CEPSTRUM APPLICATION IN DIAGNOSTICS OF MECHANICAL DEVICES

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Summary

The paper presents one of the methods of digital processing of diagnostic signals such as cepstral analysis and the possibility of its application in the investigations of sources of wave disturbances. In order to solve the problem, simulation and diagnostic research was performed. The simulation experiment consisted in generating different sample signals (by special software) containing known frequency-related information, which aimed at exploring the information generated and presented in the cepstrum characteristics and its reliability. The diagnostic experiment was carried out on an mechanical device – a Ford Transit minivan. The object was a basis for the determination of the possibilities of practical application of the cepstrum characteristics.

Keywords: DFT, spectrum, cepstrum

O MOŻLIWOŚCI ZASTOSOWANIA CEPSTRUM W DIAGNOSTYCE URZĄDZEŃ MECHANICZNYCH

Streszczenie

W artykule przedstawiono jedną z metod cyfrowego przetwarzania sygnałów diagnostycznych jaką jest analiza cepstralna i możliwość jej zastosowania w diagnostyce urządzeń mechanicznych do badania źródeł zaburzeń. Dla rozwiązania zadania przeprowadzono badania symulacyjne i diagnostyczne. Eksperyment symulacyjny polegał na generowaniu w specjalnym programie różnych sygnałów próbnych, zawierających znane informacje częstotliwościowe, w celu zbadania informacji wytwarzanych i prezentowanych w charakterystyce cepstrum oraz ich wiarygodności. Natomiast eksperyment diagnostyczny, przeprowadzony na rzeczywistym urządzeniu mechanicznym, jakim był mikrobus Ford Transit, stanowił podstawę do określenia możliwości praktycznego zastosowania charakterystyki cepstrum.

Słowa kluczowe: DFT, widmo, cepstrum

1. INTRODUCTION

An operating mechanical device emits stochastic diagnostic signals. Processing of the signals consists in determining non-random characteristics that facilitate decoding this information. One of such characteristics is a cepstrum, obtained through the DFT procedure from a spectrum of the module of a sampled signal [1, 2, 6].

The applicability of a cepstrum for diagnostic purposes of mechanical devices is hardly explored. It is thus purposeful and useful to explore how the information encoded in the cepstrum characteristics obtained through numerical signal processing is generated and presented.

To this end a numerical experiment was performed. Characteristics of a cepstrum were calculated of simulated signal models reflecting the vibroacoustic signals generated by operating mechanical devices and contain known frequency-related information. The results served the purpose of assessing the reliability of information obtained from the tests on an actual mechanical device.

2. CEPSTRUM

The Fourier transform of time realization *f(t)*, received in a finite time *T* is:

$$F(\nu) = \int_{-T/2}^{T/2} f(t) \cdot \exp(-j2\pi\nu t)dt \qquad (1)$$

where: ν [Hz] - frequency: $\nu = \omega / 2\pi$,
ω [rad/s] – angular frequency.

A cepstrum is a Fourier transform of a logarithm to base 10, spectral density of the signal *f(t),* obtained through an integral (1):

$$\bar{f}(\bar{t}) = \int_0^{\nu_g} \log |F(\nu)|^2 \cdot \exp(-j2\pi \bar{t} \nu) d\nu \qquad (2)$$

where:
$\bar{t}$ [s] - time, cepstrum argument referred to as quefrency,
ν_g [Hz] – boundary frequency of low pass filtering.

When $F(\nu)$ is a result of a feedback of several wave disturbances being the components of the signal, logarithms will separate them. Operator exp($-j2\pi\bar{t}\,\nu$) selects the components for the subsequent values of quefrency $\bar{t}$ and averages them in the frequency range [0; ν_g), in which the cepstrum was determined [1, 2].

From A/C conversions that can be notated in the form of map relation: $f(t)$: $t \in [-T/2; T/2] \rightarrow f(k)$: $k = 0, 1, 2, ..., N-1$, a series of values are obtained that are a database to perform the discrete Fourier transform:

$$F(n) = \sum_{k=0}^{N-1} f(k) \cdot \exp(-j2\pi nk/N) \qquad (3)$$

where:
N – number of signal samples.

For processing period T_e, time and frequency are discrete variables: $t_k = k \cdot T_e$, $\nu_n = n \cdot \nu_T$. The reception time $T = N \cdot T_e$ and frequency $\nu_T = 1/T = 1/N \cdot T_e$ – determine the resolution of a discrete spectrum in the range $[0; \nu_{Nyq})$, where Nyquist frequency: $\nu_{Nyq} = \nu_e/2$ is marked with index $N/2$, for: $n = 0, 1, 2, ..., N/2-1$=int. The total multiplicity $n \cdot \nu_T$ determines the set of admissible frequencies for which information may appear [3, 4].

Signals generated by operating devices are composed of harmonic components that depend on the nature of the source of wave disturbances and are independent of $n \cdot \nu_T$, which is why frequencies of many components are not within the set of the admissible ones. In the signal spectrum, the DFT procedure determines each of such components in an approximate way through an additional spectrum. It is a sum of non-existent signal components determined according to the Fourier series for the given values $n \cdot \nu_T$. A discrete signal spectrum is disturbed because the true information is transferred through false information [4, 5].

In the domain of dimensionless indexes a discrete form of a cepstrum can be obtained by changing integral (2) into sum and substituting *F(n)* [4]:

$$\bar{f}(\bar{k}) = \sum_{n=0}^{N/2-1} \log |F(n)|^2 \cdot \exp(-j2\pi \bar{k} n/N) \qquad (4)$$

Module $|F(n)|$: $n = 0, 1, ..., N/2-1$ is calculated from relation (3) in the interval [0; ν_{Nyq}). This interval determines the periodization period of the module depending on (4). In order to avoid aliasing, a cepstrum should be calculated for the quefrency: $\bar{k} = 0, 1, 2, ..., N/4-1$ [3].

3. SIMULATION EXPERIMENT

The tested signal model was generated from a computer programmed discrete spectrum, in which the determined harmonic components, carrying known information were placed in a random background with a random phase angle [3]. Figure 1 presents the discrete module spectrum composed of three harmonic components and a tracing of the signal model generated from such a spectrum.

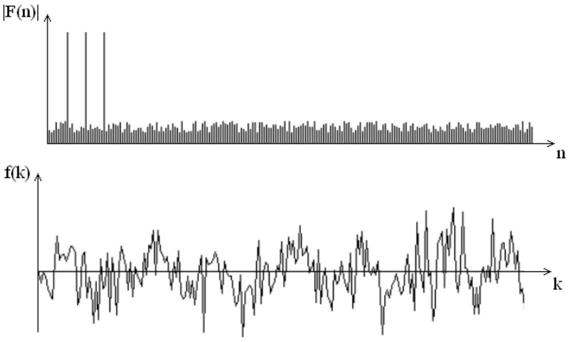

Fig. 1. The model of the signal and the spectrum

A cepstrum of four signal models was examined. The first three were generated from undisturbed spectrums, when all the components have frequencies from the set of admissible frequencies. The first signal is a single harmonic component without background. The spectrum of the second one is composed of one narrow-band local maximum and the third- two narrow-band local maximums, both in a random background (I and II simulation in table 1). The maximums may reflect an increased operating intensity of the sources of wave disturbances e.g. bearings or gear contact surfaces. Table 1 contains the parameters of the spectrums of the signal models in a dimensionless domain of indexes.

Table 1. Parameters of spectrums of the tested signals

Parameter	**Type of spectrum**					
	monoharmonic spectrum	**emission spectrum**			**actual spectrum**	
		Simulation	I emission	II emission	I emission	II emission
Location of the emission in the spectrum (centre)	32	I	32	x	32	128
		II	32	128		
Number of the emissions	1		9	9	9	9
Emission height	1		2	2	2	2
Height of the edge	1		1	1	1	1
Phase	random	random			random	
Background	zero	random			random	

The signal models were determined in the domain of dimensionless indexes. In order to compare the results of the simulation research with the diagnostics of mechanical devices, the most frequently applied parameter values of signal reception and processing were adopted. Table 2 presents these values. Figures 2 and 3, 4 and 5, 6 and 7 show the discrete spectrums and the cepstrum of these signals.

Table 2. Assumptions for the simulation of the signal models

Parameter	**Value**
Number of samples N [-]	2048
Sampling frequency ν_e [Hz]	1000
Sampling period T_e [s]	0,001
Signal reception time T [s]	2,048
Spectrum resolution ν_T [Hz]	0,49
Nyquist frequency ν_{Nyq} [Hz]	500
Resolution in cepstrum characteristics $\overline{T_e}$ [s]	0,002

For the adopted number of samples of signal N and sampling frequency ν_e, the outstanding parameters of the signal reception and processing were calculated from the following relations:

-sampling period (T_e)::

$$T_e = \frac{1}{\nu_e} \tag{5}$$

- signal reception time (T):

$$T = N \cdot T_e = \frac{N}{\nu_e} \tag{6}$$

- spectrum resolution (ν_T):

$$\nu_T = \frac{1}{T} \tag{7}$$

- Nyquist frequency (ν_{Nyq}):

$$\nu_{Nyq} = \frac{\nu_e}{2} \tag{8}$$

- resolution in the cepstrum characteristics ($\overline{T_e}$):

$$\overline{T_e} = \frac{T/2}{N/4} \tag{9}$$

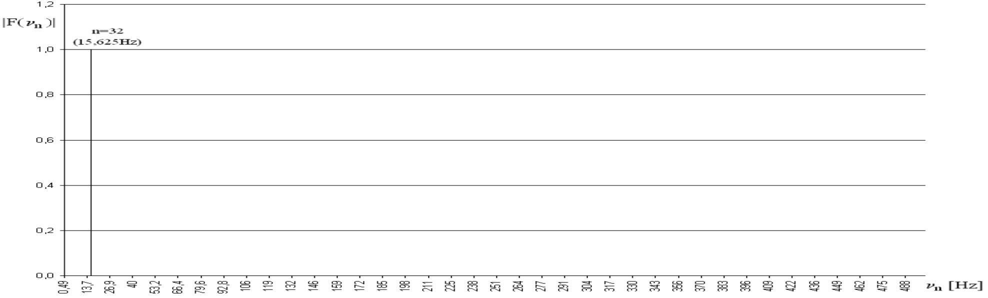

Fig. 2. Monoharmonic signal spectrum

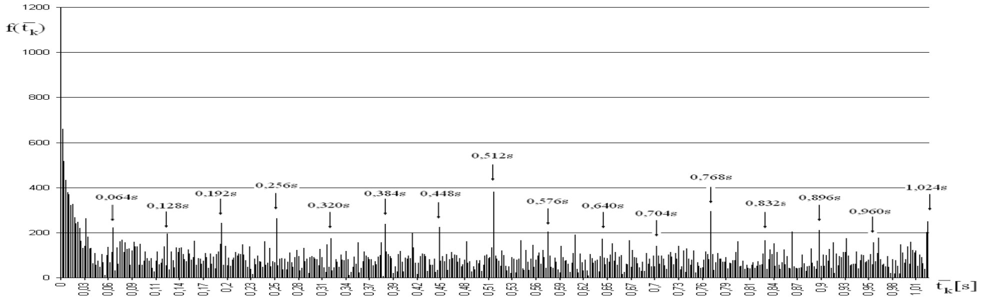

Fig. 3. Monoharmonic signal cepstrum

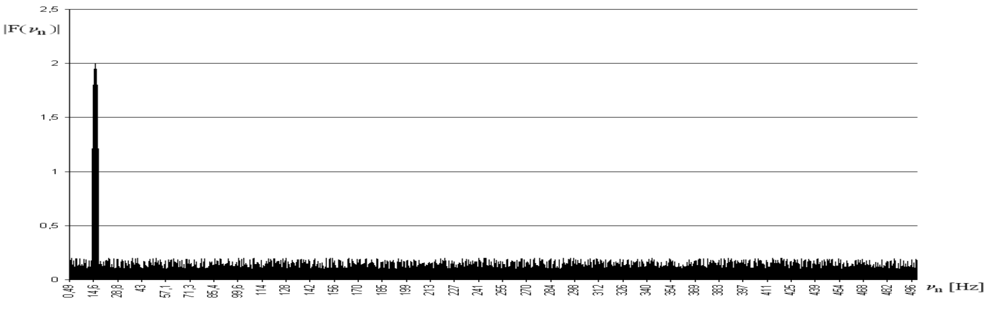

Fig. 4. A spectrum of a signal with one local maximum

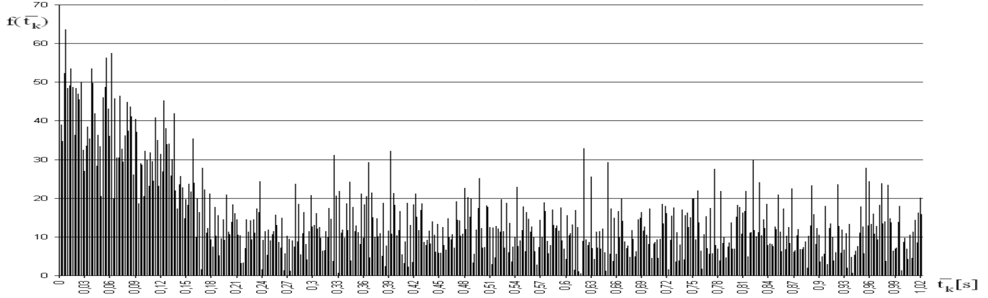

Fig. 5. A cepstrum of a signal with one local maximum

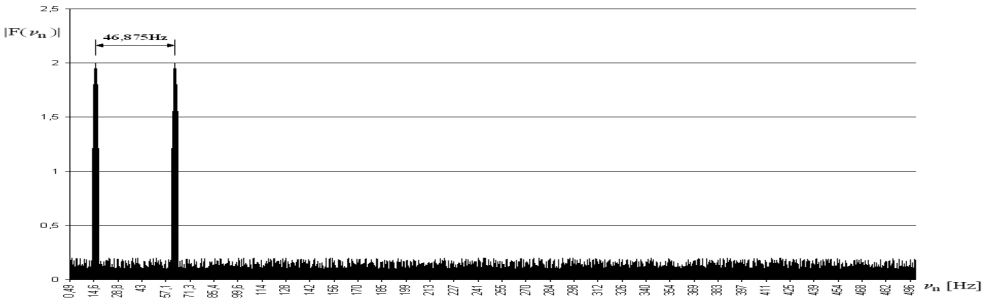

Fig. 6. A spectrum of a signal with two local maximums

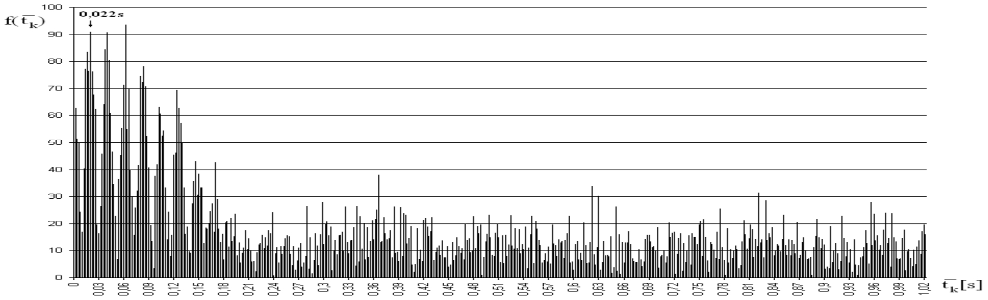

Fig. 7. A cepstrum of a signal with two local maximums

In order to reflect the disturbances of the spectrums of discrete signals occurring in reality emitted by operating mechanical devices, a model of the fourth signal was developed by shortening of the third signal by half preserving the same values of the outstanding parameters. As a consequence, the frequency of the second component of this signal falls outside of the set of admissible frequencies. Additional spectrums, representing these components disturb the discrete spectrum of this model. The parameters of the signal reduced by half are presented in table 3. The tracings of the spectrum of the fourth signal are shown in figures 8 and 9.

Table 3. Parameters of the fourth signal

Parameter	**Value**
Number of samples N [-]	1024
Sampling frequency ν_e [Hz]	1000
Sampling period T_e [s]	0,001
Signal reception time T [s]	1,024
Spectrum resolution ν_T [Hz]	0,98
Nyquist frequency ν_{Nq} [Hz]	500
Resolution in cepstrum characteristics $\overline{T}_e$ [s]	0,002

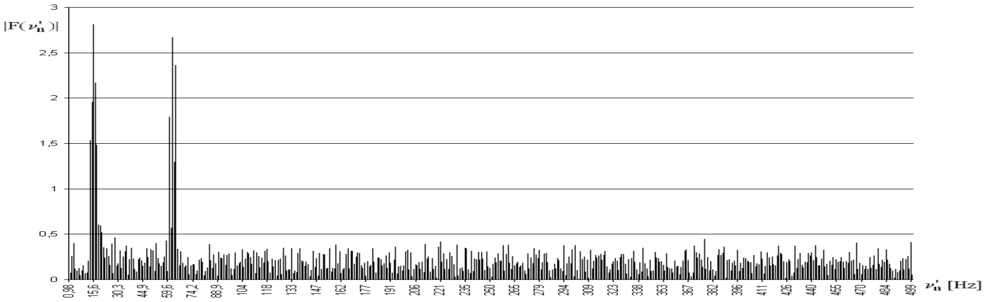

Fig. 8. A spectrum of 'actual signal'

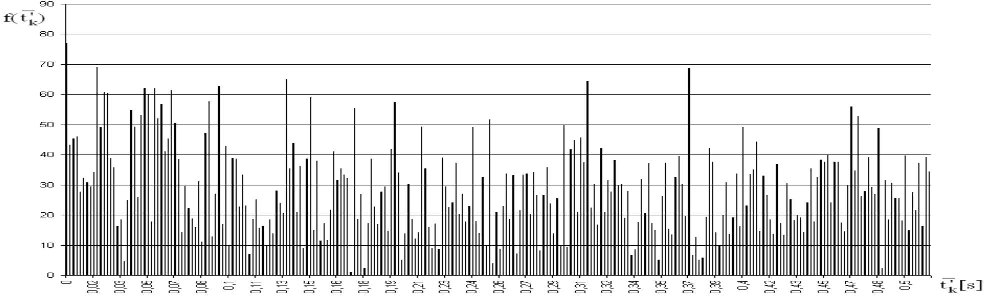

Fig. 9. A cepstrum of actual 'signal'

4. A DIAGNOSTIC EXPERIMENT

In the diagnostic experiment, performed on a minivan (Ford Transit) two signals were taken and processed into a discrete form:

1) acceleration of the vibrations of the engine block perpendicular to the axis of the crankshaft at the engine speed of 1500 rpm,
2) sound pressure level in an empty vehicle in the mid- section of the cockpit.

To receive the signals, a two-channel measurement system was used (shown in Fig. 10). It includes an accelerometer, measurement microphone, low-pass filter, pre-amplifier, connecting port, A/C converter and a PC. Table 4 presents the reception and processing parameters of the A/C vibroacoustic signals.

Fig. 10. Measurement equipment

Table 4. The parameters of reception and processing of A/C vibroacoustic signals

Parameter	**Value**
Number of samples N [-]	4096
Sampling frequency ν_e [Hz]	7000
Sampling period T_e [s]	0,0001429
Signal reception time T [s]	0,585
Spectrum resolution ν_T [Hz]	1,71
Nyquist frequency ν_{Nq} [Hz]	3500
Boundary frequency of filtering ν_g [Hz]	2048
Engine speed n [rpm]	1500

Figures 11 to 14 show the discrete spectrums and cepstrum tracings of vibroacoustic signals. On the spectrum tracings the marked local maximums are a result of the reciprocating motion of the piston assembly.

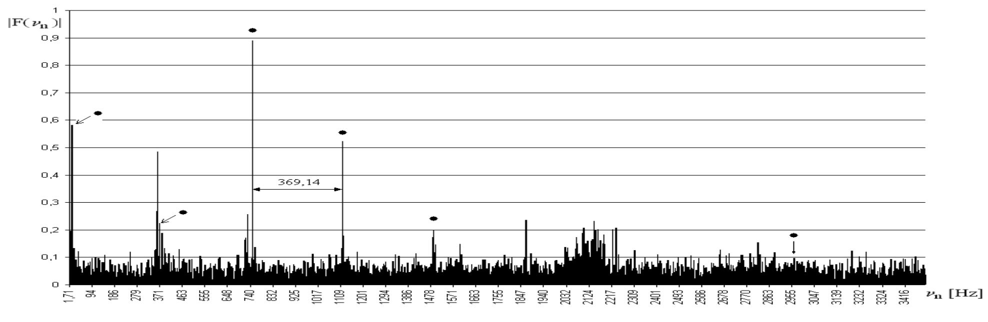

Fig. 11. Acoustic signal spectrum

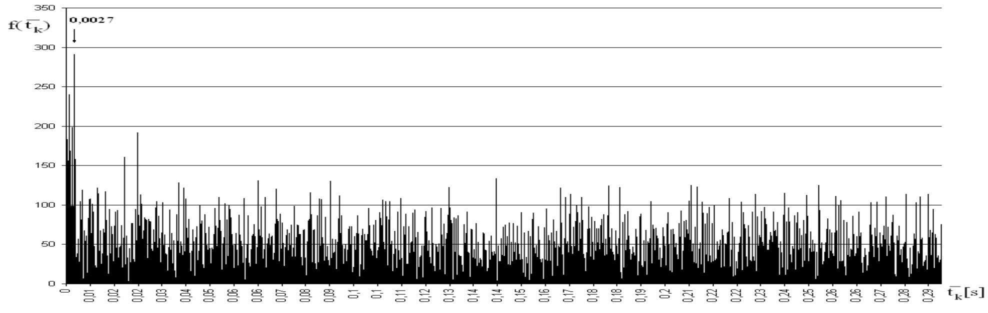

Fig. 12. Acoustic signal cepstrum

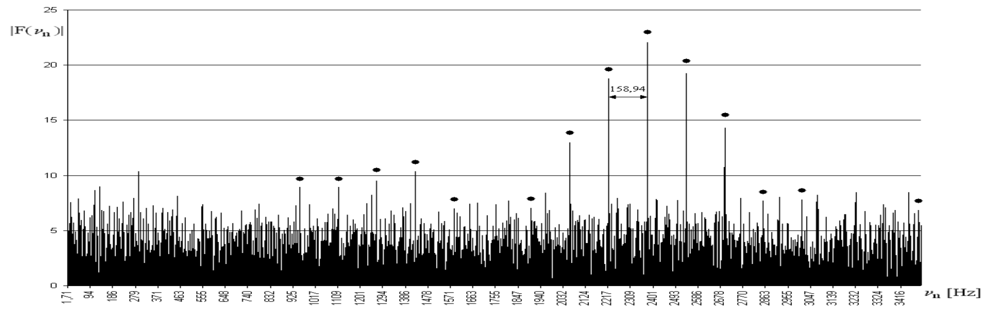

Fig. 13. Vibration signal spectrum

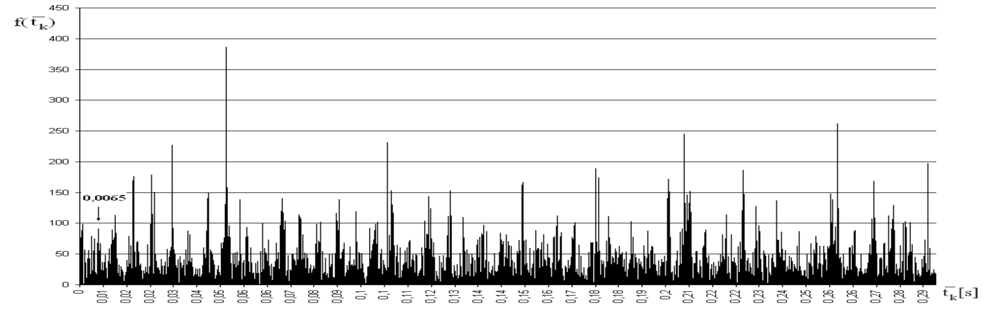

Fig. 14. Vibration signal cepstrum

5. CONCLUSIONS

The conducted research enabled a preliminary assessment of how information encoded in a numerically processed vibroacoustic signal emitted by an operating device is generated in the cepstrum characteristics.

Based on the simulation research, one can observe that in the cepstrum course the domain of time corresponds to the wave periods. Investigations of signals containing a single harmonic component without background have shown that the cepstrum procedure, apart from the simulated wave, generates harmonic components of the period equal to the total multiplicity of this wave's period, which do not exist in the signal. A similar phenomenon can be observed in the cepstrum of signals whose undisturbed discrete spectrums contain emission local maximums.

The investigations have also shown that the disturbances resulting from the existence of signal components of the frequencies that are not within the set of admissible frequencies in the discrete spectrum, have significant impact on the reliability of potential information contained in the cepstrum. A disturbance of the signal spectrum leads to a disturbance of the course of the cepstrum.

In the tested vehicle, many sources of wave disturbances were incited. The local maximums that appear in the discrete spectrums of acoustic and vibration signals prove the increased intensity of these sources. Knowing the sources of the disturbances and the conditions of operation of a device these maximums can be identified.

The tracings of the cepstrum of the acoustic and vibration signals, obtained following the diagnostic experiment, differ significantly. In the course of the cepstrum of the vibration signal, periodical local maximums occur representing the waves that were not identified in the spectrum. A phenomenon previously observed in the simulation research may be the case here; the periodical nature of the consecutive local maximums is generated by the cepstrum procedure. In the cepstrum of the acoustic signal, one cannot confirm the existence of periodical local maximums.

The performed experiments indicate a lack of reliability of the information presented in the cepstrum of vibroacoustic signals emitted by operating mechanical devices. The research proves that decoding of the diagnostic information contained in the signal related to individual sources of disturbances and their identification based on the cepstrum characteristics is impossible with this state of knowledge. The fundamental reason for this may be the unknown principle of operation of the numerical procedure of calculation of the cepstrum from a discrete spectrum or the influence of unknown disturbances (mainly acoustic) occurring during the operation of a mechanical device.

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