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ROBUSTIFYING ANALYSIS OF THE DIRECT ADAPTIVE CONTROL OF UNKNOWN MULTIVARIABLE NONLINEAR SYSTEMS BASED ON A NEW NEURO-FUZZY METHOD

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Abstract

In this paper, we are dealing with the problem of directly regulating unknown multivariable affine in the control nonlinear systems and its robustness analysis. The method employs a new Neuro-Fuzzy Dynamical System definition, which uses the concept of Fuzzy Systems (FS) operating in conjunction with High Order Neural Networks. In this way the unknown plant is modeled by a fuzzy - recurrent high order neural network structure (F-RHONN), which is of the known structure considering the neglected nonlinearities. The development is combined with a sensitivity analysis of the closed loop in the presence of modeling imperfections and provides a comprehensive and rigorous analysis showing that our adaptive regulator can guarantee the convergence of states to zero or at least uniform ultimate boundedness of all signals in the closed loop when a not-necessarily-known modeling error is applied. The existence and boundedness of the control signal is always assured by employing a method of parameter “Hopping” and “Modified Hopping”, which appears in the weight updating laws. Simulations illustrate the potency of the method showing that by following the proposed procedure one can obtain asymptotic regulation despite the presence of modeling errors. Comparisons are also made to simple recurrent high order neural network (RHONN) controllers, showing that our approach is superior to the case of simple RHONN’s.

1 Introduction

High order neural network structures can deal with imprecise data and ill-defined activities. However, subjective phenomena such as reasoning and perceptions are often regarded beyond the domain of conventional neural network theory [21]. It is interesting to note that fuzzy logic is another powerful tool for modeling uncertainties associated with human cognition, thinking and perception. Therefore, it has been established that neural networks and fuzzy inference systems are universal approximators [8, 19, 26],i.e., they can approximate any nonlinear function to any prescribed accuracy provided that sufficient hidden neurons and training data, which have to be distributed in the full operation space of the plant, or fuzzy rules are available. The neural and fuzzy approaches are most of the time equivalent, differing between each other mainly in the structure of the approximator chosen. Indeed, in order to bridge the gap between the neural and fuzzy approaches several researchers introduce adaptive schemes using a class of parameterized functions that include both neural networks and fuzzy systems [3, 10, 14, 16].

Recently, the combination of these two different technologies has given rise to $fuzzy-neural$ or $neuro-fuzzy$ approaches, that are intended to capture the advantages of both fuzzy logic and neural networks. Numerous works have shown the viability of this approach for system modeling [3, 10, 13, 14, 16]. Algorithms based upon this integration are believed to have considerable potential in the areas of expert systems, medical diagnosis, control systems, pattern recognition and system modeling.

Adaptive control theory has been an active area of research over the past years [2, 5, 7, 6, 9, 11, 17, 18, 20, 25, 27]. The identification procedure is an essential part in any control procedure. In the neuro or $neuro-fuzzy$ adaptive control two main approaches are followed. In the indirect adaptive control schemes, first the dynamics of the system are identified and then a control input is generated according to the certainty equivalence principle. In the direct adaptive control schemes [9, 5, 18] the controller is directly estimated and the control input is generated to guarantee stability without knowledge of the system dynamics. Also, many researchers focus on robust adaptive control that guarantees signal boundness in the presence of modeling errors and bounded disturbances [2, 5, 6, 7, 9, 11, 18, 25, 27].

Recently [4, 12], high order neural network function approximators (HONNF's) have been proposed for the identification of nonlinear dynamical systems of the form $\dot{x} = f(x,u)$, approximated by a Fuzzy Dynamical System. This approximation depends on the fact that fuzzy rules could be identified with the help of HONN's. The same rationale has been employed in [24, 1, 22, 23] where a neuro - fuzzy approach for the indirect and direct control of square unknown systems has been introduced assuming only parameter uncertainty.

In this paper HONN's are also used for the $neuro-fuzzy$ direct control of nonlinear dynamical systems with modeling errors and a robustifying analysis of the method is presented. From the neural network aspect, we have the alternative approximation of weighted indicator functions ensured with the help of multi high order neural networks. From the fuzzy logic aspect, the underlying fuzzy model is of Mamdani-type [15]. The structure identification of the fuzzy system is made offline based either on human expertise or on gathered data. However, the required a-priori information obtained by linguistic information or data is very limited. The only required information is an estimate of the centers of the output fuzzy membership functions. Information on the input variable membership functions and on the underlying fuzzy rules is not necessary because this is automatically estimated by the HONN's. This way the proposed method is less vulnerable to initial design assumptions.

We consider that the unknown system is of an affine in the control multivariable form and propose its approximation by a recurrent structure employing two independent fuzzy subsystems. We also assume the existence of disturbance expressed as modeling error terms depending on both input and system states. Every fuzzy subsystem is approximated from a family of HONN's, each one being related with a group of fuzzy rules. Weight updating laws are given and it is proved that when the structural identification is appropriate and the modeling error terms are within a certain region depending on the input and state values, then the error reaches zero very fast. Also, an appropriate state feedback is constructed to achieve asymptotic regulation of the output, while keeping bounded all signals in the closed loop. A novel technique of weight hopping is also introduced to assure the existence and boundedness of the control signal.

The paper is organized as follows. Section 2 presents notation and preliminaries related to the concept of fuzzy systems (FS) and the terminology used in the remaining paper, while Section 3 demonstrate the $neuro-fuzzy$ representation of the proposed algorithm. The direct neuro fuzzy regulation of affine in the control dynamical systems under the presence of modeling errors and its robustifying analysis is presented in Section 4, where the method of parameter hopping is explained and the associated weight adaptation laws are given. Finally, simulations presented in Section 5 results on the control of a well known system show off that by following the proposed procedure one can obtain asymptotic regulation in a much better way than by just simply using RHONN controllers. Finally, Section 6 concludes the work.

2 Preliminaries

Consider a nonlinear function $f(x) \in R^n$, $x \in X \subset R^n$ approximately described by a Mamdani-type Fuzzy System (FS). Let $\Omega^{l_1,l_2,\ldots,l_n}_{j_1,j_2,\ldots,j_n}$ be defined as the subset of $x \in X$ belonging to the $(j_1, j_2, \ldots, j_n)^{th}$ input fuzzy patch and pointing - through the vector field $f(\cdot)$ - to the subset which belong to the $l_1, l_2, \ldots, l_n{}^{th}$ output fuzzy patch. In other words, $\Omega^{l_1,l_2,\ldots,l_n}_{j_1,j_2,\ldots,j_n}$ contains input values x that are associated through a fuzzy rule with output values $f(x)$.

Furthermore, the FS receiving as input the $n-tuple$ of $x = (x_1, x_2, \ldots, x_n)$ gives as output an approximate of $f(x)$ using fuzzy rules and a well known fuzzy inference procedure.

Definition 1 *According to the above notation the Rule Firing Indicator Function (RFIF) or simply Indicator Function (IF) connected to* $\Omega^{l_1,l_2,\ldots,l_n}_{j_1,j_2,\ldots,j_n}$ *is defined as follows:+*

$$I^{l_1,l_2,\ldots,l_n}_{j_1,j_2,\ldots,j_n}(x(t)) = \begin{cases} \alpha(x(t)) & if\, x(t) \in \Omega^{l_1,l_2,\ldots,l_n}_{j_1,j_2,\ldots,j_n} \\ 0 & otherwise \end{cases} \tag{1}$$

where $\alpha(x(t))$ *denotes the firing strength of the rule.*

According to the standard fuzzy system description, this strength depends on the membership value of each x_i in the corresponding input membership functions $\mu_{F\,j_i}$ and more specifically [26], $\alpha(x(t)) = \min[\mu_{F\,j_1}(x_1(k)), \ldots \mu_{F\,j_n}(x_n(k))]$. Then, assuming a standard defuzzification procedure (e.g. weighted average), the functional representation of the fuzzy system can be written as

$$f(x(t)) = \sum (\bar{x}_f)^{l_1,l_2,\ldots,l_n}_{j_1,\ldots,j_n} \times \left(I'\right)^{l_1,l_2,\ldots,l_n}_{j_1,\ldots,j_n}(x(t)) \tag{2}$$

where the summation is carried out over all the available fuzzy rules. $(\bar{x}_f)^{l_1,l_2,\ldots,l_n}_{j_1,\ldots,j_n}$ is any constant vector consisting of the centers of fuzzy partitions of f determined by $l_1, l_2, \ldots, l_n$ and $(I')^{l_1,l_2,\ldots,l_n}_{j_1,\ldots,j_n}(x(t))$ is the weighted IF (WIF) defined in (1) divided by the sum of all IF participating in the summation of 2.

However, in order the approximation problem to make sense the space $y := x$ must be compact. Thus, our first assumption is the following:

Assumption 1 y := x *is a compact set.*

Notice that since $y \subset \Re^n$ the above proposition is identical to the proposition that it is closed and bounded. Also, it is noted that even if y is not compact we may assume that there is a time instant T such that $x(t)$ remains in a compact subset of y for all $t < T$; i.e. if $y_T := \{x(t) \in y, t < T\}$ We may replace proposition 1 by the following proposition

Assumption 2 y_T *is a compact set.*

Basing on the fact that functions of high order neurons are capable of approximating discontinuous functions [4] and [12] we use high order neural networks (HONN's) [22] in order to approximate the WIF. The next lemma [12] states that a HONN can approximate the WIF $(I')^{l_1,\ldots,l_n}_{j_1,\ldots,j_n}$.

Lemma 1 *Consider the indicator function* $(I')^{l_1,\ldots,l_n}_{j_1,\ldots,j_n}$ *and the family of the HONN's* $N(x(t); w, L)$. *Then for any* $\varepsilon > 0$ *there is a vector of weights* $w^{j_1,\ldots,j_n;l_1,\ldots,l_n}$ *and a number of* $L^{j_1,\ldots,j_n;l_1,\ldots,l_n}$ *high order connections such that*

$$\sup_{(x(t))\in \bar{y}} \{(I')^{l_1,\ldots,l_n}_{j_1,\ldots,j_n}(x(t)) - N(x(t); w^{j_1,\ldots,j_n;l_1,\ldots,l_n}, L^{j_1,\ldots,j_n;l_1,\ldots,l_n})\} < \varepsilon$$

where $\bar{y} \equiv y$ *if assumption 1 is valid and* $\bar{y}_T \equiv y$ *if assumption 2 is valid.*

Let us now keep $L^{j_1,\ldots,j_n;l_1,\ldots,l_n}$ constant, i.e. let us preselect the number of high order connections, and let us define the optimal weights of the HONN with $L^{j_1,\ldots,j_n;l_1,\ldots,l_n}$ high order connections as follows

$$\bar{w}^{j_1,\ldots,j_n;l_1,\ldots,l_n} := \arg \min_{w \in R^{j_1,\ldots,j_n;l_1,\ldots,l_n}} \times \left\{ \sup_{(x(t)) \in \bar{Y}} \left| (I')^{l_1,\ldots,l_n}_{j_1,\ldots,j_n}(x(t)) - N(x(t); w, L^{j_1,\ldots,j_n;l_1,\ldots,l_n}) \right| \right\}$$

and the modeling error as follows

$$\nu^{l_1,\ldots,l_n}_{j_1,\ldots,j_n}(x(t)) = (I')^{l_1,\ldots,l_n}_{j_1,\ldots,j_n}(x(t)) - N(x(t); w^{j_1,\ldots,j_n;l_1,\ldots,l_n}, L^{j_1,\ldots,j_n;l_1,\ldots,l_n})$$

It is worth noticing that from Lemma 1, we have that $\sup_{(x(t)) \in \bar{y}} \left| \nu^{l_1,\ldots,l_n}_{j_1,\ldots,j_n}(x(t)) \right|$ can be made arbitrarily small by simply selecting appropriately the number of high order connections.

Following the above notation $(I')^{l_1,l_2,\ldots,l_n}_{j_1,\ldots,j_n}$ in (2) can be approximated by $N^{l_1,l_2,\ldots,l_n}_{j_1,\ldots,j_n}(x) = N(x(t); w^{j_1,\ldots,j_n;l_1,l_2,\ldots,l_n}, L^{j_1,\ldots,j_n;l_1,l_2,\ldots,l_n})$.

So, Eq. (2) can be rewritten as

$$f(x(t)) = \sum (\bar{x}_f)^{l_1,l_2,\ldots,l_n}_{j_1,\ldots,j_n} \times N^{l_1,l_2,\ldots,l_n}_{j_1,\ldots,j_n}(x(t)) \quad (3)$$

From the above definitions and Eq. (3), it is obvious that the accuracy of the approximation of $f(x)$ depends on the approximation abilities of HONN's and on an initial estimate of the centers of the output membership functions. These centers can be obtained by experts or by off-line techniques based on gathered data. Any other information related to the input membership functions is not necessary because it is replaced by the HONN's.

Figure (1) shows the overall scheme of the proposed $neuro-fuzzy$ approximation of a function $f(x)$ depending on measurements of input variables x and a-priori knowledge of the centers of the partitions of the fuzzy output variables. When x is given as inputs to the $neuro-fuzzy$ network (input layer), the output of indicator layer gives the weighted indicator function outputs which activate the corresponding rules around a fuzzy center (rule layer). The summation of all rules at each sampling time instant gives the overall output of the function $f(x)$ (output layer).

3 Neuro-Fuzzy Representation of the Algorithm

We consider affine in the control, nonlinear dynamical systems of the form

$$\dot{x} = f(x) + g(x) \cdot u \quad (4)$$

where the state $x \in R^n$ is assumed to be completely measured, the control u is in R^q, f is an unknown smooth vector field called the drift term and g is a matrix with rows containing the unknown smooth controlled vector fields g_{ij}. The above class of continuous-time nonlinear systems are called affine, because in (4) the control input appears linear with respect to g. The main reason for considering this class of nonlinear systems is that most of the systems encountered in engineering, are by nature or design, affine.

We are using an affine in the control fuzzy dynamical system, which approximates the system in (4) and uses two fuzzy subsystem blocks for the description of $f(x)$ and $g(x)$ as follows

$$\hat{f}(x) = A\hat{x} + \sum (\bar{x}_f)^{l_1,\ldots,l_n}_{j_1,\ldots,j_n} \times (I')_f{}^{l_1,\ldots,l_n}_{j_1,\ldots,j_n}(x) \quad (5)$$

$$\hat{g}_{ij}(x) = \sum (\bar{x}_{g_{ij}})^{l}_{j_1,\ldots,j_n} \times (I')_g{}^{l}_{j_1,\ldots,j_n}(x) \quad (6)$$

where A is a $n \times n$ stable matrix which for simplicity can be taken to be diagonal as $A = diag[-a_1, -a_2, \ldots, -a_n]$, a_i positive, and the summation is carried out over the number of all available fuzzy rules. $(I')_f$, $(I')_g$ are appropriate fuzzy rule indicator functions and the meaning of indices $\bullet^{l_1,\ldots,l_n}_{j_1,\ldots,j_n}$ has already been described in Section 2.

According to Lemma 1, every indicator function can be approximated with the help of a suitable HONN. Therefore, every $(I')_f$, $(I')_g$ can be replaced with a corresponding HONN as follows

$$\hat{f}(x|W_f) = A\hat{x} + \sum (\bar{x}_f)^{l_1,\ldots,l_n}_{j_1,\ldots,j_n} \times N_f{}^{l_1,\ldots,l_n}_{j_1,\ldots,j_n}(x) \quad (7)$$

$$\hat{g}_{ij}(x|W_g) = \sum (\bar{x}_{g_{ij}})^{l}_{j_1,\ldots,j_n} \times N_g{}^{l}_{j_1,\ldots,j_n}(x) \quad (8)$$

where W_f, W_g are weights that results from adaptive laws which will be discussed later, and N_f, N_g are appropriate HONN's.

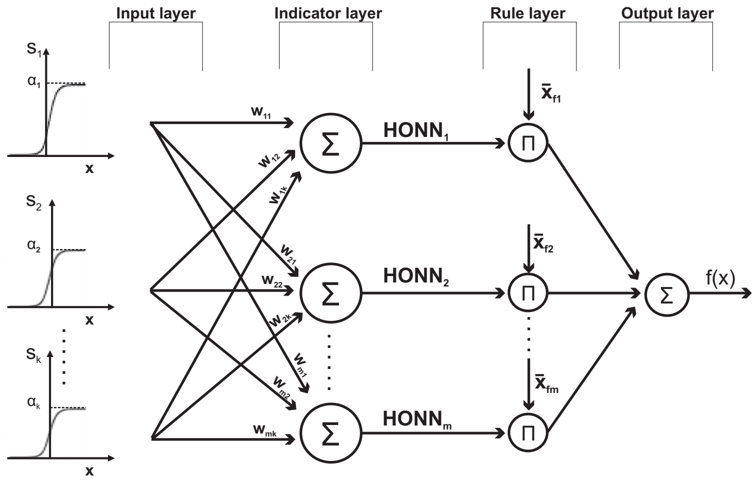

Figure 1. Overall scheme of the proposed $neuro-fuzzy$ representation which approximates function $f(x)$ based on measurements of x and a-priori knowledge of the centers $\bar{x}_f$.

So, the optimal approximation of $f(x)$ and $g(x)$ subfunctions of the dynamical system becomes

$$f(x|W_f^*) = Ax + \sum (\bar{x}_f)^{l_1,\ldots,l_n}_{j_1,\ldots,j_n} \times N^{*l_1,\ldots,l_n}_{f\,j_1,\ldots,j_n}(x) \quad (9)$$

$$g_{ij}(x|W_g^*) = \sum (\bar{x}_{g_{ij}})^{l}_{j_1,\ldots,j_n} \times N^{*l}_{g\,j_1,\ldots,j_n}(x) \quad (10)$$

In order to simplify the model structure, since some rules result in the same output partition, we could replace the NNs associated to the rules having the same output with one NN and therefore the summations in (7),(8) are carried out over the number of the corresponding output partitions. Therefore, the affine in the control fuzzy dynamical system in (5), (6) is replaced by the following equivalent affine Fuzzy - Recurrent High Order Neural Network (F-RHONN), which depends on the centers of the fuzzy output partitions $(\bar{x}_f)_l$ and $(\bar{x}_{g_{ij}})_l$

$$\dot{\hat{x}} = A\hat{x} + \sum_{l=1}^{Npf} (\bar{x}_f)_l \times N_{f_l}(x) + \sum_{i=1}^{n}\left(\sum_{j=1}^{q}\left(\sum_{l=1}^{Npg_i}(\bar{x}_{g_{ij}})_l \times N_{g_l}(x)\right)u_j\right) \quad (11)$$

Or in a more compact form

$$\dot{\hat{x}} = A\hat{x} + X_f W_f S_f(x) + X_g W_g S_g(x)u \quad (12)$$

where X_f, X_g are matrices containing the centres of the partitions of every fuzzy output variable of $f(x)$ and $g(x)$, respectively, $S_f(x), S_g(x)$ are matrices containing high order combinations of sigmoid functions of the state x and W_f, W_g are matrices containing respective neural weights according to (11). The dimensions and the contents of all the above matrices are chosen so that $X_f W_f S_f(x)$ is a $n \times 1$ vector and $X_g W_g S_g(x)$ is a $n \times q$ matrix. For notational simplicity we assume that all output fuzzy variables are partitioned to the same number, m, of partitions. Under these specifications X_f is a $n \times n \cdot m$ block diagonal matrix of the form $X_f = diag(X_{f_1}, X_{f_2}, \ldots, X_{f_n})$ with X_{f_i} being an m-dimensional row vector of the form

$$X_{f_i} = \begin{bmatrix} \bar{x}^1_{f_i} & \bar{x}^2_{f_i} & \cdots & \bar{x}^m_{f_i} \end{bmatrix}$$

or in a more detailed form

$$X_f = \begin{bmatrix} \bar{x}^1_{f_1} & \cdots & \bar{x}^m_{f_1} & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & \bar{x}^1_{f_2} & \cdots & \bar{x}^m_{f_2} & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 & \bar{x}^1_{f_n} & \cdots & \bar{x}^m_{f_n} \end{bmatrix}$$

where $\bar{x}^p_{f_i}$ with $p = 1,2,...,m$, denotes the centre of the p-th partition of f_i. Also, $S_f(x) = \begin{bmatrix} s_1(x) & \ldots & s_k(x) \end{bmatrix}^T$, where each $s_l(x)$ with $l = \{1,2,...,k\}$, is a high order combination of sigmoid functions of the state variables and W_f is a $n \cdot m \times k$ matrix with neural weights. W_f assumes the form $W_f = \begin{bmatrix} W_{f_1} & \cdots & W_{f_n} \end{bmatrix}^T$, where each W_{f_i} is a matrix $\begin{bmatrix} w^{pl}_{f_i} \end{bmatrix}_{m\times k}$. X_g is a $n \times n \cdot m \cdot q$ block diagonal matrix of the form $X_g = diag(X_{g_{1j}}, X_{g_{2j}}, \ldots, X_{g_{nq}})$ with each $X_{g_{ij}}$ $(j = 1,2,...,q,\ i = 1,2,...,n)$ being an $m-$dimensional raw vector of the form

$$X_{g_{ij}} = \begin{bmatrix} \bar{x}^1_{g_{ij}} & \bar{x}^2_{g_{ij}} & \cdots & \bar{x}^m_{g_{ij}} \end{bmatrix}$$

or in a more detailed form

$$X_g = \begin{bmatrix} X_{g_{11}} & \cdots & X_{g_{1q}} & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & X_{g_{21}} & \cdots & X_{g_{2q}} & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 & X_{g_{n1}} & \cdots & X_{g_{nq}} \end{bmatrix}$$

where $\bar{x}^k_{g_{ij}}$ denotes the center of the k-th partition of g_{ij}. W_g is a $n \cdot m \cdot q \times n \cdot q$ block diagonal matrix with $W_g = [W_{g_1}, W_{g_2}, \ldots, W_{g_{nq}}]^T$, where each W_{g_k} with $k = 1, 2, ..., n \cdot q$ is an $m-$dimensional column vector $\left[w^p_{g_k}\right]_{m\times 1}$ of neural weights. Finally, $S_g(x)$ is a $n \cdot q \times q$ matrix of the form $S_g = [S_{g_1}, S_{g_2}, \ldots, S_{g_n}]^T$, where each S_{g_i} is a diagonal $q \times q$ matrix $S_{g_i} = diag(s_i, \ldots, s_i)$ with the diagonal element $s_i(x)$ being a high order combination of sigmoid functions of the state variables.

4 Direct Robust Adaptive Neuro-Fuzzy Control

4.1 Problem Formulation

The state regulation problem is known as our attempt to force the state to zero from an arbitrary initial value by applying appropriate feedback control to the plant input. However, since the plant is considered unknown, we assume that the unknown plant can be described by the following model arriving from the neuro-fuzzy representation of (12), where the weight values W^*_f and W^*_g are unknown.

$$\dot{\hat{x}} = A\hat{x} + X_f W^*_f S_f(x) + X_g W^*_g S_g(x)u \quad (13)$$

Due to the approximation capabilities of the dynamic neural networks, we can assume with no loss of generality, that the unknown plant (4) can be completely described by a dynamical neural network plus a modeling error term $\omega(x,u)$. In other words, there exist weight values W^*_f and W^*_g such that the system (4) can be written as

$$\dot{\hat{x}} = A\hat{x} + X_f W^*_f S_f(x) + X_g W^*_g S_g(x)u + \omega(x,u) \quad (14)$$

Therefore, the state regulation problem is analyzed for the system (14) instead of (4). Since W^*_f and W^*_g are unknown, our solution consists of designing a control law $u(W_f, W_g, x)$ and appropriate update laws for W_f and W_g to guarantee convergence of the state to zero and in some cases, which will be analyzed in the following sections, boundedness of x and of all signals in the closed loop.

The following mild assumptions are also imposed on (4), to guarantee the existence and uniqueness of solution for any finite initial condition and $u \in U$.

Assumption 3 *Given a class $U \subset R^q$ of admissible inputs, then for any $u \in U$ and any finite initial condition, the state trajectories are uniformly bounded for any finite $T > 0$. Meaning that we do not allow systems processing trajectories which escape at infinite, in finite time T, T being arbitrarily small. Hence, $|x(T)| < \infty$.*

Assumption 4 *The vector fields* f, g_{ij}, $i = 1, 2, ..., n$ *are continuous with respect to their arguments and satisfy a local Lipchitz condition so that the solution $x(t)$ of (4) is unique for any finite initial condition and $u \in U$.*

4.2 Adaptive Regulation - Complete Matching

In this subsection, we present a solution to the adaptive regulation problem and investigate the modeling error effects. Assuming the presence of modeling error the unknown system can be written as (14), where $x \in \Re^n$ is the system state vector, $u \in \Re^q$ are the control inputs, X_f, X_g are $n \times n \cdot m$ and $n \times n \cdot m \cdot q$ block diagonal matrices, respectively, W^*_f is a $n \cdot m \times k$ matrix of synaptic weights and W^*_g is a $n \cdot m \cdot q \times n \cdot q$ block diagonal matrix. Finally, $S_f(x)$ is a k-dimensional vector and $S_g(x)$ is a $n \cdot q \times n$ block diagonal matrix with each diagonal element $s_i(x)$ being a high order combination of sigmoid functions of the state variables.

Define now ν as

$$\nu \stackrel{\Delta}{=} X_f W_f S_f(x) + X_g W_g S_g(x)u - \dot{x} - Ax \quad (15)$$

substituting Eq. (14) to Eq. (15) we have

$$\nu \stackrel{\Delta}{=} X_f \tilde{W}_f S_f(x) + X_g \tilde{W}_g S_g(x)u - \omega(x,u) \quad (16)$$

where $\tilde{W}_f = W_f - W_f^*$ and $\tilde{W}_g = W_g - W_g^*$. W_f and W_g are estimates of W_f^* and W_g^*, respectively and are obtained by update laws which are to be designed in the sequel. ν cannot be measured since $\dot{x}$ is unknown. To overcome this problem, we use the following filtered version of ν

$$\nu = \dot{\xi} + K\xi$$

where $K = \begin{bmatrix} k_1 & 0 & \cdots & 0 \\ 0 & k_2 & 0 & \cdots \\ \cdots & 0 & \cdots & 0 \\ 0 & \cdots & 0 & k_n \end{bmatrix}$ is a diagonal positive definite matrix. In the sequel, according to Eq. (15) we have that

$$\dot{\xi} + K\xi = -\dot{x} - Ax + X_f W_f S_f(x) + X_g W_g S_g(x)u \tag{17}$$

and after substituting Eq. (14) we have

$$\dot{\xi} = -K\xi + X_f \tilde{W}_f S_f(x) + X_g \tilde{W}_g S_g(x)u - \omega(x,u) \tag{18}$$

To implement Eq. (18), we take

$$\xi \triangleq \zeta - x \tag{19}$$

Employing Eq. (19), Eq. (17) can be written as

$$\dot{\zeta} + K\zeta = Kx - Ax + X_f W_f S_f(x) + X_g W_g S_g(x)u \tag{20}$$

with state $\zeta \in \Re^n$. This method is referred to as error filtering.

The regulation of the system can be achieved by selecting the control input to be

$$u = -\left[X_g W_g S_g(x)\right]^+ \left[X_f W_f S_f(x) + \upsilon\right] \tag{21}$$

where $[\cdot]^+$ means pseudo-inverse in Moore-Penrose sense and

$$\upsilon = (K - A)x \tag{22}$$

Thus, substituting Eq. (21), Eq. (20) becomes

$$\dot{\zeta} = -K\zeta \tag{23}$$

To continue, consider the Lyapunov candidate function

$$V = \xi^T\xi + \zeta^T\zeta + \frac{1}{2\gamma_1} tr\left\{\tilde{W}_f^T \tilde{W}_f\right\} + \frac{1}{2\gamma_2} tr\left\{\tilde{W}_g^T \tilde{W}_g\right\} \tag{24}$$

If we take the derivative of Eq. (24) with respect to time we obtain

$$\dot{V} = -\xi^T K\xi - \zeta^T K\zeta + \xi^T X_f \tilde{W}_f S_f(x) + \xi^T X_g \tilde{W}_g S_g(x)u - \xi^T \omega(x,u) + \tfrac{1}{\gamma_1} tr\left\{\dot{W}_f^T \tilde{W}_f\right\} + \tfrac{1}{\gamma_2} tr\left\{\dot{W}_g^T \tilde{W}_g\right\}$$

Hence, if we choose

$$tr\left\{\dot{W}_f^T \tilde{W}_f\right\} = -\gamma_1 \xi^T X_f \tilde{W}_f S_f(x) \tag{25}$$

$$tr\left\{\dot{W}_g^T \tilde{W}_g\right\} = -\gamma_2 \xi^T X_g \tilde{W}_g S_g(x)u \tag{26}$$

$\dot{V}$ becomes

$$\dot{V} \leq -\lambda_{min}(K)\|\xi\|^2 - \lambda_{min}(K)\|\zeta\|^2 + \|\xi\|\,\|\omega(x,u)\| \tag{27}$$

It can be easily verified that Eqs. (25) and (26) after making the appropriate operations, can be element wise written as

a) for the elements of W_f

$$\dot{w}_{f_i}^{pl} = -\gamma_1 \bar{x}_{f_i}^p \xi s_l(x) \tag{28}$$

or equivalently $\dot{W}_{f_i}^l = -\gamma_1 \left(X_{f_i}\right)^T \xi s_l(x)$ for all $i = 1,2,...,n$, $p = 1,2,...,m$ and $l = 1,2,...,k$.

b) for the elements of W_g

$$\dot{w}_{g_{ij}}^p = -\gamma_2 \bar{x}_{g_{ij}}^p \xi u_j s_i(x) \tag{29}$$

or equivalently $\dot{W}_{g_{ij}} = -\gamma_2 \left(X_{g_{ij}}\right)^T \xi u_j s_i(x)$ for all $i = 1,2,...,n$, $j = 1,2,...,q$ and $p = 1,2,...,m$.

Equations (28) and (29) can be finally written in a compact form as

$$\dot{W}_f = -\gamma_1 X_f^T \xi S_f^T(x) \tag{30}$$

$$\dot{W}_g = -\gamma_2 X_g^T \xi u^T S_g^T(x) \tag{31}$$

where ξ is a vector $\xi = (\xi_1, \xi_2, ..., \xi_n)$ and u is also a vector $u = (u_1, u_2, ..., u_n)$. Furthermore, we cannot conclude anything about the weight convergence if the existence and boundeness of signal u are not assured. So, the weight updating laws (28), (29) have to be modified by introducing a method of parameter "Hopping" or "Modified Hopping", which is explained below.

4.2.1 Introduction to the Parameter Hopping

The weight updating laws presented previously in Section 4.2 are valid when the control law signal in (21), (22) exists. Therefore, the existence of $\left[X_g W_g S_g(x)\right]^+$ has to be assured. Since the sub matrices of $S_g(x)$ are diagonal with the diagonal elements $s_i(x) \neq 0$ and X_g, W_g are block diagonal and thus linearly independent, the existence of the pseudoinverse is assured when $X_{g_{i,j+q(i-1)}} \cdot W_{g_{j+q(i-1)}} \neq 0$, $\forall i = 1,\ldots,n$ and $\forall j = 1,\ldots,q$. Therefore, $W_{g_{j+q(i-1)}}$ has to be confined such that $\left| X_{g_{i,j+q(i-1)}} \cdot W_{g_{j+q(i-1)}} \right| \geq \theta_{j+q(i-1)} > 0$, with $\theta_{j+q(i-1)}$ being a small positive design parameter (usually in the range of $[0.001, 0.01]$). In case the boundary defined by the above confinement is nonlinear the updating W_g can be modified by using a projection algorithm [9]. For notational simplicity, we can define with $a = i, j+q(i-1)$ and $b = j+q(i-1)$. However, in our case the boundary surface is linear and the direction of updating is normal for it because $\nabla [X_{g_a} \cdot W_{g_b}] = X_{g_a}$. Therefore, the projection of the updating vector on the boundary surface is of no use. Instead, using concepts from multidimensional vector geometry we modify the updating law such that, when the weight vector approaches (within a safe distance θ_b) the forbidden hyperplane $X_{g_a} \cdot W_{g_b} = 0$ and the direction of updating is toward the forbidden hyper-plane, it introduces a *hopping* which drives the weights in the direction of the updating but on the other side of the space, where here the weight space is divided into two sides by the forbidden hyper-plane. For example, let the weight updating hopping occurs at the t_h time instant. Then, if the weights at t_h^- time instant lies in the space determined by $X_{g_a} \cdot W_{g_b} < -\theta$ then, after performing hopping the weights move into the space determined by $X_{g_a} \cdot W_{g_b} > \theta$ and from t_h^+ on they continue their updating direction. This procedure is depicted in Fig. 2, where a simplified 2-dimensional representation is given. Theorem 2 below introduces this *hopping* in the weight updating law.

Lemma 2 *The updating law for the elements of $W_{g_{ij}}$ given by (29) and modified according to the Hopping method:*

$$\dot{W}_{g_b} = \begin{cases} -\gamma_2 (X_{g_a}) \xi u_j s_i(x) & if\ |X_{g_a} \cdot W_{g_b}| > \theta_b \\ & or\, X_{g_a} \cdot W_{g_b} = \pm\theta_b \\ & and\, X_{g_a} \cdot \dot{W}_{g_b} <> 0 \\ -\gamma_2 (X_{g_a}) \xi u_j s_i(x) & otherwise \\ -\frac{2\kappa^{inner}\left(X_{ga} W_{g_b} (X_{ga})^T\right)}{tr\{(X_{ga})^T X_{ga}\}} & \end{cases}$$

assures the existence of the control signal.

Proof The first part of the weight updating equation is used when the weights are at a certain distance (condition if $|X_{g_a} \cdot W_{g_b}| > \theta_b$) from the forbidden plane or at the safe limit (condition $|X_{g_a} \cdot W_{g_b}| = \pm\theta_b$) but with the direction of updating moving the weights far from the forbidden plane (condition $X_{g_a} \cdot \dot{W}_{g_b} <> 0$). In the current notation, the "$\pm$" symbol has a one to one correspondence with the "$<>$" one, meaning that "$+$" case corresponds to "$<$" case and the "$-$" case corresponds to "$>$" case.

In the second part of $\dot{W}_{g_b}$, term $-\frac{2\kappa^{inner}\left(X_{ga} W_{g_b} (X_{ga})^T\right)}{tr\{(X_{ga})^T X_{ga}\}}$ determines the magnitude of weight *hopping*, which as explained in the vectorial proof of "hopping" [22], has to be at least two times the distance of the current weight vector to the forbidden hyper-plane. In addition, the constant value κ^{inner} helps the weights to move nearby but outside the forbidden hyper planes in order to avoid the infinite hopping. Therefore, the ***existence*** of the control signal is assured because the weights never reach the forbidden plane. □

The inclusion of weight hopping in the weights updating law guarantees that the control signal does not go to infinity. Apart from that, it is also of practical use to assure that $X_g W_g S_g(x)$ does not approach even temporarily at very large values because in this case the method may become algorithmically unstable driving at the same time the control signal to zero failing to control the system. To assure that this situation does not happen we have again to assure that $|X_{g_a} \cdot W_{g_b}| < \rho_b$ with ρ_b being again a design parameter determining an external limit for $X_{g_a} \cdot W_{g_b}$. Following the same lines of thought with the case of weight hopping introduced above we could again consider the forbidden hyperplanes being defined by the equation $|X_{g_a} \cdot W_{g_b}| = \rho_b$. When the weight vector reaches one of the forbidden hyper-planes $X_{g_a} \cdot W_{g_b} = \rho_b$ and the direction of updating is toward the forbidden hyper-plane, a new *modified hopping* is introduced which moves the weights insight the restricting area. This procedure is depicted in Fig. 4, in a simplified 2-dimensional representation. The magnitude of hopping is $-\frac{\kappa^{outer}\left(X_{ga} W_{g_b} (X_{ga})^T\right)}{tr\{(X_{ga})^T X_{ga}\}}$ being determined by following again the same vectorial proof [22], with κ^{outer} a small positive number decided appropriately from the designer as will be ex-

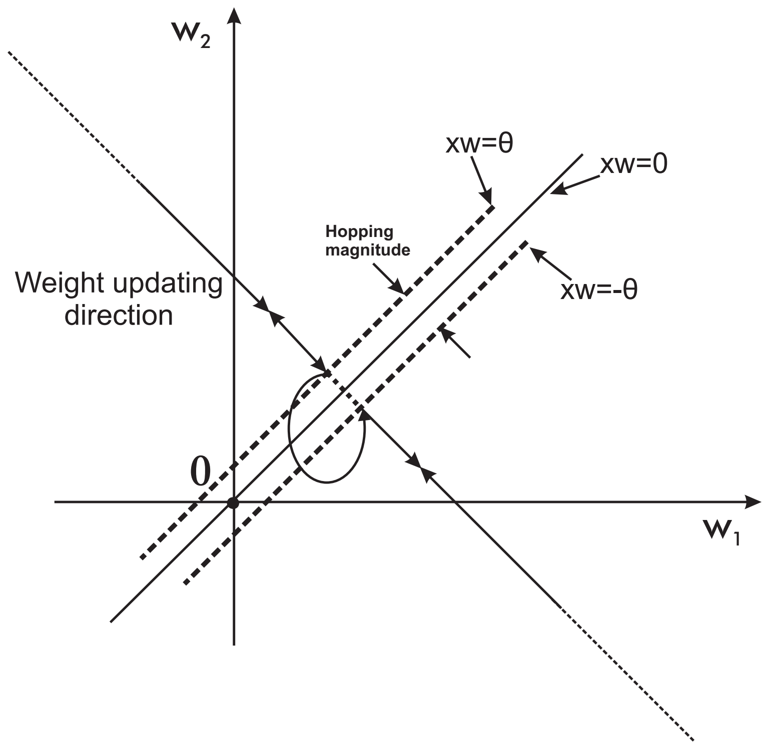

Figure 2. Pictorial Representation of parameter hopping

plained further down.

The adaptation of the weights is perpendicular to the forbidden hyperplanes. This is demonstrated using the first derivative of $X_{g_a} \cdot W_{g_b}$ in respect to the elements of W_{g_b} which is actually equal to the vector X_{g_a} of the fuzzy output centers. When the weights leave the admissible area then the hopping condition is activated and the weights come back to the permissible area as can be seen in Fig. 3. The positive constant values κ^{inner}, κ^{outer} help the designer to avoid the infinite hopping that may occur between the forbidden hyperplanes.

By performing *hopping* when $X_{g_a} \cdot W_{g_b}$ reaches either the inner or outer forbidden planes, $X_{g_a} \cdot W_{g_b}$ is confined to lie in space $P = \{X_{g_a} \cdot W_{g_b} : |X_{g_a} \cdot W_{g_b}| \leq \rho_b \; and \; |X_{g_a} \cdot W_{g_b}| > \theta_b\}$ lying between these hyper-planes. The weight updating law for W_{g_b} incorporating the two hopping conditions can now be expressed as

$$\dot{W}_{g_b} = \begin{cases} -\gamma_2\left(X_{g_a}\right)\xi u_j s_i(x) & \begin{array}{l} if\, X_{g_a} \cdot W_{g_b} \in P \\ or\, X_{g_a} \cdot W_{g_b} = (\pm\theta_b \; or \pm \\ and\, X_{g_a} \cdot \dot{W}_{g_b} <> or \\ >< 0 \end{array} \\ \begin{array}{l} -\gamma_2\left(X_{g_a}\right)\xi u_j s_i(x) - \\ -\frac{2\sigma_i \kappa^{inner}\left(X_{ga} W_{g_b} (X_{ga})^T\right)}{tr\{(X_{ga})^T X_{ga}\}} \\ -\frac{(1-\sigma_i)\kappa^{outer}\left(X_{ga} W_{g_b} (X_{ga})^T\right)}{tr\{(X_{ga})^T X_{ga}\}} \end{array} & otherwise \end{cases} \tag{32}$$

where

$$\sigma_i = \begin{cases} 0 & \begin{array}{l} if\; X_{g_a} \cdot W_{g_b} = \pm\rho_l \\ and\; X_{g_a} \cdot \dot{W}_{g_b} <> 0 \end{array} \\ 1 & otherwise \end{cases} \tag{33}$$

where again, the " $\pm$ " symbol has a one to one correspondence with the " $<>$ " one, meaning that " $+$ " case corresponds to " $<$ " case and the " $-$ " case corresponds to " $>$ " case.

At this point we can distinguish two possible cases. The complete model matching at zero case and the modeling error at zero case.

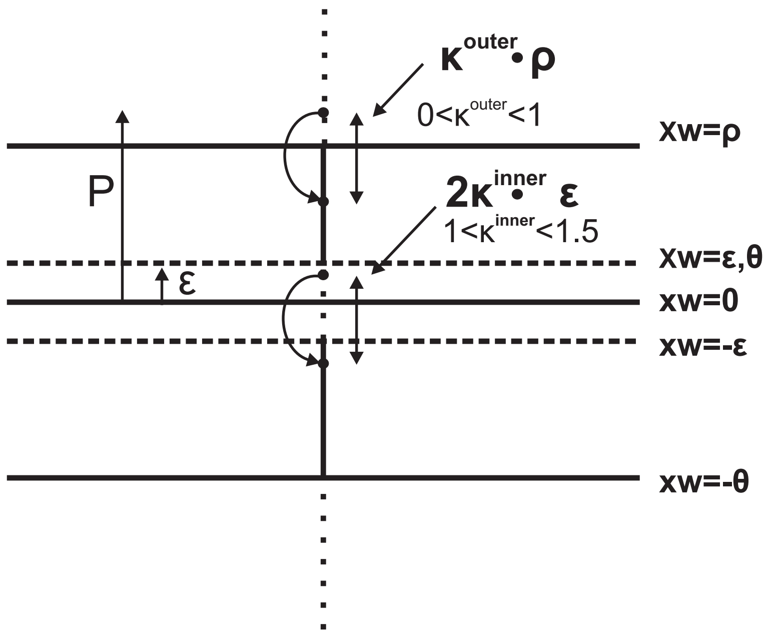

Figure 3. Inner and outer hopping at a distance which depends on an appropriate selection of κ^{inner}, κ^{outer} constant values.

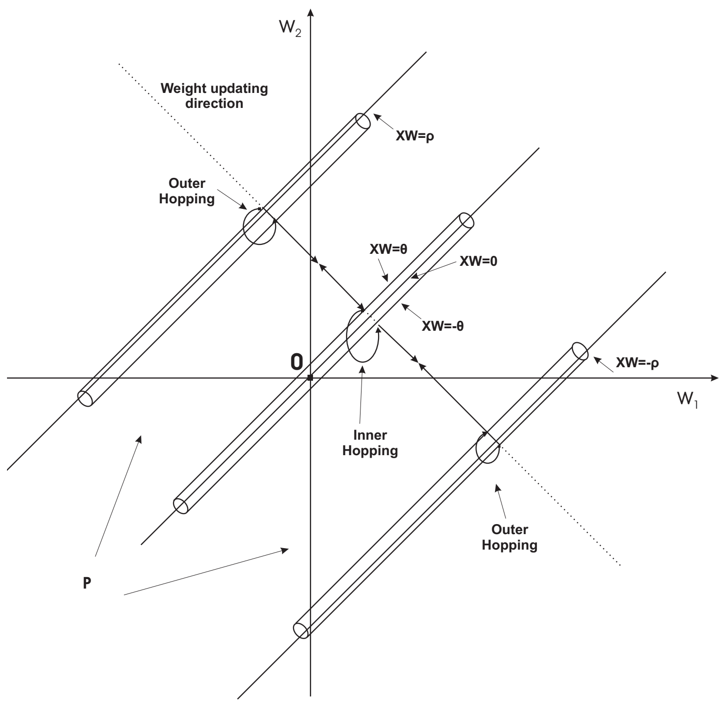

Figure 4. Pictorial Representation of inner and outer parameter hopping

4.2.2 The Complete Model Matching at Zero Case

We make the following assumption.

Assumption 5 The modeling error term satisfies

$$\|\omega(x,u)\| \leq \ell_1' \|x\| + \ell_1'' \|u\|$$

where ℓ_1' and ℓ_1'' are known positive constants.

Also, we can find an *a priori* known constant $\ell_u > 0$, such that

$$\|u\| \leq \ell_u \|x\| \tag{34}$$

and assumption 5 becomes equivalent to

$$\|\omega(x)\| \leq \ell_1 \|x\| \tag{35}$$

where

$$\ell_1 = \ell_1' + \ell_1'' \ell_u \tag{36}$$

is a positive constant.

One can easily verify that (34) is valid provided that $X_f W_f$ is uniformly bounded by a known positive constant ε_i so $X_f W_f(t)$ is confined to the set $P_2 = \{X_{f_i} \cdot W_{f_i}^l : \left|X_{f_i} \cdot W_{f_i}^l\right| \leq \varepsilon_i\}$ through the use of a hopping algorithm. In particular, the standard update law (26) is modified to

$$\dot{W}_{f_i}^l = \begin{cases} -\gamma_1 \left(X_{f_i}\right)^T \xi s_l(x) & \begin{array}{l} if\, X_{f_i} \cdot W_{f_i}^l \in P_2 \\ or\, X_{f_i} \cdot W_{f_i}^l = \pm\varepsilon_i \\ and\, X_{f_i} \cdot \dot{W}_{f_i}^l >< 0 \end{array} \\ -\gamma_1 \left(X_{f_i}\right)^T \xi s_l(x) - \\ \quad -\frac{\kappa^{outer}\left(X_{f_i} W_{f_i}^l \left(X_{f_i}\right)^T\right)}{tr\left\{\left(X_{f_i}\right)^T X_{f_i}\right\}} & otherwise \end{cases} \tag{37}$$

therefore, we have the following lemma.

Lemma 3 If the initial weights are chosen such that $X_{f_i} \cdot W_{f_i}^l(0) \in P_2$ and $X_{f_i} \cdot W_{f_i}^{*l} \in P_2$ then we have $X_{f_i} \cdot W_{f_i}^l \in P_2$ for all $t \geq 0$.

Proof The above lemma can be readily established by noting that whenever $|X_{f_i} \cdot (W_{f_i}^l)^-| \geq \varepsilon_i$ then

$$\frac{d}{dt}\left(|X_{f_i} \cdot (W_{f_i}^l)^+|^2\right) \leq 0 \tag{38}$$

which implies that after hopping occurs, the weights $(W_{f_i}^l)^+$, are directed towards the interior of P_2. For simplicity, since we will be working from now on with the time $(\cdot)^+$, we omit the + sign from the exponent. It is true that

$$\frac{d}{dt}\left(|X_{f_i} \cdot W_{f_i}^l|^2\right) = W_{f_i}^{l\,T} \dot{W}_{f_i}^l X_{f_i} X_{f_i}^T \tag{39}$$

Since $X_{f_i} X_{f_i}^T > 0$, only $W_{f_i}^{l\,T} \dot{W}_{f_i}^l$ determines the sign of the above derivative.

Employing the modified adaptive law (37), we obtain

$$\left(W_{f_i}^l\right)^T \dot{W}_{f_i}^l = -\gamma_1 \left(W_{f_i}^l\right)^T \left(X_{f_i}\right)^T \xi s_l(x) - \kappa^{outer} \varepsilon_i \frac{\left(W_{f_i}^l\right)^T W_{f_i}^l}{\left\|W_{f_i}^l\right\|} \tag{40}$$

where $\varepsilon_i = \frac{\left(X_{f_i} W_{f_i}^l \left(X_{f_i}\right)^T\right)}{tr\left\{\left(X_{f_i}\right)^T X_{f_i}\right\}}$. As concerning the second part of the above equation it is obvious that $\varepsilon_i > 0$ and $\frac{\left(W_{f_i}^l\right)^T W_{f_i}^l}{\left\|W_{f_i}^l\right\|} > 0$. So,

$$-\kappa^{outer} \varepsilon_i \frac{\left(W_{f_i}^l\right)^T W_{f_i}^l}{\left\|W_{f_i}^l\right\|} < 0.$$

Now, regarding the first part of eq. (40), we can distinguish two cases:

Case 1: $X_{f_i} \cdot W_{f_i}^l = \varepsilon_i$ and $X_{f_i} \cdot \dot{W}_{f_i}^l < 0$.

From the above notation we have that

$$X_{f_i} \dot{W}_{f_i}^l = -\gamma_1 X_{f_i} \left(X_{f_i}\right)^T \xi s_l(x) < 0 \Rightarrow \gamma_1 \xi s_l(x) > 0 \tag{41}$$

also, $X_{f_i} W_{f_i}^l \geq \varepsilon_i$ and so the first part of Eq. (40) becomes

$$-\gamma_1 \left(W_{f_i}^l\right)^T \left(X_{f_i}\right)^T \xi s_l(x) \leq -\gamma_1 \varepsilon_i \xi s_l(x)$$

according to Eq. (41)

$$-\gamma_1 \left(W_{f_i}^l\right)^T \left(X_{f_i}\right)^T \xi s_l(x) < 0$$

Case 2: $X_{f_i} \cdot W_{f_i}^l \leq -\varepsilon_i$ and $X_{f_i} \cdot \dot{W}_{f_i}^l > 0$.

From the above notation we have that

$$X_{f_i}\dot{W}^l_{f_i} = -\gamma_1 X_{f_i}\left(X_{f_i}\right)^T \xi s_l(x) > 0 \Rightarrow \gamma_1 \xi s_l(x) < 0 \tag{42}$$

also, $X_{f_i}W^l_{f_i} \leq -\varepsilon_i$ and so the first part of Eq. (40) becomes

$$-\gamma_1\left(W^l_{f_i}\right)^T \left(X_{f_i}\right)^T \xi s_l(x) \geq -\gamma_1(-\varepsilon_i)\xi s_l(x)$$

according to Eq. (42)

$$-\gamma_1\left(W^l_{f_i}\right)^T \left(X_{f_i}\right)^T \xi s_l(x) < 0$$

therefore, we finally obtain

$$\tfrac{d}{dt}\left(|X_{f_i}\cdot W^l_{f_i}(t)|^2\right) \leq 0$$

□

In the sequel, employing assumption 5, Eq. (27) becomes

$$\dot{V} \leq -\lambda_{min}(K)\|\xi\|^2 - \lambda_{min}(K)\|\zeta\|^2 + \ell_1\|\xi\|\|x\| \Rightarrow$$

$$\dot{V} \leq -\lambda_{min}(K)\left(\|\xi\|^2 + \|\zeta\|^2\right) + \ell_1\|\xi\|^2 + \ell_1\|\xi\|\|\zeta\| \Rightarrow$$

$$\dot{V} \leq -\begin{bmatrix}\|\xi\| & \|\zeta\|\end{bmatrix}\begin{bmatrix}\lambda_{min}(K)-\ell_1 & -\ell_1\\ 0 & \lambda_{min}(K)\end{bmatrix}\begin{bmatrix}\|\xi\|\\ \|\zeta\|\end{bmatrix} \tag{43}$$

Hence, if we chose $\lambda_{min}(K) \geq \ell_1$ then Eq. (43) becomes negative. Thus, we have

$$\dot{V} \leq 0. \tag{44}$$

Regarding the ***negativity*** of $\dot{V}$ we proceed with the following lemma.

Lemma 4 Based on the adaptive laws (32), (37) the additional terms introduced in the expression for $\dot{V}$, can only make $\dot{V}$ more negative.

Proof Let that $W^*_{g_b}$ contains the actual unknown values of W_{g_b} such that $\left|X_{g_a}\cdot W^*_{g_b}\right| >> \theta_b$ and that $\tilde{W}_{g_b} = W_{g_b} - W^*_{g_b}$. Then, the weight hopping can be equivalently written with respect to $\tilde{W}_{g_b}$ as $-2\kappa^{inner}\theta_b\tilde{W}_{g_b}/\|\tilde{W}_{g_b}\|$ when the inner hopping condition is activated or $-\kappa^{outer}\rho_b\tilde{W}_{g_b}/\|\tilde{W}_{g_b}\|$ when the outer hopping condition is activated. Under this consideration the modified updating law is rewritten as $\dot{W}_{g_b} = -\gamma_2\left({}^1X^i\right)^T\xi u_j s_i(x) - 2\sigma_i\kappa^{inner}\theta_b\tilde{W}_{g_b}/\|\tilde{W}_{g_b}\| - (1-\sigma_i)\kappa^{outer}\rho_b\tilde{W}_{g_b}/\|\tilde{W}_{g_b}\|$. With this updating law it can be easily verified that (43) becomes

$$\dot{V} \leq -\begin{bmatrix}\|\xi\| & \|\zeta\|\end{bmatrix}\begin{bmatrix}\lambda_{min}(K)-\ell_1 & -\ell_1\\ 0 & \lambda_{min}(K)\end{bmatrix}\begin{bmatrix}\|\xi\|\\ \|\zeta\|\end{bmatrix} - \Theta_g \tag{45}$$

with Θ_g being a positive constant expressed as

$$\Theta_g = \sigma_i\sum 2\kappa^{inner}\theta_b\left((\tilde{W}_{g_b})^T)\tilde{W}_{g_b})\right)/\|\tilde{W}_{g_b}\| + (1-\sigma_i)\kappa^{outer}\sum\rho_b\left((\tilde{W}_{g_b})^T)\tilde{W}_{g_b})\right)/\|\tilde{W}_{g_b}\| \geq 0$$

for all time, where the summation includes all weight vectors which require hopping.

Therefore, the negativity of $\dot{V}$ is actually strengthened due to the last negative term.

By using the modified updating law for $W^l_{f_i}$ the negativity of the Lyapunov function is not compromised. Indeed, the first part of the modified form of $\dot{W}^l_{f_i}$ shown in Eq. (37), is exactly the same with (26) and therefore according to the development of (26) the negativity of V is in effect. The first part is used when the weights are inside the constraint area (condition if $\left|X_{f_i}\cdot W^l_{f_i}\right| \leq \varepsilon_i$) or at the safe limit (condition $X_{f_i}\cdot W^l_{f_i} = \pm\varepsilon_i$) but with the direction of updating moving the weights towards the 'safe' region (condition $X_{f_i}\cdot\dot{W}^l_{f_i} >< 0$).

In the second part of $W^l_{f_i}$, term $-\frac{\kappa^{outer}\left(X_{f_i}W^l_{f_i}(X_{f_i})^T\right)}{tr\{(X_{f_i})^T X_{f_i}\}}$ determines the magnitude of weight *hopping*, which as explained in the vectorial proof of "hopping" [22], has to be two κ^{outer} times the distance of the current weight vector. Regarding the ***negativity*** of $\dot{V}$ we proceed as follows.

Let that $W^{*l}_{f_i}$ contains the actual unknown values of $W^l_{f_i}$ such that $\left|X_{f_i}\cdot W^{*l}_{f_i}\right| << \varepsilon_i$ and that $\tilde{W}^l_{f_i} = W^l_{f_i} - W^{*l}_{f_i}$. Then, the weight hopping can be equivalently written with respect to $\tilde{W}^l_{f_i}$ as $-\kappa^{outer}\varepsilon_i\tilde{W}^l_{f_i}/\|\tilde{W}^l_{f_i}\|$. Under this consideration the modified updating law is rewritten as $\dot{W}^l_{f_i} = -\gamma_1\left(X^i\right)^T\xi s_l(x) - \kappa^{outer}\varepsilon_i\tilde{W}^l_{f_i}/\|\tilde{W}^l_{f_i}\|$. With this updating law it can be easily verified that eq. (45) becomes

$$\dot{V} \leq -\begin{bmatrix} \|\xi\| & \|\zeta\| \end{bmatrix} \begin{bmatrix} \lambda_{min}(K)-\ell_1 & -\ell_1 \\ 0 & \lambda_{min}(K) \end{bmatrix} \begin{bmatrix} \|\xi\| \\ \|\zeta\| \end{bmatrix} -\Theta_f-\Theta_g \quad (46)$$

with Θ_f being a positive constant expressed as $\Theta_f = \sum \kappa^{outer} \varepsilon_i \left(((\tilde{W}^l_{f_i})^T) \tilde{W}^l_{f_i}) \right) / \|\tilde{W}^l_{f_i}\| \geq 0$ for all time, where the summation includes all weight vectors which require hopping. Therefore, the negativity of $\dot{V}$ is actually strengthened due to the last negative terms.

Lemma 3 imply that the hopping modifications (32), (37) guarantees boundedness of the weights, without affecting the rest of the stability properties established in the absence of hopping. □

Hence, we can prove the following theorem.

Theorem 5 The control law (21) and (22) together with the updating laws (32) and (37) guarantee the following properties

1. $\xi, \|x\|, W_f, W_g, \zeta, \dot{\xi} \in L_\infty, \quad \|\xi\| \in L_2$

2. $\lim_{t\to\infty}\xi(t) = 0, \quad \lim_{t\to\infty}\|x(t)\| = 0$

3. $\lim_{t\to\infty}\dot{W}_f(t) = 0, \quad \lim_{t\to\infty}\dot{W}_g(t) = 0$

provided that $\lambda_{min}(K) \geq \ell_1$.

Proof From Eq. (44) we have that $V \in L_\infty$ which implies $\xi, \tilde{W}_f, \tilde{W}_g \in L_\infty$. Furthermore $W_f = \tilde{W}_f + W_f^* \in L_\infty$ and $W_g = \tilde{W}_g + W_g^* \in L_\infty$. Since, $\xi = \zeta - x$ and $\zeta, \xi \in L_\infty$ this in turn implies that $\|x\| \in L_\infty$. Moreover, since V is a monotone decreasing function of time and bounded from below, $\lim_{t\to\infty}V(t) = V_\infty$ exists so by integrating $\dot{V}$ from 0 to ∞ we have

$$(\lambda_{\min}(K) - l_1)\int_0^\infty \|\xi\|^2 dt + \lambda_{\min}(K)\int_0^\infty \|\zeta\|^2 dt - \ell_1 \int_0^\infty \|\xi\|\,\|\zeta\|\, dt = |V(0) - V_\infty| < \infty$$

which implies that $\|\xi\| \in L_2$. We also have that

$$\dot{\xi} = -K\xi + X_f\tilde{W}_f S_f(x) + X_g\tilde{W}_g S_g(x)u - \omega(x).$$

Hence and since $u, \|x\| \in L_\infty$ $\dot{\xi} \in L_\infty$, the sigmoidals are bounded by definition, $\tilde{W}_f, \tilde{W}_g \in L_\infty$ and Assumption 5 hold, so since $\xi \in L_2 \cap L_\infty$ and $\dot{\xi} \in L_\infty$, applying Barbalat's Lemma [9] we conclude that $\lim_{t\to\infty}\xi(t) = 0$. Now, using the boundedness of $u, S_f(x), S_g(x), x$ and the convergence of $\xi(t)$ to zero, we have that $\dot{W}_f, \dot{W}_g$ also converge to zero. Hence and since $\zeta(t)$ also converges to zero, we have that

$$\lim_{t\to\infty} x(t) = \lim_{t\to\infty}\zeta(t) - \lim_{t\to\infty}\xi(t) = 0$$

Thus,

$$\lim_{t\to\infty}\|x(t)\| = 0.$$

□

Remark 1 *Inequality $\lambda_{min}(K) \geq \ell_1$ shows how the design constant K should be selected, in order to guarantee convergence of the state x to zero, even in the presence of modeling error terms which are not uniformly bounded* a priori*, as assumption 5 implies. The value of K becomes large as we allow for large model imperfections but K is implemented as a gain in the construction of $\dot{\zeta}$ and for practical reasons it cannot take arbitrarily large values. This leads to a compromise between the value of K and the maximum allowable modeling error terms.*

4.2.3 The Modeling Error at Zero Case

In the subsection (4.2.2), we have assumed that the modeling error term satisfies the following condition

$$\|\omega(x,u)\| \leq l_1' \|x\| + l_1'' \|u\|$$

which implies that the modeling error becomes zero when $\|x\| = 0$ and we have proven convergence of the state x to zero, plus boundedness of all signals in the closed-loop. In this subsection however, we examine the more general case which is described by the following assumption.

Assumption 6 The modeling error term satisfies

$$\|\omega(x,u)\| \leq l_0 + l_1' \|x\| + l_1'' \|u\|$$

Having made this assumption, we now allow a not-necessarily-known modeling error $l_0 \neq 0$ at zero. Furthermore, as stated previously, we can find an *a priori* known constant $l_u > 0$, such that

$$\|u\| \le l_u \|x\|$$

thus making

$$\|\omega(x,u)\| \equiv \|\omega(x)\|$$

and Assumption 6 equivalent to

$$\|\omega(x)\| \le l_0 + l_1 \|x\| \tag{47}$$

where

$$l_1 = l_1' + l_1'' l_u \tag{48}$$

is a positive constant. Employing (35), Eq. (27) becomes

$$\begin{aligned} \dot{V} &\le -\lambda_{min}(K)\|\xi\|^2 - \lambda_{min}(K)\|\zeta\|^2 \\ &\quad + \|\xi\|\left[l_0 + l_1\|x\|\right] \\ &\le -\lambda_{min}(K)\|\xi\|^2 - \lambda_{min}(K)\|\zeta\|^2 \\ &\quad + l_1\|\xi\|^2 + l_1\|\xi\|\|\zeta\| + l_0\|\xi\|. \end{aligned} \tag{49}$$

To continue, we need to state and prove the following lemma

Lemma 6 The control law

$$u = -\left[X_g W_g S_g(x)\right]^+ \left[X_f W_f S_f(x) + \upsilon\right] \tag{50}$$

$$\upsilon = (K - A)x \tag{51}$$

where the synaptic weight estimates W_f and W_g, are adjusted according to equations (32), (37) guarantee the following properties

1. $\zeta(t) \le 0, \quad \forall t \ge 0$

2. $\lim_{t\to\infty}\zeta(t) = 0$ exponentially fast provided that $\zeta(t) < 0$.

Proof Observe that if we use the control laws (50), (51), Eq. (20) becomes

$$\dot{\zeta} = -K\zeta, \quad \forall t \ge 0$$

which is a homogeneous differential equation with solution

$$\zeta(t) = \zeta(0)e^{-Kt}$$

Hence, if $\zeta(0)$ which represents the initial value of $\zeta(t)$, is chosen negative, we obtain

$$\zeta(t) \le 0 \quad \forall t \ge 0.$$

Moreover, $\zeta(t)$ converges to zero exponentially fast. □

Hence, we can distinguish the following cases:

Case 1: If $x \ge 0$ we have that $\zeta(t) \ge \xi(t)$ but $\zeta(t) \le 0, \forall t \ge 0$ which implies that $\|\zeta(t)\| \le \|\xi(t)\|$. So, we have

$$\|x\| \le \|\zeta\| + \|\xi\| \le 2\|\xi\|. \tag{52}$$

Therefore, Eq. (49) becomes

$$\begin{aligned} \dot{V} &\le -\lambda_{min}(K)\|\xi\|^2 - \lambda_{min}(K)\|\zeta\|^2 \\ &\quad + 2l_1\|\xi\|^2 + l_0\|\xi\| \qquad (53) \\ &\le -\left(\lambda_{min}(K)\|\xi\| - 2l_1\|\xi\| - l_0\right)\|\xi\| \\ &\quad - \lambda_{min}(K)\|\zeta\|^2 \le 0 \qquad (54) \end{aligned}$$

provided that

$$\|\xi\| > \frac{l_0}{\lambda_{min}(K) - 2l_1} \tag{55}$$

with $\lambda_{min}(K) > 2l_1$.

Case 2: If $x < 0$ we have that $\zeta(t) < \xi(t)$ but $\zeta(t) \le 0, \forall t \ge 0$ which implies that $\|\zeta(t)\| > \|\xi(t)\|$. So, we have

$$\|x\| \le \|\zeta\| + \|\xi\| \le 2\|\zeta\|. \tag{56}$$

Therefore, Eq. (49) becomes

$$\begin{aligned} \dot{V} &\le -\lambda_{min}(K)\|\xi\|^2 - \lambda_{min}(K)\|\zeta\|^2 \\ &\quad + 2l_1\|\xi\|\|\zeta\| + l_0\|\xi\| \qquad (57) \\ &\le -\left(\lambda_{min}(K)\|\xi\| - l_0\right)\|\xi\| \\ &\quad - \left(\lambda_{min}(K) - 2l_1\right)\|\zeta\|^2 \le 0 \qquad (58) \end{aligned}$$

provided that

$$\|\xi\| > \frac{l_0}{\lambda_{min}(K)} \tag{59}$$

and $\lambda_{min}(K) > 2l_1$.

Conclusively, $\forall x \in R^n$ the Lyapunov candidate function becomes negative when $\|\xi\| > \frac{l_0}{\lambda_{min}(K) - 2l_1}$ and $\lambda_{min}(K) > 2l_1$.

In the sequel, inequality (55) together with (52), (56) demonstrate that the trajectories of $\xi(t)$ and $x(t)$ are uniformly bounded with respect to the arbitrarily small, (since K can be chosen sufficiently large), sets Ξ and X shown below

$$\Xi = \left\{\xi(t) : \|\xi(t)\| \leq \frac{2l_0}{\lambda_{min}(K)-2l_1},\ \lambda_{min}(K) > 2l_1 > 0\right\}$$

and

$$X = \left\{x(t) : \|x(t)\| \leq \frac{2l_0}{\lambda_{min}(K)},\ \lambda_{min}(K) > 2l_1 > 0\right\}.$$

Thus, we have proven the following theorem:

Theorem 7 Consider the system (14) with the modeling error term satisfying (35). Then the control law (21), (22) together with the update laws (32) and (37) guarantees the uniform ultimate boundedness with respect to the sets

1.

$$\Xi = \left\{\xi(t) : \|\xi(t)\| \leq \frac{2l_0}{\lambda_{min}(K)-2l_1},\ \lambda_{min}(K) > 2l_1 > 0\right\}$$

2.

$$X = \left\{x(t) : \|x(t)\| \leq \frac{2l_0}{\lambda_{min}(K)},\ \lambda_{min}(K) > 2l_1 > 0\right\}$$

Furthermore,

$$\dot{\xi} = -K\xi + X_f\tilde{W}_f S_f(x) + X_g\tilde{W}_g S_g(x)u - \omega(x).$$

Hence, since the boundedness of $\tilde{W}_f$ and $\tilde{W}_g$ is assured by the use of the hopping algorithm and $\omega(x)$ owing to (35) and Theorem 7, we conclude that $\dot{\xi} \in L_\infty$.

Remark 2 *The previous analysis reveals that in the case where we have a modeling error different from zero at $\|x\| = 0$, our adaptive regulator can guarantee at least uniform ultimate boundedness of all signals in the closed loop. In particular, Theorem 7 shows that if l_0 is sufficiently small, or if the design constant K is chosen such that $\lambda_{min}(K) > 2l_1$, then $\|x(t)\|$ can be arbitrarily close to zero and in the limit as $K \to \infty$, actually becomes zero but as we stated in Remark 1, implementation issues constrain the maximum allowable value of K.*

5 Simulation Results

To demonstrate the potency of the proposed scheme we present simulation results which assume modeling errors. So, we tested the ability of the proposed direct control scheme to regulate a Dc Motor, under the presence of modeling error distinguishing two cases. The first case defined as "Complete Model Matching at Zero Case" where the modeling error depends on the states and the control inputs, while in the second case we have "Modeling Error at Zero Case" where the modeling errors depends on the states, the control inputs and a not-necessarily-known modeling error which is of the known constant value different than zero. All the simulation results present a comparison between the proposed method and a simple RHONN direct controller [21], which shows off the performance superiority of the proposed method.

5.1 Direct Control of DC Motor When We Have "Complete Model Matching at Zero Case"

We apply the proposed approach to control the speed of a 1 KW DC motor with a normalized model described by the following dynamical equations [21]

$$\begin{aligned} T_a\frac{dI_a}{dt} &= -I_a - \Phi\Omega + V_a \\ T_m\frac{d\Omega}{dt} &= \Phi I_a - K_0\Omega - m_L \\ T_f\frac{d\Phi}{dt} &= -I_f + V_f \\ \Phi &= \frac{aI_f}{1+bI_f} \end{aligned} \tag{60}$$

The states are chosen to be the armature current, the angular speed and the stator flux, $x = \left[I_a\Omega\Phi\right]$. As control inputs the armature and the field voltages, $u = \left[V_aV_f\right]$ are used. With this choice, we have

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_a}x_1 - \frac{1}{T_a}x_2x_3 \\ \frac{1}{T_m}x_1x_3 - \frac{K_0}{T_m}x_2 - \frac{m_L}{T_m} \\ -\frac{1}{T_f}\frac{x_3}{a-\beta x_3} \end{bmatrix} + \begin{bmatrix} \frac{1}{T_a} & 0 \\ 0 & 0 \\ 0 & \frac{1}{T_f} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{61}$$

which is of a nonlinear, affine in the control form.

Assuming the existence of modeling errors, we add disturbance terms in the two states x_1 and x_2 as follows

$$\omega(x_1,u_1) = 2x_1 + 2\sin(10x_1) + \sin(3u_1)$$

$$\omega(x_2,u_2) = 3x_2 + \sin(5x_2) + \sin(2u_2)$$

In many control schemes of the literature V_f is assumed constant. This may naturally occur when the field is produced by a permanent magnet or when it may be separately excited but is intentionally kept constant. This assumption may facilitate things because if V_f is constant then Φ is constant and the above nonlinear 3^{rd} order system can be linearized and reduced to a second order form having 2 states ($x_1 = I_a$ and $x_2 = \Omega$), with the value Φ being included as a constant parameter.

$$T_a \frac{dI_a}{dt} = -I_a - \Phi\Omega + V_a$$

$$T_m \frac{d\Omega}{dt} = \Phi I_a - K_0\Omega - m_L$$

In a more general case, however, V_f is not considered constant and this scheme can also be used for armature and field weakening control of the separately excited Dc motor. Moreover, if the motor characteristics are not exactly known we may consider that the nonlinear model is unknown and therefore its control can be accomplished using the proposed neuro-fuzzy approach. In this case, the regulation problem of a DC motor is translated as follows: Find a state feedback to force the angular velocity Ω and the armature current I_a to go to zero, while the magnetic flux varies.

Motivated by this simplification (2^{nd} instead of 3^{rd}), we first assume that the system is described, within a degree of accuracy, by the 2^{nd} order neuro-fuzzy system of the form (12), where $x_1 = I_a$ and $x_2 = \Omega$. So, the number of states is $n = 2$, the number of fuzzy output partitions of each f_i is $m = 5$ with the ranges of f_1 [-182.5667 , 0], f_2 [-19.3627 , 30.0566] and the depth of high order sigmoid terms $k = 5$. In this case $s_i(x)$ assume high order connection up to the second order. The number of fuzzy partitions of each g_{ii} is selected to be $m = 3$ with the ranges of g_{11} [148 , 150] and g_{22} [42 , 44], using only the first order sigmoid term.

However, in the simulations carried out, the actual system is simulated by using the complete set of equations (61). The produced control law described in (21) and (22) is applied to this system, which in turn produces states x_1, x_2, which are in the sequel used in the updating laws of the controller's weights.

We simulated a 1KW DC motor with parameter values that can be seen in Table 1 and sampling time 10^{-3} sec. In order our model to be equivalent with RHONN's regarding to other parameters we have chosen the initial values of all variables as $[I_a \quad \Omega \quad \Phi] = [1 \quad 1 \quad 0.98]$, the initial weights $W_{f_i} = [0]$, $W_{g_{ij}} = [1]$ and the updating learning rates $\gamma_1 = 0.01$ and $\gamma_2 = 25$. Also, the parameters of the sigmoidal terms were chosen to be $a_1 = 0.4$, $a_2 = 5$, $b_1 = b_2 = 1$, $c_1 = c_2 = 0$, while the diagonal elements of matrix K were $k_1 = 5$, $k_2 = 10$.

As concerning comparison abilities Figure (5) gives the evolution of the states x_1 and x_2, which are the armature current and angular velocity of the RHONN [21] (red line) and the proposed Fuzzy-RHONN model (blue line), with time respectively where we can observe that the RHONN Model has oscillations, while the Fuzzy-RHONN has smooth development going close to zero as expected. Also, figures (6) and (7) show the evolution of control inputs and disturbances for RHONN (red line) and F-RHONN (blue line) with time, respectively.

5.2 Direct Control of DC Motor When We Have "Modeling Error at Zero Case"

Assuming the existence of modeling errors again to the same Dc Motor, we add disturbance terms in the two states x_1 and x_2 as follows

$$\omega(x_1, u_1) = 3x_1 + \sin(0.1x_1) + \sin(0.1u_1) + 1$$
$$\omega(x_2, u_2) = 2x_2 + \sin(1000x_2) + \sin(1000u_2) + 1$$

We performed three different simulations with varying values for the elements of matrix K, $\kappa_i = 120, 140, 160$ with $i = 1, 2$ and $\kappa_1 = \kappa_2$.

As concerning comparison abilities figures (8), (9) and (10) give the evolution of state x_2, which is the angular velocity of the RHONN [21] (red line) and the proposed Fuzzy-RHONN (blue line) models and the disturbance, with time, respectively. It can be observed that the RHONN Model converges to zero slower when compared to the proposed adaptive control algorithm. Also, while K changes and more precisely when its values are increasing then our model converges to zero faster any time and keeps peak values constant against RHONN's which have slower convergence and bigger peak values.

Table 1. Parameter values for the DC motor.

Parameter	Value
$1/T_a$	$148.88\ sec^{-1}$
$1/T_m$	$42.91\ sec^{-1}$
K_0/T_m	$0.0129\ N \cdot m/rad$
T_f	$31.88\ sec$
m_L	0.0
a	2.6
β	1.6

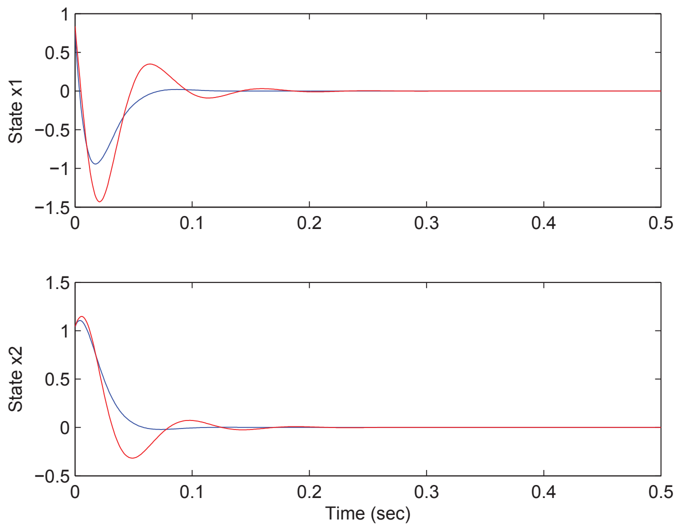

Figure 5. Evolution of armature current and angular velocity x_1 and x_2 respectively, for RHONN's (red line) and F-RHONN approach (blue line).

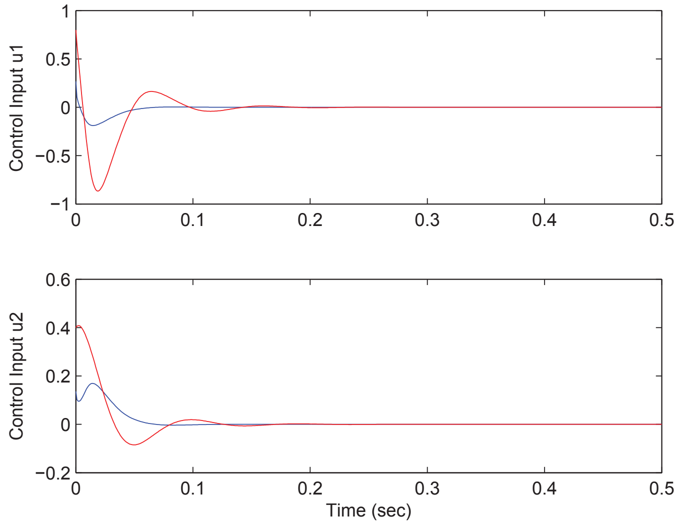

Figure 6. Evolution of control inputs u_1 and u_2 respectively, for RHONN's (red line) and F-RHONN approach (blue line).

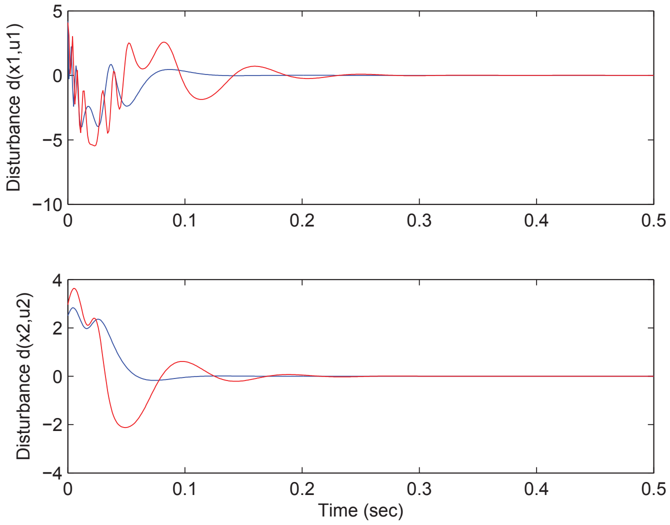

Figure 7. Evolution of disturbances $d(x_1,u_1)$ and $d(x_2,u_2)$ respectively, for RHONN's (red line) and F-RHONN approach (blue line).

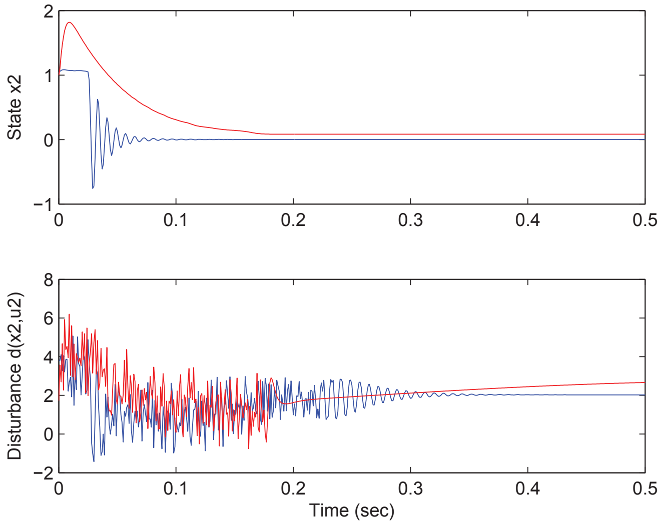

Figure 8. Evolution of angular velocity x_2 for Fuzzy-RHONN and RHONN Models when $\kappa_i = 120$.

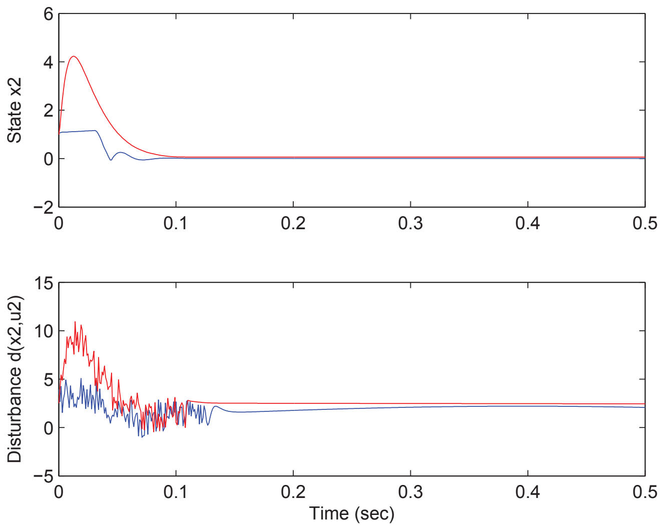

Figure 9. Evolution of angular velocity x_2 for Fuzzy-RHONN and RHONN Models when $\kappa_i = 140$.

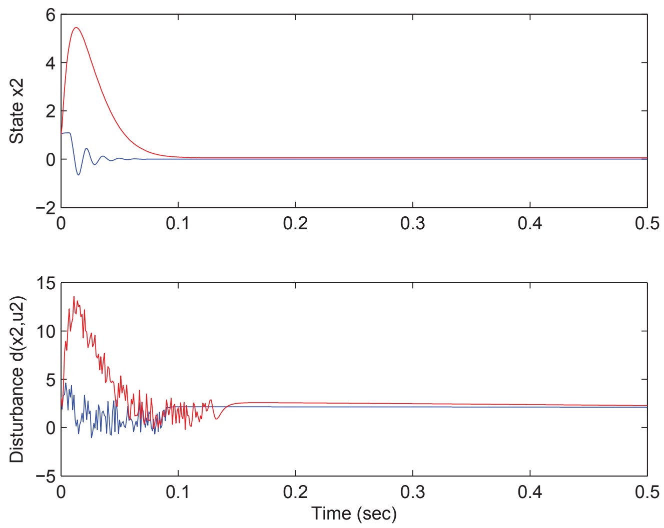

Figure 10. Evolution of angular velocity x_2 for Fuzzy-RHONN and RHONN Models when $\kappa_i = 160$.

6 Conclusion

The robustifying analysis of a direct adaptive control scheme was considered in this paper, aiming at the regulation of nonlinear unknown plants. The approach is based on a new Neuro-Fuzzy Dynamical Systems definition, which uses the concept of Fuzzy Dynamical Systems (FDS) operating in conjunction with High Order Neural Network (F-HONN's). Since the plant is considered unknown, we propose its approximation by a special form of an affine in the control fuzzy system (FDS) and in the sequel the fuzzy rules are approximated by appropriate HONN's. The fuzzy-recurrent high order neural networks are used as models of the unknown plant, practically transforming the original unknown system into a F-RHONN model which is of the known structure, but contains a number of unknown constant value parameters known as synaptic weights. The proposed scheme does not require a-priori experts' information on the number and type of input variable membership functions making it less vulnerable to initial design assumptions, is computationally very fast and thus can be used in several real-time engineering applications. Weight updating laws for the involved HONN's are provided, which guarantee that the system states reach zero exponentially fast, while keeping all signals in the closed loop bounded. A novel method of parameter hopping developed for the first time by the authors, assures the existence of the control signal and is incorporated in the weight updating law. Simulations illustrate the potency of the method in controlling an unknown nonlinear multivariable plant. Compared to simple RHONN direct control, the proposed method proves to be superior.

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