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Determination of electromechanical eigenvalues based on analysis of different disturbance waveforms of a power system

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The paper presents the calculation results of the power system state matrix eigenvalues associated with electromechanical phenomena (i.e. electromechanical eigenvalues). There is compared the accuracy of eigenvalue calculations performed on the basis of the analysis of disturbance waveforms of instantaneous power, angular speed and power angle of generators of particular generating units of the power system. The method used for eigenvalue calculations consists in approximation of the disturbance waveforms of generating units with waveforms being a superposition of modal components whose parameters depend on the searched eigenvalues and their participation factors. A hybrid optimisation algorithm, being a serial combination of genetic and gradient algorithms, is used for minimisation of the objective function defined as a mean square error between the approximated and approximating waveforms. In order to increase the calculation accuracy, computations were repeated many times. The computation results were averaged.

KEYWORDS: power system, eigenvalues associated with electromechanical phenomena, transient states, angular stability

1. Introduction

Maintaining the angular stability of a power system (PS) is one of the most important conditions of its proper work. The angular stability of a power system may be assessed with use of stability factors [1] calculated on the basis of the system state matrix eigenvalues associated with electromechanical phenomena (called *electromechanical eigenvalues* in the paper). The electromechanical eigenvalues may be calculated based on the system state equations, but then the calculation results depend on the elements of the system state matrix, and indirectly on the assumed models of the power system components and their uncertain parameters [2]. These eigenvalues can also be calculated with good accuracy on the basis of analysis of actual disturbance waveforms appearing in the system after various disturbances [3, 4, 5].

The aim of the paper is to compare the accuracy of calculating electromechanical eigenvalues on the basis of the analysis of the simulated disturbance waveforms of the instantaneous power, angular speed and power angle in PS generating units.

2. The linearised power system model

The power system model linearised at the steady operating point is described by the state equation and output equation [4, 5]:

$$\Delta\dot{\boldsymbol{X}} = \boldsymbol{A}\Delta\boldsymbol{X} + \boldsymbol{B}\Delta\boldsymbol{U}, \tag{1}$$

$$\Delta\boldsymbol{Y} = \boldsymbol{C}\Delta\boldsymbol{X} + \boldsymbol{D}\Delta\boldsymbol{U}, \tag{2}$$

where: $\Delta\boldsymbol{X}$, $\Delta\boldsymbol{U}$, $\Delta\boldsymbol{Y}$ – deviations of the vectors of: state variables, input variables and output variables, respectively. The elements of the $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{D}$ matrices are calculated for the steady operating point of the PS.

The waveforms of the output variables of the linearised PS model can be calculated directly by solving equations (1) and (2), or on the basis of eigenvalues and eigenvectors of the state matrix $\boldsymbol{A}$ [4, 5]. Assuming only single eigenvalues of the state matrix, the vectors of the state and the output variables can be expressed by the formulas [6]:

$$\Delta\boldsymbol{X}(t) = \int_0^t \boldsymbol{V}\mathrm{e}^{\boldsymbol{\Lambda}(t-\tau)}\boldsymbol{W}^{\mathrm{T}}\boldsymbol{B}\boldsymbol{u}(\tau)\mathrm{d}\tau = \int_0^t \mathrm{e}^{\boldsymbol{A}(t-\tau)}\boldsymbol{B}\boldsymbol{u}(\tau)\mathrm{d}\tau, \tag{3}$$

$$\begin{aligned}\Delta\boldsymbol{Y}(t) &= \int_0^t \boldsymbol{C}\boldsymbol{V}\mathrm{e}^{\boldsymbol{\Lambda}(t-\tau)}\boldsymbol{W}^{\mathrm{T}}\boldsymbol{B}\boldsymbol{u}(\tau)\mathrm{d}\tau + \boldsymbol{D}\boldsymbol{u}(t) = \\ &= \int_0^t \boldsymbol{C}\mathrm{e}^{\boldsymbol{A}(t-\tau)}\boldsymbol{B}\boldsymbol{u}(\tau)\mathrm{d}\tau + \boldsymbol{D}\boldsymbol{u}(t),\end{aligned} \tag{4}$$

where: $\boldsymbol{V}$, $\boldsymbol{W}$ – right-side and left-side modal matrices, the columns of which are, respectively, subsequent right-side and left-side normalised ($\boldsymbol{W}^{\mathrm{T}}{}_h\boldsymbol{V}_h = 1$) state matrix eigenvectors, $\boldsymbol{\Lambda}$ – diagonal matrix, whose main diagonal consists of subsequent state matrix eigenvalues.

The waveform of each PS output variable is a superposition of the modal components which depend on the eigenvalues and eigenvectors of the state matrix. For example, in the case of a disturbance being a Dirac pulse of the j-th input variable $\Delta U_j(t) = \Delta U\delta(t)$, the i-th output variable (at $\boldsymbol{D} = \boldsymbol{0}$ and assuming only single eigenvalues) is [4, 5]:

$$\Delta y_i(t) = \sum_{h=1}^{n} F_{ih} e^{\lambda_h(t-t_0)}\Delta U, \quad t \ge t_0, \tag{5}$$

$$F_{ih} = \boldsymbol{C}_i\boldsymbol{V}_h\boldsymbol{W}_h^{\mathrm{T}}\boldsymbol{B}_j, \tag{6}$$

where: $\lambda_h = \alpha_h + \mathrm{j}\nu_h$ – h-th eigenvalue of the state matrix, F_{ih} – participation factor of the h-th eigenvalue in the i-th output waveform, $\boldsymbol{C}_i$ – i-th row of $\boldsymbol{C}$ matrix, $\boldsymbol{V}_h$ – h-th right-side eigenvector of the state matrix, $\boldsymbol{W}_h$ – h-th left-side eigenvector of the state matrix, $\boldsymbol{B}_j$ – j-th column of $\boldsymbol{B}$ matrix, n – dimension of the state matrix. The values λ_h and F_{ih} can be real or complex.

In the case of the transient waveforms of the instantaneous power, angular speed and power angle of generators of generating units in the PS, the electromechanical eigenvalues are of decisive significance. They are complex conjugate eigenvalues whose imaginary parts correspond to the frequency range (0.1-2 Hz), hence their imaginary parts fall into the range (0.63-12.6 rad/s). These eigenvalues intervene in different ways in the output variable waveforms of particular generating units, which is related to the different values of their participation factors.

3. The method for calculations of electromechanical eigenvalues

The disturbance waveforms of generating unit output variables deviations which occur after purposeful introduction of a small disturbance to the PS were used for calculations. The assumed disturbance was a rectangular pulse in the waveform of the generating unit voltage regulator reference voltage V_{ref}. The system response to a short rectangular pulse (with properly selected height and width) is similar to the response to a Dirac pulse [4, 5, 6].

The amplitude of electromechanical swings must be large enough to enable their extraction from the waveforms recorded in PS generating units [4].

In the case of a rectangular pulse, the amplitude of electromechanical swings is approximately proportional to the surface area of the pulse. The pulse height, however, must be limited to avoid significant influence of nonlinearity and limits occurring in the power system on the output variables waveforms. The rectangular pulse duration must also be limited, because its significant extension causes increasingly growing differences in the system response to a rectangular pulse and to a Dirac impulse, which can deteriorate the electromechanical eigenvalue calculation accuracy [4]. From the performed investigations it follows that the eigenvalue calculation accuracy is satisfactory if the pulse duration does not exceed 300 ms in the case of the waveforms of the instantaneous power deviations ΔP and power angle deviations $\Delta\delta$ and does not exceed 200 ms in the case of the waveforms of the angular speed deviations $\Delta\omega$.

For safety reasons, the height ΔV_{ref} of a disturbance introduced to the voltage regulation system of a generator operating at a power plant should rather not exceed ca. 3-5% of the steady-state value of the reference voltage V_{ref}. The higher power of the generating unit disturbed, the larger amplitudes of the electromechanical swings in particular generating units. These amplitudes are also influenced by interactions between these units and the unit disturbed [4].

The output variable waveforms of a single generating unit usually contain a few modal components with significant amplitudes, associated with electromechanical eigenvalues. That is why, in order to calculate all electromechanical eigenvalues of the power system, the output variable waveforms in different generating units need to be analysed for various disturbance locations [4].

The method for calculations of electromechanical eigenvalues used in investigations consists in the approximation of the output variable deviation waveforms of particular generating units with the use of expression (5). The electromechanical eigenvalues and their participation factors are the unknown parameters of this approximation. In the approximation process, these parameters are iteratively selected to minimise the value of the objective function ε_{w} defined as a mean square error between the approximated and approximating waveform:

$$\varepsilon_{\mathrm{w}}(\boldsymbol{\lambda},\boldsymbol{F})=\sum_{k=1}^{N}\left(\Delta W_{k(\mathrm{m})}-\Delta W_{k(\mathrm{a})}(\boldsymbol{\lambda},\boldsymbol{F})\right)^{2}, \qquad (7)$$

where: $\boldsymbol{\lambda}$ – vector of electromechanical eigenvalues, $\boldsymbol{F}$ – vector of participation factors, ΔW – waveform of the deviations of the quantity analysed, k – current number of the waveform sample, N – number of samples, the index m denotes the approximated waveform, while the index a – the approximating waveform, calculated from the searched eigenvalues and participation factors with the use of expression (5). The eigenvalues with small participation factor modules in the given waveform are neglected in calculations based on this waveform.

The objective function (7) is minimised by a hybrid optimisation algorithm [4, 7, 8] consisting of serially connected genetic [7, 8, 9] and gradient [7, 8] algorithms. The results obtained from the genetic algorithm are the starting point of the gradient algorithm. The use of a genetic algorithm in the first stage of the search for the objective function minimum eliminates the problem of precise determination of the starting point, whereas the gradient algorithm used in the second stage converges faster and allows finding the minimum more accurately [4, 7]. For the genetic algorithm 50 generations, population of 20 individuals, and chromosome length of 6 bits were assumed. The selection was performed by the elite (ranking) method [7, 9], which ensures that the fittest individuals of a given generation will proceed to the next generation. The maximum number of iterations assumed for the gradient algorithm was 1000 [4].

To eliminate the effect of the fast decaying modal components associated with the real and complex eigenvalues, which are not electromechanical eigenvalues, it is convenient to start the waveform analysis after a certain time t_{p} after the disturbance occurrence [4]. In the calculations presented there was assumed t_{p} = 0.6 s for ΔP waveforms, t_{p} = 1.0 s for $\Delta\omega$ waveforms and t_{p} = 1.1 s for $\Delta\delta$ waveforms.

From the performed calculations it follows that in the waveforms of the instantaneous power deviations ΔP only the modal components associated with electromechanical eigenvalues intervene significantly (after decay of the strongly damped modal components). In the waveforms of angular speed deviations $\Delta\omega$ also the modal components associated with other electromechanical eigenvalues intervene significantly but the influence of electromechanical eigenvalues is also significant. The influence of electromechanical eigenvalues on the waveforms of power angle deviations $\Delta\delta$ is relatively small. Moreover, the steady values of the waveforms $\Delta\delta$ after the disturbance differ from the initial values of these waveforms before the disturbance. From the investigations performed, it follows that to make the correct approximation of the waveform $\Delta\omega$ possible, it is necessary (in the case of a pulse disturbance) to take into account one equivalent oscillatory modal component of a relatively low frequency which represents the influence of the neglected modal components on this waveform. Whereas for the waveform $\Delta\delta$ it is necessary to take into account two equivalent modal components: oscillatory (as for the waveform $\Delta\omega$) and aperiodic. The parameters of the equivalent modal components are also arguments of the objective function (7) and are optimised.

The waveforms ΔP are calculated based on the waveforms of voltages and currents of the generator stator (phase or axial ones – calculated by the Park transformation [10]). Appropriate measurements of waveforms $\Delta\omega$ and $\Delta\delta$ are also possible with the use of the equipment developed by the Institute of Electrical Engineering and Computer Science of the Faculty of Electrical Engineering of the Silesian University of Technology [10].

Due to the existence of the objective function local minima, where the optimisation algorithm may get stuck, the approximation process was performed repeatedly based on the same waveform. The calculation results with the objective function values larger than a certain assumed limit were rejected. As the final calculation results of the real and imaginary parts of particular eigenvalues, there were assumed the arithmetic means of the results not rejected in subsequent calculations.

In many cases the eigenvalues are calculated on the basis of an output variable waveform in two stages. In the first stage, based on a particular waveform, there are calculated the eigenvalues that have relatively large values of the real parts (i.e. small modules of the real parts), corresponding to the weakly damped modal components. Simultaneously, there are neglected the eigenvalues of smaller values of the real parts corresponding to the stronger damped modal components. In the second stage, based on the same waveform, there are calculated the eigenvalues with the smaller values of the real parts, when taking into account the eigenvalues calculated in the first stage. Additionally, in the first and second stage of the calculations the eigenvalues calculated earlier based on other instantaneous power waveforms are often assumed as known eigenvalues [4].

4. Exemplary calculations

Exemplary calculations were carried out for a 7 – machine PS CIGRE (Fig. 1). There were analysed the waveforms occurring after introducing a pulse disturbance of amplitude ΔV_{ref} = –5% V_{ref0} and duration t_{imp} = 200 ms to generating unit G4 (V_{ref0} denotes the initial value of the voltage regulator reference voltage).

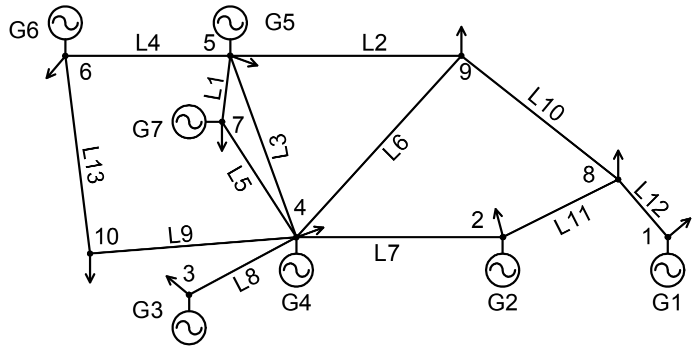

Fig. 1. The analysed 7 – machine PS CIGRE [11]

The analysed PS model was developed in the Matlab - Simulink environment. It consists of 7 models of generating units as well as the model of the network and loads. Each generating unit in the PS model consists of configurable subsystem units which allow selecting the models of the generator, excitation system, turbine and power system stabilizer (PSS). In calculations there were taken into account the models of: a GENROU synchronous generator with nonlinear magnetization characteristic [6, 7, 12, 13], a static excitation system working in the Polish Power System [6, 7], an IEEEG1 steam turbine [12, 14] and a PSS3B stabilizer [6, 12].

Eigenvalues (including electromechanical eigenvalues) of the PS state matrix can be calculated directly on the basis of the structure and parameters of the PS model in the Matlab - Simulink environment. These electromechanical eigenvalues are called *original eigenvalues* in the paper. Comparison of the eigenvalues calculated based on minimisation of the objective function (7) and the original eigenvalues is a measure of the calculation accuracy [4]. The original eigenvalues of the analysed PS CIGRE are presented in Table 1.

Table 1. The original eigenvalues of the analysed PS CIGRE

λ_1, 1/s	–0,881±j10,443	λ_2, 1/s	–0,826±10,620	λ_3, 1/s	–0,763±j9,669
λ_4, 1/s	–0,527±j8,748	λ_5, 1/s	–0,417±j7,872	λ_6, 1/s	–0,189±j6,542

4.1. Analysis of the impact of output quantity selection on the participation factor values

As it can be seen in formula (2), when assuming $\boldsymbol{D} = \mathbf{0}$, the values of the vector $\Delta \boldsymbol{Y}$ are dependent on the values of the vector $\Delta \boldsymbol{X}$ and the matrix $\boldsymbol{C}$. The waveforms of the i-th output variable are obtained by multiplying the i-th row of the matrix $\boldsymbol{C}$ and the vector $\Delta \boldsymbol{X}$. From formula (5) it is apparent that the participation factor F_{ih} also depends on the values of the elements in the i-th row of the matrix $\boldsymbol{C}$. Thus the participation factors of particular eigenvalues are different in the disturbance waveforms of particular output quantities of the PS. It also refers to different output quantities of the same generating unit.

Table 2. The participation factors of electromechanical eigenvalues in the waveforms of the PS generating unit

ΔP waveforms						
Unit	$\lvert F_1\rvert_{pu}$	$\lvert F_2\rvert_{pu}$	$\lvert F_3\rvert_{pu}$	$\lvert F_4\rvert_{pu}$	$\lvert F_5\rvert_{pu}$	$\lvert F_6\rvert_{pu}$
G1	0.0371	0.0199	0.0099	**0.3044**	0.0147	**1**
G2	**0.4681**	**1**	0.0797	**0.3043**	0.0229	**0.6920**
G3	**0.4836**	**1**	0.0512	0.0248	**0.3564**	**0.2051**
G4	**0.4361**	**1**	0.0266	0.0036	0.0084	0.0359
G5	**1**	**0.6796**	**0.2931**	0.0082	0.0878	**0.1261**
G6	**0.8528**	**1**	0.0705	0.0008	0.0131	0.0283
G7	**0.3701**	**1**	**0.6015**	0.0377	**0.2815**	**0.3885**
$\Delta\omega$ waveforms						
Unit	$\lvert F_1\rvert_{pu}$	$\lvert F_2\rvert_{pu}$	$\lvert F_3\rvert_{pu}$	$\lvert F_4\rvert_{pu}$	$\lvert F_5\rvert_{pu}$	$\lvert F_6\rvert_{pu}$
G1	0.0226	0.0119	0.0065	**0.2229**	0.0120	**1**
G2	**0.4103**	**0.8620**	0.0757	**0.3215**	0.0271	**1**
G3	**0.4918**	**1**	0.0564	0.0304	**0.4885**	**0.3438**
G4	**0.4435**	**1**	0.0293	0.0044	0.0115	0.0605
G5	**1**	**0.6684**	**0.3175**	0.0099	**0.1181**	**0.2068**
G6	**0.8672**	**1**	0.0776	0.0009	0.0180	0.0472
G7	**0.3763**	**1**	**0.6620**	0.0460	**0.3837**	**0.6439**
$\Delta\delta$ waveforms						
Unit	$\lvert F_1\rvert_{pu}$	$\lvert F_2\rvert_{pu}$	$\lvert F_3\rvert_{pu}$	$\lvert F_4\rvert_{pu}$	$\lvert F_5\rvert_{pu}$	$\lvert F_6\rvert_{pu}$
G1	0.0141	0.0073	0.0044	**0.1665**	0.01	**1**
G2	**0.2562**	**0.5296**	0.0511	**0.2401**	0.0225	**1**
G3	**0.5**	**1**	0.062	0.0369	**0.66**	**0.5596**
G4	**0.4508**	**1**	0.0322	0.0053	0.0156	0.0985
G5	**1**	**0.6575**	**0.343**	0.0119	**0.157**	**0.3311**
G6	**0.8815**	**1**	0.0853	0.0011	0.0243	0.0768
G7	**0.365**	**0.9541**	**0.6937**	0.0534	**0.4947**	**1**

Table 2 presents the relative absolute values of the participation factors $|F|_{pu}$ of PS electromechanical eigenvalues in the waveforms of the deviations of instantaneous power ΔP, angular speed $\Delta\omega$ and power angle $\Delta\delta$ of the PS generating units when introducing a disturbance to the unit G4 (in relation to the largest absolute value of the participation factors of electromechanical eigenvalues in the given waveform). The relative absolute values of the participation factors of the eigenvalues, calculated based on particular waveforms are written in bold.

From Table 2 it follows that the relative absolute values of the participation factors of particular electromechanical eigenvalues differ significantly for particular output quantities of the PS. Based on the investigations performed, it can be stated that the eigenvalues can be usually calculated with a satisfactory accuracy based on the waveforms in which absolute values of their participation factors are larger than 0.1-0.3.

4.2. Calculations of electromechanical eigenvalues

Table 3 presents the absolute errors $\Delta\lambda$ of calculations of electromechanical eigenvalues on the basis of the disturbance waveforms ΔP, $\Delta\omega$ and $\Delta\delta$ of particular PS generating units. The arithmetic means of the calculation errors are also listed in the table. These means do not take into account the results (written on a grey background) with the real or imaginary parts significantly different than the other calculation results.

From Table 3 it follows that in almost all the cases eigenvalues were calculated with the satisfactory accuracy. The accuracies of calculations based on the waveforms of all the analysed quantities were close to each other. The arithmetic means of the errors of calculating the real and imaginary parts based on the waveforms of different generating units were generally the smallest (referred to the absolute value) in the case of the waveforms $\Delta\omega$.

For example, Figs. 2 and 3 show the simulation disturbance waveforms of the deviations of the instantaneous power ΔP, angular speed $\Delta\omega$ and power angle $\Delta\delta$ of the generators in units G1 and G5 in the case of the pulse disturbance in unit G4 as well as the bands of the approximating waveforms corresponding to the non-rejected calculation results. The band of the approximating waveforms determines the range of the waveform changes in which there are contained all approximating waveforms corresponding to particular calculation results.

Fig. 4 shows the histograms of the calculation results of the real and imaginary parts of the eigenvalue λ_4, based on the simulation analysis of the output variable waveforms of unit G1 in the case of the pulse disturbance in unit G4. Due to the stochastic nature of the genetic algorithm, individual calculations are initiated for different starting points, each time selected randomly from the given search ranges [5]. The dark bars represent the results included in the further analysis, and the light bars represent the results rejected. The vertical thin solid lines in the middle of the histograms correspond to the original eigenvalues. The ranges of the real and imaginary parts of the eigenvalue λ_4 correspond to the assumed ranges of their searches.

Table 3. The absolute errors of eigenvalue calculations

ΔP waveforms						
Unit	$\Delta\lambda_1$, 1/s	$\Delta\lambda_2$, 1/s	$\Delta\lambda_3$, 1/s	$\Delta\lambda_4$, 1/s	$\Delta\lambda_5$, 1/s	$\Delta\lambda_6$, 1/s
G1	–	–	–	–0.006±j0.128	–	–0.017±j0.006
G2	0.157∓j0.587	–0.089±j0.167	–	–0.049∓j0.197	–	–0.016±j0.032
G3	0.261±j0.622	–0.106±j0.143	–	–	0.073∓j0.132	0.064±j0.050
G4	0.010∓j0.061	0.118∓j0.060	–	–	–	–
G5	–0.036±j0.210	0.091∓j0.094	0.102∓j0.221	–	–	–0.009±j0.118
G6	–0.289∓j0.186	–0.056∓j0.046	–	–	–	–
G7	0.240∓j3.126	–0.049±j0.036	0.002±j0.001	–	–0.029±j0.114	0.064±j0.106
Mean	–0.013±j0.075	–0.015±j0.024	0.052∓j0.110	–0.027∓j0.034	0.022∓j0.009	0.017±j0.062
$\Delta\omega$ waveforms						
Unit	$\Delta\lambda_1$, 1/s	$\Delta\lambda_2$, 1/s	$\Delta\lambda_3$, 1/s	$\Delta\lambda_4$, 1/s	$\Delta\lambda_5$, 1/s	$\Delta\lambda_6$, 1/s
G1	–	–	–	–0.018±j0.018	–	0.012∓j0.020
G2	–0.180∓j4.000	–0.107±j0.084	–	0.032∓j0.155	–	–0.015±j0.014
G3	–0.398∓j1.303	0.092∓j0.177	–	–	0.001±j0.093	–0.042∓j0.117
G4	–0.094∓j4.000	0.075∓j0.119	–	–	–	–
G5	0.058±j0.038	–0.027±j0.271	0.081∓j0.148	–	–0.009±j0.030	–0.014±j0.001
G6	–0.010±j0.127	0.051∓j0.058	–	–	–	–
G7	–0.317±j2.087	–0.047∓j0.025	–0.045∓j0.062	–	–0.016∓j0.010	–0.002±j0.054
Mean	0.024±j0.083	0.006∓j0.004	0.018∓j0.105	0.007∓j0.068	–0.008±j0.037	–0.012∓j0.014
$\Delta\delta$ waveforms						
Unit	$\Delta\lambda_1$, 1/s	$\Delta\lambda_2$, 1/s	$\Delta\lambda_3$, 1/s	$\Delta\lambda_4$, 1/s	$\Delta\lambda_5$, 1/s	$\Delta\lambda_6$, 1/s
G1	–	–	–	0.023∓j0.013	–	0.008∓j0.044
G2	0.210±j3.053	0.092±j0.080	–	0.167∓j3.022	–	–0.015∓j0.061
G3	0.222∓j1.613	0.011±j0.134	–	–	–0.021±j0.132	0.012∓j0.044
G4	–0.594±j1.936	0.033∓j0.104	–	–	–	–
G5	–0.092∓j0.399	–0.140∓j0.070	0.141∓j3.689	–	0.023±j0.003	0.005±j0.063
G6	–0.151∓j0.547	0.018±j0.168	–	–	–	–
G7	0.197±j2.844	0.112±j0.133	0.089∓j0.179	–	0.036±j0.082	0.011±j0.058
Mean	–0.122∓j0.473	0.021±j0.057	0.089∓j0.179	0.023∓j0.013	0.012±j0.072	0.004∓j0.006

In the case of the power angle waveforms $\Delta\delta$, there occurred numerous local minima of the objective function corresponding to the incorrect results of the calculation of electromechanical eigenvalues (see the exemplary histogram of Fig. 4c). It could be caused, among others, by the relatively small amplitudes of the modal components associated with the calculated electromechanical eigenvalues.

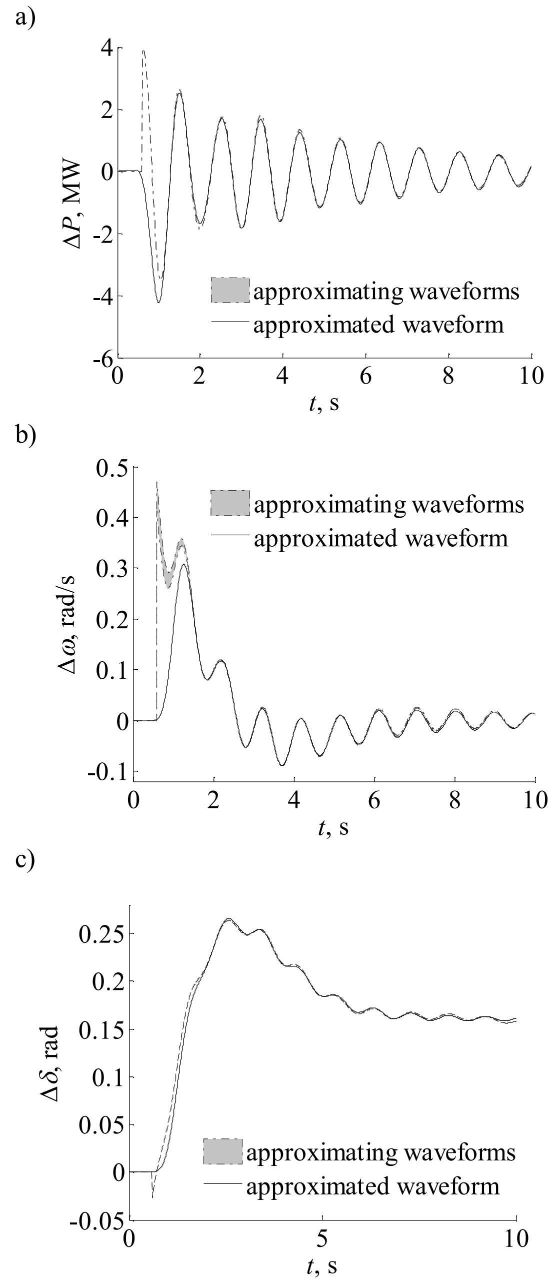

Fig. 2. Disturbance waveforms of the deviations of: instantaneous power ΔP (a), angular speed $\Delta\omega$ (b) and power angle $\Delta\delta$ (c) of generating unit G1

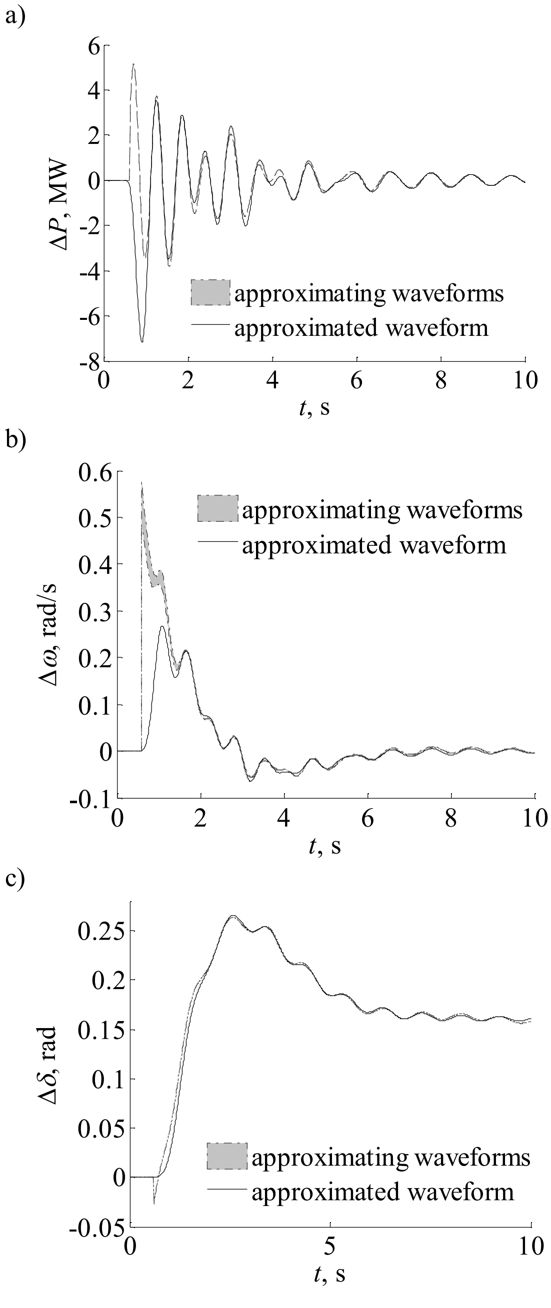

Fig. 3. Disturbance waveforms of the deviations of: instantaneous power ΔP (a), angular speed $\Delta\omega$ (b) and power angle $\Delta\delta$ (c) of generating unit G5

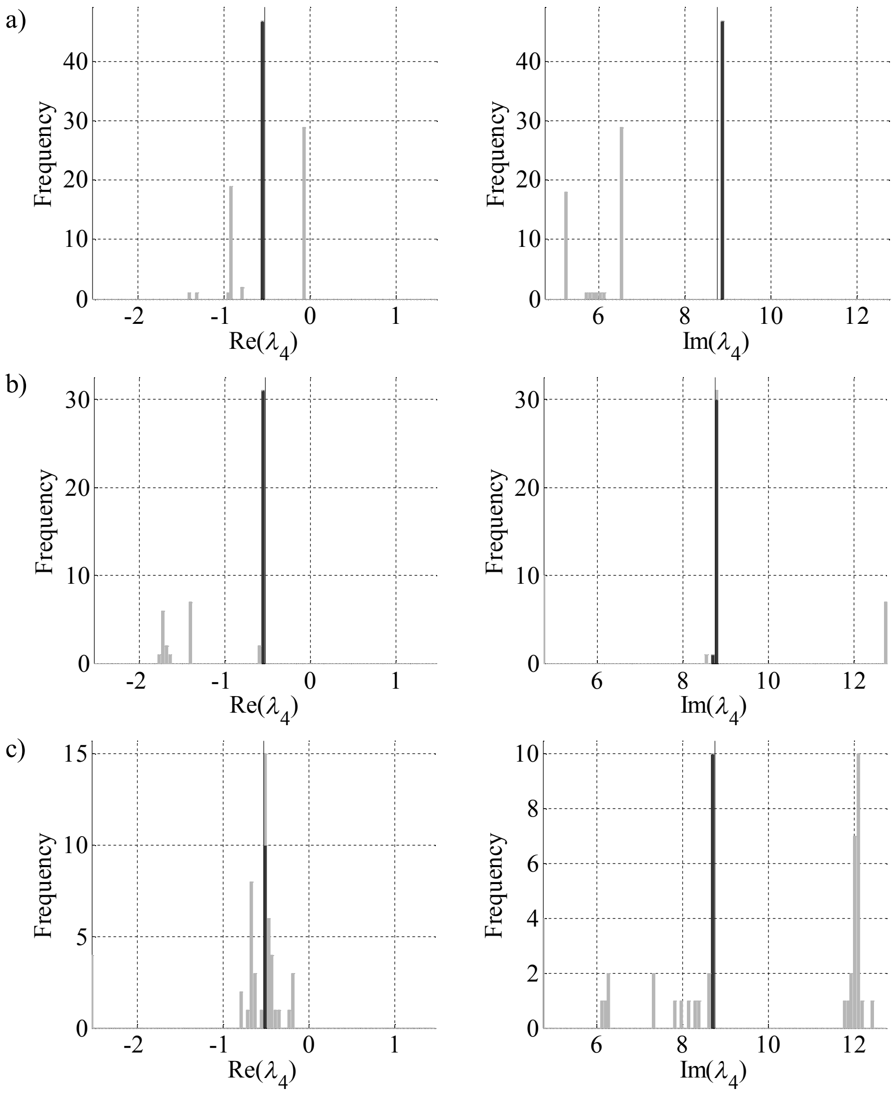

Fig. 4. Histograms of the real and imaginary parts of the eigenvalue λ_4 calculated based on the waveforms of the deviations of: instantaneous power ΔP (a), angular speed $\Delta\omega$ (b) and power angle $\Delta\delta$ (c) of generating unit G1

5. Conclusion

The investigations performed allow to draw the following conclusions:

- The investigations performed for the 7-machine CIGRE PS model prove that it is possible to determine electromechanical eigenvalues with the good accuracy based on the analysis of the waveforms of the instantaneous power, angular speed and power angle occurring after introducing a rectangular pulse disturbance in the voltage regulation system of one of generating units. The method used for calculations of eigenvalues on the basis of these waveforms works well also in the case of large PSs like the Polish Power System.

- The calculation accuracy of eigenvalues was good. They were calculated based on the analysis of most of the waveforms whose absolute values of the participation factors were large enough.
- Repeating calculations of eigenvalues with the use of the hybrid algorithm, at different starting points selected randomly at each calculation from the search range, eliminates the problem of algorithm freezing at local minima of the objective function. This proved to be particularly helpful in calculations based on the power angle waveforms. In this case there were numerous local minima of the objective function corresponding to the incorrect calculation results.
- The averaging of calculation results of eigenvalues on the basis of the analysis of the waveforms of different generating units allows increasing the calculation accuracy. This proved to be especially efficient in the case of the angular speed waveforms. In the cases when the calculation result of the eigenvalue on the basis of one waveform differed significantly from the calculation results of that eigenvalue on the basis of other waveforms, that result was rejected.

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