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MODELLING OF GROUND-SUPPORTED STORAGE CONTAINER USING FEM

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1. Introduction

Ground-supported tanks are used to store a variety of liquids, e.g. water for drinking and firefighting, petroleum, chemicals, and liquefied natural gas. The analysis and design of liquid storage tanks is, in fact, due to the high complexity of the problem, a really complicated task. Number of particular problems should be taken into consideration, for example: interaction between contained fluid and tank and interaction between tank and sub-soil. Those belong to wide range of so called fluid structure interactions (FSI). The tank-soil interaction could have a significant effect on seismic response of the tank under specific conditions [8-10].

The knowledge of forces, pressures acting onto walls and the bottom of containers, and response of liquid storage tanks plays essential role in their reliable and durable design.

2. Fluid-structure interaction by using Finite Element Method

For the fluid-structure interaction analysis, there are possible three different finite element approaches to represent fluid motion: Eulerian, Lagrangian, and mixed methods. In the Eulerian approach, velocity potential (or pressure) is used to describe the behavior of the fluid, whereas the displacement field is used in the Lagrangian approach. In the mixed approaches, both the pressure and displacement fields are included in the element formulation [2-5].

In fluid-structure interaction analyses, fluid forces are applied into the solid and the solid deformation changes the fluid domain. For most interaction problems, the computational domain is divided into the fluid domain and solid domain, where a fluid model and a solid model are defined respectively, through their material data, boundary conditions, etc. The interaction occurs along the interface of the two domains. Having the two models coupled, we can perform simulations and predictions of many physical phenomena [4,8].

In many fluid flow calculations, the computational domain remains unchanged in time. Such problems involve rigid boundaries and are suitable handled in Eulerian formulation of equilibrium equations [6-9]. In the case, where the shape of the fluid domain is expected to change significantly, modified formulation called Arbitrary Lagrangian-Eulerian (ALE) formulation was adopted to simulate the physical behavior of the domain of interest properly. The ALE description is designed to follow the boundary motions rather than the fluid particles. Thus, the fluid particles flow through a moving FE-mesh. Basically there are two different algorithms available for generation of possible moving mesh:

- remeshing of fluid domain, which is computationally expensive procedure,
- rezoning of FE-mesh of fluid domain; this procedure is quite fast while precise enough if no “dramatic” changes of fluid domain is expected.

2.1. Governing Equations

Dynamic equilibrium of fluid domain involving effect of moving mesh describes modified Navier-Stokes equations. Let us to assume temperature independent problem. Then the balance of momentum by ALE formulation is

$$\rho\left[\frac{\partial \mathbf{v}}{\partial t}+\left(\mathbf{v}-\mathbf{v}_b\right)\cdot\nabla\mathbf{v}\right]=\nabla\cdot\left(-p\,\mathbf{I}+\boldsymbol{\tau}\right)+\rho\,\mathbf{g}\ , \tag{1}$$

where ρ – density of fluid, $\mathbf{v}$ – velocity of fluid, $\mathbf{v}_b$ – velocity of moving FE-mesh, p – fluid pressure, $\mathbf{I}$ – unit matrix, $\boldsymbol{\tau}$ – stress tensor, $\mathbf{g}$– gravity acceleration.

Dynamic equilibrium of solid domain governs balance of momentum – e.g. in Cauchy form, it is

$$div\,\boldsymbol{\tau}+\rho_0\left(\mathbf{b}-\ddot{\mathbf{u}}\right)=0\,, \tag{2}$$

where: ρ_0 – density of solid in initial configuration, $\mathbf{u}$ – displacement, $\mathbf{b}$ – body load.

Together with traditional boundary conditions defined for fluid domain (pressure and velocity), additional special conditions are considered:

- free surface, the interface between fluid and gas,
- FSI boundary, common boundary between solid and fluid.

2.2. Fluid domain. Free surface. Boundary conditions.

Dynamic boundary conditions for free surface expresses a balance of forces between interactive forces of liquid and gas, and they are as follows

$$\begin{aligned}\mathbf{f}_l\cdot\mathbf{n}+\sigma K&=-\mathbf{f}_g\cdot\mathbf{n}\\ \mathbf{f}_l\cdot\mathbf{t}+\sigma K&=-\mathbf{f}_g\cdot\mathbf{t}\\ \mathbf{f}_l\cdot\mathbf{s}+\sigma K&=-\mathbf{f}_g\cdot\mathbf{s}\end{aligned} \tag{3}$$

where $\mathbf{f}_l$ and $\mathbf{f}_g$ are forces exerted by liquid and gas, respectively, $\mathbf{t}$ and $\mathbf{n}$ are tangent and normal to FSI surface, respectively, and $\mathbf{s}$ is surface tension (if present).

Kinematic boundary condition states the velocity at a point of free surface moves together with point of FE-mesh. Thus

$$\left(\mathbf{v}-\mathbf{v}_b\right)\cdot\mathbf{n}=0\,. \tag{4}$$

2.3. FSI boundary conditions

Dynamic boundary condition defines stresses at the common FSI boundary which is opposite and equal

$$\sigma_f = \sigma_s \tag{5}$$

Kinematic boundary condition assumes that velocities and displacements of FSI boundary are the same

$$\begin{aligned} \mathbf{v}_f &= \mathbf{v}_s \\ \mathbf{u}_f &= \mathbf{u}_s \end{aligned} \tag{6}$$

where indexes f and s refer to fluid and solid respectively.

2.4. Discretization by finite elements

Any of unknown physical variables in Finite Element Method is expressed in terms of nodal values instead of field value. That causes local discontinuity of the problem, but globally, with regards to whole FE model all governing equations, are satisfied.

Unknown variables (displacement, velocity and pressure) are approximated using so called shape functions **N**

$$\hat{\mathbf{u}} = \mathbf{N}\mathbf{U}, \qquad \hat{\mathbf{v}} = \mathbf{N}\mathbf{V}, \qquad \hat{\mathbf{p}} = \mathbf{N}\mathbf{P}\,, \tag{7}$$

where **U**, **V**, and **P** are nodal values of initially unknown fields.

Applying one of appropriate variation principle, governing equations are transformed into integral form, in which interpolations (7) are being easily incorporated and followingly proceeded in numerical calculation.

As the governing equations are basically nonlinear and time dependent, an appropriate linearization should be used together with a discretization in time domain. Plenty of methods by linearization and time discretization were published in the past. Software ADINA, which was used within the framework of the presented article, has implemented some of most popular of them [1,10].

3. Example and results

As an example case, we will assume the ground supported rectangular endlessly long shipping channel, with the length L = 5 m and the height H_w = 3 m. Channel surrounding walls have the uniform thickness of 0.25 m. The base slab of the channel is h = 0.4 m thick. Shipping channel is filled with water up to the height of 2.6 m. There is no roof slab structure covering the channel. This water filled tank is grounded on hard subsoil (Fig. 1).

Response of concrete open top rectangular liquid storage tanks - chipping channel was performed by application of Finite Element Method (FEM) utilizing software ADINA 8.3.1. Arbitrary-Lagrangian-Eulerian (ALE) formulation was used for the problem (Model F). Two way Fluid-Structure Interaction (FSI) techniques were used for simulation of the interaction between the structure and the fluid at the common boundary. The solid walls and base of the shipping channel was modeled by using 2D SOLID finite element under plain strain condition. The fluid inside the shipping channel was modeled by using 2D FLUID finite elements.

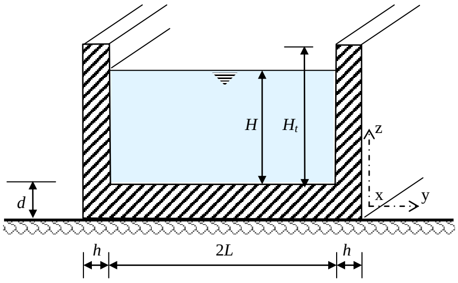

Fig. 1. Shipping channel

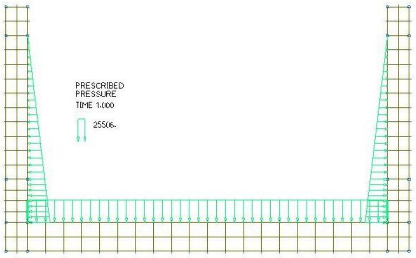

Fig. 2 Scheme of Model S

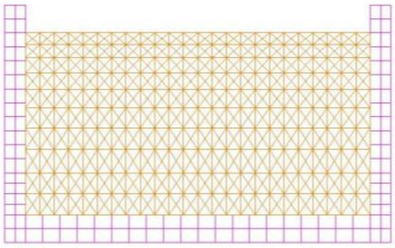

Fig. 3. Scheme of Model F - simulation FSI ALE FEM

Fig. 4. Horizontal displacement at reservoir

Fig. 5. Horizontal displacement at reservoir in time $t = 8$ s

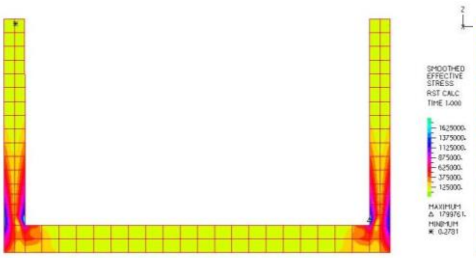

Fig. 6. Von Mises stress in reservoir

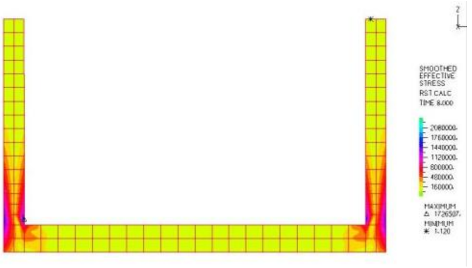

Fig. 7. Von Mises stress in time $t = 8$ s

The alternative approach of numerical simulation by application of FEM and utilizing software ADINA was considered. Only the solid walls and base (excluding physical representation of fluid field) of the shipping channel were modeled by using 2D SOLID finite element under plain strain condition. The effect of fluid interaction was simulated as the static load acting onto the walls and the bottom of the tank (Model S).

Table 1. Comparison of results by using procedure of statical FEM solution and by modeling FEM ALE FSI

	FEM	**FEM ALE FSI**
Maximal horizontal displacement at reservoirs [mm]	0.7348	1.7998
Maximal Von Mises stress in reservoirs [MPa]	0.7392	1.7265

4. Conclusion

The ground supported rectangular endlessly long open top shipping channel having the length of $L = 5$ m and the height $H_w = 3$ m was analysed. The channel was partially filled with the water, grounded on sub-soil of 30 MNm^{-3}. Basic responses of the interest were: structural deformation and stress distribution over the tank.

Response of concrete open top rectangular liquid storage tanks - chipping channel was performed by application FEM ALE FSI problem (Model F) and the alternative approach of numerical simulation by application FEM, where the effect of fluid interaction was simulated as the static load acting onto the walls and the bottom of the tank (Model S). It correlates well with peak value of maximal horizontal displacement at reservoirs and maximal Von Mises stress in reservoirs.

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Summary

Ground-supported cylindrical tanks are used to store a variety of liquids. This paper provides the theoretical background for numerical simulation by application FEM ALE FSI. Two models were used for calculating response of concrete open top rectangular liquid storage tanks - chipping channel. It was performed by application FEM (Model S) and FEM ALE FSI (Model F). The former uses an alternative approach of numerical FEM simulation where only solid domain of tank was modeled by FEM and the effect of fluid interaction was simulated as the static load acting onto the walls and the bottom of the tank. As an example case we will assume the ground supported rectangular endlessly long concrete shipping channel filled with water, without roof slab structure covering the channel and grounded on hard sub-soil.