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chatter,
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CHATTER IN CUTTING PROCESSES

Vibrations during a cuttting process are analysed by means of a frictional and a regenerative model. For this purpose a new model which contains a nonlinear friction force, defined by Rayleigh's self-excitation, is developed. Then a one degree of freedom model is examined using analytical and numerical methods. The regenerative model of cutting is separately examined in order to emphasize differences between frictional and regenerative models. Next, a mutual interaction of frictional and regenerative effects is discussed and their influence on process stability is shown. Finally, a two degree of freedom frictional model with an extra centrifugal force is investigated. Various kinds of behaviour, from regular to chaotic are shown.

1. INTRODUCTION

The cutting process is still one of the most popular and important machining processes from a technological point of view. The variety of materials and cutting methods speaks volumes for the usefulness of cutting processes in the present world. This variety involves some difficulties especially when high productivity and surface quality is demanded. One of the most frequent causes for deteriorating cutting conditions are vibrations between the tool and the workpiece, which may also decrease quality of the product and lead to faster tool wear. A fundamental reason for an appearance of vibrations called chatter is the complexity of the chip-formation process. There are four kinds of chatter mechanism: (a) dry friction effect, (b) regenerative effect, (c) mode coupling, (d) external excitation. In some papers, thermal chatter is also distinguished. Dry friction and regenerative effects are the most important, widely widespread and principal influences on the process. The former is typical for conventional cutting where vibrations are caused by a nonlinear dry friction force. Regenerative chatter is produced by workpiece geometry from the previous pass. That is typical for high speed machining (HSM). HSM is said to be High Cutting Speed (*vc*), High Spindle Speed (*n*), High Feed-rate (*vf)* and Highly Productive Machining.

The first meaningful papers treating dynamics of the cutting process were published at the beginning of last century [11],[13],[21]. Further research improved knowledge and provided more experience from investigations but the next key development in this field

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appeared when chaotic behaviour was found [4],[5]. This work opened new horizons and gave new opportunities for researchers. From this time there are a number of papers which model the cutting process as a one, two or sometimes three degree of freedom system but always a cutting force is essential for explaining a phenomenon. Different papers show models of the cutting process in which nonlinearity of cutting forces are of significant importance. Very often the force depends on an axial width of cut, a chip thickness or on cutting speed. The nonlinearity of cutting force can explain Hopf bifurcation and other nonlinear phenomena which often exist. On the other hand, a tool losing contact with a workpiece causes nonlinear behaviour as well, including chaos. This kind of non-continuity is discussed in [26] and additionally, stochastic properties of a workpiece material are introduced. Stochastic dynamics of a metal cutting process is also examined in [6]. System instability can also be caused by a workpiece profile which is generated during a previous tool pass [7],[16]. Thorough review of the current state of the art in dynamics of cutting and grinding processes, new challenges in modelling and avoiding machine tool vibrations are presented in [1]. These authors provided a significant review of literature in the related areas, which will not be repeated here.

Generally, the set of publications can be divided into either of the following groups: where chatter is produced by regenerative effect [2,3],[8],[17-19],[19,20],[22] and dry friction [9,10],,14;15],[23-26]. There is a lack of papers which treat both problems together and explain interactions between them.

This paper shows vibrations of 1 and 2 DOF models of a cutting process. The first part is devoted to models which include a new approach of cutting force using Rayleigh's self-excitation. The next part of the work covers dynamical analysis of the systems with a comparison of phenomena caused by regenerative effect and dry friction simultaneously. Additionally, the 2 DOF model is supplied with centrifugal force.

2. MODELS OF CUTTING PROCESS

Modelling of mechanical systems play a key role in research because results, to a large extent, depend on the kind of model, its correctness and on selection of proper phenomena which are essential for the considered process. At the beginning a 2 DOF model with single lumped mass is presented in Fig. 1. The workpiece motion in x and y directions is considered therefore two independent variables, in order to define the mass position, is necessary. It means that the system has 2 DOF. Simplifying this model by a projection of motion in one direction (y) a 1 DOF system is obtained.

A model of a cutting process can be reduced to a planar oscillator which is excited by the cutting force component F_z, F_y and a centrifugal force B (Fig. 1). The elastic, dissipative and inertial properties of the workpiece are denoted as k_y, k_z, c_y, c_z and m. The lumped mass m is actually a reduced mass of the workpiece received from an experiment on the basis of natural frequency of the workpiece which is fixed in a jaw chuck.

In this model stiffness k and damping c are linear. B is the inertial force deriving from the displacement of the mass centre (point C) of the workpiece in relation to the centre of rotation (point O). This displacement is produced by tool pressure and vibrations of the workpiece. The inertial force depends on the workpiece angular velocity ω, reduced mass m and displacements y and z:

$$B = m\omega^2\sqrt{y^2 + z^2} \tag{1}$$

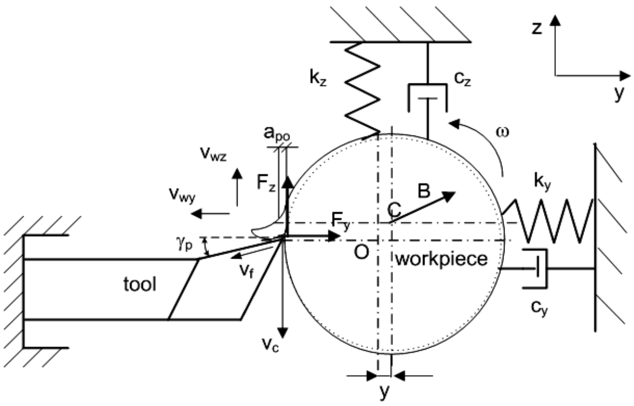

Fig. 1. 2DOF frictional model of cutting process

The nonlinear forces F_z, F_y are the most important features of the model. As it is shown in Fig. 2, the cutting force is a nonlinear function of cutting depth and speed. Therefore the cutting force F_z may be represented by the equation:

$$F_z = \begin{cases} K_z a_p{}^{\rho}(1-\alpha_z v_{wz} + \beta_z v_{wz}{}^3)H(a_p) & for\ \ v_{wz} > 0 \\ 0 & for\ \ v_{wz} \le 0 \end{cases} \tag{2}$$

where: a_p is an actual depth of cut, v_{wz} is a relative velocity between a tool and a chip in z direction, ρ, α_z, β_z, are constants for the chosen process and K_z corresponds to a cutting resistance. Since the model accounts for the instantaneous separation of the tool from the workpiece, Heaviside's function $H()$ is introduced. The expression $(1-\alpha_z v_{wz} + \beta_z v_{wz}{}^3)$ given in (2) presents the self-excited Rayleigh's term. That is a new approach in cutting force modeling. The actual depth a_p and the velocity v_{wz} are defined as follows:

$$\begin{aligned} a_p &= a_{po} - y \\ v_{wz} &= \frac{v_c - \dot{z}}{k_h} \end{aligned} \tag{3}$$

where a_{po} – is an assumed depth of cut, v_c – is a cutting speed, k_h – is a chip thickness ratio.

The motion in z direction influences a motion in y direction therefore the thrust force F_y is defined by means of the cutting force F_z in the following way:

$$F_y = F_z\left(Sgn\left(v_{wy}\right) - \alpha_y v_{wy} + \beta_y v_{wy}{}^3\right) \tag{4}$$

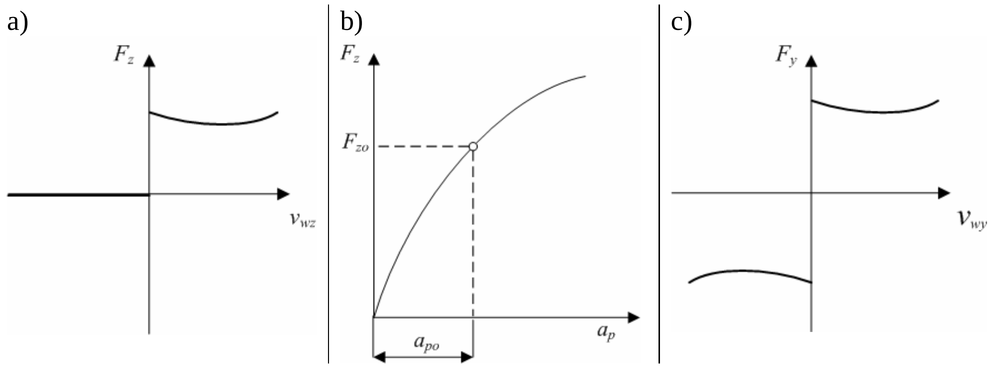

Fig. 2 Cutting force as a function of relative velocity v_{wz} (a); cutting force as a function of cutting depth a_{po} (b); thrust force as a function of relative velocity v_{wy} (c)

where, $v_{wy} = v_{wy}' - \dot{y}$, v_{wy}' - is a velocity of a chip flow in y direction when the process is static which can be understood as cutting velocity, β_y and α_y - are constants which describe the force characteristic. Sgn() means the sign function defined as follows:

$$Sgn\left(v_{wy}\right) = \begin{cases} -1 & if \quad v_{wy} < 0 \\ 0 & if \quad v_{wy} = 0 \\ 1 & if \quad v_{wy} > 0 \end{cases} \tag{5}$$

The characteristics of the proposed forces are shown in Fig. 2. The above forces consist of static and dynamic parts. The static forces F_{zo}, F_{yo} describe the process when there are not vibrations in the system, so that the process is static:

$$\begin{aligned} F_{zo} &= K_z a_{po}{}^{\rho} \left(1 - \alpha_z \frac{v_c}{k_h} + \beta_z \left(\frac{v_c}{k_h} \right)^3 \right) \\ F_{yo} &= F_{zo} \left(1 - \alpha_y v'_{wy} + \beta_y v'^{\,3}_{wy} \right) \end{aligned} \tag{6}$$

The differential equations of the system dynamics can be written as:

$$\begin{aligned} m\ddot{y} + k_y y + c_y \dot{y} &= (F_y - F_{yo}) + B\cos(\omega t) \\ m\ddot{z} + k_z z + c_z \dot{z} &= (F_z - F_{zo}) + B\sin(\omega t) \end{aligned} \tag{7}$$

To concentrate on the nonlinear phenomena produced by dry friction, the 2 DOF model is reduced to y direction (Fig. 3) and the inertial force B is neglected.

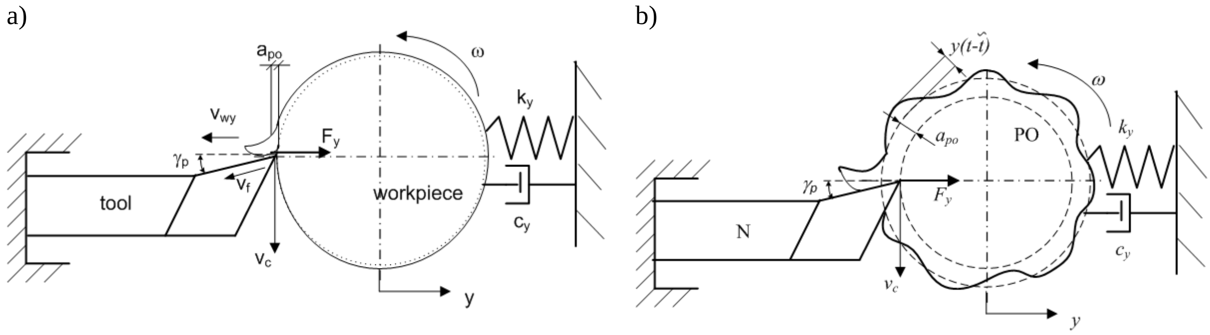

Fig. 3. 1DOF frictional model of cutting process

Now, the thrust force is F_y and is expressed as follows:

$$F_y = K_y a_p{}^{\rho}\left(Sgn\left(v_{wy}\right) - \alpha_y v_{wy} + \beta_y v_{wy}{}^3\right) H(a_p) \tag{8}$$

and differential equation of motion takes the form:

$$m\ddot{y} + k_y y + c_y \dot{y} = F_y - F_{yo} \tag{9}$$

To compare 1DOF frictional model (Fig. 3a) with a regenerative one (Fig. 3b) and also in order to explore mutual interaction between frictional and regenerative effects, time delay $\tilde{t}$ is introduced to equation (8) which changes as follows:

$$F_y = K_y\left(a_{po} - y + y\left(t - \tilde{t}\right)\right)^{\rho}\left(Sgn\left(v_{wy}\right) - \alpha_y v_{wy} + \beta_y v_{wy}{}^3\right) H(a_p) \tag{10}$$

where: $\tilde{t} = \dfrac{2\pi}{\omega}$.

The next section is devoted to analysis of these models and shows major differences and interdependences between them.

3. FRICTION and REGENERATIVE CHATTER

Both friction and regenerative effects can cause vibrations which are called “chatter”. To obtain an exact solution of the 1DOF system with dry friction, the analytical method of multiple scales is engaged. The results presented below are obtained by means of following parameter values a_{po} =3 mm, v'_{wy}=1.5 m/s, k_y =92.472 kN/m, c_y=5 kg/s, m=0.51 kg, K_y=44 kN/m, ρ =0.95, α_y=0.4, β_y=0.05. Fig. 4a shows stability diagram versus depth of cut a_{po} and cutting velocity v'_{wy}. Stable trivial (zero) solutions need a relatively high cutting speed and

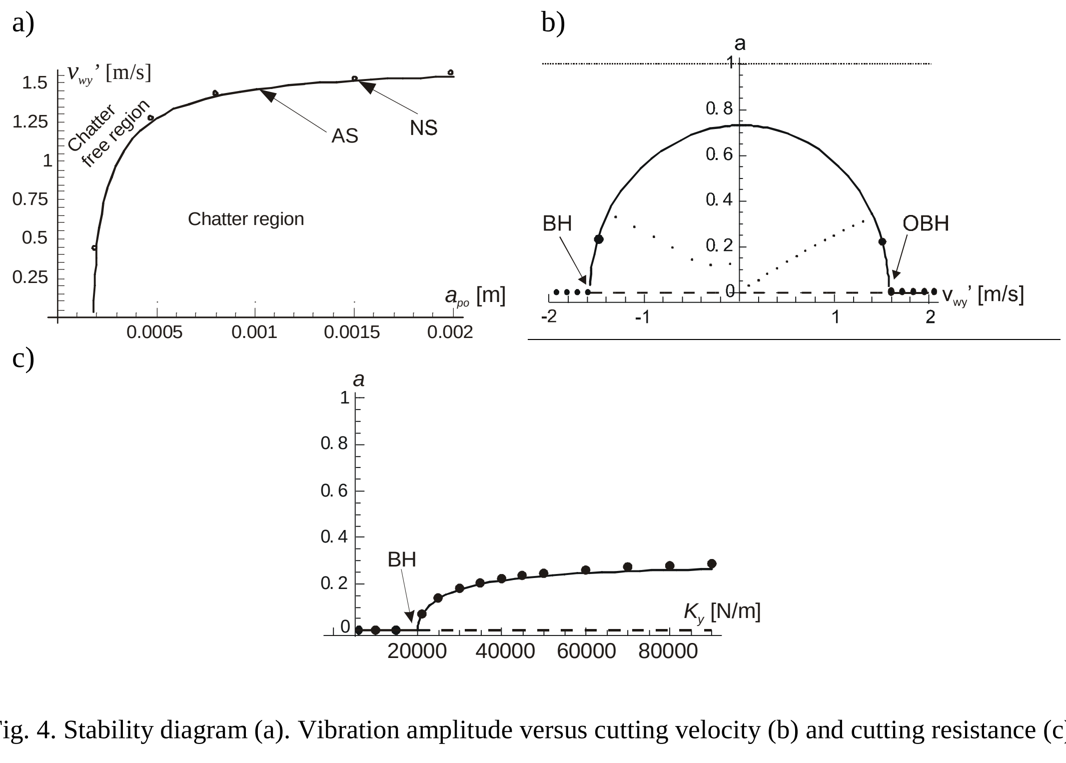

Fig. 4. Stability diagram (a). Vibration amplitude versus cutting velocity (b) and cutting resistance (c)

small cutting depth. The curve represents analytical solutions (AS) while the points – numerical ones (NS). An influence of the cutting velocity on vibrations amplitude (a) is also shown in Fig. 4b where the amplitude decreases with increasing cutting speed? and above 1.6 m/s, where the bifurcation point (OBH) is marked, only trivial solution exists. As far as cutting resistance K_y is concerned Fig. 4c, in the range below 20 kN/m stable cutting occurs. After bifurcation point (BH) the vibrations amplitude (a) increases.

Now, the additional regenerative effect is taken into account. In the case the thrust force is given by equation (10). Then the model includes both the nonlinear friction force and the regenerative effect. To focus only on the last one, it is assumed that

$$\rho=1,\ \alpha_y=0,\ \beta_y=0,\ \mathrm{H}(a_{po})=1,\ \mathrm{Sgn}(v'_{wy})=1 \tag{11}$$

thus, the linear model with delay is obtained which can be expressed by a dimensionless differential equation in the form:

$$\ddot{y}_1 + y_1 + \frac{c_y}{mp_{yo}}\dot{y}_1 = \frac{K_y}{mp_{yo}{}^2}(-y_1 + y_1(\tau - \tilde{\tau})) \tag{12}$$

where: y_1 is dimensionless displacement, p_{yo} denotes the natural vibrations frequency for the linear case and τ is dimensionless time. As a result of analytical analysis of equation (12), the stability diagram with the characteristic unstable lobes (striped region) is presented in Fig. 5a. For the cutting resistance K_y <7000 N/m only stable solutions appear irrespective of the time delay or the angular velocity ω. For K_y >7000 N/m the unstable lobes exist interchangeably with the stable (white) regions. That is rather typical, but if the parameters do not satisfy equation (11) and the thrust force is nonlinear, the unstable shaded lobes are bigger then before (see striped and shaded regions in Fig. 5a).

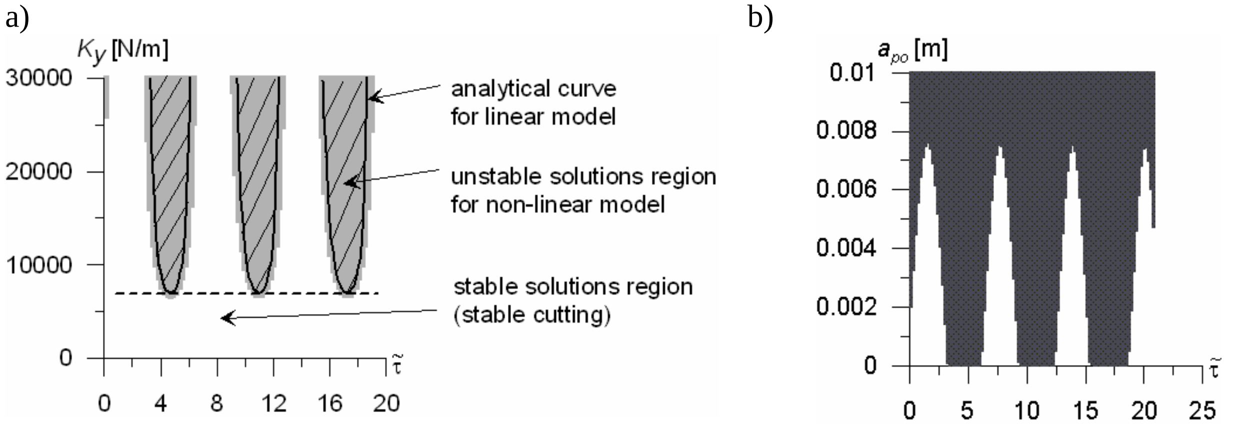

Fig. 5. Stability diagram for regenerative cutting

Considering equation (12) it is noticed that the cutting depth (a_{po}) does not influence the system dynamics. While the numerical simulation made for non-linear case gives the reverse results (Fig. 5b). An increase of the cutting depth leads to unstable regions. To sum up, adding another nonlinearity to regenerative model enlarges the unstable region.

As far as the dynamics of the cutting process, on the base of 2DOF frictional system, is concerned, it would be interesting to observe a bifurcation diagram as a function of angular velocity ω_1. Such a diagram is shown In Fig. 6 for the following parameters: c_y=c_z =5 kg/s, m =0.51 kg, K_z =44 kN/m, ρ= 0.95, α_y =0.4, β_y =0.05, α_z =0.16, β_z =0.0032, v_c =2.275 m/s, η=0.7, k_h= 2.5, a_{po} =0.003 m, v'_{wy} =1.5 m/s, k_y= k_z =992.472 kN/m.

a)
b)

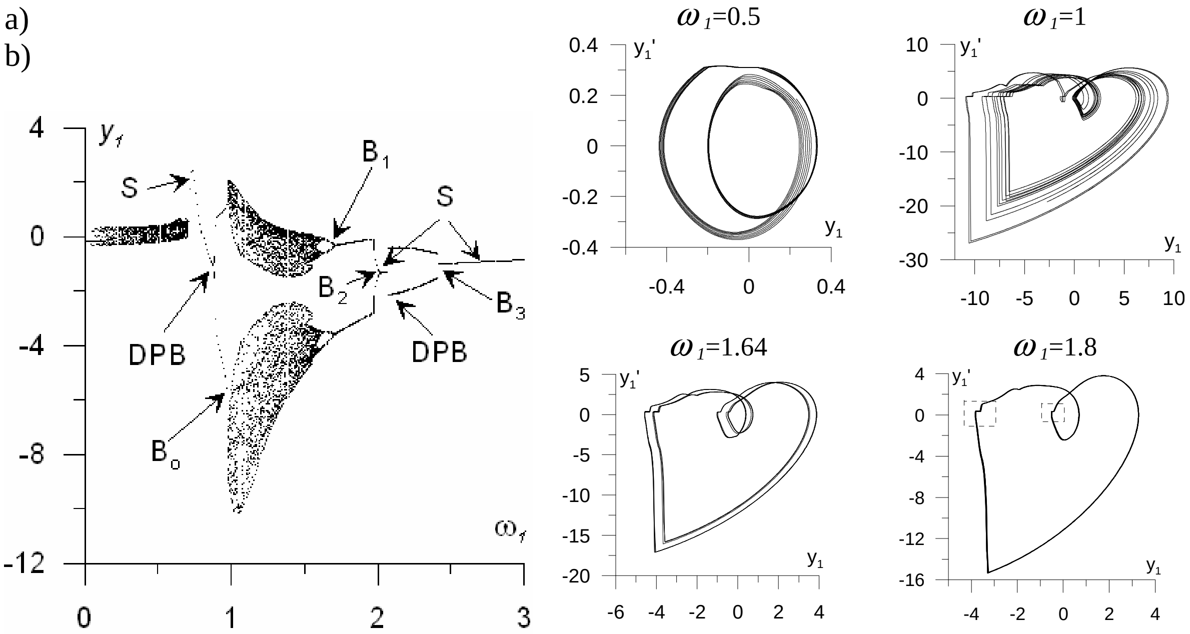

Fig. 6. Bifurcation diagram for 2 DOF system (a), space (b)

a)

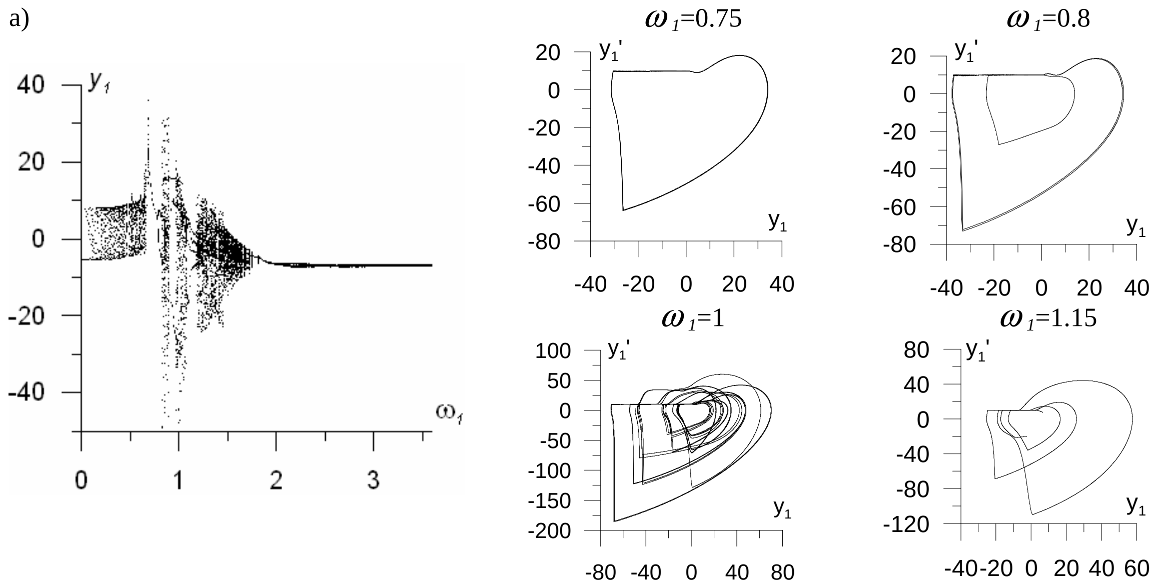

Fig. 7. Bifurcation diagram for 2DOF flexible system (a), phase space (b)

At the beginning, for small $\omega_1$periodic oscillations occur which next change into quasi periodic and after synchronisation region (S) double period bifurcation (DPB) is visible. Next 2-period vibrations undergo in quasi-periodic motion in order that after several bifurcations goes into harmonic vibrations. To obtain more information about system dynamics phase space diagrams for chosen ω_1 are made. Besides, the phenomena shown before as doubling period, a so called stick-slip effect is present as well.

By changing the stiffness coefficient of the system, completely different behaviour may occur. New values of parameters are as follows: k_y=k_z=9924.7N/m and m=5.1kg that means more flexible system. The bifurcation diagram for new set of parameters is shown in Fig. a. Various kinds of vibrations are visible starting from regular (period 1) through period 2 till chaotic vibrations for ω_1=1 and next period 4 for ω_1=1.15. Thus, decreasing system stiffness can cause chaotic behaviour.

4. CONCLUSIONS

A real cutting process has a lot of different factors which influence system dynamics. The proposed models contain only a few of them. In the paper new approach to nonlinear cutting force is developed on the basis of friction force expressed by Rayleigh's term. This element is responsible for a self-excitation of the system and as a result chatter vibrations appear. The amplitude grows when the cutting depth increases and falls down when the cutting velocity increases. Taking into consideration the fact that bigger cutting resistance produces bigger values of forces, that can suggest optimal cutting parameters.

Analysis of the pure regenerative model gives convergent outcomes in relation to others publications. However, by connecting the regenerative and friction effects one can obtain more instabilities in the system. Though, big enough cutting velocity leads to stable trivial solution (process without chatter) for the frictional model nevertheless, the regenerative model produces unstable lobes. What is more, adding nonlinear elements to the model enlarges unstable cutting regions.

Vibrations created by the dry friction phenomenon are especially dangerous in the case of flexible workpiece when chaotic behaviour is observed. Although, increasing rotational velocity stabilizes the process, one should remember to avoid unstable lobes typical for high speed machining.

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