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Brief note

EFFECT OF SUSPENDED PARTICLES ON THERMOSOLUTAL CONVECTION OF RIVLIN-ERICKSEN FLUID IN POROUS MEDIUM WITH VARIABLE GRAVITY

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The thermosolutal stability of a layer of the Rivlin-Ericksen fluid in a porous medium is considered under varying gravity conditions. It is found that for stationary convection, medium permeability and suspended particles have a destabilizing/stabilizing effect when gravity increases/decreases. The stable solute gradient has a stabilizing effect on the system.

Key words: thermosolutal convection, Rivlin-Ericksen fluid, suspended particles, porous medium, varying gravity.

1. Introduction

The first major contribution to the study of hydrodynamic stability can be found in the theoretical papers of Helmholtz [1]. The general linear stability theory for a inviscid plane parallel shear flow was investigated by Lord Rayleigh [2]. The significant contributions were made by Reynolds, Kelvin and Rayleigh. Stern [3] studied the problem of varying concentration of solute (solute is salt). Veronis [4] studied the case when the fluid layer is heated and soluted from below. Nield [5] considered the case of a fluid layer heated from below and soluted from above. The principles of thermosolutal convection are well documented by Bejan [6]. The experimental work of Bénard [7] gave rise to the problem of the onset of thermal instability in fluids heated from below. The Bénard problem and its extension were investigated by Chandrasekhar [8] and recognized by Rumford [9] and Thompson [10]. The instability of Bénard model has been a subject of interest till today and an excellent review of this work up to 1957 with special reference to its possible fields of application has been given by Ostrach [11].

Thermal stability of a fluid layer under a variable gravitational field heated from below or above is investigated analytically by Pradhan and Samal [12]. Rana [13] investigated the thermosolutal convection in Walters' fluid under varying gravity. The instability of non-Newtonian fluids (Walters', Rivlin-Ericksen, etc) has been studied by several authors ([14]-[23], [24] and [25]).

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In this paper, we have investigated the effect of suspended particles on thermosolutal convection in a Rivlin-Ericksen elastico-viscous fluid in a porous medium under varying gravity. It is the extension of the problem studied by Aggarwal and Prakash [14].

2. Mathematical model of the problem

Here we are considering a horizontal layer of an incompressible Rivlin-Ericksen fluid which is permeated with suspended particles, bounded by planes $z = 0$ and $z = d$. This fluid layer is subjected to variable gravity $\boldsymbol{g}(0,0,-g)$ where $g = \lambda g_0, g_0(>0)$ is the value of g at $z = 0$ and λ is the variable gravity parameter, can be positive or negative. This fluid layer is heated and soluted from below and flowing through a porous medium. Then the governing equations of motion and continuity for the fluid flow can be written as

$$\frac{\rho}{\varepsilon}\left[\frac{\partial \boldsymbol{q}}{\partial t}+\frac{1}{\varepsilon}(\boldsymbol{q}.\nabla)\boldsymbol{q}\right]=-\nabla p-\rho g\boldsymbol{\lambda}+\frac{KN}{\varepsilon}(\boldsymbol{q}_d-\boldsymbol{q})-\frac{1}{k_1}\left(\mu+\mu'\frac{\partial}{\partial t}\right)\boldsymbol{q}\,, \tag{2.1}$$

$$\nabla.\boldsymbol{q}=0\,. \tag{2.2}$$

The governing equations for the suspended particles (each of mass mN per unit volume) are given by

$$mN\left[\frac{\partial \boldsymbol{q}_d}{\partial t}+\frac{1}{\varepsilon}(\boldsymbol{q}_d.\nabla)\boldsymbol{q}_d\right]=KN(\boldsymbol{q}-\boldsymbol{q}_d)\,, \qquad \text{....(2.3)}$$

$$\varepsilon\frac{\partial N}{\partial t}+\nabla.(N\boldsymbol{q}_d)=0\,. \qquad \text{....(2.4)}$$

If we consider the thermal and solute equilibrium between the fluid and solute particles then equations of heat and solute conduction can be written as

$$\left[\rho C_v\varepsilon+\rho_s C_s(1-\varepsilon)\right]\frac{\partial T}{\partial t}+\rho C_v(\boldsymbol{q}.\nabla)T+mNC_{pt}\left(\varepsilon\frac{\partial}{\partial t}+\boldsymbol{q}_d.\nabla\right)T=q\nabla^2T\,, \qquad \text{....(2.5)}$$

$$\left[\rho C_v\varepsilon+\rho_s C_s(1-\varepsilon)\right]\frac{\partial C}{\partial t}+\rho C_v(\boldsymbol{q}.\nabla)T+mNC_{pt}\left(\varepsilon\frac{\partial}{\partial t}+\boldsymbol{q}_d.\nabla\right)T=q'\nabla^2C \qquad \text{....(2.6)}$$

where ρ_s, C_s are the density and specific heat of the solid material, respectively.

The linearized perturbation equations are given by

$$\frac{1}{\varepsilon}\left[\frac{\partial \boldsymbol{q}}{\partial t}\right]=-\frac{1}{\rho_0}\nabla\delta p+g(\alpha\theta-\alpha'\gamma)\lambda+\frac{KN_0}{\rho_0\varepsilon}(\boldsymbol{q}_d-\boldsymbol{q})-\frac{1}{k_1}\left(\nu+\nu'\frac{\partial}{\partial t}\right)\boldsymbol{q}\,, \qquad \text{....(2.7)}$$

$$\nabla.\boldsymbol{q}=0\,, \tag{2.8}$$

$$\left[\frac{m}{k}\frac{\partial}{\partial t}+1\right]\boldsymbol{q}_d=\boldsymbol{q}\,, \tag{2.9}$$

$$\varepsilon\frac{\partial N}{\partial t}+N_0\left(\nabla.\boldsymbol{q}_d\right)=0\,, \tag{2.10}$$

$$\left(E+h_1\varepsilon\right)\frac{\partial\theta}{\partial t}=\beta\left(w+h_1 s\right)+\kappa\nabla^2\theta\,, \tag{2.11}$$

$$\left(E'+h_1\varepsilon\right)\frac{\partial\gamma}{\partial t}=\beta'\left(w+h_1 s\right)+\kappa'\nabla^2\gamma \qquad \text{....(2.12)}$$

where $\mu,\mu',v=\dfrac{\mu}{\rho_0},v'=\dfrac{\mu'}{\rho_0},\kappa\left(=\dfrac{q}{\rho_0 C_v}\right)$ and $\kappa'\left(=\dfrac{q'}{\rho_0 C_v'}\right)$ stands for viscosity, viscoelasticity, kinematic viscosity, kinematic viscoelasticity, thermal diffusivity and analogous solute diffusivity, respectively.

Also, $\qquad h_1=f\dfrac{C_{pt}}{C_v} \qquad$ and $\qquad f=\dfrac{mN_0}{\rho_0}$.

Using the Boussinesq approximation, the linearized perturbation equations are

$$p^{-1}\left[\frac{\partial\boldsymbol{q}}{\partial t}\right]=-\nabla\delta p+B\left(\boldsymbol{q}_d-\boldsymbol{q}\right)-P_1^{-1}\left(1+A\frac{\partial}{\partial t}\right)\boldsymbol{q}+\left(R\theta-S\gamma\right)\lambda\,, \tag{2.13}$$

$$\nabla.\boldsymbol{q}=0\,, \tag{2.14}$$

$$\left[\tau\frac{\partial}{\partial t}+1\right]\boldsymbol{q}_d=\boldsymbol{q}\,, \tag{2.15}$$

$$\varepsilon\frac{\partial M}{\partial t}+\left(\nabla.\boldsymbol{q}_d\right)=0\,, \tag{2.16}$$

$$\left(E+h_1\varepsilon\right)\frac{\partial\theta}{\partial t}=\beta\left(w+h_1 s\right)+\nabla^2\theta\,, \tag{2.17}$$

$$\left(E'+h_1\varepsilon\right)\frac{\partial\gamma}{\partial t}=\beta'\left(w+h_1 s\right)+\xi\nabla^2\gamma\,. \tag{2.18}$$

3. Dispersion relation

Consider that the two free boundaries (stress free) are at uniform temperature and solute concentration, then the boundary conditions are

$$w=\frac{\partial^2 w}{\partial z^2}=0,\qquad \theta=0,\qquad \gamma=0\qquad \text{at}\qquad z=0\qquad \text{and}\qquad 1. \tag{3.1}$$

Solving the linearized perturbation equation, we get

$$\left[L_1+\frac{L_2}{P_1}\left(1+A\frac{\partial}{\partial t}\right)\right]\left[\left(E+h_1\varepsilon\right)\frac{\partial}{\partial t}-\nabla^2\right]\left[\left(E'+h_1\varepsilon\right)\frac{\partial}{\partial t}-\xi\nabla^2\right]\nabla^2 w=$$
$$=\left[\left\{R\lambda\left(E'+h_1\varepsilon\right)\frac{\partial}{\partial t}-\xi\nabla^2\right\}-S\lambda\left\{\left(E+h_1\varepsilon\right)\frac{\partial}{\partial t}-\nabla^2\right\}\right]\left(\tau\frac{\partial}{\partial t}+H_1\right)\nabla^2{}_1 w. \tag{3.2}$$

In the normal mode analysis method, perturbations are assumed in the following form

$$w=W(z)\exp\left(ik_x.x+ik_y.y+nt\right) \tag{3.3}$$

where, n is the growth rate and $k=\sqrt{k_x^2+k_y^2}$ is the resultant wave number of the disturbance.

Using Eqs (3.2) and (3.3), we get

$$\left[L_1+\frac{L_2}{P_1}(1+An)\right]\left[\left(E+h_1\varepsilon\right)n-\left(D^2-k^2\right)\right]\left[\left(E'+h_1\varepsilon\right)n-\xi\left(D^2-k^2\right)\right]\left(D^2-k^2\right)w=$$
$$=-\left[\left\{R\lambda\left(E'+h_1\varepsilon\right)n-\xi\left(D^2-k^2\right)\right\}-S\lambda\left\{\left(E+h_1\varepsilon\right)n-\left(D^2-k^2\right)\right\}\right]\left(\tau n+H_1\right)k^2 W \tag{3.4}$$

where $\quad L_1=p_1^{-1}\left(\tau n^2+Fn\right), \qquad L_2=\left(\tau n+1\right) \qquad$ and $\qquad D=\dfrac{d}{dz}.$

4. The stationary convection

For the case of stationary convection, the neutral state is obtained by substituting $n=0$, Eq.(3.4) reduces to the form

$$\frac{\xi}{P_1}\left(D^2-k^2\right)^2 W=(R\xi-S)\lambda H_1 k^2 W\,. \tag{4.1}$$

Now for the two free boundaries, all derivatives of W (even order) vanish so a proper solution of Eq.(4.1) in the lowest mode is

$$W=W_0\sin\pi z \tag{4.2}$$

where W_0 is a constant .

From Eqs (4.1) and (4.2), we obtain

$$R=\frac{\left(\pi^2+k^2\right)^2}{k^2\lambda H_1 P_1}+\frac{S}{\xi}\,. \tag{4.3}$$

This relation is called the dispersion relation.

5. Analytical analysis of instability of the system

To investigate the effects of various parameters such as the medium permeability, suspended particles, stable solute gradient and variable gravity on the instability of the system, the behavior of derivatives of relation (4.3) with respect to effecting parameters is examined.

Differentiation of Eq.(4.3) with respect to P_1 gives

$$\frac{dR}{dP_1}=-\frac{\left(\pi^2+k^2\right)^2}{k^2\lambda H_1 P_1^2}.$$

It is evident that the effect of the medium permeability is to destabilize the system when gravity increases upwards from its value g_0 (i.e., λ is positive). This destabilizing effect is in accordance with the earlier work of Aggarwal and Prakash [1]. On the other hand, a stabilizing effect is observed when gravity decreases upwards from its value g_0 (i.e., λ is negative).

On differentiation of Eq.(4.3) with respect to H_1, we obtain

$$\frac{dR}{dH_1}=-\frac{\left(\pi^2+k^2\right)^2}{k^2\lambda H_1^2 P_1}.$$

It is evident that suspended particles destabilize the system when gravity increases upwards from its value g_0 (i.e., λ is positive). This destabilizing effect is in accordance with the earlier work of Aggarwal and Prakash [14] and when gravity decreases upwards from its value g_0 (i.e., λ is negative) a stabilizing effect is observed.

On differentiation Eq.(4.3) with respect to S

$$\frac{dR}{dS}=\frac{1}{\xi}=\frac{\kappa}{\kappa'}=\frac{q}{q'}.$$

This is positive implying thereby that the stable solute gradient has a stabilizing effect on the system. This stabilizing effect is in accordance with the earlier work of Aggarwal and Prakash [14].

6. Graphical analysis of instability of system

Graphically, it is shown in Figs 1-2 that as the values of permeability and suspended particles increase, the values of the Rayleigh number decreases implying the destabilizing effects on the system. On the other hand, in Fig.3, as the values of the solute gradient increase, the values of the Rayleigh number increase which shows the stabilizing effects on the system.

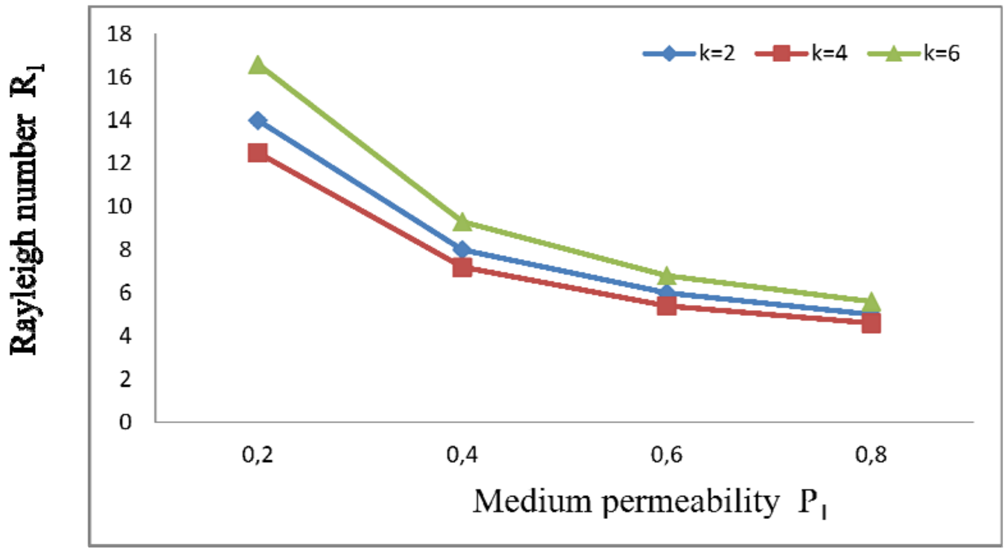

Fig.1. The Rayleigh number R_1 varies with the wave number k (2, 4, 6..) for $\lambda=2$, $H_1=10$, $S=10$, $\xi=5$ and medium permeability P_1 (=0.2, 0.4, 0.6, 0.8).

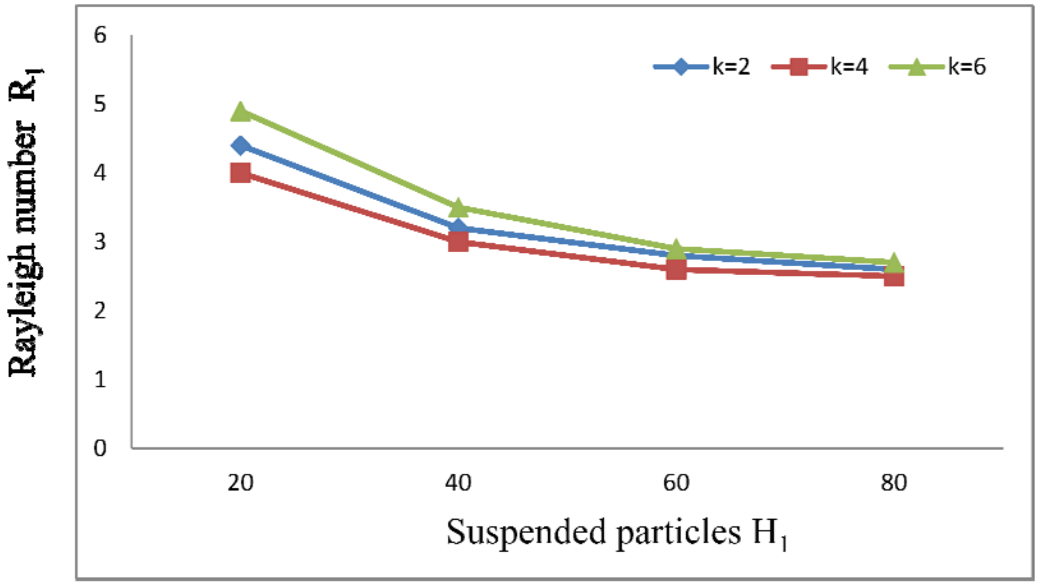

Fig.2. The Rayleigh number R_1 varies with the wave number *k* (*2*, *4*, *6*..) for $\lambda = 2$, $S=10$, $P_l=0.5$, $\xi = 5$ and suspended particle parameter H_l (=*20*, *40*, *60*, *80*).

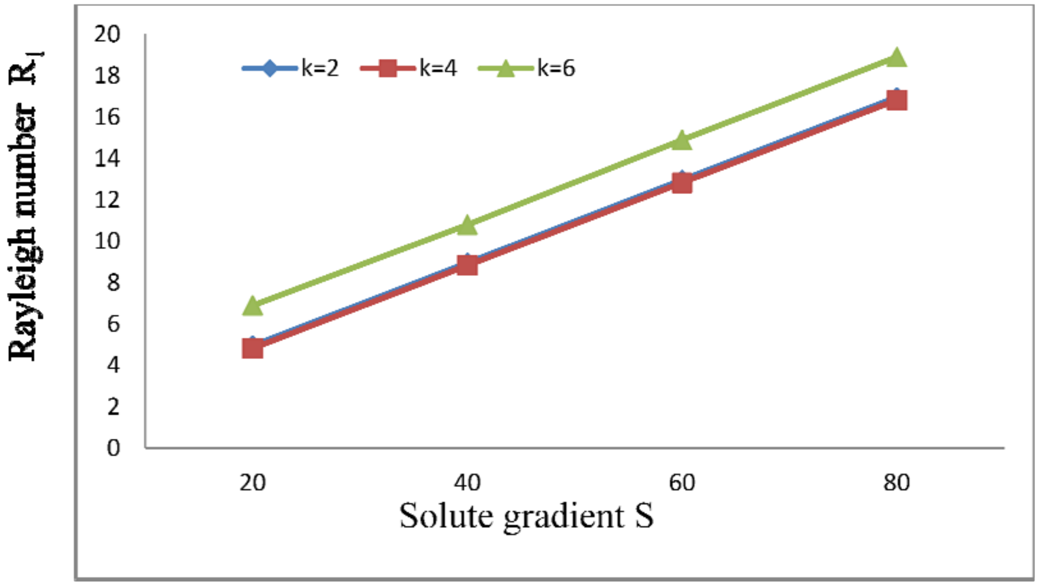

Fig.3. The Rayleigh number R_1 varies with the wave number *k* (*2*, *4*, *6*..) for $H_l=50$, $P_l=0.5$, $\lambda = 2$, $\xi = 5$ and solute gradient parameter *S* (=*20*, *40*, *60*, *80*).

7. Conclusion

In this paper, we have investigated the effect of various parameters such as suspended particles, medium permeability and stable solute gradient on thermosolutal convection in a Rivlin-Ericksen elastico-viscous fluid under varying gravity. From the analysis, it is found that suspended particles and medium permeability destabilize the system (i.e., more unstable flow). When gravity increases upwards from its value

g_0 and stabilizes the system (i.e., more stable flow) when gravity decreases upwards from its value g_0. The stable solute gradient has stabilizing effect and is independent of gravity field.

Nomenclature

C – solute concentration
C_p – heat capacity of fluid at constant pressure
C_{pt} – heat capacity of particles
g – gravitational acceleration
$\boldsymbol{g}$ – gravitational acceleration vector
K – Stokes drag coefficient
k – wave number of disturbances
N – perturbation in number density
N_0 – number density of suspended particles
p – pressure of the fluid
R – thermal Rayleigh number
S – analogous Rayleigh number
$\boldsymbol{q}(u,v,w)$ – velocity of the fluid
$\boldsymbol{q}_d(l,r,s)$ – velocity of suspended particles
q – effective thermal conductivity of pure fluid
q' – analogous effective solute conductivity
β – steady adverse temperature gradient
β' – solute gradient
γ – perturbation in solute concentration
δp – perturbation in pressure
$\delta\rho$ – perturbation in density
ε – porosity of the medium
θ – perturbation in temperature
κ – thermal diffusivity
κ' – analogous solute diffusivity
ρ – density of the fluid
ν – kinematic viscosity
ν' – kinematic viscoelasticity

References

[1] Helmholtz H. (1868): *Uber discontinuirliche Flussigkeitsbewegungen*. – Phil. Mag. Ser. 4, 36, pp.337.

[2] Rayleigh Lord (1880): *On the stability or instability of certain fluid motions*. – Proc. London Math. Soc, vol.11, pp.57-70.

[3] Stern M.E. (1960): *The 'salt-fountain' and thermohaline convection.* – Tellus 12, pp.172.

[4] Veronis G. (1965): *On finite amplitude instability inthermohaline convection*. – J. Marine Res., vol.23, pp.1-7.

[5] Nield D.A. (1967): *The thermohaline Rayleigh-Jeffreys problem.* – J. Fluid Mechanics., vol.29, pp.545.

[6] Bejan A. (1984): *Convection Heat Transfer.* – New York: Wiley Publications.

[7] Benard H. (1900): *Les tourbillons cellulaires dans une nappe liquide*. – Revue Generale des Sciences Pures et Appliqués, vol.11, pp.1261-1271 and pp.1309-1328.

[8] Chandrasekhar S. (1981): *Hydrodynamic and Hydromagnetic Stability.* – New York: Dover Publications.

[9] Rumford C. (1870): *Of the propagation of heat in fluids, the complete works of count Rumford.* – Vol.1, pp.237-400. American Academy of Arts and Sciences. Boston.

[10] Thompson J. (1882): *On a changing tessellated structure in certain liquids.* – Proc. Phill. Soc. Glasgow, vol.13, pp.464-468.

[11] Ostrach S. (1957): *Convection phenomenon in fluids heated from below*. – Trans. ASME., vol.79, pp.299-305.

[12] Pradhan G.K. and Samal P.C. (1987): *Thermal instability of fluid layer under variable body forces.* – J. Math. Anal. Appl., vol.122, pp.487-495.

[13] Rana G.C. (2013): *Thermosolutal convection in Walters' (Model B') rotating fluid permeated with suspended particles and variable gravity field in porous medium in hydromagnetics*. – Journal of Applied Fluid Mechanics, vol.06, pp.87-94.

[14] Aggarwal A.K. and Prakash K. (2007): *Thermosolutal Instability of an elastico-viscous fluid in porous medium in presence of suspended particles.* – Proceedings of the Indo-Australian Workshop and Symposium on CFD Approach on Fluid Flow, Heat and Mass Transfer and Applications in Multidisciplinary Areas, Research Publishing Services, ISBN:978-81-904262-6-8, pp.1-6.

[15] Aggarwal A.K. and Verma A. (2014): *The effect of compressibility, rotation and magnetic field on thermal stability of Walters' fluid permeated with suspended particles in porous medium*. – Thermal Science, vol.18, pp.539-550.

[16] Aggarwal A.K. and Verma A. (2008): *Thermal instability of a rotating viscoelastic fluid permeated with suspended particles in hydromagnetics*. – Continuum Mechanics, pp.302-306.

[17] Aggarwal A.K. and Makhija S. (2011): *Combined effect of magnetic field and rotation on thermal stability of couple-stress fluid heated from below in presence of suspended particles.* – International Journal of Applied Mechanics and Engineering, vol.16, pp.931-942.

[18] Aggarwal A.K. and Verma A. (2010): *Effect of rotation and magnetic field on thermal instability of a viscoelastic fluid permeated with suspended particles.* – WSEAS Transactions on Mathematics, vol.9, pp.593-602.

[19] Aggarwal A.K. and Verma A. (2012): *Effect of suspended particles, magnetic field and rotation on the thermal stability of a ferromagnetic fluid*. – International Journal of Applied Mechanics and Engineering, vol.17, pp.1109-1122.

[20] Aggarwal A.K. (2010): *Effect of rotation on thermosolutal convection in a Rivlin-Ericksen fluid permeated with suspended particles in porous medium*. – Advances in Theoretical and Applied Mechanics, vol.3, pp.177-188.

[21] Aggarwal A.K. and Makhija S. (2012): *Hall effect on thermal stability of ferromagnetic fluid in the presence of suspended particles*. – International Journal of Applied Mechanics and Engineering, vol.17, pp.349-365.

[22] Aggarwal A.K. and Verma A. (2016): *Effect of Hall currents on thermal instability of dusty couple stress fluid.* – Archives of Thermodynamics, vol.37, pp.3-18.

[23] Agarwal S. and Rana P. (2016): *Periodic and a periodic convective stability analysis of double diffusive nanofluid convection in a rotating porous layer*. – Applied Mathematics and Mechanics, Springer, vol.37, pp.215-226.

[24] Gupta U. and Aggarwal P. (2011): *Thermal instability of compressible Walter's fluid in the presence of hall currents and suspended particles*. – Thermal Science, vol.15, pp.487-500.

[25] Kumar P. and Mohan H. (2012): *Thermal instability of a heterogeneous Oldroydian viscoelastic fluid heated from below in porous medium*. – Journal of Theoretical and Applied Mechanics, vol.50, pp.943-951.

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