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Determination of optimal current in the non-ideal one-phase system with unsteady parameters

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Abstract. Some of parameters in electrical systems cannot be considered as constants. In particular, some electrical loads should be described as unsteady because of physical reasons, e.g. arc and arc-resistance furnaces. One of the method to describe parameters of such loads is description by fuzzy numbers. Moreover, this description enables to determine optimal current of the system with unsteady parameters by an application of the optimisation technique called as fuzzy mathematical programming. The optimisation problem has been presented for one-phase system with sinusoidal waveforms. The obtained solution (optimal current) can be determined as crisp one (real numbers) in frequency domain in the numerical way. The determined optimal current has minimal RMS value in this case when the power constraints do not have to be fulfilled strictly because of changes of system parameters. It means simultaneously the minimisation of power losses in the system. This solution generalizes the classical optimisation solution obtained for the systems with constant parameters. In order to illustrate the problem the appropriate example has been presented.

Key words: fuzzy numbers, optimisation, minimisation of power losses, optimal current.

1. Introduction

The description by means of fuzzy sets and fuzzy numbers is still developed in electrical circuits, e.g. in control systems for generators [1] and motors [2], as well in neuro-fuzzy networks e.g. for gas sensors [3]. In this paper the optimisation and description of unsteady electrical parameters by means of fuzzy numbers have been presented. This description makes able to take into consideration the normal changes of parameters in the electrical systems (parameters of the voltage source with inner impedance) as well as changes of load parameters caused by their physical properties. The arc and arc-resistance furnaces can be considered as examples of loads with unsteady parameters. Potential changes of electrical parameters can be described by membership functions determined in measurement processes. This description is the first step in order to determine the optimal current in the considered one-phase system. The optimal current as a solution to the optimisation problem is determined by application of mathematical programming.

The description of electrical parameters by means of fuzzy numbers has been presented by other authors in some earlier works. In the paper [4] in order to minimise power losses the goal function as well as constraints have been formulated as fuzzy sets and the solution has been determined as taking an optimal decision with the genetic algorithm technique. In the paper [5] the description by means of fuzzy sets has been adopted to solve the problem of optimal reactive power flow and minimisation of power losses in an electrical system taking into considerations their costs. The solution has been determined as taking optimal decision i.e. the determination of intersection point with maximum value of membership functions. In the paper [6] the description by means of fuzzy sets has been adopted to solve the problem of optimal reactive power flow and minimisation of power losses in electrical system taking into considerations changes of active power in the system during 24 hours. The solution of the problem is determined by operations on fuzzy numbers defined by L. Zadeh [7]. As opposed to the aforementioned papers, in this paper the potential changes of main electrical parameters are taken into consideration and mathematical programming is applicated in order to solve the problem. This approach taking into account the shapes of all considered fuzzy numbers in fuzzy constraint.

2. Formulation and solution of the problem

The considered one-phase system with sinusoidal waveforms is presented in Fig. 1.

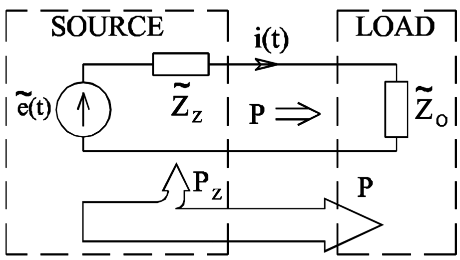

Fig. 1. The considered one-phase with source parameters described as fuzzy numbers

In this system the source voltage as well as the inner impedance of the source and the load impedance are understood as fuzzy numbers. They represent potential changes of these parameters in real situations. Taking into consideration that

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the fuzzy set A in a certain space $X = \{x\}$ is defined as the set of pairs:

$$A = \{(\mu_A(x), x)\}, \qquad \forall x \in X, \tag{1}$$

where

$$\mu_A : X \to [0,1], \qquad \mu_A(x) \in [0,1] \tag{2}$$

is a membership function, which assigns a membership degree to each element x of the space X, the fuzzy number $\widetilde{B}$ is considered as the fuzzy set in the real numbers set R, with the continuous membership function [8]:

$$\mu_B : R \to [0,1], \qquad \mu_B(x) \in [0,1]. \tag{3}$$

Taking also into consideration that the most often changes of electrical parameters can be represented as the Gauss function, the RMS value of the source voltage $\left|\widetilde{E}\right|$ as the fuzzy number can be written in the following form:

$$\left|\widetilde{E}\right| = \left\{\left(\mu_{|E|}(|E|), |E|\right)\right\} = \sum_{|E| \in \left|\widetilde{E}\right|} \mu_{|E|}/|E|, \tag{4}$$

where the sign $\sum$ has a set meaning, not arithmetic one. As above mentioned for the Gauss function the graphical representation of $\left|\widetilde{E}\right| = 400$ V supposing $\pm5\%$ potential changes has been presented in Fig. 2.

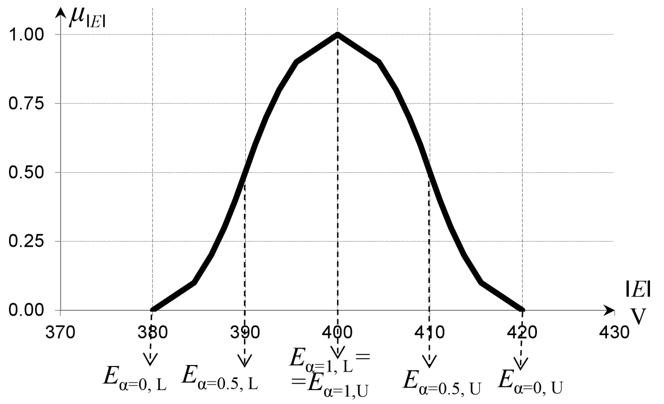

Fig. 2. The RMS value of the voltage source as the fuzzy number with marked α-cuts

The presented description can be useful in order to obtain the solution of the optimisation problem in fuzzy sense, i.e. taking into considerations the membership functions of constraints. The solution of this problem is based on the frequency domain. In order to present above mentioned fuzzy optimisation it is proper to describe fuzzy time functions of the voltage source [9]. In the paper there are considered periodical fuzzy time functions where the amplitudes (RMS values) of functions are fuzzy numbers. In particular, the voltage source is described as a fuzzy time function:

$$\widetilde{e}(t) = \sqrt{2}\left|\widetilde{E}\right| \sin(\omega t + \psi), \tag{5}$$

where $\left|\widetilde{E}\right|$ – the RMS value of the voltage source being not a real (crisp) but the fuzzy number for considered harmonics.

The application of fuzzy numbers to time waveforms description makes possible to obtain fuzzy time functions of the voltage source – Fig. 3. These time waveforms represent potential changes of a voltage source. Similarly, the inner impedance of the source can be described as a fuzzy number. Changes of source voltage and inner impedance of the source can respectively cause changes of power consumed by loads. For this reason the optimisation in electrical circuits sometimes requires to fulfill constraints in a fuzzy sense.

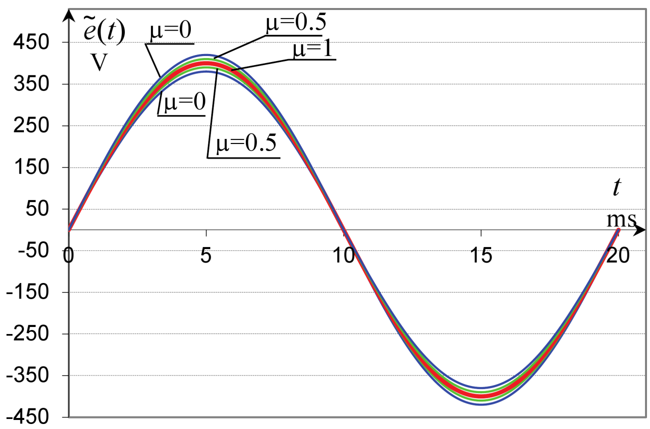

Fig. 3. The $\widetilde{e}(t)$ waveform

The optimisation problem is considered:

$$\min J = \min_i |I|^2 \tag{6}$$

under the fuzzy constraint:

$$\mathrm{Re}\left\{\widetilde{E}I^* - I\widetilde{Z}I^*\right\} \approx \widetilde{P}, \tag{7}$$

where $\widetilde{E}$, $\widetilde{Z}$, $\widetilde{P}$ – fuzzy numbers, and $\widetilde{P}$ is the active power consumed by the load before optimisation, considered as fuzzy number.

The constraint (7) can be considered as the active power consumed from a non-ideal voltage source should be the same before and after optimisation, and this constraint is the fuzzy one so it means that this condition doesn't have to be fulfilled precisely. When the voltage of the source is described as:

$$\widetilde{E} = \left|\widetilde{E}\right| e^{\mathrm{j}\psi}; \quad \psi = 0; \tag{8}$$

and current of the source can be represented as follows:

$$I = I_a + \mathrm{j}I_b \tag{9}$$

then the constraint (7) has the form:

$$\widetilde{E}I_a - \widetilde{R}_Z\left(I_a^2 + I_b^2\right) \approx \widetilde{P}. \tag{10}$$

In order to formulate and solve the problem the concept of α-cuts is used, presented in (Fig. 2). For the following values of membership function $\mu_{|E|}$ the corresponding values of $|E|$ are determined with figurative signs:

- the symbol $E_{\alpha=0,\,L}$ means the lower limit of $|E|$ for $\mu_{|E|} = \alpha = 0$;
- the symbol $E_{\alpha=0,\,U}$ means the upper limit of $|E|$ for $\mu_{|E|} = \alpha = 0$;

- the symbol $E_{\alpha=0.5,\,L}$ means the lower limit of $|E|$ for $\mu_{|E|} = \alpha = 0.5$;
- the symbol $E_{\alpha=0.5,\,U}$ means the upper limit of $|E|$ for $\mu_{|E|} = \alpha = 0.5$; etc.

Then the optimisation problem in fuzzy sense can be written as:

$$\min J = \min_i |I|^2 = \min\left(I_a^2 + I_b^2\right) \tag{11}$$

and fuzzy constraint (10) for fixed values of α-cuts has the following form:

$$\begin{aligned}
&E_{\alpha=0,\,L} I_a - R_{Z,\,\alpha=1,\,L}\left(I_a^2 + I_b^2\right) \geq P_{\alpha=0,\,L};\\
&E_{\alpha=0.1,\,L} I_a - R_{Z,\,\alpha=0.9,\,L}\left(I_a^2 + I_b^2\right) \geq P_{\alpha=0.1,\,L};\\
&\ldots\\
&E_{\alpha=0.5,\,L} I_a - R_{Z,\,\alpha=0.5,\,L}\left(I_a^2 + I_b^2\right) \geq P_{\alpha=0.5,\,L};\\
&\ldots\\
&E_{\alpha=0.9,\,L} I_a - R_{Z,\,\alpha=0.1,\,L}\left(I_a^2 + I_b^2\right) \geq P_{\alpha=0.9\,L};\\
&E_{\alpha=1,\,L} I_a - R_{Z,\,\alpha=0,\,L}\left(I_a^2 + I_b^2\right) \geq P_{\alpha=1\,L};\\
&E_{\alpha=1,\,U} I_a - R_{Z,\,\alpha=0,\,U}\left(I_a^2 + I_b^2\right) \leq P_{\alpha=1,\,U};\\
&E_{\alpha=0.9,\,U} I_a - R_{Z,\,\alpha=0.1,\,U}\left(I_a^2 + I_b^2\right) \leq P_{\alpha=0.9,\,U};\\
&\ldots\\
&E_{\alpha=0.5,\,U} I_a - R_{Z,\,\alpha=0.5,\,U}\left(I_a^2 + I_b^2\right) \leq P_{\alpha=0.5,\,U};\\
&\ldots\\
&E_{\alpha=0.1,\,U} I_a - R_{Z,\,\alpha=0.9,\,U}\left(I_a^2 + I_b^2\right) \leq P_{\alpha=0.1,\,U};\\
&E_{\alpha=0,\,U} I_a - R_{Z,\,\alpha=1,\,U}\left(I_a^2 + I_b^2\right) \leq P_{\alpha=0,\,U}.
\end{aligned} \tag{12}$$

Generally fuzzy constraint (10) can be described as:

$$\begin{aligned}
&\underset{\alpha\in[0,1]}{\forall} E_{\alpha,\,L} I_a - R_{Z,\,1-\alpha,\,L}\left(I_a^2 + I_b^2\right) \geq P_{\alpha,\,L};\\
&\underset{\alpha\in[0,1]}{\forall} E_{\alpha,\,U} I_a - R_{Z,\,1-\alpha,\,U}\left(I_a^2 + I_b^2\right) \leq P_{\alpha,\,U}.
\end{aligned} \tag{13}$$

This technique is called as fuzzy mathematical programming [8]. As the solution of the problem the optimal current (active current) of the source voltage can be determined:

$$_{opt}I = {}_{opt}I_a + \mathrm{j}\,{}_{opt}I_b \tag{14}$$

by minimisation the Lagrange's functional as follows:

$$\begin{aligned}
L(I_a, I_b, \lambda) &= I_a^2 + I_b^2\\
&+ \sum_{\alpha=0}^{1} \lambda_{\alpha,\,L}\left[E_{\alpha,\,L} I_a - R_{Z,\,1-\alpha,\,L}\left(I_a^2 + I_b^2\right) - P_{\alpha,\,L}\right]\\
&+ \sum_{\alpha=0}^{1} \lambda_{\alpha,\,U}\left[E_{\alpha,\,U} I_a - R_{Z,\,1-\alpha,\,U}\left(I_a^2 + I_b^2\right) - P_{\alpha,\,U}\right].
\end{aligned} \tag{15}$$

The minimum of the Lagrange's functional can be determined by means of Kuhn-Tucker conditions:

$$\begin{aligned}
&\frac{\partial L(I_a, I_b, \lambda)}{\partial I_a} = 0,\\
&\frac{\partial L(I_a, I_b, \lambda)}{\partial I_b} = 0,\\
&\frac{\partial L(I_a, I_b, \lambda)}{\partial \lambda} \leq 0,\\
&\underset{\alpha\in[0,1]}{\forall} \lambda_{\alpha,\,L}\frac{\partial L(I_a, I_b, \lambda)}{\partial \lambda_{\alpha,\,L}} = 0,\\
&\underset{\alpha\in[0,1]}{\forall} \lambda_{\alpha,\,U}\frac{\partial L(I_a, I_b, \lambda)}{\partial \lambda_{\alpha,\,U}} = 0
\end{aligned} \tag{16}$$

and then the optimal current can be described as:

$$\begin{aligned}
{}_{opt}I_a &= \frac{1}{2}\,\frac{\sum\limits_{\alpha=0}^{1}\lambda_{\alpha,\,L}\cdot E_{\alpha,\,L} + \sum\limits_{\alpha=0}^{1}\lambda_{\alpha,\,U}\cdot E_{\alpha,\,U}}{\left(1 + \sum\limits_{\alpha=0}^{1}\lambda_{\alpha,\,L}\cdot R_{\alpha,\,L} + \sum\limits_{\alpha=0}^{1}\lambda_{\alpha,\,U}\cdot R_{\alpha,\,U}\right)},\\
{}_{opt}I_b &= 0.
\end{aligned} \tag{17}$$

Lagrange's multipliers $\underset{\alpha\in[0,1]}{\forall}\lambda_{\alpha,\,L}$, $\underset{\alpha\in[0,1]}{\forall}\lambda_{\alpha,\,U}$ should be determined in the numerical way.

3. The example

It is considered the source voltage:

$$\widetilde{e}(t) = \sqrt{2}\left|\widetilde{E}\right|\sin(\omega t)\ \ \mathrm{V};$$

where RMS value of the voltage can change with intervals of $\pm 5\%$ and can be described as:

$$\widetilde{E} = \left|\widetilde{E}\right| e^{\mathrm{j}\psi} = 400\ \mathrm{V};\quad \psi = 0$$

with membership function as fuzzy number presented in Fig. 2. The impedance of the source is described in the frequency domain for the basic harmonic in the form as follows:

$$Z_Z = \widetilde{R}_Z + j\widetilde{X}_Z = 0.5 + \mathrm{j}1.5\,\Omega,$$

where $\widetilde{R}_Z = 0.5\,\Omega$ and $\widetilde{X}_Z = 1.5\,\Omega$ are fuzzy numbers presented in Fig. 4 and Fig. 5.

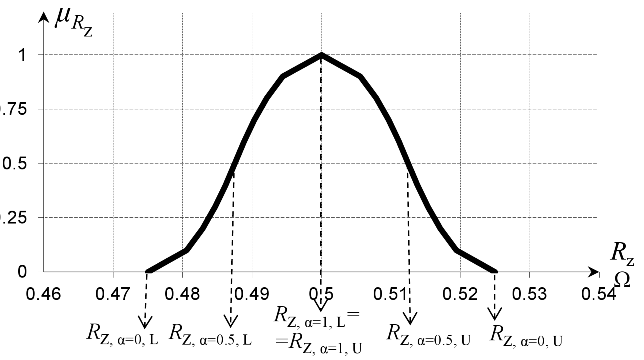

Fig. 4. The resistance of the source as the fuzzy number

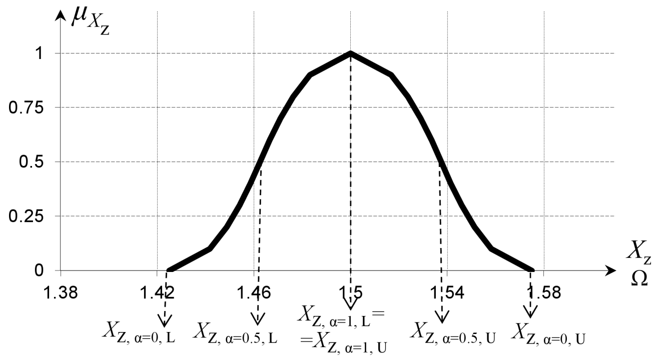

Fig. 5. The reactance of the source as the fuzzy number

The load impedance for the basic harmonic is described as follows:

$$Z_O = \widetilde{R}_O + j\widetilde{X}_O = \widetilde{5} + \mathrm{j}\widetilde{3}\,\Omega,$$

where $\widetilde{R}_O = \widetilde{5}\,\Omega$ and $\widetilde{X}_O = \widetilde{3}\,\Omega$ are fuzzy numbers presented in Fig. 6 and Fig. 7.

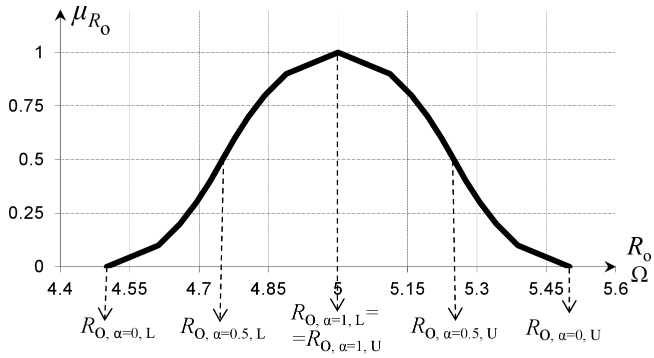

Fig. 6. The resistance of the load as the fuzzy number

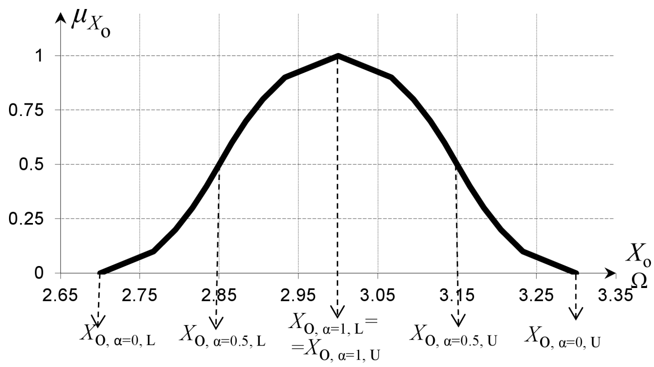

Fig. 7. The reactance of the load as the fuzzy number

Before optimisation, according to changes of the voltage source (voltage and inner impedance) and load impedance, the active power has the values as presented in Fig. 8, where characteristic points have following values:

$$P_{\alpha=0,L} = 13\,222 \text{ W}; \quad P_{\alpha=0.5,L} = 14\,472 \text{ W};$$
$$P_{\alpha=1,L} = P_{\alpha=1,U} = 15\,842 \text{ W};$$
$$P_{\alpha=0.5,U} = 17\,347 \text{ W}; \quad P_{\alpha=0,U} = 19\,006 \text{ W}.$$

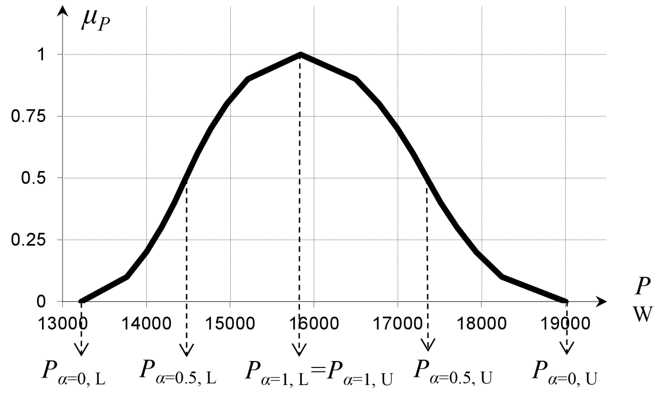

Fig. 8. The obtained values of active power before optimisation

After optimisation the optimal (active) current can be obtained as:

$${}_{opt}I_a = 41.8 \text{ A}; \quad {}_{opt}I_b = 0;$$

and the RMS value: ${}_{opt}|I| = 41.8$ A.

Before minimisation for characteristic values of active power values: $P_{\alpha=0,\,L}$, $P_{\alpha=0.5,\,L}$, $P_{\alpha=1,\,L} = P_{\alpha=1,\,U}$, $P_{\alpha=0.5,\,U}$, $P_{\alpha=0,\,U}$, the corresponding RMS values of the current can be determined as:

$$|I_{\alpha=0,\,L}| = 49.03 \text{ A};$$
$$|I_{\alpha=0.5,\,L}| = 52.5 \text{ A};$$
$$|I_{\alpha=1,\,L}| = |I_{\alpha=1,\,U}| = 56.29 \text{ A};$$
$$|I_{\alpha=0.5,\,U}| = 60.43 \text{ A};$$
$$|I_{\alpha=0,\,U}| = 64.99 \text{ A}.$$

So, the obtained optimal (active) current ${}_{opt}|I| = 41.8$ A has RMS value smaller than the smallest one before optimisation $|I_{\alpha=0,\,L}| = 49.03$ A. Moreover, determined optimal current assures reactive power compensation (imaginary part of this current ${}_{opt}I_b = 0$). It means that optimal currents assure the proper flow of active power and no reactive power consumed from voltage source. In that case when the optimal current can be kept in the system, the active power will be always obtained within the power constraint as presented in Fig. 9 (the inner Gauss function within the considered active power).

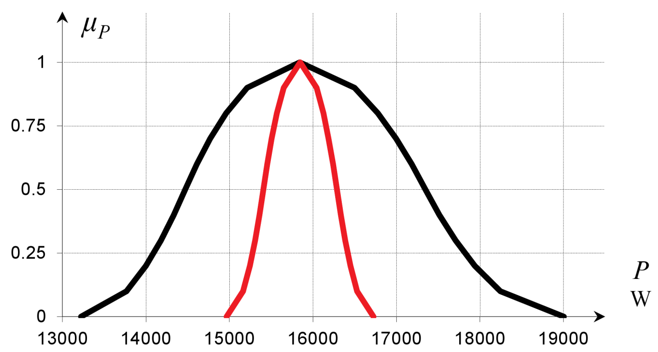

Fig. 9. The obtained values of active power – the internal shape of Gauss function

4. Conclusions

The proposed methods of optimisation and description by the application of fuzzy numbers make possible to describe unsteady parameters of voltage sources and loads in electrical systems as well as to determine the optimal current of the source in one-phase system taking into consideration the problem of minimisation of power losses. This current has the minimal RMS value and it is determined as the result of the optimisation method taking into considerations the additional constraint: active power generated by the voltage source is the same before and after optimisation. The solution (optimal current) is determined as the crisp one (real numbers) in frequency domain. Precisely it can be determined in the numerical way.

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