for asymmetric sandwich with HMX for two cases of tampers: (*N*/*C*)=1 and (*N*/*C*)=10

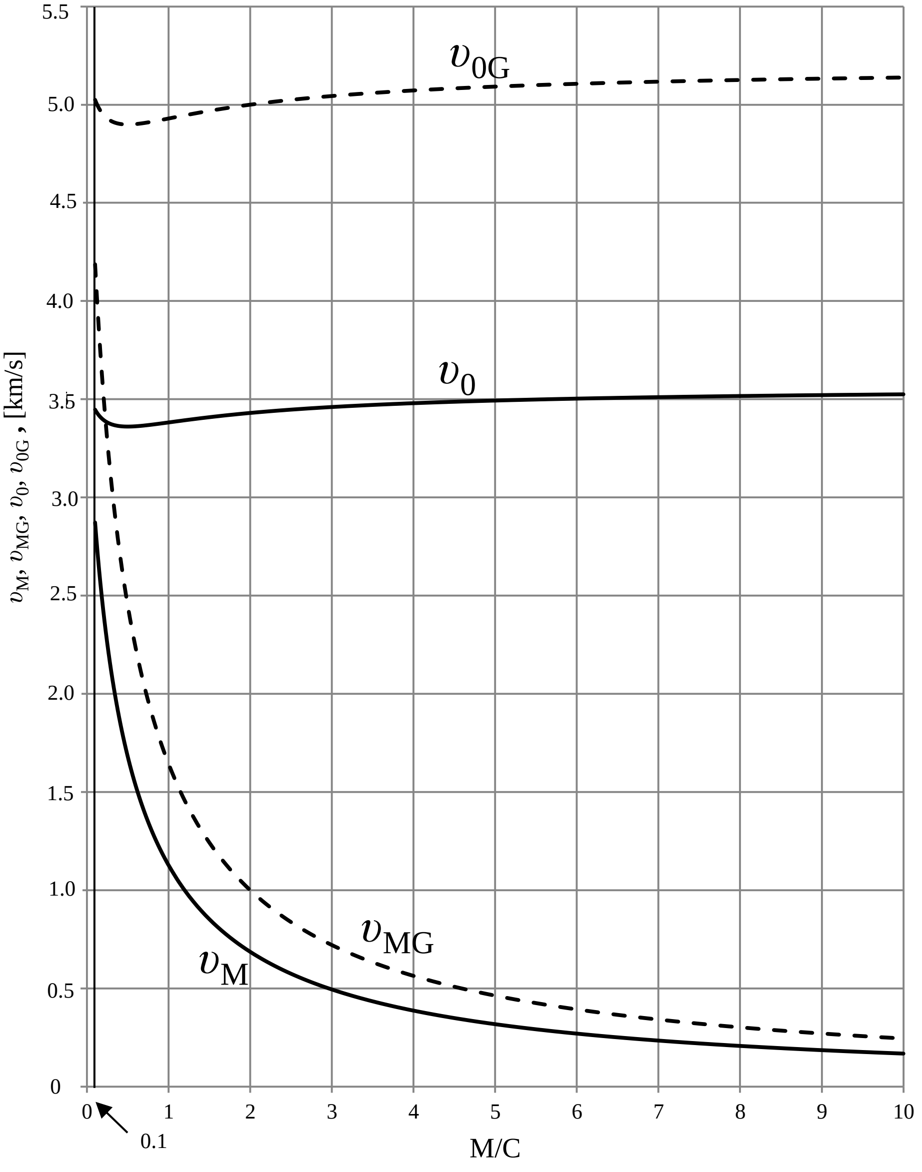

Fig. 4. υ_M, υ_{MG}, υ_0 and υ_{0G} versus M/C for an open-faced sandwich (N/C=0) with explosive – TNT

The velocities υ_M, υ_{MG}, υ_N and υ_{NG} are plotted as a function of variable M/C in Figure 3. These graphs are drawn for an asymmetric sandwich with a selected value of the parameter: (N/C)=1 and (N/C)=10, and with the explosive HMX.

The solid lines present the results obtained by expressions (3.10) – (3.12), on the contrary, the broken lines depicted the results calculated by means of Gurney formulas (1.1).

As it can be seen, the velocities υ_M and υ_{MG} intensively decrease along with an increase in *M*/*C*, particularly within the scope of a small value of *M*/*C*. This phenomenon is caused by string inequality $N/C >> M/C$ that occurs in the considered case. On the contrary, at this relation between *M*/*C* and *N*/*C*, velocities υ_N and υ_{NG} insignificantly increase along with an increase in *M*/*C*.

The results obtained by means of Gurney method for AS are more satisfying on the average about 25%.

Figure 4 plots the metal plate velocities υ_M and υ_{MG} as a function of *M*/*C* for an open-faced sandwich (N/C=0). This figure shows also the free surface velocities υ_0 and υ_{0G} as a function of *M*/*C* for this sandwich. The above mentioned velocities have been calculated by means of expressions (3.13) and (3.14) and by Gurney formulas (1.1) for (N/C)=0. In these calculations, the explosive TNT has been applied. The results obtained by means of Gurney method are considerably more satisfying in excess of 50%.

Similar graphs are presented in Fig. 5 for a symmetric sandwich (M/C=N/C). In this case, the velocities υ_{MG} are higher by about 45% than υ_M.

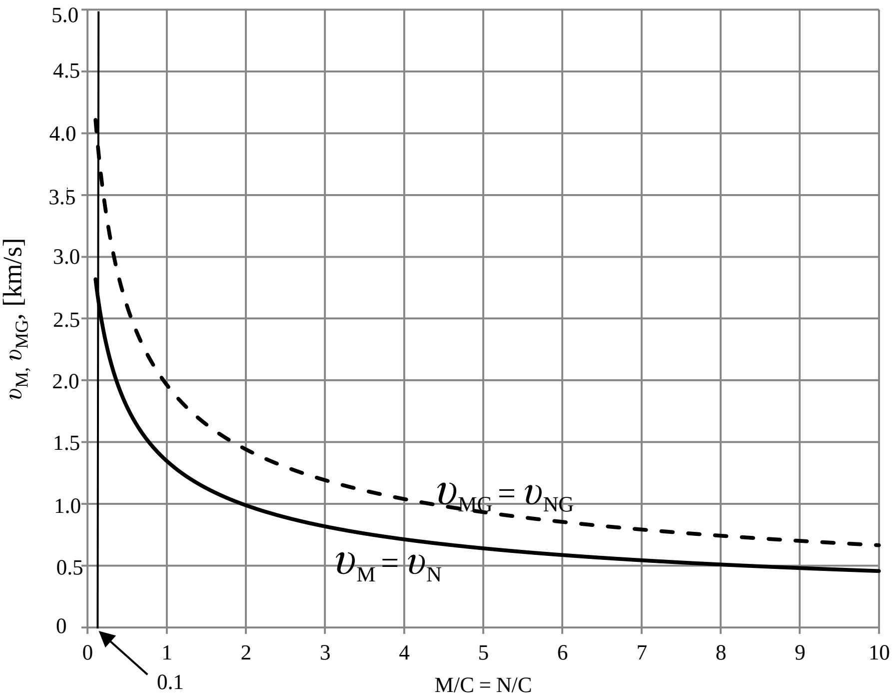

Fig. 5. υ_M and υ_{MG} versus *M*/*C* for symmetric sandwich (M/C=N/C) with explosive – TNT

5. CONCLUSIONS

To conclude, some of the above mentioned results of calculations have been compared with motion parameters of the plate launched by detonation products, recently published in [18]. In this paper, a boundary value problem of the explosion-driven plate which is part of an open – faced sandwich (N=0) has been resolved by means of a wave method. The expansion of the detonation products has been considered using propagation rarefaction waves. Figure 6 shows the variations of the relative displacement, $\bar{u} = U/l$, and the relative velocity, $\bar{\upsilon} = v/c_e$, of the metal plate in terms of the relative time , $\eta = c_e t/l$, in the initial period of launching for selected values of $x = M/C$. The horizontal lines mark the values of the relative velocities calculated by means of formulas (1.1) - $\bar{\upsilon}_{MG}$ and (3.10) - $\bar{\upsilon}_M$ for N=0. As it can be seen, the relative velocity $\bar{\upsilon}_M$ more precisely approximates the initial average value of the true velocity of the launched plate, than $\bar{\upsilon}_G$, as it should have been expected.

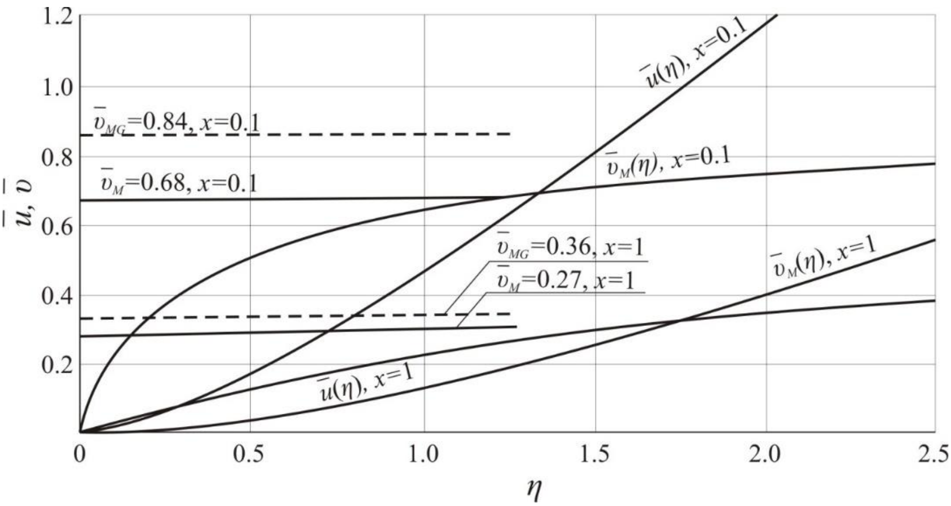

Fig. 6. Variation of the relative displacement and velocity of metal plate versus the relative time η, for selected values of x, high explosive – HMX

In the all considered cases, there occur essential differences between the results obtained by Gurney formulas (1.1) and by means of expressions (3.10) – (3.12), derived in this paper. Representing Gurney theory results are more satisfying about several dozen per cent (from 20% to 60%).

These differences are caused by the Gurney assumption of a linear velocity profile in the Lagrangian coordinates of constant density of detonation gases products, the results significantly deviate from a conventional gas dynamic theory.

This assumption introduces the largest error in configurations involving a free explosive surface such as the open-faced sandwich for which the Gurney method may overestimate~~s~~ the metal plate velocity.

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