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A variant of the Narayana coding scheme*

by

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Abstract: In this paper we use second order variant of the Narayana sequence to frame a universal coding scheme. Earlier, the Narayana series has been used by Kirthi and Kak to represent universal coding. This paper provides an extension, based on the paper by Kirthi and Kak. The second order variant Narayana universal coding is used in source coding as well as in cryptography.

Keywords: Fibonacci sequence, Zeckendorf's representation, Narayana sequence, universal code, Narayana code straight line

1. Introduction

In the today's technology driven world, we have been witnessing a great change, consisting in the dramatic development of information technology. But, by the very same token, a very relevant and important question has arisen concerning the security of data in communication systems. Security of sending and receiving information has indeed become a major problem in the present digital world. Lots of cyber crime are being observed across the world. The tug-of-war is continuously going on between the code makers and the code breakers. Facing this challenging situation, the scientists are also continuously trying to improve the coding systems, so that they might become a truly hard nut to crack.

In coding theory, generally, the source message is mapped into binary codewords of different lengths through the universal code, where the applied probability distribution is monotonic. There are several universal and non-universal codes. Elias codes (Gamma, Delta & Omega), Fibonacci universal code, Levenshtein code, or byte coding are good examples of universal codes, whereas non-universal codes include unary coding, which is used in Elias code, Rice coding used in the FLAC audio code, Huffman coding, and Golomb code, which

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encompasses Rice coding and unary coding as special cases, etc. (see, e.g., Thomas, 2007; Platos et al., 2007; Buschmann and Bystrykh, 2013; Malvar, 2006).

Peter Elias introduced a universal code known as Elias gamma code. It is one of the simplest universal codes, and it is widely used in digital communications. It helps to encode the positive integers, whose upper bound cannot be determined beforehand. The merit of this coding is that the time requirement for compression and decompression algorithms for cases where decompression time is a critical issue are advantageous for this coding (Elias, 1975; Filmus, 2013).

Goldbach conjecture states that every even integer greater than 2 can be expressed as a sum of two primes. Inverse sequence may be used to frame a universal code in terms of Goldbach conjecture Thus, an even number may be decomposed into two constituent primes and we can encode the ordinal numbers of these primes with an Elias code. Elias "alpha" code has the simplest length specifications, and the codes, based on Goldbach conjecture may be considered as the extension of "alpha" code. The only difference is the way of termination.

Then, the so-called Fibonacci code is a universal code, by which positive integers are encoded in the form of binary codewords. It is an example of representation of integers based on Fibonacci numbers. Each codeword ends with “11” and contains no other instances of “11” before the end. The Fibonacci code is very close to the Zeckendorf representation, a positional numeral system that uses Zeckendorf’s theorem and has the property that no number has a representation including consecutive 1s. The Fibonacci codeword for a particular integer is exactly the integer’s Zeckendorf representation with the order of its digits reversed and an additional “1” appended to the end. Fibonacci universal code has a useful property that sometimes makes it attractive in comparison to other universal codes. Namely, with this code, it is easier to recover data from a damaged stream. With most other universal codes, if a single bit is altered, none of the data that come after it will be correctly read. On the other hand, with Fibonacci universal coding, a changed bit may cause one token to be read as two, or cause two tokens to be read incorrectly as one, but reading a 0 from the stream will stop the errors from propagating further. Since the only stream that has no 0 in it is the stream of 11 tokens, the total edit distance between a stream damaged by a single bit error and the original stream is at most three (Basu and Prasad, 2010).

In his famous book "Ganita Kaumudi" Pandit Narayana (1325-1400) wrote, in particular, on sequences, which are closely related to Fibonacci sequences. Fibonacci describes his sequence using the famous rabbit metaphor, whereas Narayana describes his sequence by proposing the cows-in-the-field number development. It is now widely used in data coding and cryptography. In 2016, Kirthi and Kak (2016) presented a method of universal coding based on the Narayana sequence.

Agarwal, Agarwal and Saxena (2015) made use of the Fibonacci sequence to encode the plain text by changing it into Unicode. They use it as a cipher text. For encryption and decryption of data Dubey, Verma and Gaur (2017) utilized the genetic algorithm. To enhance the data security, an improved cryptographic technique has been applied by Gupta, Singh and Gupta (2012). They used block cipher method in sending confidential data. Cryptographic models, also based on Fibonacci sequence, have been presented by Mukherjee and Samantha (2014), as well as Raphael and Sundaram (2012), where they utilized Fibonacci sequence through Unicode symbols to ensure high level security. In creation of the cipher text, emphasis has been placed by them on two level security structure. In their cryptographic model Singh, Sisodaya and Ahmed (2014) made reference to some products of k-Fibonacci and k-Lucas numbers and tried to investigate the correlation between them. Basu and Prasad (2010) proposed long range variations on the Fibonacci universal code by using Gopala-Hemchandra sequence. The presence of multiple representations of the same integer allows for producing a codebook that appears larger than it actually is. This provides a great cryptographic advantage that facilitates overcoming of difficulties in the decoding procedure, i.e. strengthens the security aspect of the coding theory. Buschmann and Bystrykh (2013) used error-correcting barcodes for multiplexed DNA sequencing. But none of them derives the Narayana code straight line.

The Narayana code straight line is very useful in cryptographic security. The Narayana code straight line has unique second order variant Narayana universal codeword, which importantly strengthens the security in cryptography due to the formation of straight lines. Therefore, the ability to form Narayana code universal straight line is a token of benchmark in the improvement of security within the cryptographic domain.

This paper studies the second order variant of Narayana sequence to design a universal coding scheme. We derive Narayana code straight line and observe that each Narayana code straight line has a unique second order variant Narayana universal codeword. It is very important and significant observation, as we can strengthen the security in sending information by using second order variant Narayana coding scheme due to the formation of straight lines.

The present paper is organised in a total of six sections. Section 1 provides the introduction along with the associated research works from the literature. Section 2 defines some preliminaries, third section describes the variant of Narayana coding scheme, which is considered here. Then, very short Sections 4 and 5 conclude the paper with indication of the novelty of the results presented, a summary and some future research directions, while the paper closes with the list of references.

2. Preliminaries

2.1. Fibonacci sequence

Fibonacci number $F(k)$ is defined by the second order linear recurrence relation

$$F(k+1) = F(k) + F(k-1), \tag{1}$$

where $F(1) = 1$, $F(2) = 2$.

2.2. Zeckendorf's representation

Zeckendorf's theorem states that "Every positive integer has a unique representation as the sum of non consecutive Fibonacci numbers" (Zeckendorf, 1972). In other words, every positive integer n has a unique representation of the form $n = \sum_{k=1}^{l} a_k F(k)$ where $a_k \in \{0, 1\}$, such that the string $a_1 a_2 a_3 \ldots$ does not contain any consecutive 1's and $F(k)$ are Fibonacci numbers. This representation is defined as Zeckendorf's representation (Daykin, 1960). Therefore, while the recursive nature of Fibonacci numbers allowa some integers to have multiple representations using the above process, e.g. 10 can be represented as $F(2) + F(3) + F(4)$ or $F(2) + F(5)$, then in Zeckendorf's representation, it is uniquely represented by $F(2) + F(5)$.

While the term "Zeckendorf representation" is properly used only in reference to the standard Fibonacci sequence, we will use it when discussing similar representations of numbers based on the variants of Fibonacci sequence. In 1960, Daykin (Daykin, 1960) proved that only the standard Fibonacci sequence $F(k)$ gives a unique Zeckendorf's representation for all positive integers.

2.3. Narayana sequence

Narayana's sequence can be explicitly written down as follows

$1, 1, 1, 2, 3, 4, 6, 9, 13, 19, 28, 41, 60, 88, 129, 189, 277, 406, 595, 872, 1278, 1873, 2745, 4023, 5896, 8641, 12664, 18560, 27201, 39865, 58425, 85626, 125491, 183916, 269542, 395033, 578949, 848491, 1243524, 1822473, 2670964, 3914488, 5736961, 8407925$

this sequence of numbers being described by the recurrence relation

$$N(k+1) = N(k) + N(k-2), \tag{2}$$

with the initial terms $N(0) = N(1) = N(2) = 1$.

To explain the series Narayana proposed his famous cows and calves model. How it is obtained? The generation table is given below.

The family $N(k) = N(k-1) + N(k-m)$ with $N(k) = 1$ for $n = 0, \ldots, m-1$ can be generated by considering the sums:

Table 1. Generation of the Narayana sequence

1	1	1	1	1	1	1	1	1	1	1	1	1	...
			1	2	3	4	5	6	7	8	9	10	...
						1	3	6	10	15	21	28	...
									1	4	10	20	...
												1	...
													...
1	1	1	2	3	4	6	9	13	19	28	41	60	...

3. A variant of the Narayana coding scheme

A more general Narayana sequence $N_a(k)$ is given by

$$a, b, c, a+c, a+b+c, a+b+2c, 2a+b+3c, 3a+2b+4c, \text{and so on,}$$
$$\text{with } a=1, b=2 \text{ and } c=3. \tag{3}$$

The Narayana universal code can be obtained by the generalisation of Fibonacci universal code. To do this, we have to map the source code, represented by the positive integers, into variable length codewords. Thomas (2007) described the respective procedure in details.

A variant of Narayana coding scheme can be obtained by defining a second order variation of the Narayana sequence, $VN_a(k)$, such that $b = 3 - a$ and $c = 1 - a$. This yields $VN_a(0) = a$ $(a \in Z)$, $VN_a(1) = 3 - a$, $VN_a(2) = 1 - a$ and for $k \geq 3$, $VN_a(k) = VN_a(k-1) + VN_a(k-3)$ (Kirthi et al, 2016).

In the light of the above definition, we get a variant of the Narayana series

$$VN_{-2}(n) \text{ as } \{-2, 5, 31, 6, 9, 10, 16, 25, \ldots\},$$

and

$$VN_{-5}(n) = \{-5, 8, 6, 1, 9, 15, 16, 25, 40, \ldots\}, \text{ and so on.}$$

In this paper, we study the question for what values of n the second order variant Narayana universal code is available for $a \leq -1$. All the positive integers are not encoded by the Narayana sequence. The Narayana Universal codewords for some positive integers are displayed in the Tables 2 and 3 at the end of this paper. In these tables, N/A indicates that the Narayana universal codeword does not exist or is not available for the given integer.

By considering (a, n) as a point in the (x, y) plane, from Tables 2 and 3 we conclude that:

1. For $a \leq -1$, there is a straight line $y = 1$ such that the points $(a, 1)$ lie on this line, yielding the Narayana codeword 00011.

2. For $a \leq -1$, there is a straight line $y = 4$ such that the points $(a, 4)$ lie on this line, yielding the Narayana codeword 100011.
3. For $a \leq -1$, there are four straight lines $y + x = i$, for $i = 1, 3, 4, 5$ such that the four points $(a, 1-a)$, $(a, 3-a)$, $(a, 4-a)$, $(a, 5-a)$ lie on these lines for $i = 1, 3, 4, 5$, respectively, yielding the respective Narayana codewords 0011, 011, 000011 and 101011.
4. For $a \leq -1$, there are four straight lines $y + 2x = i$, for $i = 5, 6, 7, 10$ such that the four points $(a, 5 - 2a)$, $(a, 6 - 2a)$, $(a, 7 - 2a)$, $(a, 10 - 2a)$ lie on these lines for $i = 5, 6, 7, 10$, respectively, yielding the respective Narayana codewords 0000011, 00000011, 00010011, and 10001011.
5. For $a \leq -1$, there are four straight lines $y + 3x = i$, for $i = 7, 9, 10, 11$ such that the four points $(a, 7 - 3a)$, $(a, 9 - 3a)$, $(a, 10 - 3a)$, $(a, 11 - 3a)$ lie on these lines for $i = 7, 9, 10, 11$, respectively, yielding the respective Narayana codewords 00100011, 01000011, 000000011, and 000100011.
6. For $a \leq -1$, there are three straight lines $y + 4x = i$, for $i = 11, 13, 14$ such that the three points $(a, 11-4a)$, $(a, 13-4a)$, $(a, 14-4a)$ lie on these lines for $i = 11, 13, 14$, respectively, yielding the respective Narayana codewords 001000011, 010000011 and 000010011.
7. For $a \leq -1$, there are two straight lines $y + 5x = i$, for $i = 15, 16$ such that the two points $(a, 15 - 5a)$, $(a, 16 - 5a)$ lie on these lines for $i = 15, 16$, respectively, yielding the respective Narayana codewords 0001000011 and 000010011.
8. For $a \leq -1$, there are three straight lines $y + 6x = i$, for $i = 16, 18, 19$ such that the three points $(a, 16-6a)$, $(a, 18-6a)$, $(a, 19-6a)$ lie on these lines for $i = 16, 18, 19$, respectively, yielding the respective Narayana codewords 0010000011, 0100000011 and 0000100011.

Now we define another sequence, $W(k)$, by

$$W(k) = W(k-1) + W(k-3), \tag{4}$$

where $W(0) = -1, W(1) = W(2) = 1$.

Theorem 1 $VN_a(k) = VN_0(k) - aW(k-1), \quad k \geq 1$.

Proof: $\{a, 3-a, 1-a, 1, 4-a, 5-2a, 6-2a, 10-3a, 15-5a, 21-8a, \dots\}$ represents the sequence $VN_a(k)$.

Therefore, we can write

$$VN_a(1) = a = 0 - a(-1) = VN_0(1) - aW(0)$$

and

$$VN_a(2) = 3 - a(1) = VN_0(2) - aW(1).$$

Thus, the result is true for $k = 1$ and 2.

Let the result be true for $k = 1, 2, 3, \ldots, m$. Then,

$VN_a(m-2) = VN_0(m-2) - aW(m-3)$ and $VN_a(m) = VN_0(m) - aW(m-1)$.

Therefore,

$$\begin{aligned} & VN_a(m+1) = VN_a(m) + VF_a(m-2) \\ & = VN_0(m-2) - aW(m-3) + VN_0(m) - aW(m-1) \\ & = (VN_0(m-2) + VN_0(m)) - a(W(m-3) + W(m-1)) \\ & = VN_0(m+1) - aW(m-1). \end{aligned}$$

Hence, by induction, we can write

$$VN_a(k) = VN_0(k) - aW(k-1), \quad k \geq 1. \tag{5}$$

□

THEOREM 2 *The point (a, n) gives the Narayana codeword $a_1 a_2 \ldots a_l 1$ if and only if the point (a, n) satisfies the equation of the straight line*

$$y + \{\sum_{k=1}^{l} a_k VN_0(k)\}x = \sum_{k=1}^{l} a_k W(k-1)$$

where $a_k \in \{0, 1\}$ and the string $a_1 a_2 \ldots a_l$ does not contain any consecutive 1's.

PROOF: Let (a, n) give the codeword $a_1 a_2 \ldots a_l 1$. Then

$$n = \sum_{k=1}^{l} a_k VN_a(k) = \sum_{k=1}^{l} a_k VN_0(k) - a \sum_{k=1}^{l} a_k W(k-1),$$

by Theorem 1. The point

$$(a, \sum_{k=1}^{l} a_k W(k-1) - a \sum_{k=1}^{l} a_k VN_0(k))$$

lies on the straight line

$$y + \{\sum_{k=1}^{l} a_k VN_0(k)\}x = \sum_{k=1}^{l} a_k W(k-1).$$

Hence, the point (a, n) lies on the straight line

$$y + \{\sum_{k=1}^{l} a_k VN_0(k)\}x = \sum_{k=1}^{l} a_k W(k-1).$$

Conversely, let the point (a, n) lie on the straight line

$$y + \{\sum_{k=1}^{l} a_k VN_0(k)\}x = \sum_{k=1}^{l} a_k W(k-1).$$

Therefore, for the point (a, n) we have

$$n = \sum_{k=1}^{l} a_k VN_0(k) - a \sum_{k=1}^{l} a_k W(k-1) = \sum_{k=1}^{l} a_k VN_a(k),$$

by Theorem 1.

Hence, for $a \leq -1$, the Gopala-Hemchandra codeword of (a, n) is $a_1 a_2 \dots a_l$, where $a_k \in \{0, 1\}$ and the string $a_1 a_2 \dots a_l$ does not contain any consecutive 1's. □

Corollary 1 *$(a, VN_a(k))$ lies on the straight line $y + VN_0(k)x = W(k-1)$.*

Definition 1 *If all the integral points (a, n) for $a \leq -1$, $n \geq 1$ on a straight line have second order variant Narayana universal codeword, then we call this construct the Narayana code straight line. Otherwise it is called Non-Narayana code straight line.*

Note 1 *The point (a, n), satisfying more than one Narayana code straight line equation, does not have unique second order variant Narayana universal codeword.*

Note 2 *The point (a, n), lying at the intersection of the Narayana code straight line and the Non-Narayana code straight line, gives the second order variant Narayana universal codeword corresponding to the Narayana code straight line.*

Note 3 *Each Narayana code straight line has unique second order variant Narayana universal codeword.*

4. Objective and novelty

The objective of the present paper is to develop a highly secure coding scheme by using Narayana sequence.

The novelty of this paper is to frame the Narayana code straight line as an essential component of the approach, which appears to be very useful in improving the cryptography security.

5. Conclusion

In this paper a universal coding scheme has been derived using Narayana sequence. We also form a straight line named as Narayana Straight Line to strengthen the security in the cryptography. We hope that our proposed model will help to reveal a new dimension in the arena of coding theory. The model proposed here may be extended in future by the findings of deeper properties of straight line or formation of other forms of curves.

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Table 2. Second order variant Narayana universal Code

n	NY_{-1}	NY_{-2}	NY_{-3}	NY_{-4}	NY_{-5}	NY_{-6}	NY_{-7}	NY_{-8}	NY_{-9}	NY_{-10}
1	00011	00011	00011	00011	00011	00011	00011	00011	00011	00011
2	0011	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
3	N/A	0011	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
4	011	100011	0011	100011	100011	100011	100011	100011	100011	100011
5	000011	011	N/A	0011	N/A	N/A	N/A	N/A	N/A	N/A
6	101011	000011	011	N/A	0011	N/A	N/A	N/A	N/A	N/A
7	0000011	101011	000011	011	N/A	0011	N/A	N/A	N/A	N/A
8	00000011	N/A	101011	000011	011	N/A	0011	N/A	N/A	N/A
9	00010011	0000011	N/A	101011	000011	011	N/A	0011	N/A	N/A
10	00100011	00000011	N/A	N/A	101011	000011	011	N/A	0011	N/A
11	N/A	00010011	0000011	N/A	N/A	101011	000011	011	N/A	0011
12	01000011	N/A	00000011	N/A	N/A	N/A	101011	00011	011	N/A
13	000000011	00100011	00010011	0000011	N/A	N/A	N/A	101011	000011	011
14	000100011	10001011	N/A	00000011	N/A	N/A	N/A	N/A	101011	000011
15	001000011	01000011	N/A	00010011	0000011	N/A	N/A	N/A	N/A	101011
16	N/A	000000011	00100011	N/A	00000011	0000011	N/A	N/A	N/A	N/A
17	010000011	000100011	N/A	N/A	00010011	00000011	N/A	N/A	N/A	N/A
18	000010011	N/A	01000011	10001011	N/A	00010011	N/A	N/A	N/A	N/A
19	N/A	001000011	000000011	00100011	N/A	N/A	0000011	N/A	N/A	N/A
20	0000000011	N/A	000100011	N/A	10001011	N/A	00000011	N/A	N/A	N/A
21	0001000011	010000011	N/A	01000011	N/A	N/A	00010011	0000011	N/A	N/A
22	0010000011	000010011	N/A	000000011	00100011	10001011	N/A	00000011	N/A	N/A
23	N/A	N/A	001000011	000100011	N/A	N/A	N/A	00010011	0000011	N/A
24	0100000011	N/A	N/A	N/A	01000011	N/A	10001011	N/A	00000011	N/A
25	0000100011	0000000011	010000011	N/A	000000011	00100011	N/A	N/A	00010011	0000011
26	N/A	0001000011	000010011	N/A	000100011	N/A	N/A	10001011	N/A	00000011
27	0000010011	N/A	N/A	001000011	N/A	N/A	N/A	N/A	N/A	00010011
28	00000000011	0010000011	N/A	N/A	N/A	000000011	N/A	N/A	10001011	N/A
29	00010000011	N/A	N/A	010000011	N/A	000100011	N/A	N/A	N/A	N/A
30	00100000011	0100000011	0000000011	000010011	N/A	N/A	01000011	N/A	N/A	10001011
31	N/A	0000100011	0001000011	N/A	001000011	N/A	000000011	00100011	N/A	N/A
32	01000000011	N/A	N/A	N/A	N/A	N/A	000100011	N/A	N/A	N/A
33	00001000011	N/A	N/A	N/A	010000011	N/A	N/A	01000011	N/A	N/A
34	N/A	0000010011	0010000011	N/A	000010011	N/A	N/A	000000011	00100011	N/A
35	00000100011	0000000011	N/A	0000000011	N/A	001000011	N/A	000100011	N/A	N/A
36	00000010011	00010000011	0100000011	0001000011	N/A	N/A	N/A	N/A	01000011	N/A
37	00010010011	N/A	0000100011	N/A	N/A	010000011	N/A	N/A	000000011	00100011
38	00100010011	00100000011	N/A	N/A	N/A	000010011	N/A	N/A	000100011	N/A
39	N/A	N/A	N/A	N/A	N/A	N/A	001000011	N/A	N/A	01000011
40	01000010011	01000000011	N/A	0010000011	0000000011	N/A	N/A	N/A	N/A	000000011
41	000000000011	00001000011	0000010011	N/A	0001000011	N/A	010000011	N/A	N/A	000100011
42	000100000011	N/A	00000000011	0100000011	N/A	N/A	000010011	001000011	N/A	N/A
43	001000000011	N/A	N/A	0000100011	N/A	N/A	N/A	N/A	N/A	N/A
44	N/A	00000100011	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
45	010000000011	00000010011	N/A	N/A	N/A	0000000011	N/A	010000011	N/A	N/A
46	000010000011	00010010011	00100000011	N/A	0010000011	0001000011	N/A	000010011	N/A	N/A
47	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
48	000001000011	00100010011	01000000011	0000010011	0100000011	N/A	N/A	N/A	N/A	N/A
49	000000100011	N/A	00001000011	00000000011	0000100011	N/A	N/A	N/A	010000011	N/A
50	000100100011	01000010011	N/A	00010000011	N/A	N/A	0000000011	N/A	000010011	N/A

Table 3. Second order variant Narayana universal Code

n	NY_{-11}	NY_{-12}	NY_{-13}	NY_{-14}	NY_{-15}	NY_{-16}	NY_{-17}	NY_{-18}	NY_{-19}	NY_{-20}
1	00011	00011	00011	00011	00011	00011	00011	00011	00011	00011
2	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
3	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
4	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
5	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
6	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
7	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
8	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
9	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
10	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
11	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
12	0011	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
13	N/A	0011	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
14	011	N/A	0011	N/A	N/A	N/A	N/A	N/A	N/A	N/A
15	000011	011	N/A	0011	N/A	N/A	N/A	N/A	N/A	N/A
16	N/A	000011	011	N/A	0011	N/A	N/A	N/A	N/A	N/A
17	N/A	N/A	000011	011	N/A	0011	N/A	N/A	N/A	N/A
18	N/A	N/A	N/A	000011	011	N/A	0011	N/A	N/A	N/A
19	N/A	N/A	N/A	N/A	000011	011	N/A	0011	N/A	N/A
20	N/A	N/A	N/A	N/A	N/A	000011	011	N/A	0011	N/A
21	N/A	N/A	N/A	N/A	N/A	N/A	000011	011	N/A	0011
22	N/A	N/A	N/A	N/A	N/A	N/A	N/A	000011	011	N/A
23	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	000011	011
24	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	000011
25	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
26	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
27	0000011	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
28	00000011	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
29	00010011	0000011	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
30	N/A	0001011	0000011	N/A	N/A	N/A	N/A	N/A	N/A	N/A
31	N/A	010011	0000011	N/A	N/A	N/A	N/A	N/A	N/A	N/A
32	N/A	N/A	0001011	N/A	N/A	N/A	N/A	N/A	N/A	N/A
33	N/A	N/A	00010011	0000011	N/A	N/A	N/A	N/A	N/A	N/A
34	N/A	N/A	N/A	0001011	N/A	N/A	N/A	N/A	N/A	N/A
35	N/A	N/A	N/A	00010011	0000011	N/A	N/A	N/A	N/A	N/A
36	N/A	N/A	N/A	N/A	0001011	N/A	N/A	N/A	N/A	N/A
37	N/A	N/A	N/A	N/A	00010011	0000011	N/A	N/A	N/A	N/A
38	N/A	N/A	N/A	N/A	N/A	00000011	N/A	N/A	N/A	N/A
39	N/A	N/A	N/A	N/A	N/A	00010011	0000011	N/A	N/A	N/A
40	N/A	N/A	N/A	N/A	N/A	N/A	00000011	N/A	N/A	N/A
41	N/A	N/A	N/A	N/A	N/A	N/A	00010011	0000011	N/A	N/A
42	000000011	N/A	N/A	N/A	N/A	N/A	N/A	00000011	N/A	N/A
43	000100011	00100011	N/A	N/A	N/A	N/A	N/A	00010011	0000011	N/A
44	N/A	0100011	N/A	N/A	N/A	N/A	N/A	N/A	00000011	N/A
45	N/A	01000011	N/A	N/A	N/A	N/A	N/A	N/A	00010011	0000011
46	N/A	000000011	00100011	N/A	N/A	N/A	N/A	N/A	N/A	00000011
47	N/A	000100011	N/A	N/A	N/A	N/A	N/A	N/A	N/A	00010011
48	N/A	N/A	01000011	N/A	N/A	N/A	N/A	N/A	N/A	N/A
49	N/A	N/A	000000011	00100011	N/A	N/A	N/A	N/A	N/A	N/A
50	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A