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Fluctuations of Multi-section Aboveground Pipeline Region Under the Influence of Moving Diagnostic Piston

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Abstract

A mathematical model of transverse fluctuations of the pipeline straight section is considered in this article. Such fluctuations occur during the movement of diagnostic piston in the pipeline. The analysis is based on the method of generalized displacements. This method provides setting modes of links with distributed parameters according to the boundary conditions. Diagnostic piston is considered as a solid in the calculation model. The equations of mechanical systems motion are derived by the Lagrange scheme equations of the second kind. As the result, we illustrate the influence of the mechanical system parameters and the speed of the piston on the pipeline section deflections, bending moments and stresses in the pipe.

Keywords: mechanical fluctuations, multi-section region of the pipeline, diagnostic piston

1. Introduction

Analysis of oscillatory phenomena in mechanical systems under the action of moving loads is an important problem of modern dynamics of machines and structures [1–8, 10–12]. Its practical value is explained by the need to improve methods of lifting and transport systems calculation, mechanical transmission with flexible links, pipelines, bridges etc. Problem solving of such systems dynamics using continuum computational models is reduced to the integration of partial differential equations with moving boundary conditions [3, 6, 10, 12]. Mathematical models in the form of integral equations are used in studies of fluctuations of rods and filaments of variable length [7].

Building of closed analytical solutions of the equations of motion for these cases is associated with considerable mathematical difficulties. It is only feasible for relatively simple systems. Links of such systems have constant elastic-inertial parameters, and the laws of motion of the boundary conditions are given. Analysis of dynamic processes in real load carrying structures appropriate to perform with the use of mechanical sampling units, which greatly simplifies the problem solving. Method of generalized displacements [9] is quite effective. This method is based on presetting of modes of lengthy items. It allows to describe oscillatory processes by ordinary differential equations through the use of amplitude functions coefficients as generalized coordinates. This approach was successfully tested in the study of the dynamics of continuum-discrete mechanical systems with lengthy load carrying elements. In this work, it develops on the example of transverse fluctuations of multi-section part of the pipeline under the influence of moving diagnostic piston.

2. Mathematical model of bending fluctuations of multi-section part of the pipeline under the influence of moving diagnostic piston

The mechanical system that includes multi-section part of a pipeline and intelligent moving piston is shown schematically in figure 1, where l – the total length of the part; l_1, l_2, …, l_p – distance from the left edge of the area corresponding to the intermediate supports; m_k, J_k, c_k, ν_k (k = 1, 2, …, p) – mass, central moments of inertia, stiffness and viscous friction coefficients of reference sites; m, J – mass and central moment of inertia of the diagnostic piston; v – speed of the piston, which we assume constant; xOy – coordinate system, where we analyse bending fluctuation of the pipeline; x_m – coordinate of the mass center of the diagnostic piston. Density and modulus of elasticity of pipe material are designated as ρ and E; area and the axial moment of inertia of the cross-section tube as A and I_z.

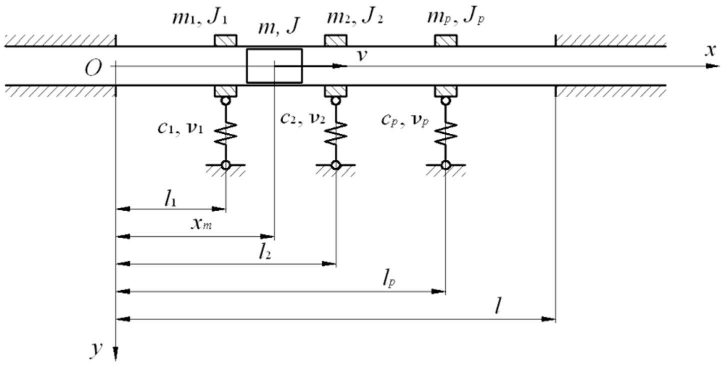

Figure 1. Diagram of the above-ground sections of the pipeline with a moving diagnostic piston

The function, that describes the curved axis of the pipeline section, is presented in the form

$$y(x,t) = \sum_{i=1}^{n} Y_i(t) \cdot \psi_i(x), \tag{1}$$

where t – time; $y(x,t)$ – deflection of the pipeline; $\psi_i(x)$ – shapes of oscillations, which must be chosen so that the boundary conditions are fulfilled at the ends of sections; $Y_i(t)$ – amplitude coefficients; n – number of degrees of mechanical system freedom, which is equal to the number of discounted modes of pipeline.

We set depending $\psi_i(x)$ as its own form of transverse fluctuation of rod with pinched ends,

$$\psi_i(x) = \psi_{1i}(x) - \frac{\psi_{1i}(l)}{\psi_{2i}(l)} \cdot \psi_{2i}(x) \quad (i=1,\, 2,\, \ldots,\, n), \tag{2}$$

where

$$\psi_{1i}(x) = \cos k_i x - \operatorname{ch} k_i x\, ; \quad \psi_{2i}(x) = \sin k_i x - \operatorname{sh} k_i x\, . \tag{3}$$

Results of $k_i l$, which are calculated for lower ten own forms:

i	1	2	3	4	5	6	7	8	9	10
$k_i l$	4,730	7,853	10,996	14,137	17,279	20,420	23,562	26,704	29,846	32,989

Taking the generalized coordinate values $Y_i(t)$ (i = 1, 2, …, n), we apply the Lagrange equations of the second kind to describe the motion of a mechanical system,

$$\frac{d}{dt}\left(\frac{\partial \mathrm{T}}{\partial \dot{q}_i}\right) - \frac{\partial \mathrm{T}}{\partial q_i} + \frac{\partial \Pi}{\partial q_i} + \frac{\partial \Phi}{\partial \dot{q}_i} = Q_i \qquad (i = 1,\, 2,\, \ldots,\, n), \tag{4}$$

where T and Π – kinetic and potential energies; q_j – generalized coordinate; Φ – Rayleigh function; Q_j – non-conservative generalized force.

The kinetic energy is written in the form of

$$\begin{aligned}
\mathrm{T} = {} & \frac{\rho A}{2}\int_0^l \left(\frac{\partial y(x,t)}{\partial t}\right)^2 dx + \frac{\rho I_z}{2}\int_0^l \left(\frac{\partial^2 y(x,t)}{\partial x \partial t}\right)^2 dx + \\
& + \frac{m}{2}\left\{\left[\frac{dx_m(t)}{dt}\right]^2 + \left[\left(\frac{\partial y(x,t)}{dx}\right)_{x=x_m} \cdot \frac{dx_m(t)}{dt} + \frac{\partial y(x_m,t)}{\partial t}\right]^2\right\} + \\
& + \frac{J}{2}\left[\frac{\partial}{\partial x_m}\left(\left(\frac{\partial y(x,t)}{\partial x}\right)_{x=x_m}\right) \cdot \frac{dx_m(t)}{dt} + \left(\frac{\partial^2 y}{\partial x \partial t}\right)_{x=x_m}\right]^2 + \\
& \frac{1}{2}\sum_{k=1}^{p} m_k \left[\frac{\partial y(l_k,t)}{\partial t}\right]^2 + \frac{1}{2}\sum_{k=1}^{p} J_k \left[\left(\frac{\partial^2 y(x,t)}{\partial x \partial t}\right)_{x=l_k}\right]^2 .
\end{aligned} \tag{5}$$

The potential energy of pipe section deformation is expressed as

$$\Pi = \frac{EI_z}{2}\int_0^l \left(\frac{\partial^2 y(x,t)}{\partial x^2}\right)^2 dx + \frac{1}{2}\sum_{k=1}^{p} c_k \left[y(l_k,t)\right]^2 . \tag{6}$$

Rayleigh function, which is used to calculate the energy dissipation of fluctuations

$$\Phi = \frac{\nu I_z}{2}\int_0^l \left[\frac{\partial^3 y(x,t)}{\partial x^2 \partial t}\right]^2 dx + \frac{1}{2}\sum_{k=1}^{p} \nu_k \left[\frac{\partial y(l_k,t)}{\partial t}\right]^2 , \tag{7}$$

where ν – coefficient of hysteresis material deviation from Hooke's law.

Considering the relationships (1), we summarize the expressions (5) – (7) to the following form

$$
\mathrm{T} = \frac{\rho A}{2}\int_0^l \left[\sum_{i=1}^{n} \dot{Y}_i(t)\cdot\psi_i(x)\right]^2 dx + \frac{\rho I_z}{2}\int_0^l \left[\sum_{i=1}^{n} \dot{Y}_i(t)\cdot\psi_i{}'(x)\right]^2 dx +
$$

$$
+\frac{m}{2}\left\{[v(t)]^2 + \left[v(t)\sum_{i=1}^{n} Y_i(t)\cdot\psi_i{}'(x_m) + \sum \dot{Y}_i(t)\cdot\psi_i(x_m)\right]^2\right\} +
$$

$$
+\frac{J}{2}\left[v(t)\sum_{i=1}^{n} Y_i(t)\cdot\psi_i{}''(x_m) + \sum_{i=1}^{n} \dot{Y}_i(t)\cdot\psi_i{}'(x_m)\right]^2 +
$$

$$
\frac{1}{2}\sum_{k=1}^{p} m_k\left[\sum_{i=1}^{n}\dot{Y}_i(t)\psi_i(l_k)\right]^2 + \frac{1}{2}\sum_{k=1}^{p} J_k\left[\sum_{i=1}^{n}\dot{Y}_i(t)\psi_i'(l_k)\right]^2;
$$

$$
\Pi = \frac{EI_z}{2}\int_0^l \left[\sum_{i=1}^{n} Y_i(t)\cdot\psi_i{}''(x)\right]^2 dx + \frac{1}{2}\sum_{k=1}^{p} c_k\left[\sum_{i=1}^{n} Y_i(t)\psi_i(l_k)\right]^2;
$$

$$
\Phi = \frac{\nu I_z}{2}\int_0^l \left[\sum_{i=1}^{n} \dot{Y}_i(t)\cdot\psi_i{}''(x)\right]^2 dx + \frac{1}{2}\sum_{k=1}^{p} \nu_k\left[\sum_{i=1}^{n} \dot{Y}_i(t)\psi_i(l_k)\right]^2. \qquad (8)
$$

We transform relationships (8) to a suitable form for the equations of mechanical system motion,

$$
\mathrm{T} = \frac{\rho A}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}\dot{Y}_i\dot{Y}_j + \frac{\rho I_z}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}\dot{Y}_i\dot{Y}_j +
$$

$$
+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}(ma_{mij} + Jb_{mij})\dot{Y}_i\dot{Y}_j + v(t)\sum_{i=1}^{n}\sum_{j=1}^{n}(md_{mij} + Je_{mij})Y_i\dot{Y}_j +
$$

$$
+\frac{[v(t)]^2}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}(mb_{mij} + Jc_{mij})Y_iY_j + \frac{m[v(t)]^2}{2} +
$$

$$
+\frac{1}{2}\sum_{k=1}^{p}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(m_k a_{kij} + J_k b_{kij}\right)\dot{Y}_i\dot{Y}_j;
$$

$$
\Pi = \frac{EI_z}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}Y_iY_j + \frac{1}{2}\sum_{k=1}^{p}\sum_{i=1}^{n}\sum_{j=1}^{n} c_k a_{kij}\, Y_iY_j;
$$

$$
\Phi = \frac{\nu I_z}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}\dot{Y}_i\dot{Y}_j + \frac{1}{2}\sum_{k=1}^{p}\sum_{i=1}^{n}\sum_{j=1}^{n} \nu_k a_{kij}\, \dot{Y}_i\dot{Y}_j, \qquad (9)
$$

where

$$a_{ij} = \int_0^l \psi_i(x)\psi_j(x)dx\,; \quad b_{ij} = \int_0^l \psi_i'(x)\psi_j'(x)dx\,; \quad c_{ij} = \int_0^l \psi_i''(x)\psi_j''(x)dx\,;$$

$$a_{mij} = \psi_i(x_m)\psi_j(x_m)\,; \quad b_{mij} = \psi_i'(x_m)\psi_j'(x_m)\,; \quad c_{mij} = \psi_i''(x_m)\psi_j''(x_m)\,;$$

$$d_{mij} = \psi_i'(x_m)\psi_j(x_m)\,; \quad e_{mij} = \psi_i''(x_m)\psi_j'(x_m)\,;$$

$$a_{kij} = \psi_i(l_k)\psi_j(l_k)\,; \quad b_{kij} = \psi_i'(l_k)\psi_j'(l_k)\,. \tag{10}$$

Differentiating the expression (9) and substituting the obtained results to the equality (4)

$$A\ddot{Y} + B\dot{Y} + CY = Q\,, \tag{11}$$

where $\dot{Y}$ – matrix-column of generalized coordinates,

$$Y = \mathrm{col}\left[Y_1(t), Y_2(t), ..., Y_n(t)\right];$$

A, B, C – square matrices,

$$A_{ij} = \rho A a_{ij} + \rho I_z b_{ij} + m a_{mij} + Y b_{mij} + \sum_{k=1}^{p}\left(m_k a_{kij} + J_k b_{kij}\right), \quad B_{ij} = \nu I_z c_{ij} + \sum_{k=1}^{p} \nu_k a_{kij}\,,$$

$$C_{ij} = \dot{v}(t)(m d_{mij} + Y e_{mij}) - \left[N(t)\right]^2 (m b_{ij} + Y c_{mij}) + E I_z c_{ij} + \sum_{k=1}^{p} c_k a_{kij}\,;$$

Q – matrix-column of generalized forces,

$$Q = \mathrm{col}\left[Q_1, Q_2, ..., Q_n\right].$$

To determine the generalized forces of the system, we write the vertical movement of the gravity center of the diagnostic piston as

$$y(x_m, t) = \sum_{i=1}^{n} Y_i \psi_i(x_m)\,. \tag{12}$$

Virtual work of weight force of piston follows the relationship

$$\delta A = mg \cdot \delta y(x_m, t)\,, \tag{13}$$

where $\delta y(x_m, t)$ – virtual displacement, which is found with considering (12),

$$\delta y(x_m, t) = \sum_{i=1}^{n} \frac{\partial y(x_m, t)}{\partial Y_i}\delta Y_i = \sum_{i=1}^{n} \psi_i(x_m)\delta Y_i\,, \tag{14}$$

moreover, δY_i (i = 1, 2, …, n) – variations of the generalized coordinates.

With taking into account (13), (14) write the virtual work as

$$\delta A = mg\sum_{i=1}^{n}\psi_i(x_m)\delta Y_i = \sum_{i=1}^{n}Q_i\delta Y_i \,. \tag{15}$$

As follows from the relationship (15), generalized forces are defined by the dependencies

$$Q_i = mg\psi_i(x_m)\,. \tag{16}$$

Thus, the non-stationary bending vibration of the aboveground pipeline section under the moving diagnostic piston are described by the differential equations (11), solution of which perform with consideration the expressions of the generalized forces (16) and the corresponding initial conditions. If at t = 0 the mechanical system is at resting state, then the value of the generalized coordinates and their time derivative are equal to zero, i.e.,

$$Y_i(0) = 0\,;\quad V_i(0) = \dot{Y}_i(0) = 0 \quad (i\text{=1, 2, …, } n). \tag{17}$$

For the application of the widespread software for solving this task, transform the system of differential equations (11) to the normal form of the Cauchy:

$$\dot{X} = D(t,x)\,, \tag{18}$$

where X, $D(t, x)$ – matrix-column,

$$X = \mathrm{col}(Y, V)\,;$$

$$D(t,x) = \mathrm{col}\left[V,\, A^{-1}(-BV - CY + Q)\right].$$

Thus, the analysis of dynamic phenomena in the mechanical system reduces to the solving the Cauchy problem for a system of $2n$ differential equations (18) with taking into account the dependencies for determination the modes of the pipeline section (2), (3), the generalized forces (16) and the initial conditions (17). After finding the generalized coordinates $Y_i(t)$ (i = 1, 2, …, n) determine pipeline section deflection by the formula (1) and bending moments – by the ratio

$$M(x,t) = EI_z\sum_{i=1}^{n}Y_i(t)\cdot\psi_i''(x)\,, \tag{19}$$

which follows directly from the theory of technical bending.

Taking into account dependencies (19), the maximum bending stress in the cross section of the pipeline is calculated as

$$\sigma(x,t) = \frac{M(x,t)}{W_z} = \frac{Ed}{2}\sum_{i=1}^{n}Y_i(t)\cdot\psi_i''(x)\,,$$

where W_z and d – the resistance moment and the outer diameter of tube cross-section.

3. The calculation results of the dynamic processes and their analysis

The dynamic phenomenas in the multi-section straight pipeline area with a length of 47 m, an outer diameter of 529 mm and a wall thickness of 10 mm during the passage of the diagnostic piston with mass of 1200 kg at a speed of 5 m/s illustrate the graphical dependencies on the Fig. 2.

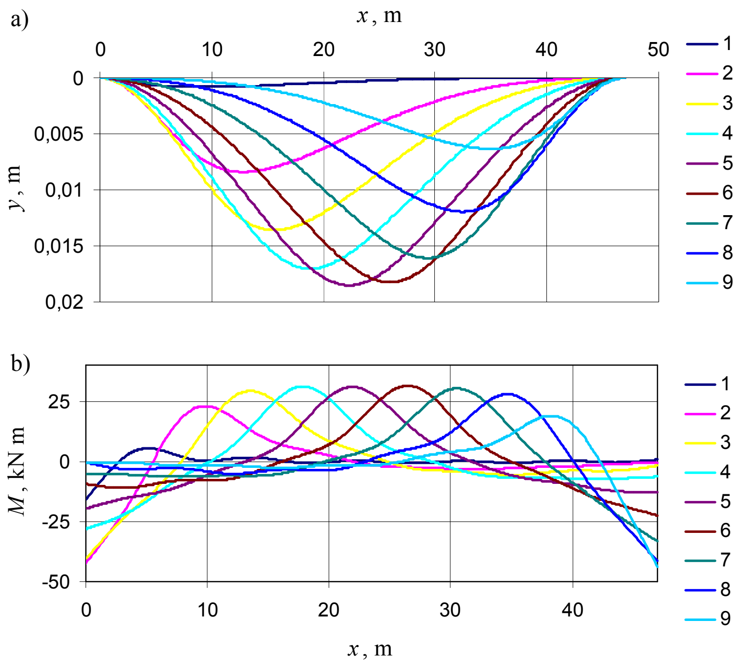

Figure 2. The dependencies of deflections (a) and bending moments (b) from the longitudinal coordinates of the pipeline section. Curves 1–9 correspond to the time points: 0,94 s; 1,88 s; 2,82 s; 3,76 s; 4,7 s; 5,64 s; 6,58 s; 7,52 s; 8,46 s

During the calculation accept that the ends of the pipeline sections are strangulated, in addition, the pipe is based on five intermediate pillars with coordinates l_1 = 7.833 m; l_2 = 15.667 m; l_3 = 23.500 m; l_4 = 31.333 m; l_5 = 39.167 m. The supporting nodes have equal masses m_i = 200 kg, moments of inertia J_i = 12 kg·m2, stiffness $c_i = 2{\cdot}10^5$ N/m and the friction coefficients $\nu_i = 2{\cdot}10^3$ Ns/m, where i = 1, 2, ..., 5. Curves 1, 2, ..., 9 in Fig. 2 correspond to the time points when the diagnostic piston has passed the way 0,1 l; 0,2 l; ...; 0,9 l.

The greatest deflection value of 18.613 mm has the point on the axis of the pipe with coordinate x = 23.400 m at the time moment t = 4,675 s. Significant bending moments that may affect on the strength of the pipeline, arise as in outer cross sections of the area, as well as in cross sections located in the middle of the area. The largest absolute value of the bending moment was 43.814 kN·m and appeared in cross section with coordinate

x = 47 m at the time t = 8,016 s. The maximum bending stress reached value of 21.102 MPa. Note that in the absence of intermediate pillars maximum deflection is 55.944 mm, the maximum absolute value of the bending moment – 83.343 kN·m, the maximum stress – 40.139 MPa. Thus, the installation of the intermediate pillars contributes significantly on stress and strain reducing of the aboveground sections of the pipelines.

The built mathematical model of the bending vibration of the multi-section constructions makes it possible to choose required number of the intermediate pillars and rational values of their stiffness during the design of the aboveground pipeline sections for ensuring the strength of the pipe and supporting constructions. In the operation of the built pipelines proposed calculation algorithm can be used to determine the permissible speed of the diagnostic piston.

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