*Computer Applications in Electrical Engineering*
*Vol. 13* *2015*

# Analysis of positivity and stability of time-varying continuous-time linear systems and electrical circuits

Tadeusz Kaczorek
Bialystok University of Technology
15-351 Bialystok, ul. Wiejska 45D, e-mail: kaczorek@isep.pw.edu.pl

The positivity and stability of a class of time-varying continuous-time linear systems and electrical circuits are addressed. Sufficient conditions for the positivity and asymptotic stability of the system are established. It is shown that there exists a large class of positive and asymptotically stable electrical circuits with time-varying parameters. The Lyapunov method is extended to positive nonlinear systems. Examples of positive electrical circuits are presented.

KEYWORDS: positive, linear, time-varying, system, electrical circuit, stability, Lyapunov method, test.

## 1. Introduction

A dynamical system is called positive if its trajectory starting from any nonnegative initial state remains forever in the positive orthant for all nonnegative inputs. An overview of state of the art in positive theory is given in the monographs [1, 5]. Variety of models having positive behavior can be found in engineering, economics, social sciences, biology and medicine, etc..

The positivity and stability of fractional time varying discrete-time linear systems have been addressed in [7, 10, 11] and the stability of continuous-time linear systems with delays in [12]. The fractional positive linear systems have been analyzed in [3, 4, 14-17].The positive electrical circuits and their reachability have been considered in [6, 9] and the controllability and observability in [2]. The stability and stabilization of positive fractional linear systems by state-feedbacks have been analyzed in [13, 14]. The Hurwitz stability of Metzler matrices has been investigated in [14, 15, 18].

In this paper positivity and stability of a class of time-varying continuous-time linear systems and electrical systems will be addressed.

The paper is organized as follows. In section 2 the solution to the scalar time-varying linear system and some stability tests of positive continuous-time linear systems are recalled. Sufficient conditions for the positivity and asymptotic stability of a class of time-varying continuous-time linear systems and electrical systems are established in section 3. In the same section the Lyapunov method is extended to positive nonlinear systems. The positive and asymptotically stable electrical circuits with time-varying parameter are addressed in section 4. Concluding remarks are given in section 5.

The following notation will be used: $\Re$ - the set of real numbers, $\Re^{n\times m}$ - the set of $n\times m$ real matrices, $\Re_+^{n\times m}$ - the set of $n\times m$ matrices with nonnegative entries and $\Re_+^n = \Re_+^{n\times 1}$, $M_n$ - the set of $n\times n$ Metzler matrices (real matrices with nonnegative off-diagonal entries), $I_n$ - the $n\times n$ identity matrix, $T$ – denotes the transposition of matrix (vector).

## 2. Preliminaries

Consider the scalar time-varying continuous-time linear system

$$\dot{x}(t) = -a(t)x(t) + b(t)u(t),\ t\in[0,+\infty) \tag{1}$$

where $x(t)$ and $u(t)$ are the state and input of the system and $a(t)$, $b(t)$ are continuous-time functions.

**Lemma 1.** The solution of (1) for given initial condition $x_0 = x(0)$ and input $u(t)$ has the form

$$x(t) = e^{-\int a(t)dt}x_0 + \int_0^t e^{-\int a(t-\tau)dt} b(\tau)u(\tau)d\tau\ . \tag{2}$$

**Proof.** Using (2) and (1) we obtain

$$-a(t)x(t)+b(t)u(t) = -a(t)\left[e^{-\int a(t)dt}x_0 + \int_0^t e^{-\int a(t-\tau)dt}b(\tau)u(\tau)d\tau\right] + b(t)u(t)$$

$$= -a(t)e^{-\int a(t)dt}x_0 - a(t)\int_0^t e^{-\int a(t-\tau)dt}b(\tau)u(\tau)d\tau + b(t)u(t) = \dot{x}(t).$$

□

Consider the autonomous continuous-time linear system with constant coefficients

$$\dot{x}(t) = Ax(t)\ , \tag{3}$$

where $x(t)\in\Re^n$ is the state vector and $A=[a_{ij}]\in M_n$.

**Theorem 1.** [15] The positive system (3) is asymptotically stable if and only if one of the following equivalent conditions is satisfied:

1) All coefficients of the characteristic polynomial

$$\det[I_n s - A] = s^n + a_{n-1}s^{n-1} + ... + a_1 s + a_0\ , \tag{4}$$

are positive, i.e. $a_k > 0$ for $k = 0,1,\ldots,n-1$.

2) All principal minors $M_k$, $k=1,...,n$ of the matrix $-A$ are positive, i.e.

$$M_1 = -a_{11} > 0,\ \ M_2 = \begin{vmatrix} -a_{11} & -a_{12} \\ -a_{21} & -a_{22} \end{vmatrix} > 0, ..., M_n = \det[-A] > 0 \tag{5}$$

3) The diagonal entries of the matrices

$$A_{n-k}^{(k)} \text{ for } k = 1,\dots,n-1 \tag{6a}$$

are negative, where $A_{n-k}^{(k)}$ are defined as follows:

$$A_n^{(0)} = A = \begin{bmatrix} a_{11}^{(0)} & \dots & a_{1,n}^{(0)} \\ \vdots & \dots & \vdots \\ a_{n,1}^{(0)} & \dots & a_{n,n}^{(0)} \end{bmatrix} = \begin{bmatrix} a_{11}^{(0)} & b_{n-1}^{(0)} \\ c_{n-1}^{(0)} & A_{n-1}^{(0)} \end{bmatrix}, \quad A_{n-1}^{(0)} = \begin{bmatrix} a_{22}^{(0)} & \dots & a_{2,n}^{(0)} \\ \vdots & \dots & \vdots \\ a_{n,2}^{(0)} & \dots & a_{n,n}^{(0)} \end{bmatrix},$$

$$b_{n-1}^{(0)} = [a_{12}^{(0)} \quad \dots \quad a_{1,n}^{(0)}], \quad c_{n-1}^{(0)} = \begin{bmatrix} a_{21}^{(0)} \\ \vdots \\ a_{n,1}^{(0)} \end{bmatrix} \tag{6b}$$

and

$$A_{n-k}^{(k)} = A_{n-k}^{(k-1)} - \frac{c_{n-k}^{(k-1)} b_{n-k}^{(k-1)}}{a_{k+1,k+1}^{(k-1)}} = \begin{bmatrix} a_{k+1,k+1}^{(k)} & \dots & a_{k+1,n}^{(k)} \\ \vdots & \dots & \vdots \\ a_{n,k+1}^{(k)} & \dots & a_{n,n}^{(k)} \end{bmatrix} = \begin{bmatrix} a_{k+1,k+1}^{(k)} & b_{n-k-1}^{(k)} \\ c_{n-k-1}^{(k)} & A_{n-k-1}^{(k)} \end{bmatrix},$$

$$A_{n-k-1}^{(k)} = \begin{bmatrix} a_{k+2,k+2}^{(k)} & \dots & a_{k+2,n}^{(k)} \\ \vdots & \dots & \vdots \\ a_{n,k+2}^{(k)} & \dots & a_{n,n}^{(k)} \end{bmatrix}, \quad b_{n-k-1}^{(k)} = [a_{k+1,k+2}^{(k)} \quad \dots \quad a_{k+1,n}^{(k)}], \quad c_{n-k-1}^{(k)} = \begin{bmatrix} a_{k+2,k+1}^{(k)} \\ \vdots \\ a_{n,k+1}^{(k)} \end{bmatrix} \tag{6c}$$

for $k = 1,\dots,n-1$.

4) All diagonal entries of the upper (lower) triangular matrix

$$, \ \tilde{A}_l = \begin{bmatrix} \tilde{a}_{11} & 0 & \dots & 0 \\ \tilde{a}_{21} & \tilde{a}_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{n,1} & \tilde{a}_{n,2} & \dots & \tilde{a}_{n,n} \end{bmatrix} \tag{7}$$

are negative, i.e. $\tilde{a}_{kk} < 0$ for $k = 1,\dots,n$ and the matrices $\tilde{A}$ has been obtained from the matrix $A$ by the use of elementary row operation [5, 14].

5) There exists a strictly positive vector $\lambda = [\lambda_1 \ \dots \ \lambda_n]^T$, $\lambda_k > 0$, $k = 1,\dots,n$ such that $A\lambda < 0$.

The elementary row operations for time-varying systems are the following:

1) Multiplication of the $i$th row by a real number $c(t)$. This operation will be denoted by $L[i \times c(t)]$.
2) Addition to the $i$th row (column) of the $j$th row (column) multiplied by a real number $c(t)$. This operation will be denoted by $L[i + j \times c(t)]$.
3) Interchange of the $i$th and $j$th rows (columns). This operation will be denoted by $L[i, j]$.

## 3. Positive time-varying stable continuous-time linear systems

Consider the time-varying linear system

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) \tag{8a}$$

$$y(t) = C(t)x(t) + D(t)u(t) \tag{8b}$$

where $x(t) \in \Re^n$, $u(t) \in \Re^m$, $y(t) \in \Re^p$ are the state, input and output vectors and $A(t) \in \Re^{n\times n}$, $B(t) \in \Re^{n\times m}$, $C(t) \in \Re^{p\times n}$, $D(t) \in \Re^{p\times m}$ are real matrices with entries depending continuously on time and $\det A(t) \neq 0$ for $t \in [0,+\infty)$.

**Definition 1.** The system (8) is called positive if $x(t) \in \Re_+^n$, $y(t) \in \Re_+^p$, $t \in [0,+\infty)$ for any initial conditions $x_0 \in \Re_+^n$ and all inputs $u(t) \in \Re_+^m$, $t \in [0,+\infty)$.

It is assumed that $A(t) \in M_n$ with negative diagonal entries and nonnegative off diagonal entries for all $t \in [0,+\infty)$.

**Theorem 2.** The time-varying linear system (8) with upper triangular form

$$A_u(t) = \begin{bmatrix} -a_{11}(t) & a_{12}(t) & \dots & a_{1,n}(t) \\ 0 & -a_{22}(t) & \dots & a_{2,n}(t) \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & -a_{n,n}(t) \end{bmatrix} \in M_n(t), \tag{9a}$$

or lower triangular form

$$A_l(t) = \begin{bmatrix} -a_{11}(t) & 0 & \dots & 0 \\ a_{21}(t) & -a_{22}(t) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{n,1}(t) & a_{n,2}(t) & \dots & -a_{n,n}(t) \end{bmatrix} \in M_n(t) \tag{9b}$$

with negative diagonal entries for $t \in [0,+\infty)$ and

$$B(t) \in \Re_+^{n\times m},\ C(t) \in \Re_+^{p\times n},\ D(t) \in \Re_+^{p\times m},\ t \in [0,+\infty) \tag{10}$$

is positive and asymptotically stable.

**Proof.** For the matrices $A(t)$ and $B(t)$ using (8a) and (9) we obtain

$$\dot{x}_n(t) = -a_{nn}(t)x_n(t) + \sum_{k=1}^{m} b_{nk}(t)u_k(t) \tag{11}$$

where

$$x(t) = [x_1(t)\ \ \dots\ \ x_n(t)]^T,\ u(t) = [u_1(t)\ \ \dots\ \ u_m(t)]^T,\ B(t) = \begin{bmatrix} b_{11}(t) & \dots & b_{1,m}(t) \\ \vdots & \dots & \vdots \\ b_{n,1}(t) & \dots & b_{n,m}(t) \end{bmatrix}. \tag{12}$$

By Lemma 1 the solution of (11) has the form

$$x_n(t) = e^{-\int a_{n,n}(t)dt} x_{n0} + \sum_{k=1}^{m} \int_0^t e^{-\int a_{n,n}(t)(t-\tau)dt} b_{nk}(\tau) u_k(\tau) d\tau \tag{13}$$

and $x_n(t) \in \Re_+$, $t \in [0,+\infty)$ for all $x_{0n} \in \Re_+$ and $x_k(t) \in \Re_+$ for $t \in [0,+\infty)$.

Similarly, form (8a) and (9) we obtain

$$\dot{x}_{n-1}(t) = e^{-\int a_{n-1,n-1}(t)dt} x_{n-1,0} + \int_0^t e^{-\int a_{n-1,n-1}(t)(t-\tau)dt} [a_{n-1,n}(\tau) x_n(\tau) + \sum_{k=1}^{m} b_{n-1,k}(\tau) u_k(\tau)] d\tau . \tag{14}$$

From (13) we have $x_{n-1}(t) \in \Re_+$ for $t \in [0,+\infty)$ since $x_n(t) \in \Re_+$ for $t \in [0,+\infty)$. Continuing this procedure we obtain

$$x_k(t) \in \Re_+ \text{ for } k = 1,2,\dots,n \text{ and } t \in [0,+\infty) \tag{15}$$

and any nonnegative initial conditions and inputs.

From (8b) it follows that $y(t) \in \Re_+^p$, $t \in [0,+\infty)$ if the conditions (9) and (10) are satisfied for any nonnegative initial conditions and all nonnegative inputs.
If the matrix (9) has negative diagonal entries then its all eigenvalues are negative function for $t \in [0,+\infty)$ and from (2) for $u(t) = 0$ it follows that $\lim_{t \to \infty} x(t) = 0$ for all $x_0 \in \Re_+^n$. □

**Remark 1.** To check the asymptotic stability of the time-varying continuous-time linear system (2.1) the Theorem 1 can be used.
The system is asymptotically stable if one of the equivalent conditions of Theorem 1 is satisfied for all $t \in [0,+\infty)$.

**Example 1.** Consider the time-varying continuous-time linear system (1) with the matrices

$$A_l(t) = \begin{bmatrix} -e^{-t} & 0 & 0 \\ 1 & -1 & 0 \\ e^{-t} & 0 & -e^{-t} \end{bmatrix}, \quad B(t) = \begin{bmatrix} 2 + 2.2e^{-t} + \sin t \\ 1 + 1.2e^{-t} \\ e^{-t} \end{bmatrix}, \tag{16}$$
$$C(t) = [0.1 \quad 1 + 0.5\sin t \quad 2e^{-t}], \quad D(t) = [0].$$

From (16) it follows that the system is positive and asymptotically stable since $A_l(t) \in M_3(t)$, $B(t) \in \Re_+^3$, $C(t) \in \Re_+^{1\times 3}$ for $t \in [0,+\infty)$.

From (16) we have

$$\begin{aligned} \dot{x}_1(t) &= -e^{-t} x_1(t) + (2 + 2.2e^{-t} + \sin t) u(t), \\ \dot{x}_2(t) &= x_1(t) - x_2(t) + (1 + 1.2e^{-t}) u(t), \\ \dot{x}_3(t) &= e^{-t} x_1(t) - e^{-t} x_3(t) + e^{-t} u(t). \end{aligned} \tag{17}$$

Using Lemma 1 we can find in sequence the positive solution of the equation (17).

Consider the positive continuous-time nonlinear system

$$\dot{x} = Ax + f(x), \tag{18}$$

where $x = x(t) \in \Re^n$, $A \in M_n$, $f(x) \in \Re_+^n$ is a continuous and bounded vector function and $f(0) = 0$.

**Definition 2.** The positive continuous-time nonlinear system (18) is called asymptotically stable in the region $D \in \Re_+^n$ if $x(t) \in \Re_+^n$, $t \ge 0$ and

$$\lim_{t \to \infty} x(t) = 0 \text{ for any finite } x_0 \in D \in \Re_+^n . \tag{19}$$

To test the asymptotic stability of the positive system (18) the Lyapunov method will be used. As a candidate of Lyapunov function we choose

$$V(x) = c^T x > 0 \text{ for } x = x(t) \in \Re_+^n,\ t \ge 0 \tag{20}$$

where $c \in \Re_+^n$ is a vector with strictly positive components $c_k > 0$ for $k = 1,...,n$.

Using (20) and (18) we obtain

$$\dot{V}(x) = c^T \dot{x} = c^T [Ax + f(x)] < 0 \tag{21}$$

for

$$Ax + f(x) < 0 \text{ for } x \in D \in \Re_+^n,\ t \ge 0 \tag{22}$$

since $c \in \Re_+^n$ is strictly positive vector.

Therefore, the following theorem has been proved.

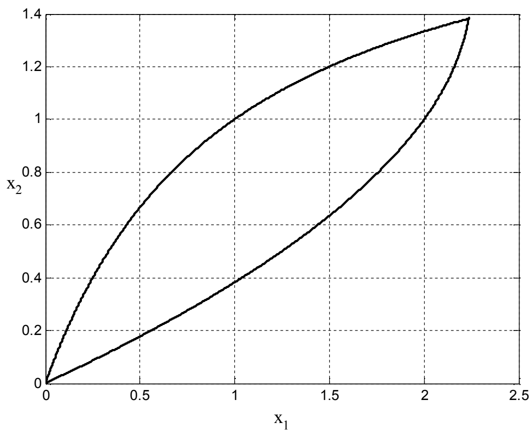


Fig. 1. Stability region (inside the curved line)

**Theorem 3.** The positive continuous-time nonlinear system (18) is asymptotically stable in the region $D \in \Re_+^n$ if the condition (22) is satisfied.

**Example 2.** Consider the nonlinear system (18) with

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix},\ A = \begin{bmatrix} -2 & 1 \\ 1 & -3 \end{bmatrix},\ f(x) = \begin{bmatrix} x_1 x_2 \\ x_2^2 \end{bmatrix}. \tag{23}$$

The nonlinear system (18) with (23) is positive since $A \in M_2$ and $f(x) \in \Re_+^2$ for all $x \in \Re_+^2$, $t \ge 0$.

In this case the condition (22) is satisfied in the region $D$ defined by

$$D := \{x_1, x_2\} = \begin{bmatrix} -2x_1 + x_2 + x_1x_2 \\ x_1 - 3x_2 + x_2^2 \end{bmatrix} < \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{24}$$

From (24) we have

$$x_1(2 - x_2) > x_2 > 0 \ \text{ and } \ 0 \le x_1 < (3 - x_2)x_2. \tag{25}$$

The region $D$ is shown on the Fig. 1.

By Theorem 2 the positive nonlinear system (18) with (23) is asymptotically stable in the region (24).

## 4. Positive time-varying linear circuits

First let us consider a simple time-varying electrical circuit shown in Fig. 2 with given resistance $R(t)$, inductance $L(t)$ depending on time $t$, and source voltage $e(t)$.

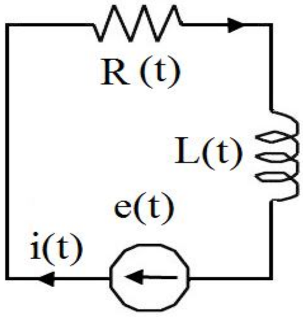


Fig. 2. Electrical circuit

Using Kirchhoff's law, we can write the equation

$$e(t) = \left[ R(t) + \frac{dL(t)}{dt} \right] i(t) + L(t)\frac{di(t)}{dt}, \tag{26}$$

which can be written in the form

$$\frac{di(t)}{dt} = -a(t)i(t) + b(t)e(t), \tag{27a}$$

where

$$a(t) = \frac{1}{L(t)}\left[ R(t) + \frac{dL(t)}{dt} \right], \ b(t) = \frac{1}{L(t)}. \tag{27b}$$

Using the formula (2) we can find the solution $i(t) \in \Re_+$, $t \in [0, +\infty)$ to the equation (27a) for given positive resistance $R(t)$ positive inductance $L(t)$ and nonnegative source voltage $e(t)$.

Therefore, the electrical circuit is a positive and asymptotically stable system if the resistance and inductance are positive functions of $t$ and $e(t) \in \Re_+$, $t \in [0,+\infty)$.

**Example 3.** Consider the time-varying electrical circuit shown in Fig. 3 with given nonzero resistances $R_1(t)$, $R_2(t)$, $R_3(t)$, inductance $L(t) > 0$, capacitance $C(t) > 0$ and source voltage $e(t) \geq 0$ for $t \in [0,+\infty)$.

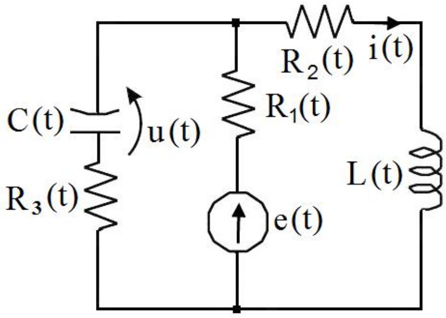


Fig. 3. Fractional time-varying electrical circuit

Using Kirchhoff's laws, we can write the equation

$$\begin{aligned} e(t) &= R_1(t)\left[i(t) + C(t)\frac{du(t)}{dt}\right] + R_3(t)\left[C(t)\frac{du(t)}{dt}\right] + u(t), \\ e(t) &= R_1(t)\left[i(t) + C(t)\frac{du(t)}{dt}\right] + L(t)\frac{di(t)}{dt} + R_2(t)i(t), \end{aligned} \tag{28}$$

The equations (28) can be written in the form

$$\frac{d}{dt}\begin{bmatrix} i(t) \\ u(t) \end{bmatrix} = A(t)\begin{bmatrix} i(t) \\ u(t) \end{bmatrix} + B(t)e(t) \tag{29a}$$

where

$$\begin{aligned} A(t) &= \begin{bmatrix} 0 & [R_1(t)+R_3(t)]C(t) \\ L(t) & R_1(t)C(t) \end{bmatrix}^{-1} \begin{bmatrix} -R_1(t) & -1 \\ -R_1(t)-R_2(t) & 0 \end{bmatrix} \\ &= \begin{bmatrix} \dfrac{R_1^2(t)}{[R_1(t)+R_3(t)]L(t)} - \dfrac{R_1(t)+R_2(t)}{L(t)} & -\dfrac{R_1(t)}{[R_1(t)+R_3(t)]L(t)} \\ -\dfrac{R_1(t)}{[R_1(t)+R_3(t)]C(t)} & -\dfrac{1}{[R_1(t)+R_3(t)]C(t)} \end{bmatrix}, \end{aligned} \tag{29b}$$

$$B(t) = \begin{bmatrix} 0 & [R_1(t)+R_3(t)]C(t) \\ L(t) & R_1(t)C(t) \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \dfrac{R_3(t)}{[R_1(t)+R_3(t)]L(t)} \\ \dfrac{1}{[R_1(t)+R_3(t)]C(t)} \end{bmatrix}.$$

From (29b) it follows that $A(t) \in M_2$ if and only if $R_1(t) = 0$ for $t \in [0,+\infty)$. Therefore, the electrical circuit is a fractional positive time-varying system if and only if $R_1(t) = 0$ for $t \in [0,+\infty)$.

Now let us consider electrical circuit shown on Fig. 4 with given conductances $G_k(t),\ k = 0,1,\ldots,n$ depending on time $t$, inductances $L_i,\ i = 2,4,\ldots,n_2$, capacitances $C_j,\ j = 1,3,\ldots,n_1$ and source voltages $e_1(t), e_2(t),\ldots,e_n(t)$. We shall show that this electrical circuit is a positive time-varying linear system.

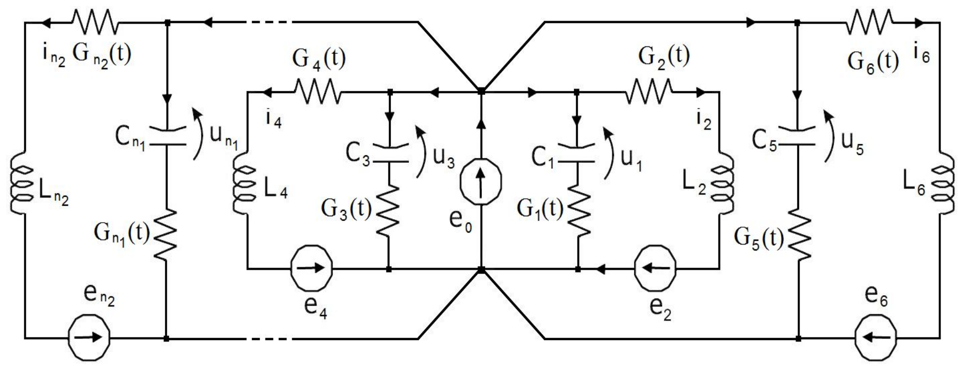


Fig. 4. Electrical circuit

Using the Kirchhoff's law we can write the equations

$$e_1(t) = \frac{C_k}{G_k(t)}\frac{du_k(t)}{dt} + u_k(t) \text{ for } k = 1,3,\ldots,n_1, \tag{30a}$$

$$e_1(t) + e_k(t) = L_k\frac{di_k(t)}{dt} + \frac{i_k(t)}{G_k(t)} + u_k(t) \text{ for } k = 2,4,\ldots,n_2, \tag{30b}$$

which can be written in the form

$$\frac{d}{dt}\begin{bmatrix} u(t) \\ i(t) \end{bmatrix} = A(t)\begin{bmatrix} u(t) \\ i(t) \end{bmatrix} + B(t)e(t), \tag{31a}$$

where

$$u(t) = \begin{bmatrix} u_1(t) \\ u_3(t) \\ \vdots \\ u_{n_1}(t) \end{bmatrix},\quad i(t) = \begin{bmatrix} i_2(t) \\ i_4(t) \\ \vdots \\ i_{n_2}(t) \end{bmatrix},\quad e(t) = \begin{bmatrix} e_1(t) \\ e_3(t) \\ \vdots \\ e_n(t) \end{bmatrix},\quad (n = n_1 + n_2) \tag{31b}$$

and

$$A(t) = \text{diag}\left[-\frac{G_1(t)}{C_1}, -\frac{G_3(t)}{C_3}, \ldots, -\frac{G_{n_1}(t)}{C_{n_1}}, -\frac{1}{G_2(t)L_2}, -\frac{1}{G_4(t)L_4}, \ldots, -\frac{1}{G_{n_2}(t)L_{n_2}}\right],$$

$$B(t) = \begin{bmatrix} B_1(t) \\ B_2 \end{bmatrix}, \quad B_1(t) = \begin{bmatrix} \frac{G_1(t)}{C_1} & 0 & 0 & \ldots & 0 \\ \frac{G_3(t)}{C_3} & 0 & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ldots & \vdots \\ \frac{G_{n_1}(t)}{C_{n_1}} & 0 & 0 & \ldots & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} \frac{1}{L_2} & \frac{1}{L_2} & 0 & \ldots & 0 \\ \frac{1}{L_4} & 0 & \frac{1}{L_4} & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{1}{L_{n_2}} & 0 & 0 & \ldots & \frac{1}{L_{n_2}} \end{bmatrix}. \tag{31c}$$

The electrical circuit is positive and asymptotically stable time-varying linear system since all diagonal entries of the matrix $A(t)$ are negative functions of $t \in [0,+\infty)$ and the matrix $B$ has nonnegative entries for $t \in [0,+\infty)$. The solution $\begin{bmatrix} u(t) \\ i(t) \end{bmatrix}$ of the equation (29) can be found using Lemma 1.

## 5. Concluding remarks

The positivity and asymptotic stability of a class of time-varying continuous-time linear systems and electrical circuits have been addressed. Sufficient conditions for the positivity and asymptotic stability of the system have been established (Theorem 2). It has been shown that there exists a large class of positive and asymptotically stable electrical circuits with time-varying parameters. The Lyapunov method has been extended to positive nonlinear systems (Theorem 3). The considerations have been illustrated by positive and asymptotically stable electrical circuits. The consideration can be extended to fractional time-varying linear systems and fractional electrical circuits.

**Acknowledgment**

This work was supported under work S/WE/1/11.

**References**

[1] Farina L., Rinaldi S., Positive Linear Systems; Theory and Applications, J. Wiley, New York 2000.

[2] Kaczorek T., Controllability and observability of linear electrical circuits, Electrical Review, Vol. 87, No. 9a, pp. 248-254, 2011.

[3] Kaczorek T., Fractional positive continuous-time linear systems and their reachability, Int. J. Appl. Math. Comput. Sci., Vol. 18, No. 2, pp. 223-228, 2008.

[4] Kaczorek T., Fractional standard and positive descriptor time-varying discrete-time linear systems, Progress in Automation, Robotics and Measuring Techniques, Vol. 350, Springer International Publishing, pp. 101-112, 2015.
[5] Kaczorek T., Positive 1D and 2D Systems, Springer Verlag, London 2002.
[6] Kaczorek T., Positive electrical circuits and their reachability, Archives of Electrical Engineering, Vol. 60, No. 3, pp. 283-301, 2011 and also Selected classes of positive electrical circuits and their reachability, Monograph Computer Application in Electrical Engineering, Poznan University of Technology, Poznan 2012.
[7] Kaczorek T., Positive descriptor time-varying discrete-time linear systems and their asymptotic stability, TransNav, the International Journal on Marine Navigation and Safety of Sea Transportation, Vol. 9, No. 1, pp. 83-89, 2015.
[8] Kaczorek T., Positive linear systems consisting of n subsystems with different fractional orders, IEEE Trans. Circuits and Systems, Vol. 58, No. 6, pp. 1203-1210, 2011.
[9] Kaczorek T., Positivity and reachability of fractional electrical circuits, Acta Mechanica et Automatica, Vol. 5, No. 2, pp. 42-51, 2011.
[10] Kaczorek T., Positivity and stability of fractional descriptor time-varying discrete-time linear systems, AMCS, 2015 (in Peress).
[11] Kaczorek T., Positivity and stability of time-varying discrete-time linear systems, Intelligent Information and Database Systems, Lecture Notes in Computer Science, Vol. 9011, *Springer International Publishing*, pp. 295-303, 2015.
[12] Kaczorek T., Stability of positive continuous-time linear systems with delays, *Bull. Pol. Acad. Sci. Techn.*, vol. 57, no. 4, 2009, 395-398.
[13] Kaczorek T., Stability and stabilization of positive fractional linear systems by state-feedbacks, *Bull. Pol. Acad. Sci. Techn.*, vol. 58, no. 4, 2010, 517-554.
[14] Kaczorek T., Selected Problems of Fractional System Theory, *Springer Verlag* 2011.
[15] Kaczorek T., New stability tests of positive standard and fractional linear systems, *Circuits and Systems*, no. 2, pp. 261-268, 2011.
[16] Ostalczyk P., Epitome of the Fractional Calculus, Theory and its Applications in Automatics, *Technical University of Lodz Press*, Lodz, 2008 (in Polish).
[17] Podlubny I., Fractional Differential Equations, *Academic Press*, San Diego, 1999.
[18] Narendra K.S., Shorten R., Hurwitz Stability of Metzler Matrices, *IEEE Trans. Autom. Contr.*, Vol. 55, no. 6 June 2010, 1484-1487.

*(Received: 2. 10. 2015, revised: 1. 12. 2015)*