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THE EXTENT OF THE UNCONFINED AQUIFER BASED ON THE DEMPSTER-SHAFER THEORY ON THE EXAMPLE OF POSTGLACIAL SANDUR AREA

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Abstract. The research aimed to present an application of the technique based on the Dempster-Shafer theory for the determination of the shallow unconfined aquifer extent in a nonparametric (probabilistic) scale. The geology of research area is predominantly composed of the Pleistocene postglacial sediments. Only unconfined aquifers were taken into account in the study described in these paper. The resulting image showed a map of the aquifers' extents in a probabilistic scale i.e. in a range between 0 (the lack of the aquifer, which is confirmed by research) and 1 (confirms the occurrence of the aquifer proved by research). Data analyses were carried out in the Geographic Information System. All the data were imported to the IDRISI. The Dempster-Shafer probability theory supported by the module BELIEF of IDRISI software was applied to the algebra of pixel maps.

Key words: unconfined aquifer, shallow aquifer, Dempster-Shafer theory, probabilistic.

INTRODUCTION

The extent of hydrogeological elements are represented in the cartographic studies based on the point or exploration performed in the field. With respect to distances, those limits are of probable course, more or less similar to the real boundary. Error assessment of graphic presentation of the hydrogeological elements, such as extent of aquifer has not been expressed in values yet. Hydrogeology cartography offers diversified studies, due to the reliability of data used. It is connected with the accuracy and likelihood of estimation of the extent of groundwater bodies and their amounts. Information about reliability of hydrogeologic studies is especially useful to readers from other speciality. Maps with nonparametric scale gives the reader easy readable data about quality of source information from area of research.

In the environmental studies, the proper use of the information (or the lack of the information) leads to searching for way to represent this kind of data. It is argued that the application of Boolean logic (the all-or-nothing system) in the GIS design causes some problems (Leung, Leung, 1993).

An aquifer is an example of a "poorly-defined"[2] object. This is due to the lack of information on its extent (especially for the confined aquifer) and various parameters of the aquifer due to lithological changes within the aquifer.

For describing "poorly-defined" objects, we can use one of the multi-valued logic such as "fuzzy logic" (Zadeh, 1965), probability methods such as Bayesian theory or Dempster-Shafer theory (Shafer, 1976; Klir, Yuan, 1995; Eastman, 1999b). This paper attempts to evaluate the extent of the unconfined aquifer in a nonparametric – probabilistic scale with help of Dempster-Shafer theory.

THE RESEARCH AREA

The main study objective was to evaluate the probability that the shallow unconfined aquifer could be found in each pixel location in a surface represented in the studied area. The shallow aquifer is here defined as the aquifer of minimum 2 m thick, and at the depth less then 15 m from surface.

The research area of 305 km^2 in the east part of the Pomeranian Lakeland in Poland was chosen for testing this proce-

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[2] "Poorly-defined" objects are due to dichotomy rules of Aristotelian logic objects which are difficult to be assigned to a specific class of objects.

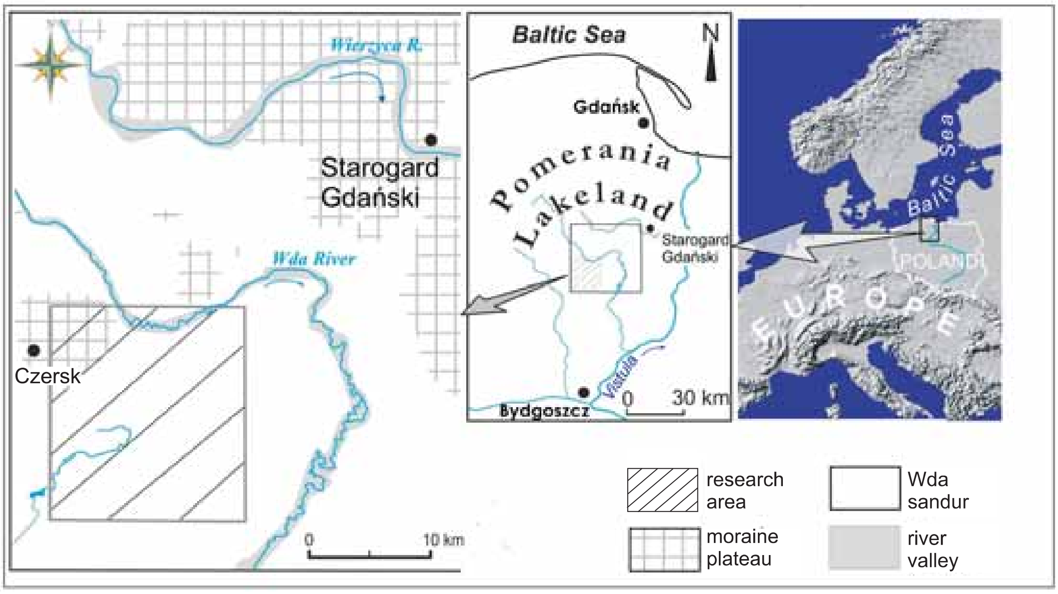

Fig. 1. Location of the research area

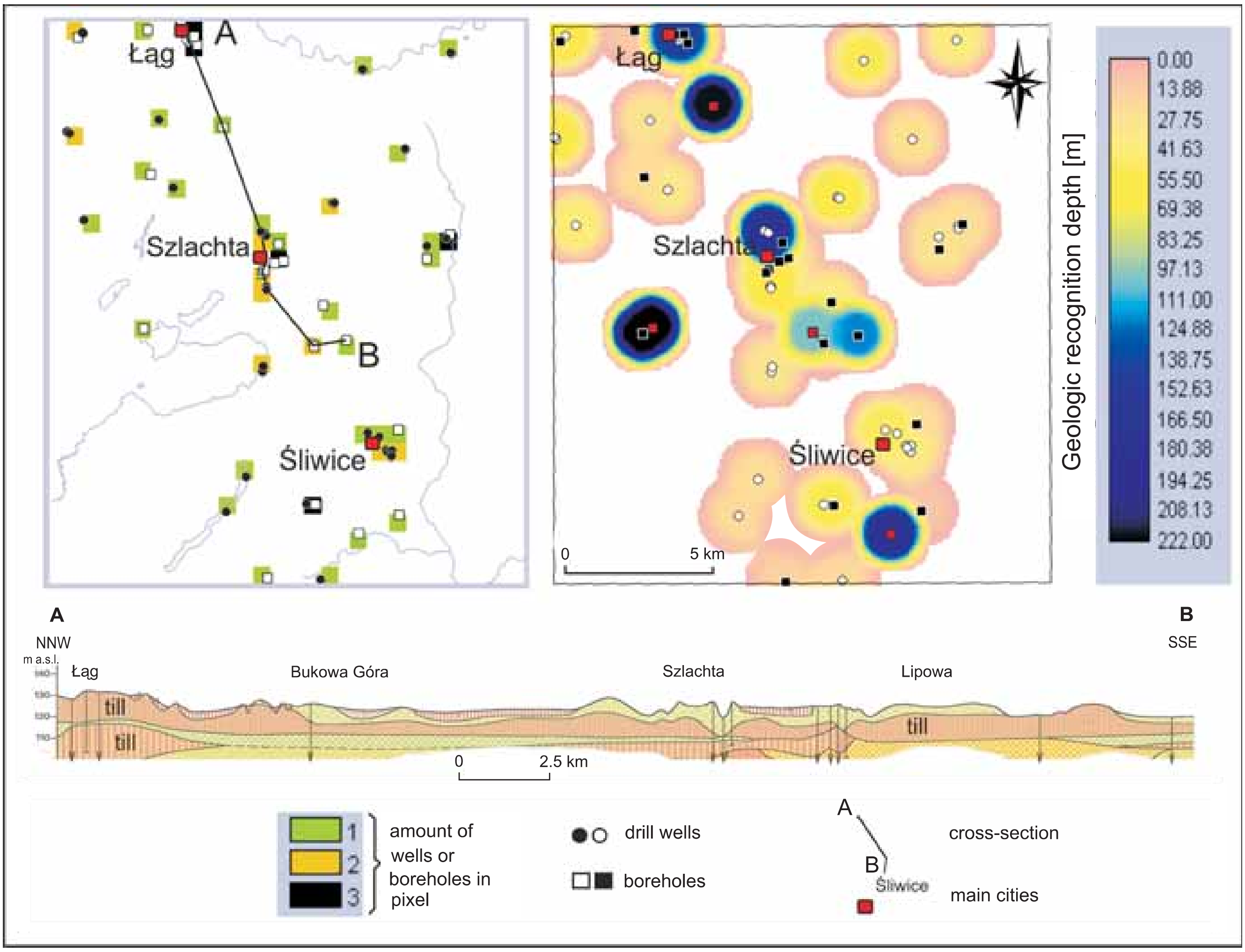

Fig. 2. Location of "hydrogeological recognition pixels" on the research area (on the left), geologic recognition depth (on the right) and cross-section (Trzepla, Drozd, 2005)

dure (Fig. 1). This area lies within border of sheet of geological map. The relief of the studied area is characterized by forms of fluvioglacial from the last (Veichselian) glaciation. The main form is outwash sediments (the Wda sandur) and in little part a moraine plateau. Only Cenozoic water bearing strata have been recognized within the log wells. The Pleistocene water bearing layers form the major aquifer for the studied area. It consists of one unconfined aquifer and two or three confined aquifer.

The hydrogeologic recognition of research area is weak. In study area there are 25 "hydrogeological recognition pixels" where drilled wells exist with logged geology profile and hydrogeologic parameters (Fig. 2). Each pixel has 0.36 km^2 area, so it gives about 3% of total area of research.

METHODOLOGY

The Dempster-Shafer theory is an extension of Bayesian probability theory. This theory makes a distinction between probability and ignorance and it allows for the expression of ignorance in uncertainty management (Lee *et al.*, 1987; Klir, Yuan, 1995). The basic assumptions of Dempster-Shafer theory are that ignorance exists in the body of knowledge, and that the belief for hypothesis is not necessary to the complement of the belief for its negation. By using the "belief functions" to represent the uncertainty of hypothesis, the theory releases some of the axioms of probability theory. The resulting system becomes a superclass of probability theory. However, it suffers from the need for large numbers of probability assignments and from the need for independence assumptions (Malczewski, 1999). Unlike Bayesian probability analysis, D-S theory explicitly recognizes the possibility of ignorance in the evaluation, i.e. the incompleteness of knowledge or evidence in the hypothesis (Eastman, 1999a).

The research objective was performed on IDRISI raster based software program. In IDRISI, the BELIEF module (Fig. 7) can be used to implement the Dempster-Shafer theory. BELIEF constructs and stores the current state of knowledge for the full hierarchy of hypotheses formed from a frame of discernment. BELIEF first requires that the basic elements in the frame of discernment be defined. As the basic elements are entered, all hypotheses in the hierarchical structure will be created in the hypothesis list. For each line of evidence entered, basic probability assignment images (in the form of real number images with a 0–1 range) are required with an indication of their supported hypothesis.

THE DEVELOPMENT OF KNOWLEDGE BASE

The research question guides us to define the frame of discernment – it includes two elements [present] and [absent]. The hierarchical combination of all possible hypotheses, therefore, includes [present], [absent] and [present, absent]. We are most interested in the result generated for the hypothesis [present] which mean existing of unconfined aquifer. The final results produced for the hypothesis [present] are dependent on how all evidence relate together in the process of aggregation.

Given knowledge about existing boring wells, dug wells and given expert knowledge about the occurrences of aquifers, each evidence is transformed into a raster layer representing likelihood that an unconfined aquifer exists. The aggregated evidence produces results that are used to predict the presence of an aquifer and evaluate the impact of each line of evidence to the total body of knowledge.

Several pixel maps of elements which confirm or deny the occurrence of the only unconfined aquifer were prepared for this study. At the beginning, each map included separately: point or area data in a scale 0 and 1. In the next stage, the information on each map was changed in order to work out membership functions. As a result, the pixel map with values from 0 to 1 was obtained. Finally, all the maps (information layers) were put to the BELIEF module and probability map was compiled regarding Dempster-Shafer theory.

Data input for the unconfined aquifer

To analyze the extent of the unconfined aquifer there is a need to collect the information about boring wells (as the best indicator of aquifers, or it's lack), dug wells and more indirect evidence of occurrences for the unconfined aquifer (springs, rivers, lakes, the area of extent of alluvial or outwash deposits). There is high probability that the unconfined aquifer will be inside or close to these forms.

For estimating the extent of the unconfined aquifer in a probabilistic scale the following data was selected:

- location of boring wells and boreholes from database of Hydrogeological Map of Poland (Prussak, 2000; Fert *et al.*, 2005; Herbich, 2005); the geologic profile of wells or boreholes was taken from Central Hydrogeological Data Bank (Cabalska *et al.*, 2005);
- location of dug wells from supplement of Hydrogeological Map of Poland – first groundwater horizons (Jankowski, Walczak, 2005);
- area of the extent of the moraine plateau and outwash (from Detailed Geological Map of Poland 1:50,000);
- course of main rivers and boundary of lakes.

CREATING PROBABILITY MAPS (FUZZYFICATION)

The stage of fuzzyfication is a procedure, which allows for converting a discrete/dichotomic image (binary map or bitmap) into pixel images (pixel map) with a probabilistic scale (from zero to one). The reliability of the obtained maps depends on the applied parameters of fuzzyfication controlled by a membership function (see Figs. 3–6). For this study the following assumptions were taken:

Probability for background

Initially, for the whole map area the background value was assumed as constant 0.5. That means there is no proof of existents of unconfined aquifer and there is no evidence for the lack of the aquifer in research area.

Membership function for drilled wells

Wells are the best point markers of the aquifer. The point map with locations of the wells (with unconfined aquifer) were converted to raster (pixel) image and all the pixels in which well (or wells) were located, obtained the value: one. Also for these features, the area in the close vicinity of the wells should obtained high likehood of occurrence of an aquifer.

However it is difficult to find proper way to extrapolate hydrogeological information from point where drill well exist to close neighborhood. Finally radius of 300 m was used around pixel representing drill wells, so the pixel values representing probability of unconfined conditions are high in the close area and decreased down to the level of the background (Fig. 3). The above distance was established subjectively as the optimal one after analyzing the hydrogeological conditions from the research area.

Membership function for wells and boreholes with lack of unconfined aquifer

The value "0" was assigned for the pixels where boreholes exist and unconfined aquifer is not noticed. Also wells which extract water form a deeper (confined) aquifer were put to this set of data. In the vicinity of those pixels, probability increases from "0" to the value of background for the range 300 m (Fig. 4). The above distance was also established subjectively as the optimal one after analyzing the geological and hydrogeological conditions (Fig. 5) from the research area.

Membership function for the area of outwash sediments and moraine plateau

In the research area of the extend the unconfined aquifer is associated with fluvioglacial outwash. The area of the outwash extent was digitized from the Detailed Geological Map of Poland in a scale 1:50,000 (Ber, 2005; Trzepla, Drozd,

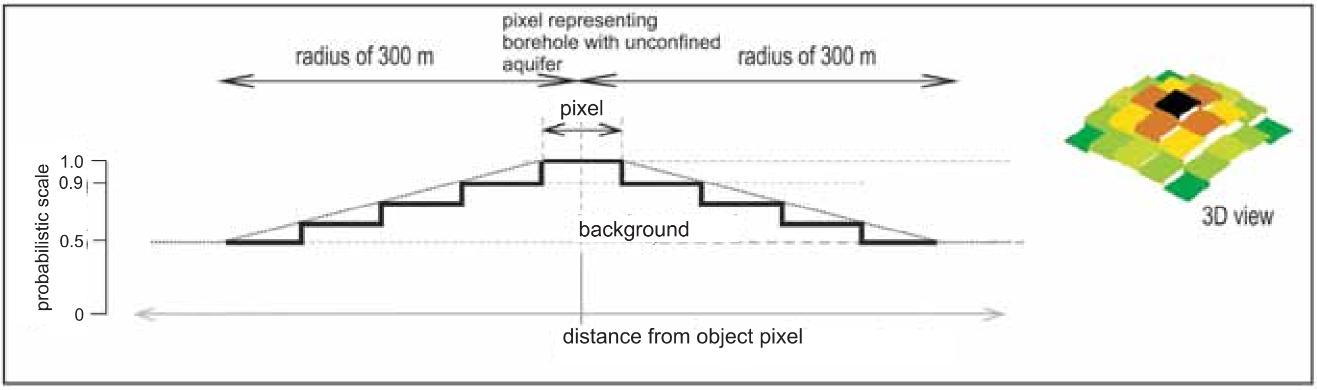

Fig. 3. Graph of the membership function for drilled wells with unconfined aquifer

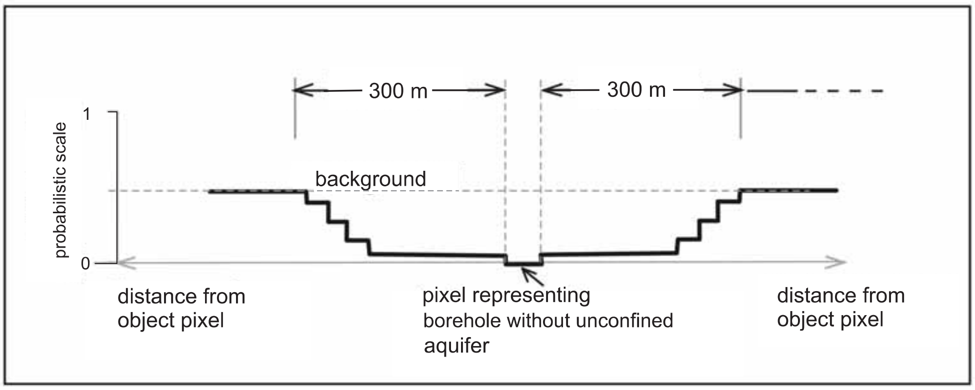

Fig. 4. Graph of the membership function for boreholes and drilled wells without unconfined aquifer

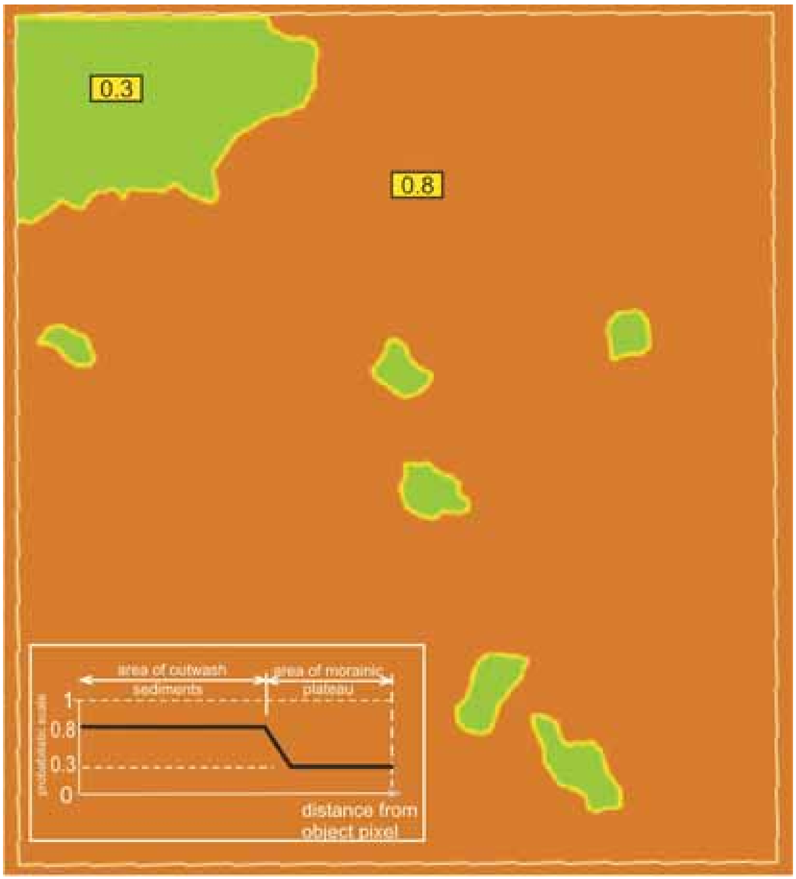

Fig. 5. Map of extend area of outwash and moraine plateau in the research area with graph of the membership function

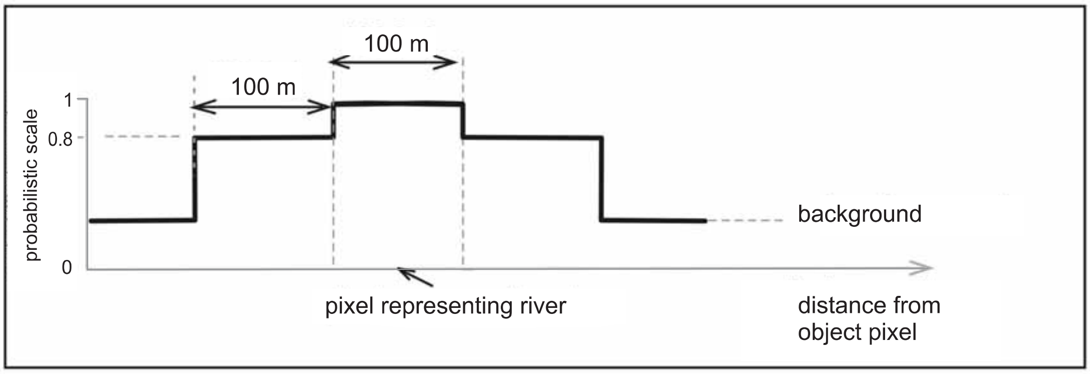

Fig. 6. Graph of the membership function for hydrologic objects

2005). The rest of the area was classified as a logic negation, which means the area without sand sediments on the terrain surface (i.e. moraine plateau). Arbitrarily the value "0.8" was assigned to all the pixels which represent the area of outwash sediments and the river valley (Fig. 5). For the remaining area a constant value "0.3" was established *a priori*.

Membership function for area in the vicinity rivers and lakes

Rivers and lakes are hydrologic objects with frequent connection to the aquifer, especially the unconfined aquifer. Close to a river or a lake there are often sand sediments with the aquifer, therefore, this vicinity to water indicates the plausibility for the aquifer. Only rivers that are longer than 5 km and lakes with the area bigger than 1 ha were analyzed.

After data analysis, the author found that there should be higher likelihood (the value of about 0.8) in the zone 100 m from the river bank or lake shores (Fig. 6).

Location of wells and boreholes was taken from the first edition of the Hydrogeological Map of Poland (HMP) in a scale 1:50,000 (Prussak, 2000). The HMP is a map, stored in GIS system as a multisheet map in database format. All data is kept in information vector layers, which contain among others: topographic situation, well and spring locations, type of the aquifer, water quality classes, aquifers pollution risk classes, land use and hydrodynamic information, e.g. hydraulic head, groundwater flow directions (Herbich, 2005). Location and parameters of dug wells was taken from the supplement of HMP database with the first groundwater horizon (Jankowski, Walczak, 2005).

Stage of calculating

All prepared pixel maps were put to the BELIEF module (Fig. 7). After processing data in the BELIEF module a set of maps was generated. These were maps of the degree to which evidence provided concrete support for the hypothesis (belief) and the degree to which that evidence did not disprove the hypothesis (plausibility). The probabilistic map of the extent of the unconfined aquifer in a research area was shown in the Figure 8.

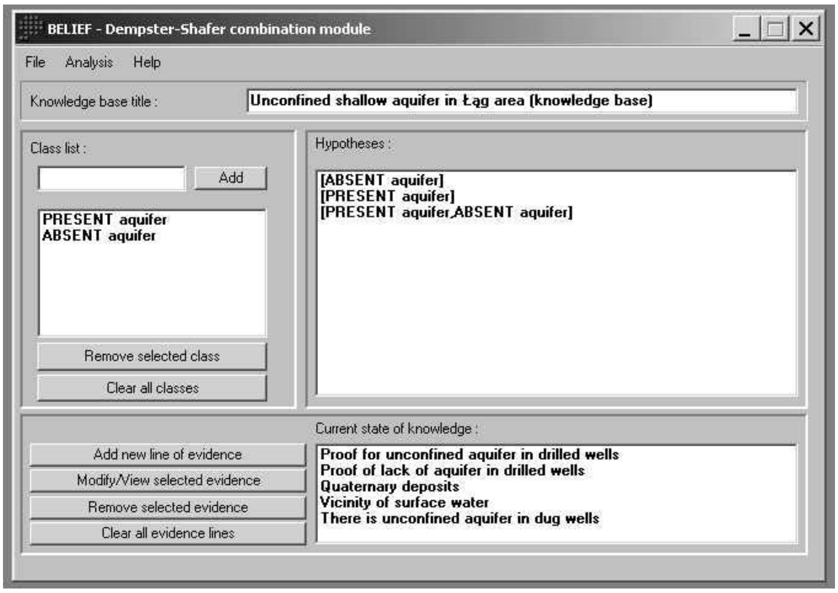

Fig. 7. The window of BELIEF module of IDRISI program

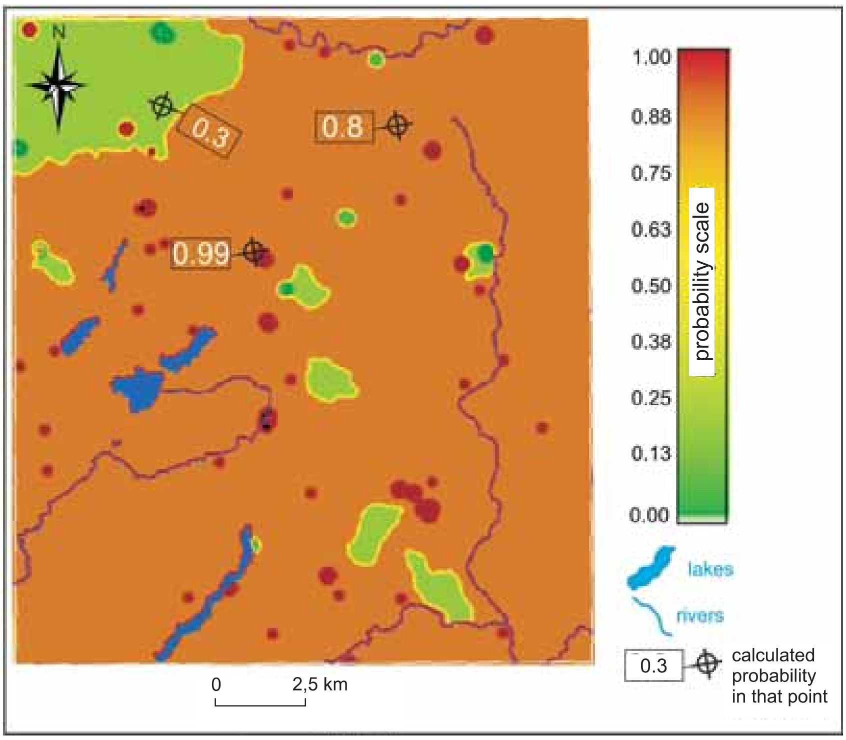

Fig. 8. The probabilistic map of the extent of the shallow unconfined aquifer in the research area

SUMMARY AND CONCLUSIONS

The limits of geological and hydrogeological structure presented on the cartographic studies contain often significant errors due to poor geologic and hydrogeologic recognition. The purpose of the methodology presented here is to produce a probabilistic information layer of the extent of the unconfined aquifer in the study area (Fig. 8). It is an attempt to use Dempster-Shafer theory in hydrogeology. Taking into account the fuzzy set theory, the author calculates probability occurrence of shallow unconfined groundwater based upon hydrogeological elements (especially drilled wells). Additional information for the probabilistic map are derived from hydrological and geomorphological investigations. The accuracy of such map is largely determined by the established membership functions.

The generated maps may be regarded as a supplement to a classic set of information concerning hydrogeology and which provides a new form of a map layer. The statistical description of the pixel value on the result map may be used for the assessment of reliability of hydrogeological model and as decision support for sustainable groundwater management.

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