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A FRAMEWORK FOR AN INVENTORY MODEL FOR DETERIORATING ITEMS WITH EXPIRATION UNDER TRAPEZOIDAL-TYPE DEMAND AND PARTIAL BACKLOGGING

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ABSTRACT. **Background:** As in the case of deteriorating items, expiration plays a major role in inventory management, a generalized approach is studied based on an inventory model for deteriorating items with expiration dates.
Methods: In this paper, the demand rate during the cycle time is assumed to be trapezoidal. Shortages are allowed and partially backlogged. An inventory replenishment strategy is formulated for trapezoidal demand rates and backlogging rates in general for retailers; with the support of fundamental concepts of calculus.
Results: The method is illustrated with the support of numerical examples along with sensitivity analysis with respect to major parameters.
Conclusion: This generalized approach provides a platform to develop a strategy using different demand functions and backlogging rates.

Key words: Inventory, deteriorating items with expiration, trapezoidal demand rate, partial backlogging.

INTRODUCTION

In the present business environment, the effect of a deterioration on items cannot be ignored in the inventory system. Deterioration is defined as any process which decreases the present value or utility of an item and prevents it from being used for its original use because of continuous spoilage, degradation, evaporation. Such items not only deteriorate but also have their expiration dates. For example, fruits, vegetables and other foodstuffs deteriorate due to spoilage occuring during storage, while electronic goods and photographic film deteriorate because of a gradual loss of utility with time. A significant deterioration occurs during routine storage periods, and as a result, the loss must be taken into account while formulating models. Therefore, many researchers have considered the effect of deterioration when developing inventory policies. An exponentially decaying inventory model was first developed by Ghare and Schrader [1963], who observed that some items shrink with time by a proportion which can be estimated using a negative exponential function of time. Whitin [1957] considered fashion goods to go out of fashion at the end of some period. At the initial studies in this field, most models were developed with a constant deterioration rate, such as Aggarwal [1978], Bhunia and Maiti [1999] etc. Recently, however, many researchers have formulated models for deteriorating inventory systems in different scenarios. A literature survey by Nahmias [1982], Raafat [1991], Shah and Shah [2000], Goyal and Giri [2001] and Bakker et al. [2012] produced an up-to-date review of deteriorating inventory models. However, none

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of the above articles were formulated for deteriorating items with expiration. Wang et al. [2014], Chen et al. [2013] and Shah et al. [2014] considered expiration of a deteriorating item into account.

For the first time, Hill [1995] formulated an inventory model with a ramp-type demand rate. In the case of ramp-type demand rate, the rate of demand increases linearly at the beginning, then remains constant until the end of the replenishment cycle. Such a demand pattern is mostly observed in new brand consumer goods which are likely to be introduced in market. The demand rate of such products is generally an increasing function of time to some extent, then it becomes constant. Many researchers have studied inventory models with ramp-type demand. Cheng and Wang [2009] extended this idea from ramp-type demand to trapezoidal-type demand. Cheng et al. [2011] extended the model to deteriorating items and by allowing shortages, with partial backlogging.

Here, we consider an inventory system for deteriorating items with expiration dates or with a maximum life time during the ordering cycle. For products like fashionable commodities, mobile phones, drugs and other with a short life cycle, the rate of demand increases at the beginning up to a particular level, then stabilizes and become constant. After some time, it starts decreasing due to either the presence of a competitive product or expiration. i.e. the demand rate is assumed to be a continuous trapezoidal function of time. At present, during the stockout period, depending upon the waiting time for the next replenishment, some customers are willing to wait, while others may be impatient and go elsewhere as the waiting time increases. To take account of the effect of shortages, we assume shortages are allowed and partially backlogged with a backlogging rate depending upon the waiting time for the next replenishment. By analyzing this inventory system, we propose an algorithm to define the optimal replenishment policy with the assumption mentioned above. The mathematical formulation is supported by numerical examples and sensitivity analysis is carried out with respect to the major parameters. The outline for the rest of this article is as follows: Section 2 deals with a list of assumptions with notations; section 3 comprises a mathematical formulation of the problem. To support this formulation, the problem is illustrated numerically in section 4. Sensitivity analysis is carried out in section 5. The authors' conclusions are presented in section 6.

ASSUMPTIONS AND NOTATIONS

Notations and assumptions used in the formulation of a model in mathematical form and considered in this article are given below.

1. The inventory system deals with a single item. The replenishment rate is infinite and the lead time is zero or negligible. The planning horizon is assumed to be infinite.
2. Function I(t) represents the level of inventory at any point in time t, where $0 \le t \le T$
3. T is the length of fixed ordering cycle.
4. The demand rate is a positive and trapezoidal type function of time is defined as follows:

$$D(t) = \begin{cases} f(t) \ ; & 0 \le t \le \lambda_1 \\ D_0 \ ; & \lambda_1 \le t \le \lambda_2 \\ g(t) \ ; & \lambda_2 \le t \le T \end{cases} \qquad (1)$$

i.e. $t=\lambda_1$ is the time when increasing demand function *f(t)* turn out to be a constant demand function D_0. And $t = \lambda_2$ is the time when constant demand function D_0 becomes a decreasing function $g(t)$

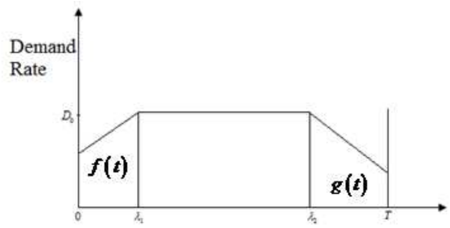

Fig. 1. Trapezoidal type function for demand
Rys. 1. Trapezoidalny typ funkcji popytu

5. During cycle time item does not only deteriorate, but it has maximum life, say *m*.

Therefore, we define the deterioration rate as:

$$\theta(t)=\frac{1}{1+m-t},\quad 0<t\leq m. \qquad (2)$$

Remark: If $t=m$ then item deteriorates completely as $\theta(t)=1$. Also consider either $T<m$ or $m\leq T$. Here, $T<m$ shows the inventory will deteriorate completely after the cycle time, i.e. it can be used to satisfy demand up to the cycle time with an increasing rate of deterioration. However in case of $m\leq T$ inventory deteriorates completely before or up to the cycle time. This case needs to be given greater importance when setting up the cycle time.

6. t_1 is the time at which the level of inventory becomes zero during the ordering cycle.
7. Shortages are allowed and partially backlogged. During stockout period, the backlogging rate is variable depending upon the length of the waiting time for the next refill. The proportion of customers who willing to accept backlogging at time t with a waiting time $(T-t)$ for the next refill. So, we assume backlogging rate as $B(t)$ during $t_1\leq t\leq T$.
8. I_M is the maximum level of inventory for each ordering cycle. And I_B is total number of units backordered. Hence, economic order quantity is $Q=I_M+I_B$
9. We consider: C_d is the cost of deterioration/unit, C_h is the holding cost/unit /time unit, C_b is the backorder cost /unit, C_L is cost due to lost sale/unit.
10. $TC_i(t_1)$, $i=1,2,3$ is the average total cost of an inventory system under different circumstances, respectively.

MATHEMATICAL MODEL

In this section, a mathematical model for the inventory system is developed from the retailer's viewpoint. It is assumed that the inventory system starts with no shortages. The flow of an inventory during ordering cycle is expressed in the Figure 2. At time $t=0$, replenishment occurs and the retailer's opening inventory level is I_M units. This maximum level of inventory reduces to zero at $t=t_1$ with combined effect of deterioration and demand. Shortages occurs during (t_1,T), during this stockout period, demand is partially backlogged with rate B(t).

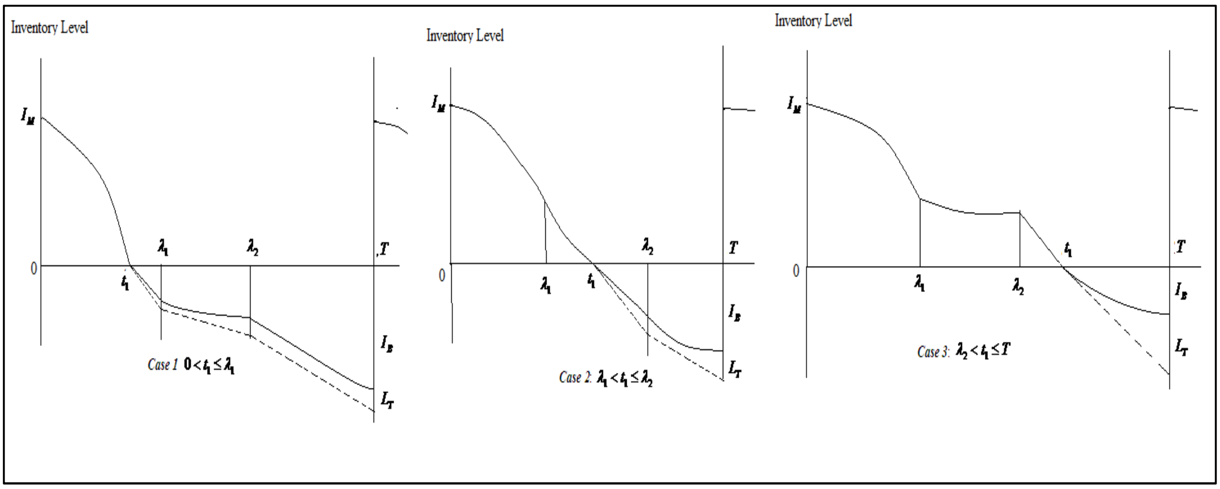

Fig. 2. Inventory level during ordering cycle with different cases
Rys. 2. Poziom zapasów w trakcie cyklu zamawiania w rozpatrywanych sytuacjach

Next, we consider three different cases, which depend on the values of t_1, λ_1 and λ_2. These three cases are summarized as follows:

Case 1: $0 < t_1 \le \lambda_1$

From (2), the differential equations which govern the inventory model are as follows:

$$\frac{dI(t)}{dt} = \begin{cases} -\theta(t) I(t) - f(t) & ; \quad 0 < t < t_1 \\ -B(t) f(t) & ; \quad t_1 < t < \lambda_1 \\ -B(t) D_0 & ; \quad \lambda_1 < t < \lambda_2 \\ -B(t) g(t) & ; \quad \lambda_2 < t < T \end{cases} \tag{4}$$

Using boundary conditions solution of equations (4) is,

$$I(t) = \begin{cases} (1+m-t)\int_t^{t_1} \frac{f(x)}{1+m-x} dx & ; 0 < t < t_1 \\ -\int_{t_1}^{t} B(x) f(x) dx & ; t_1 < t < \lambda_1 \\ -\int_{t_1}^{\lambda_1} B(x) f(x) dx - D_0 \int_{\lambda_1}^{t} B(x) dx & ; \lambda_1 < t < \lambda_2 \\ -\int_{t_1}^{\lambda_1} B(x) f(x) dx - D_0 \int_{\lambda_1}^{\lambda_2} B(x) dx - \int_{\lambda_2}^{t} B(x) g(x) dx & ; \lambda_2 < t < T \end{cases} \tag{5}$$

Now, using condition, $I(0) = I_M$, we have an initial level of inventory as:

$$I_M = (1+m)\int_0^{t_1} \frac{f(x)}{1+m-x} dx \tag{6}$$

The total amount of deterioration during cycle time is,

$$D_T = I_M - \int_0^{t_1} D(x) dx = \int_0^{t_1} \frac{xf(x)}{1+m-x} dx \tag{7}$$

The total amount of inventory H_T present during the interval $[0, t_1]$ is,

$$H_T = \int_0^{t_1} I(t) dt = \int_0^{t_1} (1+m-t)\left(\int_t^{t_1} \frac{f(x)}{1+m-x} dx\right) dt \tag{8}$$

The total shortages occurring during time duration $[t_1, T]$ are,

$$B_T = -\int_{t_1}^{T} I(t)\, dt$$

$$= \int_{t_1}^{\lambda_1}\left[\int_{t_1}^{t} B(x) f(x) dx\right] dt + \int_{\lambda_1}^{\lambda_2}\left[\int_{t_1}^{\lambda_1} B(x) f(x) dx + D_0 \int_{\lambda_1}^{t} B(x) dx\right] dt$$

$$+\int_{\lambda_2}^{T}\left[\int_{t_1}^{\lambda_1} B(x)\, f(x)\,dx + D_0\int_{\lambda_1}^{\lambda_2} B(x)\,dx + \int_{\lambda_2}^{t} B(x)\, g(x)\,dx\right] dt \tag{9}$$

Total loss of sales during $[t_1, T]$ is,

$$L_T = \int_{t_1}^{T}\left(1-B(x)\right)D(x)\,dx$$

$$= \int_{t_1}^{\lambda_1}\left(1-B(x)\right) f(x)\,dx + \int_{\lambda_1}^{\lambda_2}\left(1-B(x)\right)D_0\,dx + \int_{\lambda_2}^{T}\left(1-B(x)\right) g(x)\,dx \tag{10}$$

Maximum backordered inventory during $[t_1, T]$ is,

$$I_B = -I(T) = \int_{t_1}^{\lambda_1} B(x)\, f(x)\,dx + D_0\int_{\lambda_1}^{\lambda_1} B(x)\,dx + \int_{\lambda_2}^{T} B(x)\, g(x)\,dx \tag{11}$$

As $Q = I_M + I_B$

$$= (1+m)\int_{0}^{t_1}\frac{f(x)}{1+m-x}\,dx + \int_{t_1}^{\lambda_1} B(x)\, f(x)\,dx + D_0\int_{\lambda_1}^{\lambda_2} B(x)\,dx + \int_{\lambda_2}^{T} B(x)\, g(x)\,dx \tag{12}$$

Hence, the total average cost per time unit can be defined as follows:

$$C_1(t_1) = \frac{1}{T}\left[A + C_pQ + C_dD_T + C_hH_T + C_bB_T + C_LL_T\right]$$

$$= \frac{A}{T} + \frac{C_P}{T}\left[(1+m)\int_{0}^{t_1}\frac{f(x)}{1+m-x}\,dx + \int_{t_1}^{\lambda_1} B(x)\, f(x)\,dx + D_0\int_{\lambda_1}^{\lambda_2} B(x)\,dx + \int_{\lambda_2}^{T} B(x)\, g(x)\,dx\right]$$

$$+ \frac{C_d}{T}\left[\int_{0}^{t_1}\frac{xf(x)}{1+m-x}\,dx\right] + \frac{C_h}{T}\int_{0}^{t_1}(1+m-t)\left(\int_{t}^{t_1}\frac{f(x)}{1+m-x}\,dx\right)dt$$

$$+ \frac{C_b}{T}\left[\int_{t_1}^{\lambda_1}\left[\int_{t_1}^{t} B(x)\, f(x)\,dx\right]dt + \int_{\lambda_1}^{\lambda_2}\left[\int_{t_1}^{\lambda_1} B(x)\, f(x)\,dx + D_0\int_{\lambda_1}^{t} B(x)\,dx\right] dt\right]$$

$$+ \frac{C_b}{T}\left\{\int_{\lambda_2}^{T}\left[\int_{t_1}^{\lambda_1} B(x)\, f(x)\,dx + D_0\int_{\lambda_1}^{\lambda_2} B(x)\,dx + \int_{\lambda_2}^{t} B(x)\, g(x)\,dx\right] dt\right\}$$

$$+ \frac{C_L}{T}\int_{t_1}^{\lambda_1}\left(1-B(x)\right) f(x)\,dx + \int_{\lambda_1}^{\lambda_2}\left(1-B(x)\right)D_0\,dx + \int_{\lambda_2}^{T}\left(1-B(x)\right) g(x)\,dx \tag{13}$$

Case 2: $\lambda_1 < t_1 \le \lambda_2$

$$\frac{dI(t)}{dt} = \begin{cases} -\theta(t)\,I(t) - f(t) & ; \quad 0 < t < \lambda_1 \\ -\theta(t)\,I(t) - D_0 & ; \quad \lambda_1 < t < t_1 \\ -B(t)\,D_0 & ; \quad t_1 < t < \lambda_2 \\ -B(t)\,g(t) & ; \quad \lambda_2 < t < T \end{cases} \tag{14}$$

Using the boundary conditions solution of equations (14) is,

$$I(t)=\begin{cases}(1+m-t)\left[\int_t^{\lambda_1}\frac{f(x)}{1+m-x}dx+D_0\log\left(\frac{1+m-\lambda_1}{1+m-t_1}\right)\right]; & 0<t<\lambda_1\\ D_0\log\left(\frac{1+m-t}{1+m-t_1}\right) & ;\ \lambda_1<t<t_1\\ -D_0\int_{t_1}^{t}B(x)dx & ;\ t_1<t<\lambda_2\\ -D_0\int_{t_1}^{\lambda_2}B(x)dx-\int_{\lambda_2}^{t}B(x)g(x)dx & ;\ \lambda_2<t<T\end{cases} \tag{15}$$

Now, using the condition, $I(0)=I_M$, we have the initial level of inventory as:

$$I_M=(1+m)\left[\int_0^{\lambda_1}\frac{f(x)}{1+m-x}dx+D_0\log\left(\frac{1+m-\lambda_1}{1+m-t_1}\right)\right] \tag{16}$$

The total amount of deterioration during the cycle time is,

$$D_T=I_M-\int_0^{t_1}D(x)dx=D_0\left[\log\left(\frac{1+m-\lambda_1}{1+m-t_1}\right)-(t_1-\lambda_1)\right]+\left[\int_0^{\lambda_1}\frac{xf(x)}{1+m-x}dx\right] \tag{17}$$

The total amount of inventory H_T present during the interval $[0,\ t_1]$ is,

$$H_T=\int_0^{t_1}I(t)dt=\int_0^{\lambda_1}I(t)dt+\int_{\lambda_1}^{t_1}I(t)dt$$

$$=\int_0^{\lambda_1}(1+m-t)\left\{\int_t^{\lambda_1}\frac{f(x)}{1+m-x}dx+D_0\log\left(\frac{1+m-\lambda_1}{1+m-t_1}\right)\right\}dt$$

$$+\int_{\lambda_1}^{t_1}D_0\left\{\log\left(\frac{1+m-t}{1+m-t_1}\right)\right\}dt \tag{18}$$

The total shortages occurring during the time duration $[t_1,T]$ are,

$$B_T=-\int_{t_1}^{T}I(t)\,dt$$

$$=\int_{t_1}^{\lambda_2}\left\{D_0\int_{t_1}^{t}B(x)dx\right\}dt+\int_{\lambda_2}^{T}\left\{D_0\int_{t_1}^{\lambda_2}B(x)dx-\int_{\lambda_2}^{t}B(x)g(x)dx\right\}dt \tag{19}$$

The total loss of sales during $[t_1,T]$ is,

$$L_T=\int_{t_1}^{T}(1-B(x))D(x)dx\quad=\int_{t_1}^{\lambda_2}(1-B(x))D_0dx+\int_{\lambda_2}^{T}(1-B(x))g(x)dx \tag{20}$$

Maximum backordered inventory during $[t_1,T]$ is,

$$I_B=-I(T)=D_0\int_{t_1}^{\lambda_2}B(x)dx+\int_{\lambda_2}^{T}B(x)g(x)dx \tag{21}$$

As $Q=I_M+I_B=\left[\int_0^{\lambda_1}\frac{(1+m)f(x)}{1+m-x}dx+D_0(1+m)\log\left(\frac{1+m-\lambda_1}{1+m-t_1}\right)\right]$

$$+D_0\int_{t_1}^{\lambda_2} B(x)\,dx+\int_{\lambda_2}^{T} B(x)\,g(x)\,dx \tag{22}$$

Hence, the total average cost per time unit can be defined as follows:

$$C_2(t_1)=\frac{1}{T}\left[A+C_pQ+C_dD_T+C_hH_T+C_bB_T+C_LL_T\right]$$

$$=\frac{A}{T}+\frac{C_P}{T}\left[\int_0^{\lambda_1}\frac{(1+m)\,f(x)}{1+m-x}dx\;+D_0\,(1+m)\log\left(\frac{1+m-\lambda_1}{1+m-t_1}\right)+D_0\int_{t_1}^{\lambda_2}B(x)\,dx\right]$$

$$+\frac{C_P}{T}\left[\int_{\lambda_2}^{T}B(x)\,g(x)\,dx\right]+\frac{C_d}{T}\left[D_0\left[\log\left(\frac{1+m-\lambda_1}{1+m-t_1}\right)-(t_1-\lambda_1)\right]+\left[\int_0^{\lambda_1}\frac{xf(x)}{1+m-x}dx\right]\right]$$

$$+\frac{C_h}{T}\left[\int_0^{\lambda_1}(1+m-t)\left\{\int_t^{\lambda_1}\frac{f(x)}{1+m-x}dx\;+D_0\log\left(\frac{1+m-\lambda_1}{1+m-t_1}\right)\right\}dt+\int_{\lambda_1}^{t_1}D_0\left\{\log\left(\frac{1+m-t}{1+m-t_1}\right)\right\}dt\right]$$

$$+\frac{C_b}{T}\left[\int_{t_1}^{\lambda_2}\left\{D_0\int_{t_1}^{t}B(x)\,dx\right\}dt+\int_{\lambda_2}^{T}\left\{D_0\int_{t_1}^{\lambda_2}B(x)\,dx-\int_{\lambda_2}^{t}B(x)\,g(x)\,dx\right\}dt\right]$$

$$+\frac{C_L}{T}\left[\int_{t_1}^{\lambda_2}\big(1-B(x)\big)D_0dx+\int_{\lambda_2}^{T}\big(1-B(x)\big)g(x)\,dx\right] \tag{23}$$

Case 3: $\lambda_2 < t_1 \le T$

$$\frac{dI(t)}{dt}=\begin{cases}-\theta(t)I(t)-f(t) & ;\quad 0<t<\lambda_1\\ -\theta(t)I(t)-D_0 & ;\quad \lambda_1<t<\lambda_2\\ -\theta(t)I(t)-g(t) & ;\quad \lambda_2<t<t_1\\ -B(t)\,g(t) & ;\quad t_1<t<T\end{cases} \tag{24}$$

Using the boundary conditions solution of equations (25) is,

$$I(t)=\begin{cases}(1+m-t)\left[\int_t^{\lambda_1}\frac{f(x)}{1+m-x}dx+D_0\int_{\lambda_1}^{\lambda_2}\frac{1}{1+m-x}dx+\int_{\lambda_2}^{t_1}\frac{g(x)}{1+m-x}dx\right]; & 0<t<\lambda_1\\ (1+m-t)\left[\int_t^{\lambda_2}D_0\frac{1}{1+m-x}dx+\int_{\lambda_2}^{t_1}\frac{g(x)}{1+m-x}dx\right] & ;\lambda_1<t<\lambda_2\\ (1+m-t)\int_t^{t_1}\frac{g(x)}{1+m-x}dx & ;\lambda_2<t<t_1\\ -\int_{t_1}^{t}B(x)\,g(x)\,dx & ;t_1<t<T\end{cases} \tag{25}$$

Now, using the condition, $I(0)=I_M$, we have an initial level of inventory as:

$$I_M=\left[\int_0^{\lambda_1}\frac{(1+m)\;f(x)}{1+m-x}dx+D_0(1+m)\log\left(\frac{1+m-\lambda_1}{1+m-t_1}\right)+\int_{\lambda_2}^{t_1}\frac{(1+m)\;g(x)}{1+m-x}dx\right] \tag{26}$$

The total amount of deterioration during the the cycle time is,

$$D_T = I_M - \int_0^{t_1} D(x)\,dx$$

$$= \left[\int_0^{\lambda_1} \frac{x f(x)}{1+m-x} dx + D_0 \left\{ (1+m)\log\left(\frac{1+m-\lambda_1}{1+m-t_1}\right) - (\lambda_2 - \lambda_1) \right\} + \int_{\lambda_2}^{t_1} \frac{x g(x)}{1+m-x} dx \right] \quad (27)$$

The total amount of inventory H_T present during the interval $[0, t_1]$ is,

$$H_T = \int_0^{t_1} I(t)\,dt = \int_0^{\lambda_1} I(t)\,dt + \int_{\lambda_1}^{\lambda_2} I(t)\,dt + \int_{\lambda_2}^{t_1} I(t)\,dt$$

$$= \int_0^{\lambda_1} \left\{ (1+m-t) \left[\int_t^{\lambda_1} \frac{f(x)}{1+m-x} dx + D_0 \int_{\lambda_1}^{\lambda_2} \frac{1}{1+m-x} dx + \int_{\lambda_2}^{t_1} \frac{g(x)}{1+m-x} dx \right] \right\} dt$$

$$+ \int_{\lambda_1}^{\lambda_2} \left\{ (1+m-t) \left[\int_t^{\lambda_2} D_0 \frac{1}{1+m-x} dx + \int_{\lambda_2}^{t_1} \frac{g(x)}{1+m-x} dx \right] \right\} dt \quad (28)$$

$$+ \int_{\lambda_2}^{t_1} \left\{ (1+m-t) \int_t^{t_1} \frac{g(x)}{1+m-x} dx \right\} dt$$

The total shortages occurring during time duration $[t_1, T]$ are,

$$B_T = -\int_{t_1}^{T} I(t)\,dt = \int_{t_1}^{T} \left\{ \int_{t_1}^{t} B(x) g(x)\,dx \right\} dt \quad (29)$$

The total loss of sales during $[t_1, T]$ is,

$$L_T = \int_{t_1}^{T} (1 - B(x)) D(x)\,dx = \int_{t_1}^{T} (1 - B(x)) g(x)\,dx \quad (30)$$

Maximum backordered inventory during $[t_1, T]$ is,

$$I_B = -I(T) = \int_{t_1}^{T} B(x) g(x)\,dx \quad (31)$$

As $Q = I_M + I_B = \left[\int_0^{\lambda_1} \frac{(1+m)\ f(x)}{1+m-x} dx + D_0 (1+m)\log\left(\frac{1+m-\lambda_1}{1+m-t_1}\right) + \int_{\lambda_2}^{t_1} \frac{(1+m)\ g(x)}{1+m-x} dx \right]$

$$+ \int_{t_1}^{T} B(x) g(x)\,dx \quad (32)$$

Hence, the total average cost per time unit can be defined as follows:

$$C_3(t_1) = \frac{1}{T}\left[A + C_p Q + C_d D_T + C_h H_T + C_b B_T + C_L L_T \right]$$

$$= \frac{C_p}{T} \left[\int_0^{\lambda_1} \frac{(1+m)\ f(x)}{1+m-x} dx + D_0 (1+m)\log\left(\frac{1+m-\lambda_1}{1+m-t_1}\right) + \int_{\lambda_2}^{t_1} \frac{(1+m)\ g(x)}{1+m-x} dx + \int_{t_1}^{T} B(x) g(x)\,dx \right]$$

$$+ \frac{A}{T} + \frac{C_d}{T} \left[\left[\int_0^{\lambda_1} \frac{x f(x)}{1+m-x} dx + D_0 \left\{ (1+m)\log\left(\frac{1+m-\lambda_1}{1+m-t_1}\right) - (\lambda_2 - \lambda_1) \right\} + \int_{\lambda_2}^{t_1} \frac{x g(x)}{1+m-x} dx \right] \right]$$

$$+\frac{C_h}{T}\left[\begin{array}{l}\int_0^{\lambda_1}\left\{(1+m-t)\left[\int_t^{\lambda_1}\frac{f(x)}{1+m-x}dx+D_0\int_{\lambda_1}^{\lambda_2}\frac{1}{1+m-x}dx+\int_{\lambda_2}^{t_1}\frac{g(x)}{1+m-x}dx\right]\right\}dt\\+\int_{\lambda_1}^{\lambda_2}\left\{(1+m-t)\left[\int_t^{\lambda_2}D_0\frac{1}{1+m-x}dx+\int_{\lambda_2}^{t_1}\frac{g(x)}{1+m-x}dx\right]\right\}dt+\int_{\lambda_2}^{t_1}\left\{(1+m-t)\int_t^{t_1}\frac{g(x)}{1+m-x}dx\right\}dt\end{array}\right]$$

$$+\frac{C_b}{T}\left[\int_{t_1}^{T}\left\{\int_{t_1}^{t}B(x)g(x)dx\right\}dt\right]+\frac{C_L}{T}\left[\int_{t_1}^{T}\left(1-B(x)\right)g(x)dx\right] \qquad (33)$$

From (13), (24) and (33), we can define the total average cost of inventory per time unit over time duration $[0,T]$ as follows:

$$TC(t_1)=\begin{cases}C_1(t_1) & 0<t_1<\lambda_1\\ C_2(t_1) & \lambda_1<t_1<\lambda_2\\ C_3(t_1) & \lambda_2<t_1<T\end{cases} \qquad (34)$$

To derive the optimal ordering policy, we need to locate the optimal value of t_1, say t_1^*, which minimizes the total average cost of an inventory system. To locate the optimal value t_1^* we follow the algorithm given below:

Algorithm:

Step 1: Compute t_1 by solving $\frac{dC_1(t_1)}{dt_1}=0$. If $t_1\in(0,\lambda_1)$ & $\frac{d^2C_1(t_1)}{dt_1^2}>0$, then $C_1(t_1)$ is minimum, else go to step 2.

Step 2: Compute t_1 by solving $\frac{dC_2(t_1)}{dt_1}=0$. If $t_1\in(\lambda_1,\lambda_2)$ & $\frac{d^2C_2(t_1)}{dt_1^2}>0$, then $C_2(t_1)$ is minimum, else go to step 3.

Step 3: Compute t_1 by solving $\frac{dC_3(t_1)}{dt_1}=0$. If $t_1\in(\lambda_2,T)$, $\frac{d^2C_3(t_1)}{dt_1^2}>0$, then $C_3(t_1)$ is minimum.

Step 4: Evaluate the optimal values of order quantity Q and the total average cost of the inventory system TC at the optimal value of t_1.

NUMERICAL EXAMPLE

To validate the mathematical model proposed, we consider the following examples for each case. Costs are defined in $ and time in months.

Example 1 ($0<t_1\le\lambda_1$): We consider $f(t)=a_1+b_1t$, $g(t)=a_2-b_2t$ and $B(t)=e^{-\delta(T-t)}$ Abad (1996) Also, $a_1=100$, $b_1=5,\ a_2=220,\ b_2=10,\ A=200,\ C_d=3,$ $C_h=10,\ C_b=5,\ C_L=10,\ D_0=120,$ $T=12,\ m=12,\ \lambda_1=4,\ \lambda_2=10,\ \delta=0.05$ which gives optimal value $t_1^*=2.803836502$ This leads EOQ $Q=1201.83$ units and $TC(t_1)=2148.20439$. The graphical relationship between the total cost of the inventory system with respect to t_1 time is given in Figure 3. Convexity validates optimality.

Example 2 ($\lambda_1 < t_1 \le \lambda_2$): We consider $f(t) = a_1 + b_1 t$, $g(t) = a_2 - b_2 t$ and $B(t) = e^{-\delta(T-t)}$ Also, $a_1 = 100,\ b_1 = 5,\ a_2 = 155,\ b_2 = 10,\ A = 200,\ C_d = 3,\ C_h = 10,\ C_b = 5,\ C_L = 10,\ D_0 = 105,\ T = 12,\ m = 12,\ \lambda_1 = 1,\ \lambda_2 = 5,\ \delta = 0.05$ which gives optimal value $t_1^* = 2.035152959$. This leads EOQ $Q = 835.12$ units and $TC(t_1) = 1888.19093$. The graphical relationship between the total cost of the inventory system with respect to t_1 time is given in Figure 4. Convexity shows optimality again.

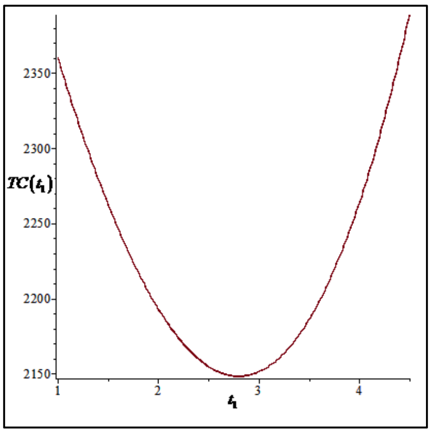

Fig. 3. $t_1 \to TC(t_1)$ (Example 1)

Rys. 3. $t_1 \to TC(t_1)$ (Przykład 1)

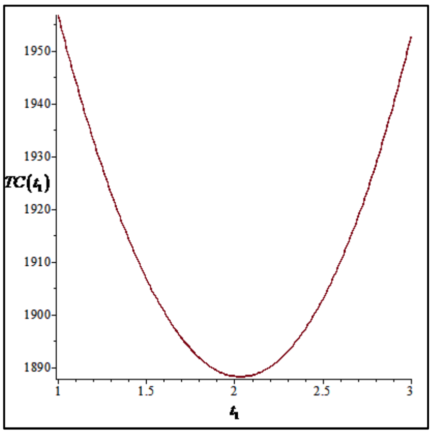

Fig. 4. $t_1 \to TC(t_1)$ (Example 2)

Rys. 4. $t_1 \to TC(t_1)$ (Przykład2)

Example 3 ($\lambda_2 < t_1 \le T$): We consider $f(t) = a_1 + b_1 t$, $g(t) = a_2 - b_2 t$ and $B(t) = e^{-\delta(T-t)}$ Also, $a_1 = 100,\ b_1 = 5,\ a_2 = 125,\ b_2 = 10,\ A = 200,\ C_d = 3,\ C_h = 10,\ C_b = 5,\ C_L = 10,\ D_0 = 105,\ T = 12,\ m = 12,\ \lambda_1 = 1,\ \lambda_2 = 2,\ \delta = 0.05$ which gives optimal value $t_1^* = 2.803836505$. This leads EOQ $Q = 674.97$ units and $TC(t_1) = 1323.41104$. The graphical relationship between the total cost of the inventory system with respect to t_1 time is given in Figure 5. Again optimality can be observed.

Example 4 To illustrate different approaches, we consider $f(t) = a_1 t^{b_1}$, $g(t) = a_2 t^{-b_2}$ be nonlinear demand functions and $B(t) = (1 + \delta(T-t))^{-1}$ (Ouyang 2005) where $a_1 = 30,\ b_1 = 2,\ a_2 = \frac{10}{3}\ b_2 = 6,\ A = 200,\ C_d = 3,\ C_h = 10,\ C_b = 5,\ C_L = 10\ D_0 = 120,\ T = 12,\ m = 12\ \lambda_1 = 2,\ \lambda_2 = 6,\ \delta = 0.05$. Hence optimal value $t_1^* = 2.922957278$ leads EOQ $Q = 529.70$ units and $TC(t_1) = 1288.1846$. The graphical relationship between the total cost of the inventory system with respect to t_1 time is given in Figure 6 to present optimality conditions.

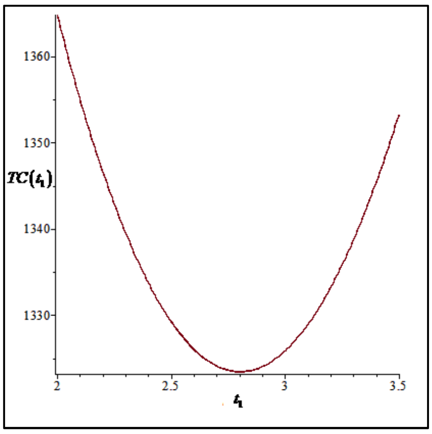

Fig. 5. $t_1 \to TC(t_1)$ (Example 3)

Rys. 5. $t_1 \to TC(t_1)$ (Przykład 3)

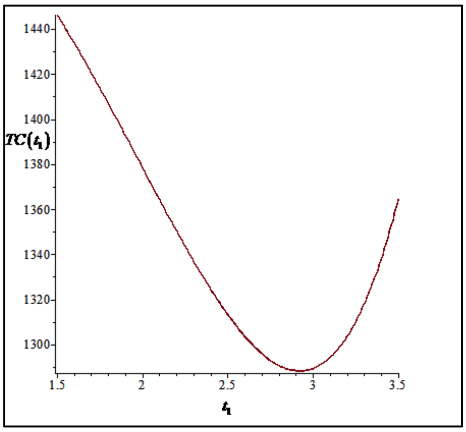

Fig. 6. $t_1 \to TC(t_1)$ (Example 4)

Rys. 6. $t_1 \to TC(t_1)$ (Przykład 4)

SENSITIVITY ANALYSIS

Variation with respect to different parameters are presented in the following tables.

Using Table 1 and Table 2 we may observe following:

Increase in the value of t_1 is due to an increment in m,C_b&C_L and for other parameter it decreases. Moreover, an increase in value of TC due to all costs per unit per time unit and for rest it decreases. Similarly, an increase in the value of Q due to C_b&C_L only. The variations are the same pattern for all cases.

Table 1. Variation with respect to parameters m , δ and C_d

Tabela 1. Wariancja w odniesieniu do parametrów m , δ oraz C_d

Parameters	m			δ			C_d		
	8	10	12	0.05	0.10	0.15	3	5	7
t_1	2.636	2.734	2.804	2.804	2.183	1.710	2.804	2.769	2.735
TC	2169.61	2157.04	2148.20	2148.20	1870.11	1647.42	2148.20	2154.59	2160.78
Q	1208.6	1204.8	1201.8	1201.8	980.84	799.21	1201.8	1199.3	1196.8

Table 2. Variation with respect to parameters C_h , C_b and C_L

Tabela 2. Wariancja w odniesieniu do parametrów C_h , C_b oraz C_L

Parameters	C_h			C_b			C_L		
	10	12	14	3	5	7	8	10	12
t_1	2.804	2.445	2.166	1.983	2.804	3.489	2.756	2.804	2.851
TC	2148.20	2215.32	2266.12	1483.48	2148.20	2730.11	2111.87	2148.20	2184.20
Q	1201.8	1176.3	1157.7	1145.9	1201.8	1255.6	1198.3	1201.8	1205.3

CONCLUSIONS

In this paper, we study an inventory system for deteriorating items with a trapezoidal-type demand rate. From market information, we observed that many items do not merely deteriorate but also have a maximum lifetime. We propose a general model for deteriorating items with maximum lifetime. Shortages are allowed and partially backlogged. By replacing different demand functions, variations may be observed in different scenarios. The optimal ordering policy is derived with generalized approach and illustrated through numerical examples with the assumption that the maximum lifetime is either less or equal to the total cycle time. The sensitivity analysis is conducted with respect to the various parameters. This paper provides an interesting area for further study of such type of models. One might consider problemsrelated to (1) inventory systema with a finite replenishment rate (2) imperfect production systems for future research.

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MODEL POZIOMÓW ZAPASÓW ASORTYMENTÓW ULEGAJĄCYCH ZUŻYCIU O TRAPEZOIDALNYM TYPIE POPYTU Z CZĘŚCIOWYM ZAPASEM BEZPIECZEŃSTWA

STRESZCZENIE. **Wstęp:** Uwzględniając fakt, że w przypadku produktów łatwopsujących się, okres przydatności odgrywa istotną rolę przy zarządzaniu zapasem, Praca prezentuje ogólne podejście do modelu zarządzania zapasem dla tego typu asortymentów, posiadających daty przydatności do spożycia.

Metody: Założono, że popyt jest typu trapezoidalnego. Dozwolone zostały braki oraz założono częściowo zapasy bezpieczeństwa. Sformułowano metodę uzupełniania zapasów dla popytu trapezoidalnego do stosowania w handlu detalicznym.

Wyniki: Prezentowaną metodą wsparto przedstawieniem przykładów liczbowych oraz analizą wrażliwości przy wsparciu ich prezentacją graficzną w odniesieniu do podstawowych kryteriów.

Wnioski: Zaprezentowane ogólne podejście dostarcza podstawy do tworzenia strategii stosujące różne funkcje popytu i różne zapasy bezpieczeństwa.

Słowa kluczowe: zapasy, towary podlegające zużyciu z terminami przydatności, trapezoidalny typ popytu, częściowe zapasy bezpieczeństwa.

EIN MODELL FÜR DIE NIVEAUS VON VERDERBLICHEN SORTIMENT-VORRÄTEN VOM TAPEZOIDFÖRMIGEN NACHFRAGETYPUS MIT EINEM TEILHAFTEN SICHERHEITSBESTAND

ZUSAMMENFASSUNG. Einleitung: Angesichts der Tatsache, dass bei den leichtverderblichen Produkten das Mindesthaltbarkeitsdatum beim Bestandsmanagement eine große Rolle spielt, stellt die Arbeit eine allgemeine Vorgehensweise an das Bestandsmanagement-Modell für eine solche Art von Sortimenten, die Mindesthaltbarkeitsdaten besitzen, dar.

Methoden: Es wurde angenommen, dass die Nachfrage einen trapezoidförmigen Typus aufweist. Es wurden bei der Analyse Mängel zugelassen und man ging vom teilhaften Sicherheitsbestand aus. Es wurde eine Methode für die Ergänzung von Vorräten für die trapezoidförmigen Nachfrage für die Anwendung im Kleinhandel formuliert.

Ergebnisse: Die projizierte Methode wurde mit einer Anführung von Zahlenbeispielen sowie mit der Empfindlichkeitsanalyse für deren Unterstützung anhand einer grafischen Darstellung in Bezug auf die grundlegenden Kriterien unterstützt.

Fazit: Die projizierte allgemeine Vorgehensweise bildet eine Grundlage für die Erstellung von Strategien, die verschiedene Nachfragefunktionen und unterschiedliche Sicherheitsbestände in Anspruch nehmen.

Codewörter: Vorräte, verderbliche Waren mit Mindesthalbarkeitsdaten, trapezoidförmiger Nachfragetypus, teilhafte Sicherheitsbestände.

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